WW production at NNLO+PS

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CERN & LAPTh Annecy

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*ongoing work with M. Wiesemann and G. Zanderighi
introduction and outline

- vector boson pair production
  - access to anomalous gauge couplings.
  - background for several searches, for instance $H \rightarrow WW$.

- as shown in previous talk, the current experimental precision already demands for predictions that go beyond NLO(+PS) accuracy.
- NNLO corrections are certainly needed, and resummation too, in corners of phase-space.

this talk: matching NNLO and PS for $pp \rightarrow W^+W^-$, using **MiNLO** and **MATRIX**

(a) method: (improved) **MiNLO**
(b) NNLO input: **MATRIX**
(c) results [preliminary]

[the literature on VV production is vast, so I will refrain from trying to summarize it]
Multiscale Improved NLO

- original goal: method to a-priori choose scales in multijet NLO computation
- non-trivial task: hierarchy among scales can spoil accuracy (large logs can appear, without being resummed)
- how: correct weights of different NLO terms with CKKW-inspired approach (without spoiling formal NLO accuracy)
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- \( \Delta(q_T, m_V) \)
- \( \Delta(q_T, q_T) \)
- \( \Delta(q_T, \bar{\mu}_R) \)
- \( \Delta(q_T, \bar{\mu}_F) \)
- \( \bar{\mu}_R = q_T \)
- \( \log \Delta_f(q_T, m_V) = -\int_{q_T^2}^{m_V^2} \frac{dq^2}{q^2} \frac{\alpha_s(q^2)}{2\pi} \left[A_f \log \frac{m_V^2}{q^2} + B_f \right] \)
- \( \Delta_f^{(1)}(q_T, m_V) = -\frac{\alpha_s}{2\pi} \left[\frac{1}{2} A_{1,f} \log^2 \frac{m_V^2}{q_T^2} + B_{1,f} \log \frac{m_V^2}{q_T^2} \right] \)
- \( \bar{\mu}_F = q_T \)
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\]

- Sudakov FF included on \( V + j \) Born kinematics

- MiNLO-improved VJ yields finite results also when 1st jet is unresolved \( (q_T \to 0) \)
- \( \tilde{B}_{\text{MiNLO}} \) allows extending the validity of VJ–POWHEG [called “VJ−MiNLO” hereafter]
formal accuracy of VJ-MiNLO for inclusive observables carefully investigated.  

possible to improve VJ-MiNLO such that inclusive NLO is recovered (NLO\(^{(0)}\)), without spoiling NLO accuracy of \(V+j\) (NLO\(^{(1)}\)):

\[
\text{MiNLO'}: \text{NLO+PS merging, without merging scale}
\]

accurate control of subleading small-\(p_T\) logarithms is needed:
- include \(B_2\) (NNLL) coefficient in MiNLO-Sudakov.
- set scales in \(R\), \(V\) and subtraction terms equal to \(q_T\) (boson transverse momentum).
- without the above requirements, spurious \(\alpha_s^{3/2}\) terms show up in \(\sigma_{NLO}^{(0)}\) upon integration over \(q_T\).
formal accuracy of VJ-MiNLO for inclusive observables carefully investigated. [Hamilton et al. 1212.4504]

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for color-singlet production \(X\), the above procedure is general, and (almost) process independent.

<table>
<thead>
<tr>
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a generalization of the MiNLO’ approach for processes with jets at LO has also been proposed (but here we are not using it). [Frederix,Hamilton ’15]
A MiNLO’ generator that merges $WW$ and $WW + 1$ jet at NLO+PS was obtained a while ago:

- **POWHEG** $WWJ$ generator obtained ex-novo using interfaces to Madgraph and Gosam 2.0

  [Campbell et al. 1202.547; Luisoni et al. 1306.2542; Cullen et al. 1404.7096]

- starting from the Drell-Yan case, we extracted the $B_2^{(WW)}$ term from the virtual ($V^{(WW)}$) and Born ($B^{(WW)}$) contributions of $pp \rightarrow WW$.

  - for Drell-Yan, $V^{(V)}$ and $B^{(V)}$ are proportional, hence $B_2^{(V)}$ is just a number.

  - in $pp \rightarrow WW$, this is no longer true: $B_2^{(WW)} = B_2^{(WW)}(\Phi_{WW})$:

    - for $q\bar{q}$-initiated color singlet production, $B_2$ has the form

      $B_2 = -2\gamma^{(2)} + \beta_0 C_F \zeta_2 + 2(2C_F)^2 \zeta_3 + \beta_0 H_1(\Phi)$

- $H_1(\Phi)$ (process-dependent part of $B_2$) extracted on an event-by-event basis: projection of $\Phi_{WWJ}$ onto $\Phi_{WW}$, used FKS ISR mapping (smooth collinear limit).

- for validation and results, see paper from ’16.
importance of NNLO for diboson production

...clear example where plots speak for themselves...

- NNLO results in these plots: MATRIX

[Grazzini,Kallweit,Wiesemann -'17]
The **MATRIX** framework

[Grasso, Kallweit, MW '17]

<table>
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<td><strong>OPENLOOPS</strong></td>
</tr>
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<td>(Collier, CutTOols, ...)</td>
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<tr>
<td><strong>Dedicated 2-loop codes</strong></td>
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**MUNICH**

MUlti-chaNnel Integrator at Swiss (CH) precision

$q_T$ subtraction $\leftrightarrow$ $q_T$ resummation

**MATRIX**

MUNICH Automates $q_T$ Subtraction and Resummation to Integrate X-sections.

[slide from talk by M. Wiesemann, CERN, Feb '18]
q_T-subtraction formalism, in a nutshell

\[ d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \left[ d\sigma_{(N)LO}^{F+\text{jet}} - d\sigma_{(N)NLO}^{\text{CT}} \right] \]

- subtraction term known from resummation, and process independent (apart from LO dependence).
- hard-collinear function: can be extracted from 2-loops amplitudes.
- so far, extensively used for color-singlet production at NNLO.
**qT-subtraction** formalism, in a nutshell [Catani,Grazzini ’07]

\[
\frac{d\sigma_{(N)NLO}^{F}}{d\sigma_{(N)LO}^{F}} = \mathcal{H}_{(N)NLO}^{F} \otimes d\sigma_{LO}^{F} + \left[ d\sigma_{(N)LO}^{F + \text{jet}} - d\sigma_{(N)NLO}^{CT} \right]
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---

as shown next, for NNLOPS, one needs

\[
\left( \frac{d\sigma}{d\Phi_B} \right)_{NNLO} \quad \text{fully differential in the Born phase space}
\]

- as shown next, for NNLOPS, one needs

we used **MATRIX**: 2-loops amplitudes from **VVAMP** [Gehrmann et al. ’15]

- tree-level and 1-loop from **OPENLOOPS** [Cascioli et al. ’11]

---

we have **NOT** included the $gg$ loop-induced channel

- it’s about 30% of the NNLO correction.
starting from a MiNLO’ generator, it’s possible to match a PS simulation to NNLO.

XJ–MiNLO’ (+POWHEG) generator gives X-XJ @ NLOPS:

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by construction NNLO accuracy on inclusive observables; 

to reach NNLOPS accuracy, need to be sure that the reweighting doesn’t spoil the NLO accuracy of XJ-MiNLO in 1-jet region; 

notice: formally works because no spurious $O(\alpha S)$ terms in X-XJ @ NLOPS (relative to $\sigma_X$).
NNLO+PS for color-singlet production

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- reweighting (differential on $\Phi_B$) of “MiNLO-generated” events:

\[
W(\Phi_B) = \frac{\left( \frac{d\sigma}{d\Phi_B} \right)_{\text{NNLO}}}{\left( \frac{d\sigma}{d\Phi_B} \right)_{\text{XJ-MiNLO'}}}
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notice: formally works because no spurious $\mathcal{O}(\alpha_S^{1.5})$ terms in X-XJ @ NLOPS (relative to $\sigma_X$).
**WW at NNLO+PS, in practice**

- $pp \to e^- \bar{\nu}_e \mu^+ \nu_\mu$: $\Phi_B$ is 9-dimensional [impossible]

- choose variables, drop dependence upon $(\ell, \nu_\ell)$ invariant masses (fairly flat)

$$\frac{d\sigma}{d\Phi_B} = \frac{d^9 \sigma}{dp_{T,W} - dy_{WW} d\Delta y_{W+W-} d\cos \theta_{W+}^{CS} d\phi_{W+}^{CS} d\cos \theta_{W-}^{CS} d\phi_{W-}^{CS} d m_{W+} d m_{W-}}$$

- use “Collins-Soper” angles for both $W$ decays

$$\frac{d\sigma}{d\Phi_B} = \frac{9}{256 \pi^2} \sum_{i=0}^{8} \sum_{j=0}^{8} A B_{i,j} f_i(\theta_{W-}^{CS}, \phi_{W-}^{CS}) f_j(\theta_{W+}^{CS}, \phi_{W+}^{CS})$$

$$A B_{i,j} = A B_{i,j}(p_{T,W-}, y_{WW}, \Delta y_{W+W-})$$

- final complexity: **81 triple-differential distributions** at NNLO [doable]
**WW at NNLO+PS: validation**

[ER, Wiesemann, Zanderighi, preliminary]

- $m_W$ distribution well reproduced also off from peak.
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- validated also other “Born” observables, as well as angular dependence (Collins-Soper angles) [not shown].
expected patterns in inclusive and exclusive observables.

- the jet-vetoed cross-section is particularly important
  (e.g. for Higgs studies, but also to just measure $WW$ production).
jet-veto cross section.

- fiducial cuts almost identical to ATLAS analysis [1702.04519]
- in ATLAS paper, jet-veto at 25/30 GeV.
ongoing remarkable progress of NNLO computations: try to match them with parton showers.

for color-singlet-production, **POWHEG+MiNLO** allows to do that.

shown for the first time (preliminary) results for $W W$ production at **NNLO+PS** ($pp \rightarrow e^- \bar{\nu}_e \mu^+ \nu_\mu$).

Next steps:
- finish paper, release code;
- other diboson processes...;
- loop-induced gluonic channels;
- find more efficient method;
ongoing remarkable progress of NNLO computations: try to match them with parton showers.

for color-singlet-production, **POWHEG+MiNLO** allows to do that. [other methods are possible]

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Next steps:
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- find more efficient method;

Thanks for your attention!
All off-shell and single-resonant diagrams included. Full matrix-element with leptonic decays.

worked in the 4F scheme: no interference with $Wt$ and $t\bar{t}$.

for same-family leptons, "$Z(\rightarrow \ell\bar{\ell})Z(\rightarrow \nu\ell\bar{\nu}\ell)$" not included:

- will be part of $ZZ$ generator;
- interference between $WW$ and $ZZ$ shown to be extremely small; [Melia et al. 1107.5051]

option to include/exclude fermionic loop corrections (at most 1-2% difference in tails, x2 difference in speed).
NNLOPS: technical details

- Variants for reweighting $W(\Phi_B)$ are also possible:

$$W(\Phi_B, p_T) = h(p_T) \frac{\int d\sigma^\text{NNLO}_A \delta(\Phi_B - \Phi_B(\Phi))}{\int d\sigma^\text{MiNLO}_A \delta(\Phi_B - \Phi_B(\Phi))} + (1 - h(p_T))$$

$$d\sigma_A = d\sigma \ h(p_T), \quad d\sigma_B = d\sigma \ (1 - h(p_T)), \quad h(p_T) = \frac{(\beta M)^2}{(\beta M)^2 + p_T^2}$$

- freedom to distribute “NNLO/NLO K-factor” only over medium-small $p_T$ region
  - $h(p_T)$ controls where the NNLO/NLO K-factor is distributed
    (in the high-$p_T$ region, there is no improvement in including it)
  - $\beta$ cannot be too small, otherwise resummation spoiled:
    for Higgs, chosen $\beta = 1/2$; for DY, HW, WW, $\beta = 1$

- in practice, we used

$$W(\Phi_B, p_T) = h(p_T) \frac{\int d\sigma^\text{NNLO}_B \delta(\Phi_B - \Phi_B(\Phi))}{\int d\sigma^\text{MiNLO}_B \delta(\Phi_B - \Phi_B(\Phi))} + (1 - h(p_T))$$

- one gets exactly $(d\sigma/d\Phi_B)_{\text{NNLOPS}} = (d\sigma/d\Phi_B)_{\text{NNLO}}$
  - chosen $h(p_T^{j1})$