Electric conductivity of a hadron gas

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Introduction

In order to understand the dynamical behavior of heavy-ion collisions, it is important to study transport coefficients, which characterize the behavior of the system out of equilibrium. After the quark-gluon plasma is formed in a heavy ion collision it rapidly cools down and undergoes a phase transition to the hadronic phase. This gas is still hot and the particles interact with each other. There are several applications of the electric conductivity. It is possible to relate the soft photon rates for low p_T to it. Furthermore for hydrodynamical calculations one needs transport coefficients as an input parameter. Knowing the electric conductivity for both hadronic and QGP matter gives a better understanding of their nature.

Model: SMASH

Simulating **M**any **A**ccelerating **S**trongly-interacting **H**adrons [1]
Hadronic transport approach which effectively solves Boltzmann equation

$$p^\mu \partial_\mu f_i(x,p) + m_i F^\alpha \partial^p_\alpha f_i(x,p) = C^i_{coll}\,.$$

It uses a geometric collision criterion for particle collision

$$d_{coll} < \sqrt{\frac{\sigma_{tot}}{\pi}}$$

In order to calculate the electric conductivity we use a box with periodic boundary conditions. When initialising the Box, the momentum of the particles are sampled with a Boltzmann distribution.

Method

In order to extract the transport coefficient we apply the well established Green-Kubo formalism which is based on linear response theory. The electric conductivity can be calculated in the following way

$$\sigma_{el} = \frac{V}{T}\int_0^\infty \langle j_i(0) j_i(t)\rangle\,,$$

where V is the volume of the system, T the temperature of the box in equilibrium and $\langle j_i(0) j_i(t)\rangle$ is the so-called auto-correlation function of the electric current. The auto-correlation function can be calculated in the following way

$$\langle j_i(t) j_i(t_l)\rangle = \frac{1}{s_{max}}\sum_{s=0}^{s_{max}} j_i(t_s) j_i(t_s+t_l) \quad t = \{t_1, t_2, ..., t_K\} \quad s_{max} = K - l\,,$$

where K denotes the total number of time steps. It is possible to make a simple ansatz to this function, which has an exponential shape as seen in Figure 2

$$\langle j_i(0) j_i(t)\rangle = C(0) e^{-\frac{t}{\tau}}\,.$$

That implies that the electric conductivity can be computed the following way:

$$\sigma_{el} = \frac{V\tau C(0)}{T}\,.$$

Where C(0) is the correlation function at the origin and τ the inverse slope parameter of the correlation function.

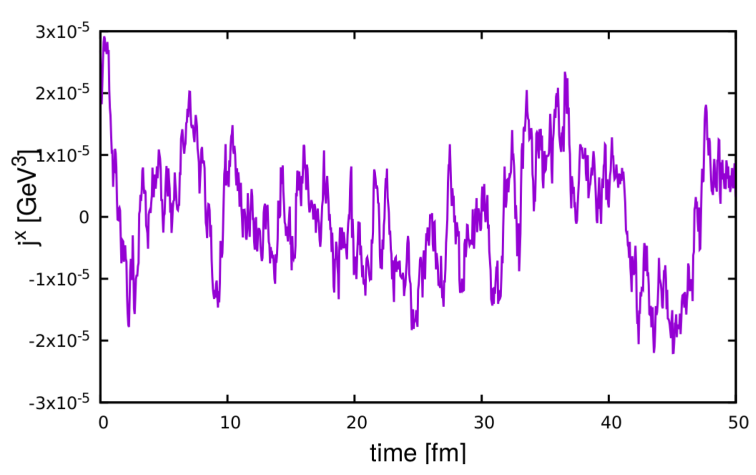

Figure 1: Electric current of a box filled with pions interacting with a constant isotropic cross section. Since there is no external force the value fluctuates around zero.

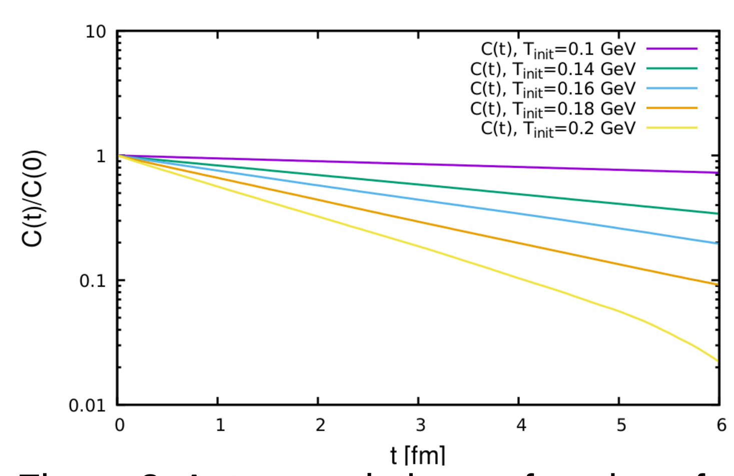

Figure 2: Auto-correlation as function of time for different initialisation temperatures. The system contains only pions interacting via a constant isotropic cross section.

Relaxation time

Fit parameter τ of the correlation function can be related to the relaxation time of the system, which is the time the system needs to equilibrate.
In SMASH the interaction between two particles is modeled via the formation and decay of a resonance. Since they are treated as real particles, they propagate in space until they decay (Figure 3, right side). In analytic calculations the cross section is only modeled with the shape of the resonance (Figure 3, left side). Since we are comparing our results to analytic calculation, this effect is not negligible as seen in [4].

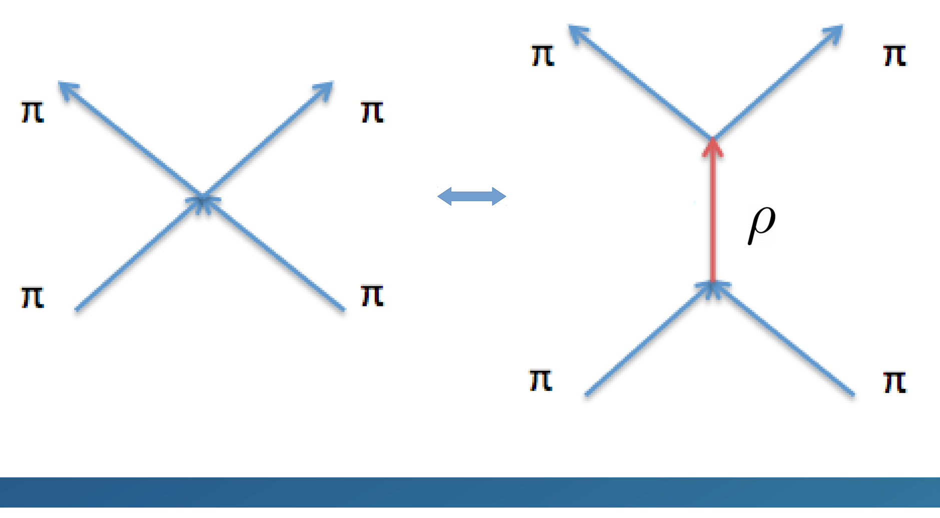

Figure 3: Sketch of the two descriptions

Correlation function at the origin

First value of the correlation function at the origin is well defined in thermodynamic equilibrium and can be calculated analytically in the following way

$$\langle j_i(0) j_i(0)\rangle V = \sum_{a=1}^{N_s} \frac{g_a q_a^2 e^2}{6\pi^2}\int_0^\infty dp \frac{p^4}{m_a^2+p^2} e^{-\frac{\sqrt{m_a^2+p^2}}{T}}\,.$$

Where the sum runs over all particle species with degeneracy *g*, electric charge *q* and mass *m*.
This comparison is a good cross-check to the numerical results from SMASH. Figure 4 (right) and Figure 5 (right) show very good agreement to the analytic calculation.

Results

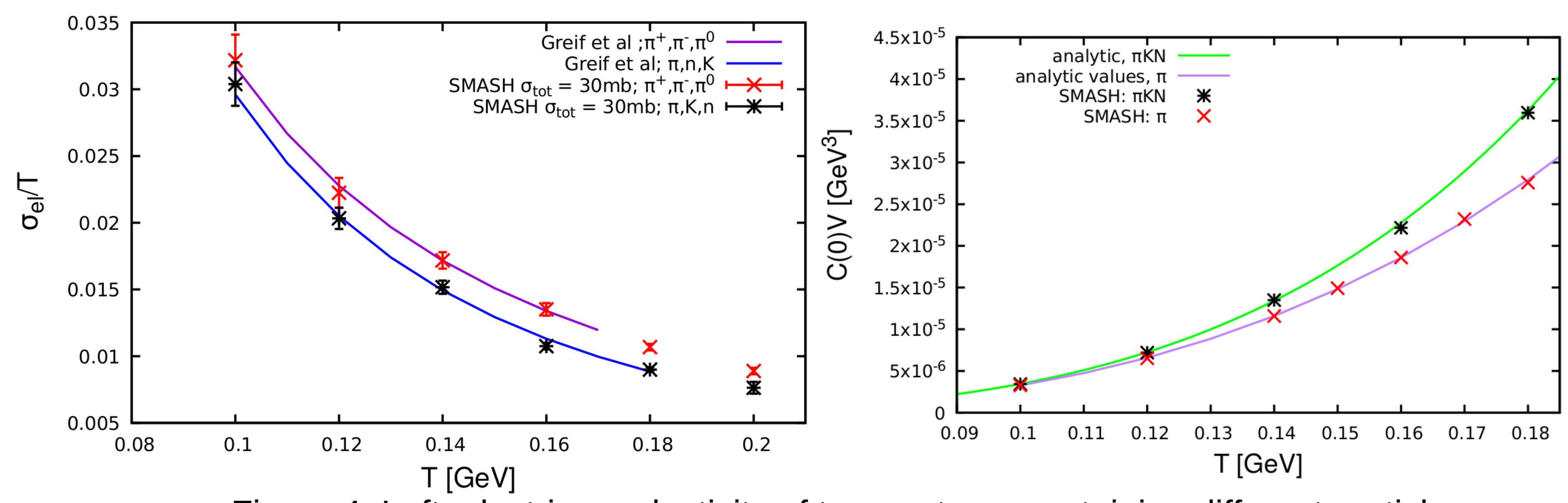

Figure 4: Left: electric conductivity of two systems containing different particle species interacting with a constant isotropic cross section, compared to analytic calculation taken from [3]. Right: C(0) of these systems compared to analytic values.

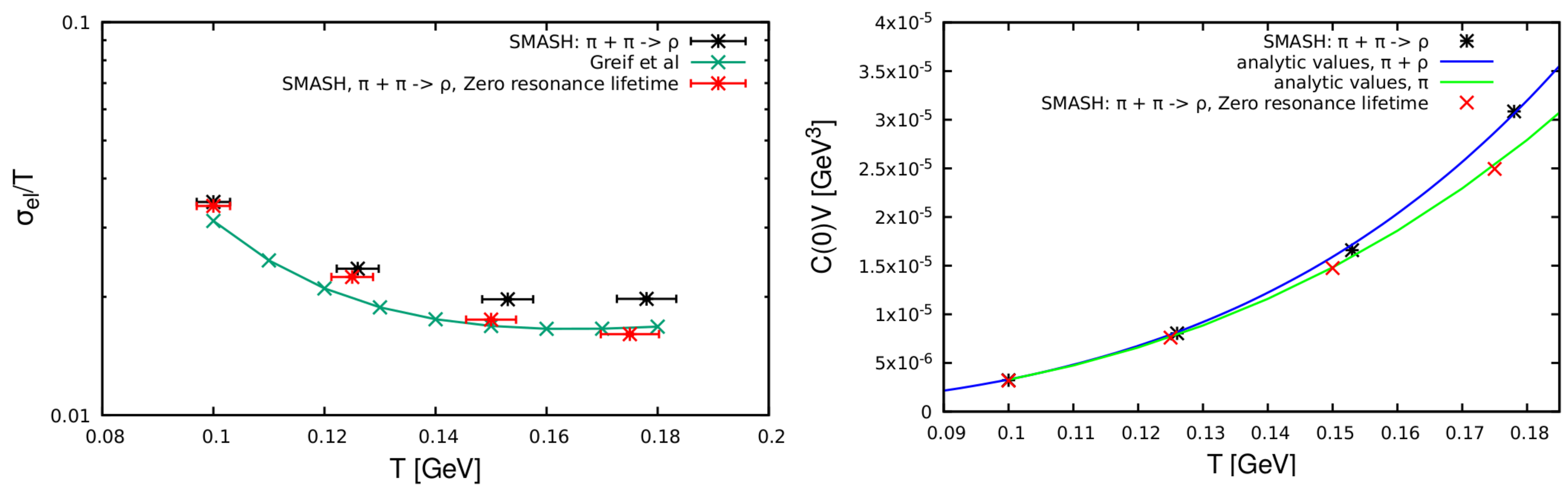

Figure 5: Left electric conductivity of a pion gas interacting via a rho resonance compared to analytic calculation taken from [3]. Comparison of the effect of a zero resonance lifetime and a non-zero resonance lifetime on the electric conductivity. Right: Comparison of C(0) for these systems.

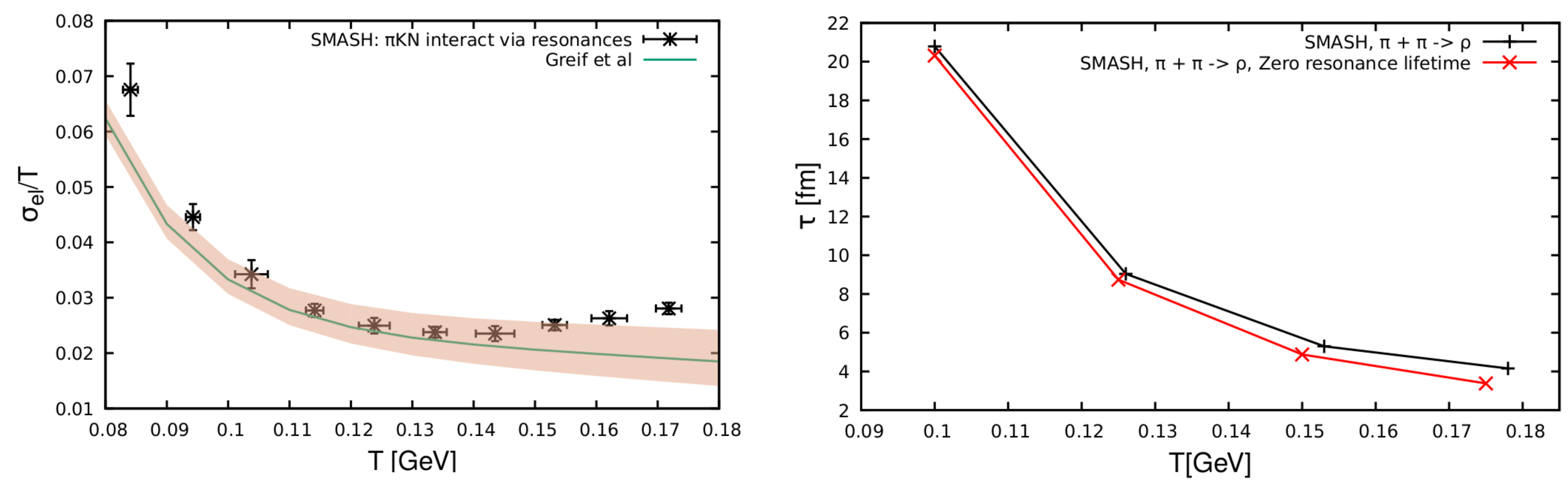

Figure 6: Left: Electric conductivity of a gas, containing π, K and N interacting via Δ, ρ and K^* compared to analytic calculation taken from [3]. Right: Comparison of fit parameter τ of two systems from Figure 5 (left).

Conclusion and Outlook

In this work the electric conductivity of a hot hadron gas has been calculated using the Green-Kubo formalism.
In comparison with the results in [3], we have excellent agreement with analytic calculation for systems interacting with constant isotropic cross sections. And reasonable agreement for results of resonance gas between SMASH and analytical calculation.
Future work would include going to non-zero baryochemical potential and investigate more on the differences between the results from this work and the analytical calculation from [3].

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