# Complete one-loop electroweak corrections to polarized $e^+e^-$ scattering in SANC

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**CLIC Worksop 2018**
CERN, 24.01.2017

# Outline

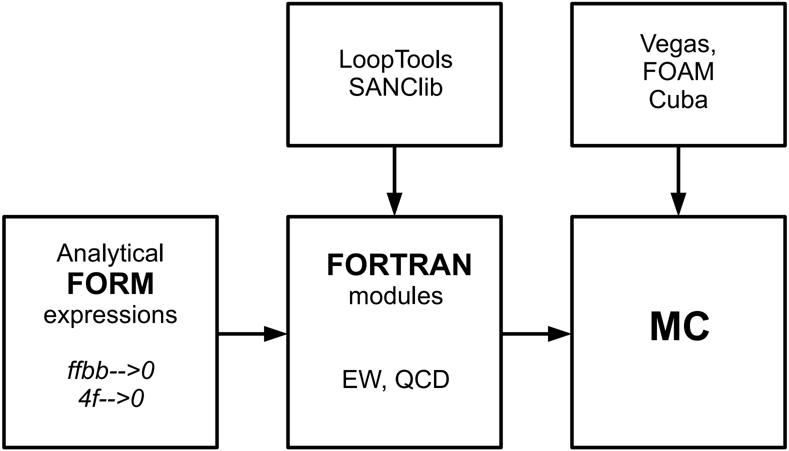
- Introduction to SANC
- SANC branch for processes with polarized $e^+e^-$ beams
- Polarized Bhabha scattering $e^+e^- \to e^+e^-$ at NLO EW
- Preliminary results for polarized $e^+e^- \to Z\gamma$ at NLO EW
- Conclusions and plans

# Introduction to SANC

The project SANC has been developing since 2001.The main results published in CPC (v.174, 2006 and v.177, 2007) and recent developement in J.Phys.Conf.Ser. 762 (2016) no.1, 012062. The project webpage is `http://sanc.jinr.ru`.

- The SANC system implements calculations of complete one-loop QCD and EW corrections for various processes at the partonic level
- All calculations are performed within the OMS (on-mass-shell) renormalization scheme in the $R_\xi$ gauge
- Cross-sections of the processes at hadron level obtained by convoluting the partonic level cross-sections with PDFs
- The list of processes implemented in the `MCSANC` integrator includes Drell-Yan processes (inclusive), associated Higgs and gauge boson production and single-top quark production in s- and t-channel (v1.01 – CPC 184 (2013) 2343-2350), photon-induced contribution, EW corrections beyond NLO approximation to DY (v1.20 – JETP Lett. 103 (2016) no.2, 131-136)

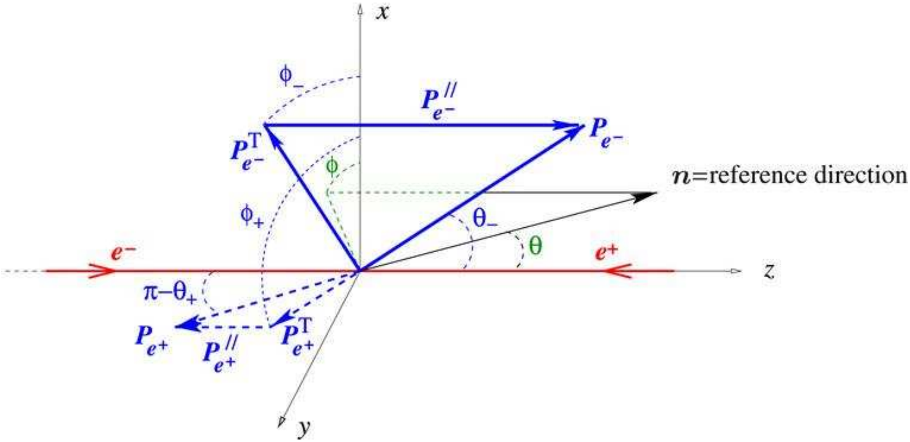
# The SANC framework scheme



# SANC for processes with polarized beams

- NLO EW corrections for polarized $e^+e^-$ scattering:
  - Bhabha scattering (arXiv:1801.00125)
  - $e^+e^- \to Z\gamma$ (preliminary results)
  - $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$ (in progress)
  - $e^+e^- \to t\bar{t}$ (in progress)
  - $e^+e^- \to ZH$ (in progress)
  - $e^+e^- \to \gamma\gamma$ (in progress)
  - $e^+e^- \to ZZ$ (in progress)
  - $e^+e^- \to f\bar{f}\gamma$ (future plans)
  - $e^+e^- \to f\bar{f}H$ (future plans)
- NLO EW corrections for polarized $\gamma\gamma$ scattering:
  - $\gamma\gamma \to \gamma\gamma$ (future plans)
  - $\gamma\gamma \to Z\gamma$ (future plans)
  - $\gamma\gamma \to ZZ$ (future plans)

# Decomposition of the $e^{\pm}$ polarization vector




Ref. Phys. Rept. 460 (2008) 131–243

# Matrix element squared

$$
\begin{aligned}
|\mathcal{M}|^2 = \quad & \frac{1}{4}\Bigg\{(1-P^{||}_{e^-})(1+P^{||}_{e^+})|F_{LR}|^2 + (1+P^{||}_{e^-})(1-P^{||}_{e^+})|F_{LR}|^2 \\
& +(1-P^{||}_{e^-})(1-P^{||}_{e^+})|F_{LL}|^2 + (1+P^{||}_{e^-})(1-P^{||}_{e^+})|F_{RR}|^2 \\
& -2P^T_{e^-}P^T_{e^+}\Big[cos(\phi_- - \phi_+)Re(F_{RR}F^*_{LL}) + cos(\phi_- + \phi_+ - 2\phi)Re(F_{LR}F^*_{RL}) \\
& +sin(\phi_- + \phi_+ - 2\phi)Im(F_{LR}F^*_{RL}) + sin(\phi_- - \phi_+)Im(F_{RR}F^*_{LL})\Big] \\
& +2P^T_{e^-}\Big[cos(\phi_- - \phi)\Big((1-P_{e^+})Re(F_{RL}F^*_{LL}) + (1+P_{e^+})Re(F_{RR}F^*_{LR})\Big) \\
& -sin(\phi_- - \phi)\Big((1-P_{e^+})Im(F_{RL}F^*_{LL}) + (1+P_{e^+})Im(F_{RR}F^*_{LR})\Big)\Big] \\
& -2P^T_{e^+}\Big[cos(\phi_+ - \phi)\Big((1-P_{e^-})Re(F_{LR}F^*_{LL}) + (1+P_{e^-})Re(F_{RR}F^*_{RL})\Big) \\
& -sin(\phi_+ - \phi)\Big((1-P_{e^-})Im(F_{LR}F^*_{LL}) + (1+P_{e^-})Im(F_{RR}F^*_{RL})\Big)\Big]\Bigg\},
\end{aligned}
$$

where $F_{LL}$, $F_{RR}$,$F_{LR}$,$F_{RL}$ — helicity amplitudes.

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# Longitudinally-polarized beams

With longitudinally-polarized beams, cross-sections at an $e^+e^-$ collider can be subdivided into four parts:

$$\begin{aligned} \sigma_{P_{e^-}P_{e^+}} &= \frac{1}{4}\Big\{(1+P_{e^-})(1+P_{e^+})\,\sigma_{RR} + (1-P_{e^-})(1-P_{e^+})\,\sigma_{LL} \\ &+ (1+P_{e^-})(1-P_{e^+})\,\sigma_{RL} + (1-P_{e^-})(1+P_{e^+})\,\sigma_{LR}\Big\}, \end{aligned}$$

where $\sigma_{RL}$ stands for the cross-section if the $e^-$ beam is completely right-handed polarized ($P_{e^-} = +1$) and the $e^+$ beam is completely left-handed polarized ($P_{e^+} = -1$). The cross-sections $\sigma_{LR}$, $\sigma_{RR}$ and $\sigma_{LL}$ are defined analogously.

Ref. Phys. Rept. 460 (2008) 131–243

# Polarized Bhabha scattering at one-loop

Here we present complete one-loop EW corrections to Bhabha scattering $e^+e^- \to e^+e^-$ with longitudinally polarized initilal particles

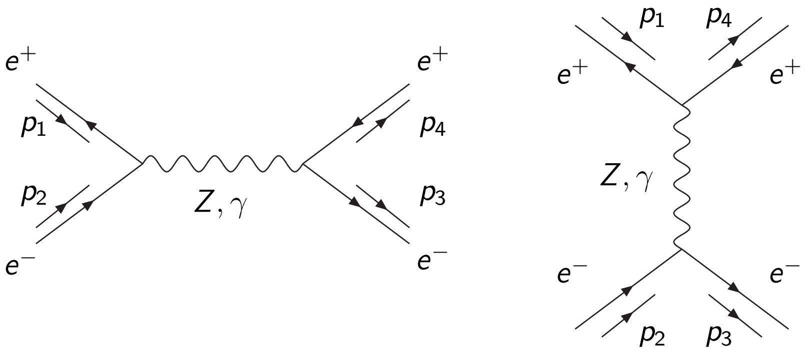
The cross-section of this process at one-loop can be devided into four parts:

$$\sigma^{\text{1-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

where $\sigma^{\text{Born}}$ — Born level cross-section, $\sigma^{\text{virt}}$ — contribution of virtual(loop) corrections, $\sigma^{\text{soft}}$ — contribution due to soft photon emission, $\sigma^{\text{hard}}$ — contribution due to hard photon emission (with energy $E_\gamma > \omega\frac{\sqrt{s}}{2}$).

Auxiliary parameters $\lambda$ ("photon mass") and $\omega$ cancel out after summation.

# Bhabha scattering: Born-level diagrams



# Polarized Bhabha scattering: HA for Born and Virtual parts

At one-loop level we have six non-zero HAs (four independent):

$$\mathcal{H}_{++++} = \mathcal{H}_{----} = -2e^2\,\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi^t_Z\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right],$$

$$\mathcal{H}_{+-+-} = \mathcal{H}_{-+-+} = -e^2\,c_-\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi^s_Z\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right],$$

$$\mathcal{H}_{+--+} = -e^2\,c_+\Big(\Big[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi^s_Z\left(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e\mathcal{F}^Z_{QL}(s,t,u)\right)\Big]$$

$$+\frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) + \chi^t_Z\left(\mathcal{F}^Z_{LL}(t,s,u) - 2\delta_e\mathcal{F}^Z_{QL}(t,s,u)\right)\right]\Big),$$

$$\mathcal{H}_{-++-} = -e^2\,c_+\left(\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u)\right] + \frac{s}{t}\left[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u)\right]\right),$$

where $c_+ = 1+\cos\theta,\quad c_- = 1-\cos\theta,$

$$\chi^s_Z = \frac{1}{4s^2_W c^2_W}\,\frac{s}{s-M^2_Z+iM_Z\Gamma_Z},\quad \chi^t_Z = \frac{1}{4s^2_W c^2_W}\,\frac{t}{t-M^2_Z},\quad \delta_e = v_e - a_e = 2s^2_W,$$

$$\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{\gamma}_{QQ}(a,b,c) + \chi^a_Z\delta^2_e\mathcal{F}^Z_{QQ}(a,b,c).$$

We get the Born level HAs by replacing $\mathcal{F}^Z_{LL}\to 1$, $\mathcal{F}^Z_{QL}\to 1$, $\mathcal{F}^Z_{QQ}\to 1$ and $\mathcal{F}^{\gamma}_{QQ}\to 1$.

# Polarized Bhabha scattering: soft photon contribution

The soft photon contribution contains the infrared divergences which compensate the infrared divergences of the one-loop QED corrections.

This soft photon correction can be calculated analytically and to be factorized to Born cross section. The polarization dependence is contained in $\sigma^{\mathrm{Born}}$.

$$\begin{aligned}
\sigma^{\mathrm{soft}}(\lambda,\omega) \qquad &= -\sigma^{\mathrm{Born}}\frac{\alpha}{\pi}\Biggl\{\left(1+\ln\left(\frac{m_e^2}{s}\right)\right)^2+\ln\left(-\frac{u}{s}\right)^2-\ln\left(-\frac{t}{s}\right)^2 \\
&-2\mathrm{Li}_2\left(-\frac{u}{s}\right)+2\mathrm{Li}_2\left(-\frac{t}{s}\right)+4\mathrm{Li}_2\left(1\right) \\
&-1+2\ln\left(\frac{4\omega^2}{\lambda}\right)\left[1+\ln\left(\frac{m_e^2}{s}\right)-\ln\left(\frac{t}{u}\right)\right]\Biggr\}.
\end{aligned}$$

# Polarized Bhabha scattering: Monte Carlo generator

We created Monte Carlo generator of unweighted events for the polarized Bhabha scattering $e^+e^- \to e^+e^-$ with complete one-loop EW corrections and with possibility to produce events in standard Les Houches format.

This generator uses adaptive algorithm `mFOAM` (CPC 177:441-458,2007) which is a part of `ROOT` program.

# Setup for tuned comparison

We performed a tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results WHIZARD program. The contributions of soft and virtual parts were compared with the results of AÏtalk program

Input parameters:

$$\alpha^{-1}(0) = 137.03599976,$$
$$M_W = 80.4514958 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$
$$m_e = 0.51099907 \text{ MeV}, \quad m_\mu = 0.105658389 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$
$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$
$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts:

$$|\cos\theta| < 0.9,$$
$$E_\gamma > 1 \text{ GeV} \quad \text{(for comparison of hard Bremsstrahlung)}.$$

# $e^+e^- \to e^+e^-$: WHIZARD vs SANC (Born)

| $P_{e^-}, P_{e^+}$ | 0, 0 | -0.8, 0 | -0.8, -0.6 | -0.8, 0.6 |
|---|---|---|---|---|
| $\sqrt{s} = 250$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 56.677(1) | 57.774(1) | 56.272(1) | 59.276(1) |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 56.677(1) | 57.775(1) | 56.272(1) | 59.275(1) |
| $\sqrt{s} = 500$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 14.379(1) | 15.030(1) | 12.706(1) | 17.355(1) |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 14.379(1) | 15.030(1) | 12.706(1) | 17.354(1) |
| $\sqrt{s} = 1000$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 3.6792(1) | 3.9057(1) | 3.0358(1) | 4.7756(1) |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 3.6792(1) | 3.9057(1) | 3.0358(1) | 4.7755(1) |

# $e^+e^- \to e^+e^-$: WHIZARD vs SANC (hard)

| $P_{e^-}, P_{e^+}$ | 0, 0 | -0.8, 0 | -0.8, -0.6 | -0.8, 0.6 |
|---|---|---|---|---|
| $\sqrt{s} = 250$ GeV | | | | |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 48.62(1) | 49.58(1) | 48.74(1) | 50.40(1) |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 48.65(1) | 49.56(1) | 48.78(1) | 50.44(1) |
| $\sqrt{s} = 500$ GeV | | | | |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 15.14(1) | 15.81(1) | 13.54(1) | 18.07(1) |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 15.12(1) | 15.79(1) | 13.55(1) | 18.11(2) |
| $\sqrt{s} = 1000$ GeV | | | | |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 4.693(1) | 4.976(1) | 3.912(1) | 6.041(1) |
| $\sigma^{\text{hard}}_{e^+e^-}$, pb | 4.694(1) | 4.975(1) | 3.913(1) | 6.043(1) |

# $e^+e^- \to e^+e^-$: AI̊talk vs SANC (virtual+soft)

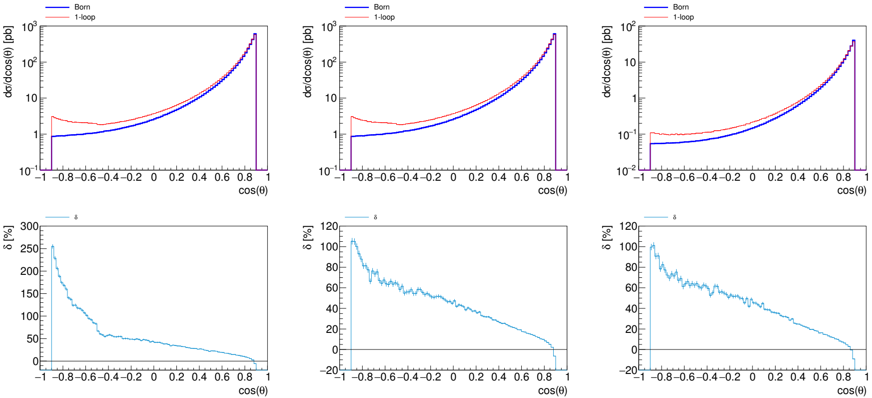
| $\cos\theta$ | $\sigma^{\text{Born}}_{e^+e^-}$, pb | $\sigma^{\text{Born+virt+soft}}_{e^+e^-}$, pb |
|---|---|---|
| $-0.9$ | $2.16999 \cdot 10^{-1}$<br>$2.16999 \cdot 10^{-1}$ | $1.93445 \cdot 10^{-1}$<br>$1.93445 \cdot 10^{-1}$ |
| $-0.5$ | $2.61360 \cdot 10^{-1}$<br>$2.61360 \cdot 10^{-1}$ | $2.38707 \cdot 10^{-1}$<br>$2.38707 \cdot 10^{-1}$ |
| 0 | $5.98142 \cdot 10^{-1}$<br>$5.98142 \cdot 10^{-1}$ | $5.46677 \cdot 10^{-1}$<br>$5.46677 \cdot 10^{-1}$ |
| $+0.5$ | $4.21273 \cdot 10^{0}$<br>$4.21273 \cdot 10^{0}$ | $3.81301 \cdot 10^{0}$<br>$3.81301 \cdot 10^{0}$ |
| $+0.9$ | $1.89160 \cdot 10^{2}$<br>$1.89160 \cdot 10^{2}$ | $1.72928 \cdot 10^{2}$<br>$1.72928 \cdot 10^{2}$ |
| $+0.99$ | $2.06556 \cdot 10^{4}$<br>$2.06555 \cdot 10^{4}$ | $1.90607 \cdot 10^{4}$<br>$1.90607 \cdot 10^{4}$ |
| $+0.999$ | $2.08236 \cdot 10^{6}$<br>$2.08236 \cdot 10^{6}$ | $1.91624 \cdot 10^{6}$<br>$1.91624 \cdot 10^{6}$ |
| $+0.9999$ | $2.08429 \cdot 10^{8}$<br>$2.08429 \cdot 10^{8}$ | $1.91402 \cdot 10^{8}$<br>$1.91402 \cdot 10^{8}$ |

# $e^+e^- \to e^+e^-$: Born vs 1-loop

| $P_{e^-}$, $P_{e^+}$ | 0, 0 | -0.8, 0 | -0.8, -0.6 | -0.8, 0.6 |
|---|---|---|---|---|
| $\sqrt{s} = 250$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 56.677(1) | 57.775(1) | 56.272(1) | 59.275(1) |
| $\sigma^{\text{1-loop}}_{e^+e^-}$, pb | 61.55(1) | 59.72(3) | 61.02(3) | 58.44(3) |
| $\delta$, % | 8.59(2) | 3.37(5) | 8.45(5) | -1.42(5) |
| $\sqrt{s} = 500$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 14.379(1) | 15.030(1) | 12.706(1) | 17.354(1) |
| $\sigma^{\text{1-loop}}_{e^+e^-}$, pb | 15.436(7) | 14.441(7) | 13.501(6) | 15.40(1) |
| $\delta$, % | 7.35(5) | -3.92(5) | 6.26(5) | -11.29(5) |
| $\sqrt{s} = 1000$ GeV | | | | |
| $\sigma^{\text{Born}}_{e^+e^-}$, pb | 3.6792(1) | 3.9057(1) | 3.0358(1) | 4.7755(1) |
| $\sigma^{\text{1-loop}}_{e^+e^-}$, pb | 3.862(2) | 3.609(2) | 3.148(1) | 4.067(3) |
| $\delta$, % | 4.98(5) | -7.60(5) | 3.70(5) | -14.84(6) |

# $e^+e^- \to e^+e^-$: **distributions on** $\cos\theta$

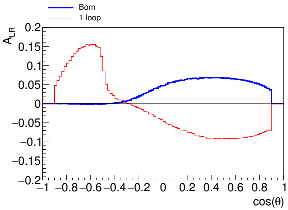
$\sqrt{s} = 250$ GeV $\quad$ $\sqrt{s} = 500$ GeV $\quad$ $\sqrt{s} = 1000$ GeV



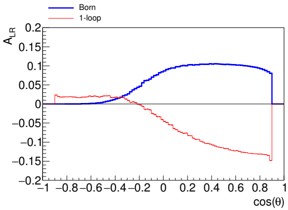
# $e^+e^- \to e^+e^-$: $A_{LR}$ distributions on $\cos\theta$

$$A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$

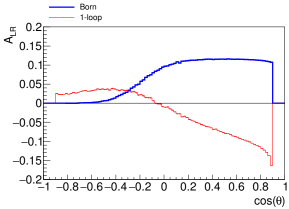
$\sqrt{s} = 250$ GeV



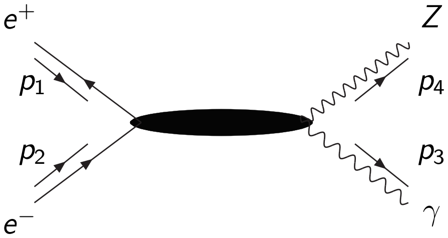

$\sqrt{s} = 500$ GeV




$\sqrt{s} = 1000$ GeV



# $e^+e^- \to Z\gamma$




The expressions fo HA and the results for unpolarized case were published in Eur.Phys.J.C54:187-197,2008

Here we present the preliminary results for 1-loop corrections taking into account the effect of polarization of $e^+e^-$ beams.

# $e^+e^- \to Z\gamma$: WHIZARD vs SANC

| $\sqrt{s}$, GeV | 250 | 500 | 1000 |
|---|---|---|---|
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 45.28(1) | 10.55(1) | 2.694(5) |
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 45.27(1) | 10.52(1) | 2.697(1) |
| $\sigma_{Z\gamma}^{\text{hard}}$, pb | 22.2(1) | 6.53(2) | 2.01(1) |
| $\sigma_{Z\gamma}^{\text{hard}}$, pb | 22.4(1) | 6.48(2) | 1.98(1) |

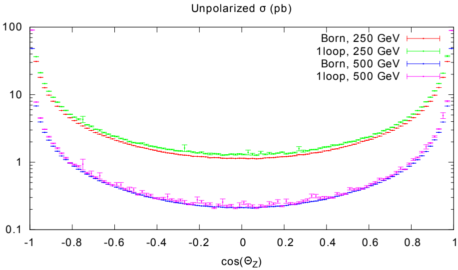
| $P_{e^-}$, $P_{e^+}$ | 0, 0 | -0.8, 0 | -0.8, 0.6 | -0.8, -0.6 |
|---|---|---|---|---|
| $\sqrt{s} = 250$ GeV | | | | |
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 45.28(1) | 53.15(1) | 79.48(3) | 26.78(1) |
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 45.27(1) | 53.14(1) | 79.50(1) | 26.78(1) |
| $\sigma_{Z\gamma}^{\text{hard}}$, pb | 22.2(1) | 27.0(1) | 40.1(1) | 13.3(1) |
| $\sigma_{Z\gamma}^{\text{hard}}$, pb | 22.4(1) | 26.4(1) | 40.2(1) | 12.5(1) |

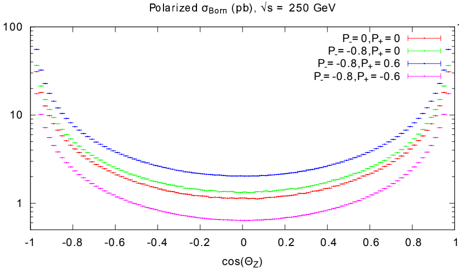
# $e^+e^- \to Z\gamma$: Born vs 1-loop (preliminary)

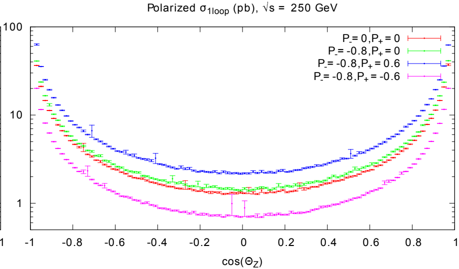
| $\sqrt{s}$ | 250 | 500 | 1000 |
|---|---|---|---|
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 15.7038(6) | 3.3858(3) | 0.81958(3) |
| $\sigma_{Z\gamma}^{\text{1-loop}}$, pb | 24.37(1) | 5.23(6) | 1.237(3) |
| $\delta$, % | 55.20(6) | 54.4(2) | 50.9(4) |

| $P_{e^-}$, $P_{e^+}$ | 0, 0 | -0.8, 0 | -0.8, 0.6 | -0.8, -0.6 |
|---|---|---|---|---|
| $\sqrt{s} = 250$ GeV | | | | |
| $\sigma_{Z\gamma}^{\text{Born}}$, pb | 15.7038(6) | 18.520(4) | 28.174(2) | 8.870(3) |
| $\sigma_{Z\gamma}^{\text{1-loop}}$, pb | 24.37(1) | 28.00(1) | 42.13(2) | 13.53(1) |
| $\delta$, % | 55.20(6) | 51.11(8) | 49.55(7) | 52.57(9) |

# $e^+e^- \to Z\gamma$: distributions on $\cos\theta_Z$ (preliminary)



# Conclusions

- We developed SANC basis for processes with polarized $e^+e^-$ beams taking into account complete one-loop EW corrections
- We created the SANC modules for polarized Bhabha scattering with complete one-loop EW corrections. Based on these modules Monte Carlo generator [arXiv:1801.00125] of unweighted events was created with possibility to produce events in Les Houches Event format
- Preliminary results for polarized $e^+e^- \to Z\gamma$ were presented

# Plans

- to include in SANC Monte Carlo generator the following processes:
  - $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$
  - $e^+e^- \to t\bar{t}$
  - $e^+e^- \to ZH$
  - $e^+e^- \to \gamma\gamma$
  - $e^+e^- \to ZZ$
  - $e^+e^- \to f\bar{f}\gamma$
  - $e^+e^- \to f\bar{f}H$
  - $\gamma\gamma \to \gamma\gamma$, $\gamma\gamma \to Z\gamma$, $\gamma\gamma \to ZZ$
- to extend the functionality of this generator with actual energy spectrum of initial particles and with transverse polarization