# Parton orbital angular momentum at small-x

Yoshitaka Hatta

(Yukawa institute, Kyoto U)

YH, Yuya Nakagawa, Bo-Wen Xiao, Feng Yuan, Yong Zhao, PRD95 (2017) 114032
YH, Dong-Jing Yang, PLB781 (2018) 213

# The proton spin problem

The proton has spin ½.

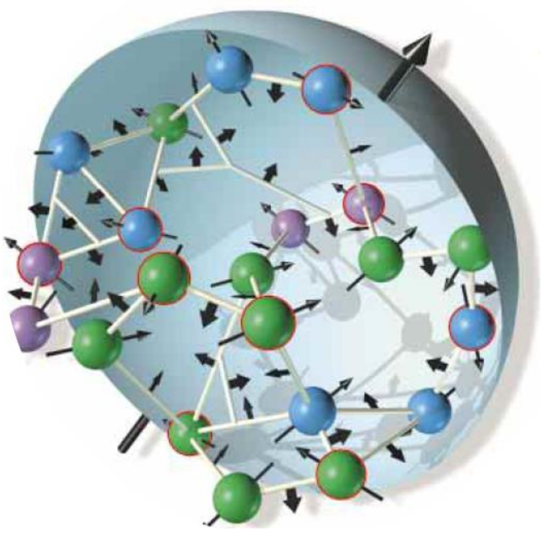
The proton is not an elementary particle.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Orbital angular Momentum (OAM)



In the quark model, $\Delta\Sigma = 1$

with relativistic effects, $\Delta\Sigma \approx 0.7$

EMC result (1987) $\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14$

Current value $\Delta\Sigma = 0.25 \sim 0.3$

# Global analysis for $\Delta G$

from H. Gao's talk on Monday

$$\int_{0.05}^{1} dx\Delta g(x,Q^2=10\text{GeV}^2) = 0.20^{+.06}_{-.07} \qquad \text{DSSV++}$$

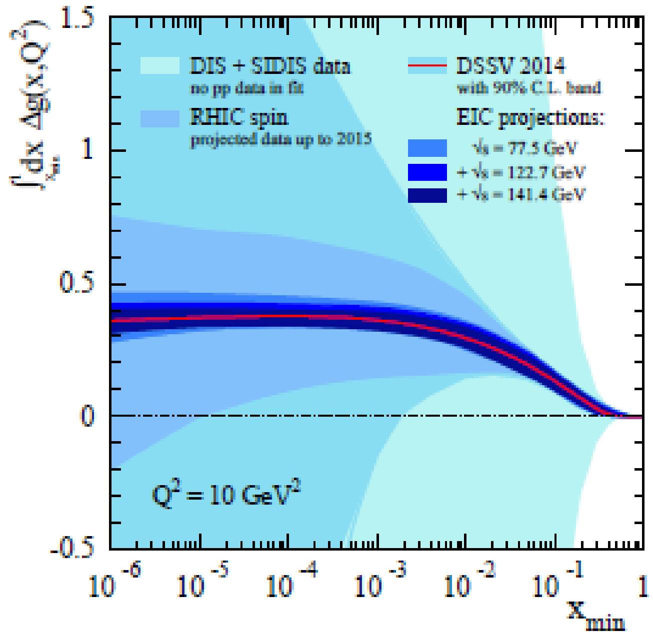
$$\int_{0.05}^{0.2} dx\Delta g(x,Q^2=10\text{GeV}^2) = 0.17\text{+-}0.06 \qquad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.8} dx\Delta g(x,Q^2 = 1\ \text{GeV}^2) = 0.5\text{+-}0.4 \qquad \text{JAM15}$$

HUGE uncertainty from the small-x region

Maybe $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G$ , after all?

What's the role of OAM?




Achenauer, Sassot, Stratmann, 1509.06489

# Wigner distribution and OAM

Wigner distribution → Phase space distribution of quarks and gluons
Naturally defines the orbital angular momentum.

$$L^{q,g} = \int dx \int d^2 b_\perp d^2 k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

Lorce, Pasquini, (2011);
YH (2011)

`PDF' for OAM

$$L^{q,g}(x) = \int d^2 b_\perp d^2 k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^{q,g}(x, \vec{b}_\perp, \vec{k}_\perp)$$

How does $L^{q,g}(x)$ behave as $x \to 0$ ?

I give two arguments that $L_g(x) \approx -\Delta G(x)$ at small-x.

# Twist structure of OAM distributions

YH, Yoshida (2012)

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2}$$
$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)} .$$

Genuine twist-three part

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x') + E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X, x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^3(x_1 - x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1, x_2)\mathcal{P}\frac{2x_1 - x_2}{x_1^3(x_1 - x_2)^2}$$

# OAM at small-x

Gluon Wigner distribution

$$xW(x,\vec{k}_\perp,\vec{b}_\perp)$$
$$= \int_{\Delta_\perp} e^{i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \int_{z^-,z_\perp} e^{ixP^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp} \langle P-\Delta/2|\mathrm{Tr}F^{+i}(-z/2)U^{[+]}F^{+}_{i}(z/2)U^{[-]}|P+\Delta/2\rangle$$

Approximate $e^{ixP^+z^-} \approx 1$

$$\boxed{xW(x,\vec{k}_\perp,\vec{b}_\perp) \approx \frac{2N_c}{\alpha_s}\int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{k}_\perp\cdot\vec{r}_\perp}\left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) S(\vec{r}_\perp,\vec{b}_\perp)}$$

YH, Xiao, Yuan (2016)

$$S(\vec{r}_\perp,\vec{b}_\perp) = \left\langle \frac{1}{N_c}\mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger\left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right)\right\rangle_x$$ ``Dipole S-matrix”

# Where is spin dependence?

$$S(x,\Delta_\perp,q_\perp) \equiv \int d^2x_\perp d^2y_\perp e^{iq_\perp\cdot(x_\perp-y_\perp)+i(x_\perp+y_\perp)\cdot\frac{\Delta_\perp}{2}} \left\langle P+\frac{\Delta}{2}\left|\frac{1}{N_c}\mathrm{Tr}\left[U(x_\perp)U^\dagger(y_\perp)\right]\right|P-\frac{\Delta}{2}\right\rangle$$

$$= P(x,\Delta_\perp,q_\perp) + iq_\perp\cdot\Delta_\perp O(x,|q_\perp|) \;\; + q_\perp \times S_\perp O_s(x,q_\perp)$$

"Pomeron" "odderon" "spin-dependent odderon"

Jian Zhou (2013)

$S(x,\Delta_\perp,q_\perp)$ cannot depend on the longitudinal spin
…forbidden by PT symmetry

Lesson: All information about spin is lost in the eikonal approximation.

$$e^{ixP^+z^-} \approx 1$$

# OAM as a next-to-eikonal effect

YH, Nakagawa, Xiao, Yuan, Zhao (2017)

Go to next-to-eikonal

$$e^{ixP^+z^-} \approx 1 + ixP^+z^- \quad \Longrightarrow \quad W = W_0 + \delta W$$

$$\delta W(x, \Delta_\perp, q_\perp, S) = \frac{4P^+}{g^2(2\pi)^3} \int d^2x_\perp d^2y_\perp e^{i(q_\perp + \frac{\Delta_\perp}{2})\cdot x_\perp + i(-q_\perp + \frac{\Delta_\perp}{2})\cdot y_\perp}$$
$$\times \left\{ \int_{-T}^{T} dz^- \left(q_\perp^i - \frac{\Delta_\perp^i}{2}\right) \left\langle \mathrm{Tr}\left[ U_{Tz^-}(x_\perp) \overleftarrow{D}_i U_{z^- -T}(x_\perp) U^\dagger(y_\perp) \right] \right\rangle \right.$$
$$\left. + \int_{-T}^{T} dz^- \left(q_\perp^i + \frac{\Delta_\perp^i}{2}\right) \langle \mathrm{Tr}\left[ U(x_\perp) U_{-Tz^-}(y_\perp) D_i U_{z^- T}(y_\perp) \right] \rangle \right\}.$$

Can have spin-dependent matrix element. Involves half-infinite Wilson lines

# Unintegrated polarized gluon distribution

$$ix\Delta G(x,q_\perp)\frac{S^+}{P^+} \equiv 2\int \frac{d^2z_\perp dz^-}{(2\pi)^3 P^+} e^{-ixP^+z^- + iq_\perp\cdot z_\perp} \left\langle PS \left| \epsilon_{ij} F^{+i}\left(\frac{z}{2}\right) U_- F^{+j}\left(-\frac{z}{2}\right) U_+ \right| PS \right\rangle$$

$$\approx \frac{4P^+}{g^2(2\pi)^3}\int d^2x_\perp d^2y_\perp e^{i(q_\perp+\frac{\Delta_\perp}{2})\cdot x_\perp + i(-q_\perp+\frac{\Delta_\perp}{2})\cdot y_\perp}$$

$$\times\epsilon_{ij}\left\{ q_\perp^j \int_{-\infty}^{\infty} dz^- \left\langle \mathrm{Tr}\left[ U_{\infty z^-}(x_\perp)\overleftarrow{D}_i U_{z^- -\infty}(x_\perp) U^\dagger(y_\perp)\right]\right\rangle \right.$$

$$\left. + q_\perp^i \int_{-\infty}^{\infty} dz^- \left\langle \mathrm{Tr}\left[ U(x_\perp) U_{-\infty z^-}(y_\perp) D_j U_{z^- \infty}(y_\perp)\right]\right\rangle \right\}$$

Exactly the same matrix element appears.
→ Linear relation between $\Delta G(x)$ and $L_g(x)$

$$L_g(x) \approx -\Delta G(x) + \cdots$$

YH, Nakagawa, Xiao, Yuan, Zhao (2017)

# `DGLAP' equation for OAM

cf. Hagler, Schafer (1998)

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z) \end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta q(x/z) \\ \Delta G(x/z) \end{pmatrix},$$

$$\hat{P}_{qq}(z) = C_F\left(\frac{z(1+z^2)}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right),$$

$$\hat{P}_{qg}(z) = n_f z(z^2+(1-z)^2),$$

$$\hat{P}_{gq}(z) = C_F(1+(1-z)^2),$$

$$\hat{P}_{gg}(z) = 6\frac{(z^2-z+1)^2}{(1-z)_+} + \frac{\beta_0}{2}\delta(z-1),$$

$$\Delta\hat{P}_{qq}(z) = C_F(z^2-1),$$

$$\Delta\hat{P}_{qg}(z) = n_f(1-3z+4z^2-2z^3),$$

$$\Delta\hat{P}_{gq}(z) = C_F(-z^2+3z-2),$$

$$\Delta\hat{P}_{gg}(z) = 6(z-1)(z^2-z+2),$$

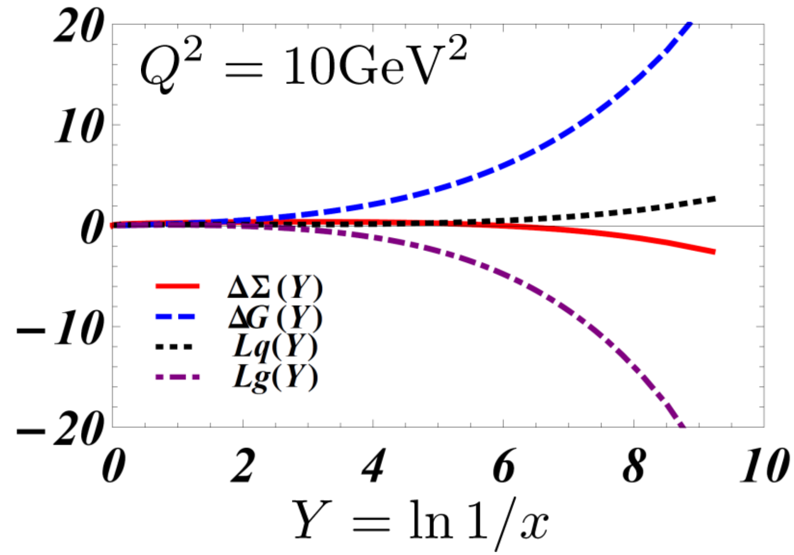
# Numerical results

YH, Yang (2018)

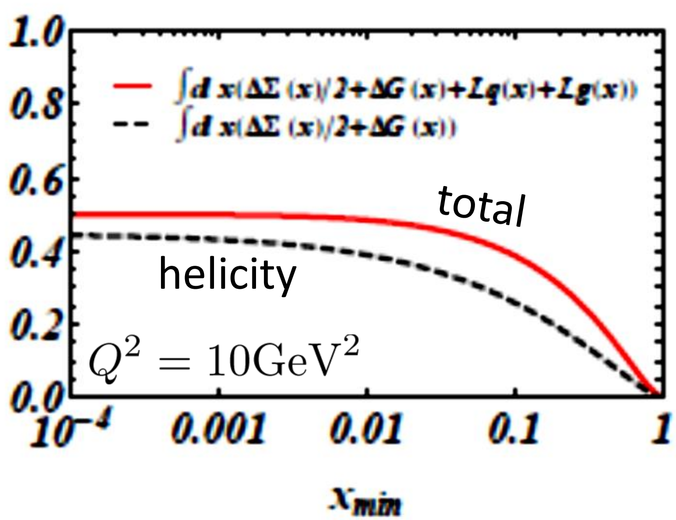
`Democratic model'

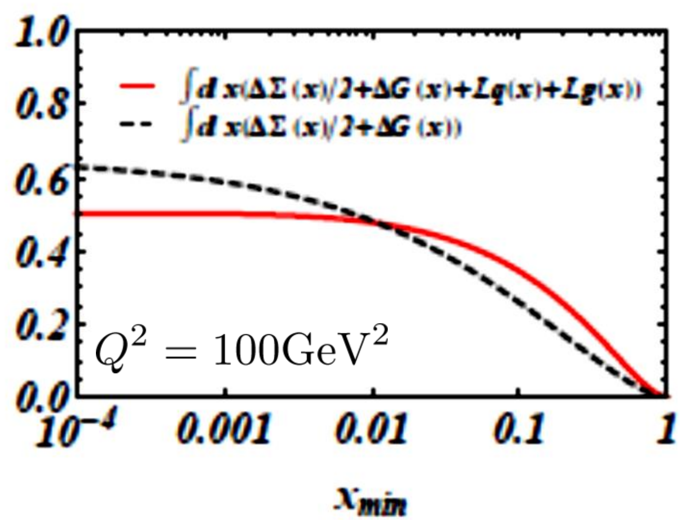
$$\Delta\Sigma(x, Q_0^2) = \frac{1}{4}, \quad \Delta G(x, Q_0^2) = \frac{1}{8},$$

$$L_q(x, Q_0^2) = \frac{1}{8}, \quad L_g(x, Q_0^2) = \frac{1}{8},$$




$$\int_{x_{min}}^{1} dx \left(\frac{1}{2}\Delta\Sigma(x) + \Delta G(x) + L_q(x) + L_g(x)\right)$$

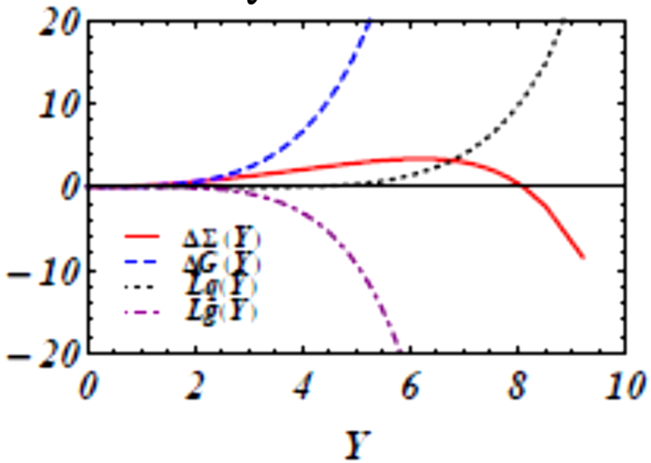


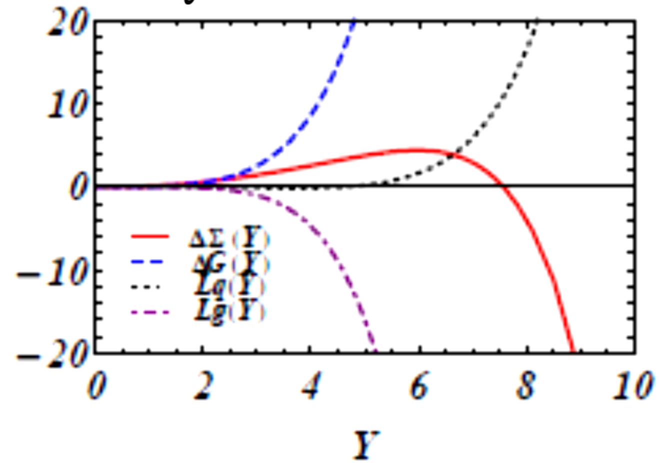
`Helicity dominance model'

$$\Delta\Sigma(x,Q_0^2) = A_q x^{-0.3}(1-x)^3, \quad \Delta G(x,Q_0^2) = A_g x^{a_g}(1-x)^3$$
$$L_q(x,Q_0^2) = L_g(x,Q_0^2) = 0,$$

$Q^2 = 10\mathrm{GeV}^2$

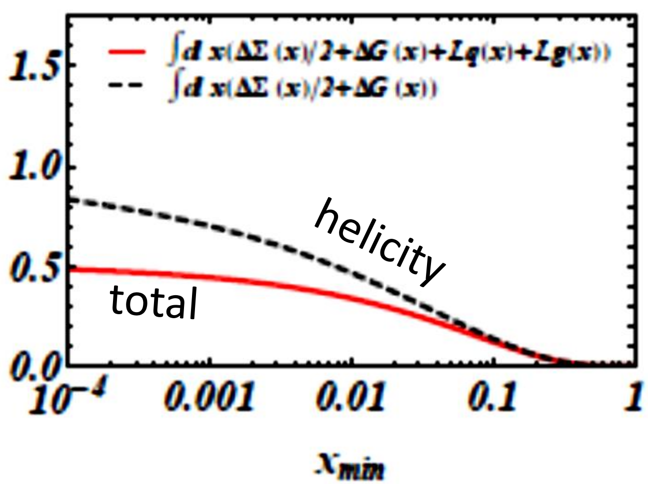


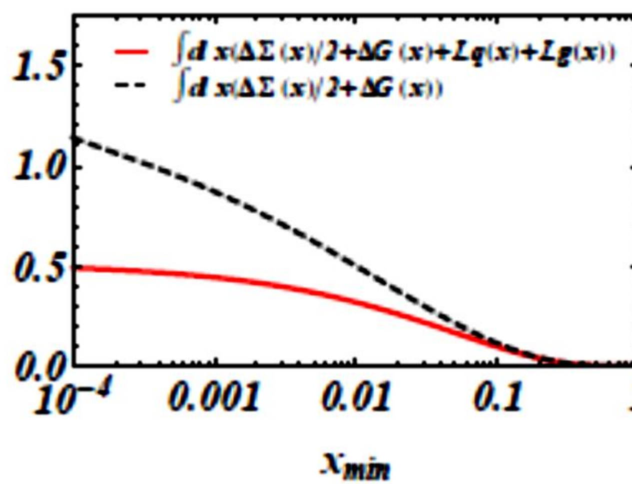

$Q^2 = 100\mathrm{GeV}^2$




$a_g = -0.6$

$$\int_{x_{min}}^{1} dx \left(\frac{1}{2}\Delta\Sigma(x) + \Delta G(x) + L_q(x) + L_g(x)\right)$$



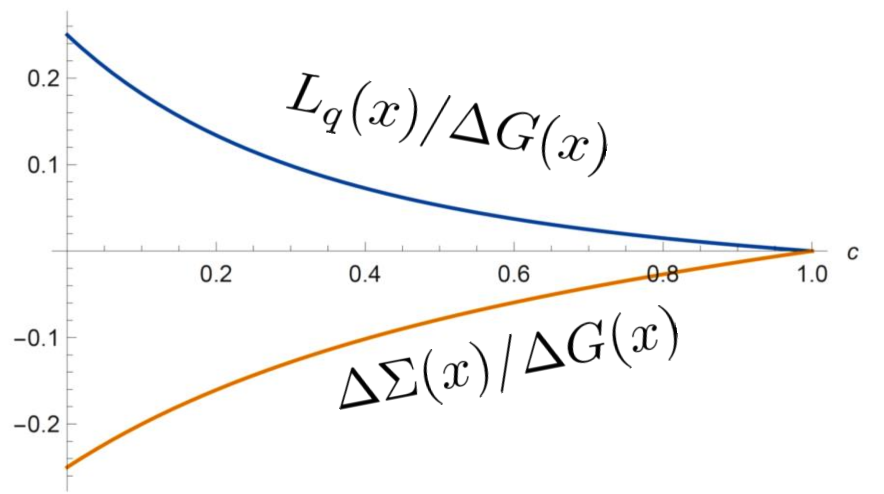
# Analytical insights

Use the ansatz $$L_g(x, Q^2) = A(Q^2)\frac{1}{x^{c}} \qquad \Delta G(x, Q^2) \approx B(Q^2)\frac{1}{x^{c}}$$

$$\frac{L_g(x, Q^2)}{\Delta G(x, Q^2)} \approx -\frac{2}{c+1}$$

$$\frac{\Delta\Sigma(x)}{\Delta G(x)} \approx -n_f \frac{1-c}{c(1+c)\left[6\left(-H_{c-1} + \frac{1}{c} - \frac{1}{1+c}\right)\frac{\beta_0}{2}\right]}$$

$$L_q(x) \approx -\frac{\Delta\Sigma(x)}{1+c}$$




At small-x, helicity and OAM distributions are related to each other by the Regge parameter $c$.

# Conclusion

- OAM is the holy grail of spin physics. A lot of recent progress, still there's a long way to go.
- x-distribution for OAM can be defined. Significant cancelation between OAM and helicity at small-x.
- Beyond `DGLAP': Resum double logarithms $\left(\alpha_s \ln^2 \frac{1}{x}\right)^n$

  →talk by Matt Sievert on Thursday
- Experimental observable for $L_{q,g}(x)$?

  → talk by Shohini Bhattacharya on Thursday