Hydrodynamic results of a Principal Component Analysis at $\sqrt{s_{NN}} = 2.76$ TeV

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Abstract

We perform a principal component analysis (PCA) of two-particle correlations in event-by-event hydrodynamic simulations of Pb+Pb collisions at the Large Hadron Collider. PCA is a statistical technique for extracting the dominant components in fluctuating data. It was suggested to apply it to relativistic collisions [1] in order to extract the information from event-by-event fluctuations from the two-particle correlation matrix. A generalization was proposed in [2]. Its connection to initial geometry was studied in [3, 4]. Here we make a comparison with the data recently presented by the CMS collaboration [5] for elliptic and triangular flows as well as factorization breaking [9].

Introduction

In Heavy Ion Collisions the initial conditions fluctuate between events. Principal Component Analysis extracts the fluctuations of a data set, therefore it can be a tool to investigate the physics of the initial conditions and the models that describe them.

To simulate the hydrodynamics of the Quark Gluon Plasma, NeXSPheRIO [7] was used. It uses NeXus [6] initial conditions and solves 3+1 dimensional perfect fluid equations with the Smoothed Particle Hydrodynamics method.

Anisotropic Flow

Event Plane

The distribution of particles can be expanded in a Fourier series:

$$\frac{dN}{p_T dp_T\, d\eta\, d\phi} = \frac{dN}{2\pi\, p_T dp_T\, d\eta} \sum_{n=-\infty}^{\infty} V_n(p_T,\eta) e^{in\phi}, \quad (1)$$

$$V_n(p_T,\eta) = v_n(p_T,\eta) e^{in\psi_n(p_T,\eta)} \quad (2)$$

$$= \frac{1}{N(p_T,\eta)} \sum_j e^{in\phi_j} = \frac{1}{N} Q_n. \quad (3)$$

Here, the sum is over particles in a given interval of p_T and η in one event. Q_n is called the flow vector.

Two-Particle Correlation

The distribution of particle pairs is determined by the single particle distribution, if non-flow effects are neglected:

$$\frac{dN_{\text{pairs}}}{d^3p_a d^3p_b} \approx \frac{dN}{d^3p_a} \frac{dN}{d^3p_b}. \quad (4)$$

It can also be expanded in a Fourier series:

$$\left\langle \frac{dN_{\text{pairs}}}{d^3p_a d^3p_b} \right\rangle \propto \sum_n V_{n\Delta}(p_a,p_b) e^{in(\phi_a-\phi_b)}, \quad (5)$$

$$V_{n\Delta}(p_a,p_b) = \langle V_n(p_a) V_n^*(p_b) \rangle = \langle v_n(p_a) v_n(p_b) e^{in(\psi_{a_n}-\psi_{b_n})} \rangle. \quad (6)$$

Here, the brackets indicate an average over events.

Principal Component Analysis

A set of measurements $\{f_i\}$ of a random function f can be described by the mean μ and the noise $\{\epsilon_i\}$.

$$f_i(x) = \mu(x) + \epsilon_i(x). \quad (7)$$

With the covariance function, which is symmetric and non-negative,

$$G(x,y) = \text{Cov}(f(x), f(y)), \quad (8)$$

a linear operator that satisfies an eigenvalue equation can be defined:

$$\int G(x,y)\phi_\alpha(y) dy = \lambda_\alpha \phi_\alpha(x). \quad (9)$$

The covariance and the noise can be expanded in this eigenvector basis:

$$G(x,y) = \sum_\alpha \lambda_\alpha \phi_\alpha(x) \phi_\alpha(y), \quad (10)$$

$$\epsilon_i(x) = \sum_\alpha \xi_{i\alpha} \phi_\alpha(x), \quad (11)$$

where $\xi_{i\alpha}$ are coefficients with zero mean and variance λ_α, different for each measurement (event).

Writing $V_{n\Delta}$ in terms of the flow vector, Q_n, the two-particle correlation coefficient can be interpreted as a covariance function:

$$G(x,y) \to V_{n\Delta}(p_a,p_b) = \langle Q_n(p_a) Q_n^*(p_b) \rangle. \quad (12)$$

In practice, the momentum is discretized. Therefore, the covariance function is, in fact, a covariance matrix:

$$V_{n\Delta}(p_a,p_b) = \left\langle \Big(Q_n(p_a) - \langle Q_n(p_a)\rangle\Big) \Big(Q_n(p_b) - \langle Q_n(p_b)\rangle\Big) \right\rangle. \quad (13)$$

Since the event plane is random, $\langle Q_n \rangle$ is zero for $n > 0$, resulting in:

$$V_{n\Delta} = \begin{pmatrix} \langle Q_n(p_1) Q_n^*(p_1) \rangle & \cdots & \langle Q_n(p_1) Q_n^*(p_N) \rangle \\ \vdots & \ddots & \vdots \\ \langle Q_n(p_N) Q_n^*(p_1) \rangle & \cdots & \langle Q_n(p_N) Q_n^*(p_N) \rangle \end{pmatrix}. \quad (14)$$

From eq. 10, $V_{n\Delta}$ can be written in terms of its eigenvectors. And so the modes can be defined as:

$$V_n^{(\alpha)}(p) \equiv \sqrt{\lambda_\alpha} \phi_\alpha(p). \quad (15)$$

Since the flow coefficients, V_n, are the variance (noise) of Q_n, as a consequence from eq. 11, they can be expressed by the eigenvectors of the covariance matrix. And if there is only one mode, it must be equal to V_n. So, the first component, with the highest eigenvalue, should be the anisotropic flow and the others must be related to fluctuations.

Since the flow vector is not normalized, to compare to the usual flow coefficients, the modes are divided by $\langle V_0 \rangle$.

$$v_n^{(\alpha)} = \frac{V_n^{(\alpha)}}{\langle V_0 \rangle} \quad (16)$$

Factorization Breaking

In particular, the Pearson correlation coefficient can be estimated with the leading and sub-leading modes, if $V_n^{(1)} \gg V_n^{(2)}$:

$$r_n(p_a,p_b) = \frac{V_{n\Delta}(p_a,p_b)}{\sqrt{V_{n\Delta}(p_a,p_a) V_{n\Delta}(p_b,p_b)}} \quad (17)$$

$$\simeq 1 - \frac{1}{2} \left| \frac{V_n^{(2)}(p_a)}{V_n^{(1)}(p_a)} - \frac{V_n^{(2)}(p_b)}{V_n^{(1)}(p_b)} \right|^2 \quad (18)$$

If correlations are due to flow, $|r| \leq 1$, and if they factorize, $|r| = 1$.

Results

In addition to comparisons with CMS data [5, 9], figures 1, 2 and 3 show comparisons with v_n calculated using event plane and r_n without PCA approximation, to ensure consistency.

To analyze the transverse momentum dependence, the distributions were integrated over pseudorapidity.

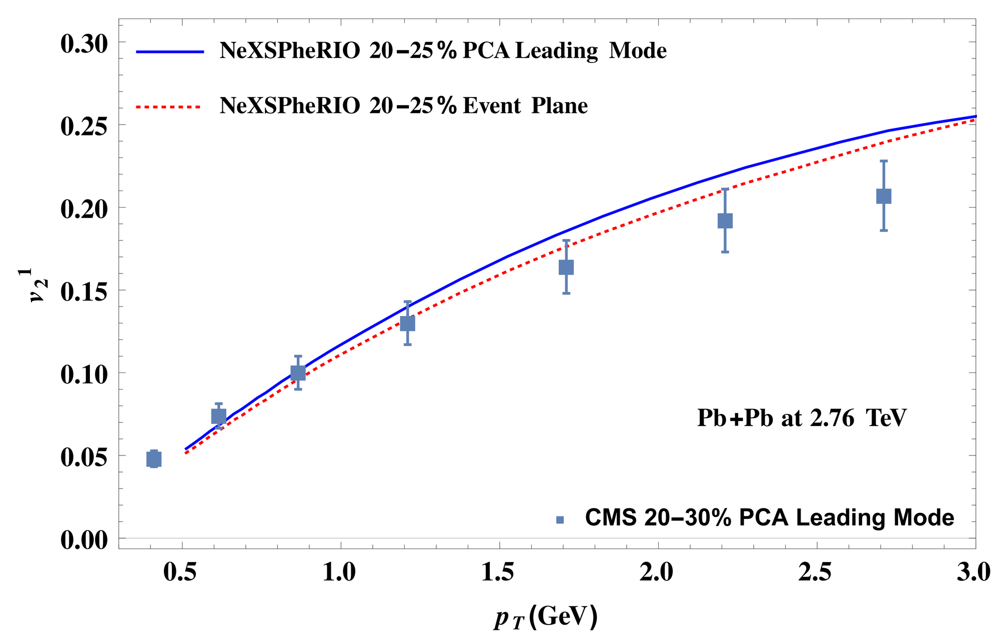

Figure 1: First component and event plane calculation of v_2 from simulations of Pb+Pb collisions at 2.76TeV with 20-25% centrality, and first component from CMS data of Pb+Pb at 2.76TeV with 20-30% centrality.

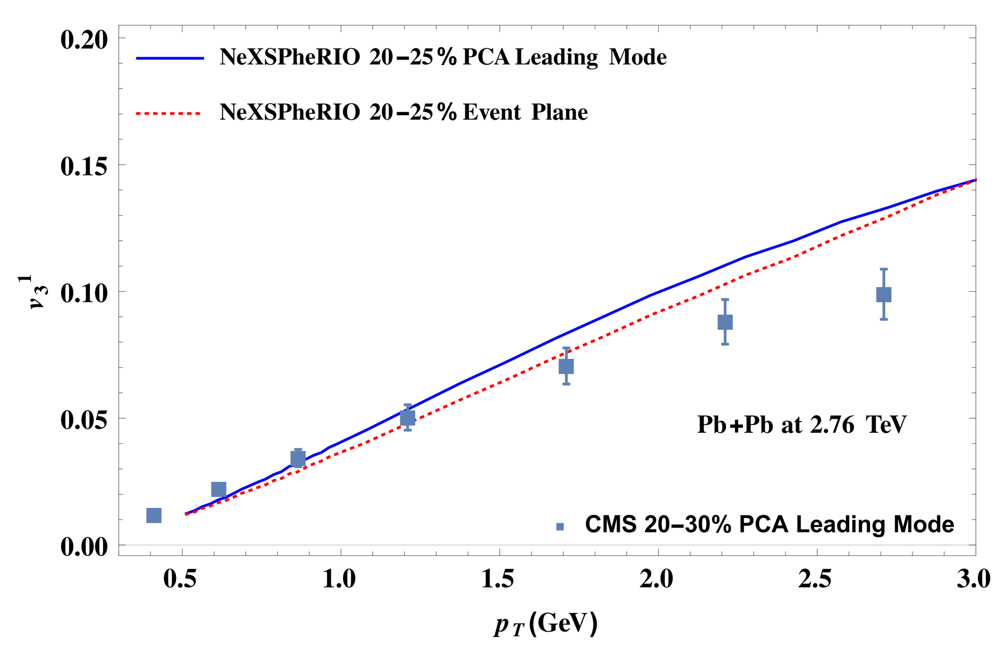

Figure 2: First component and event plane calculation of v_3 from simulations of Pb+Pb collisions at 2.76TeV with 20-25% centrality, and first component from CMS data of Pb+Pb at 2.76TeV with 20-30% centrality.

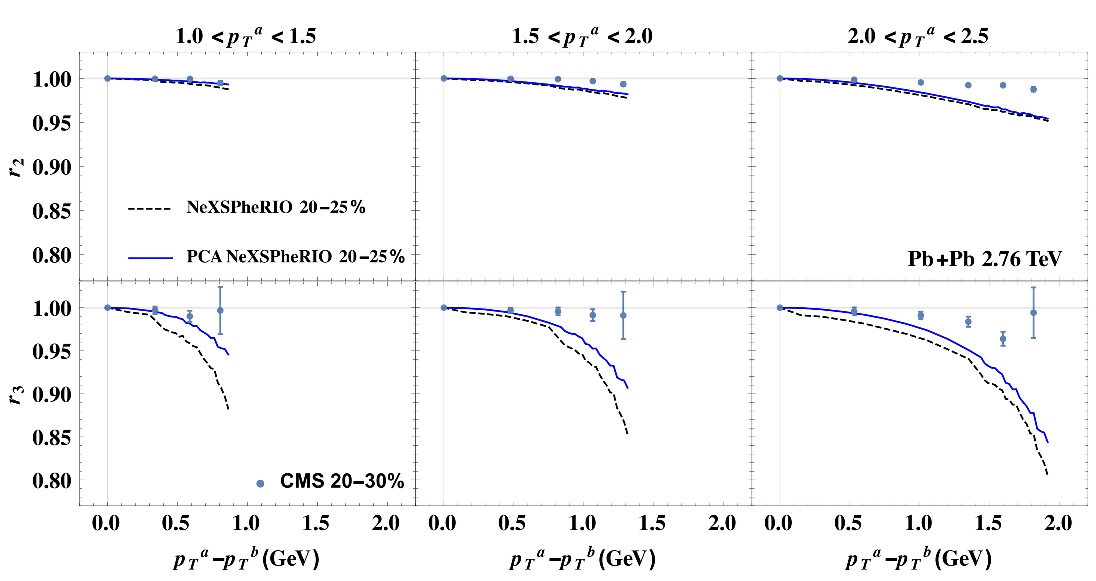

Figure 3: Factorization breaking estimated by PCA and calculated exactly from simulations of Pb+Pb collisions at 2.76TeV with 20-25% centrality, and calculated from CMS data of Pb+Pb at 2.76TeV with 20-30% centrality.

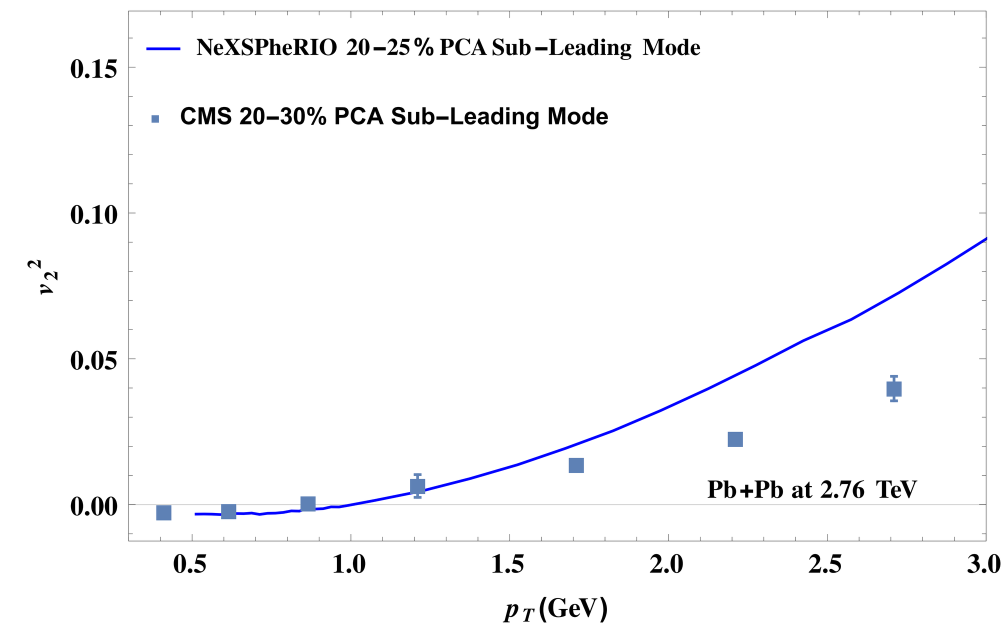

Figure 4: Second component of v_2 from simulations of Pb+Pb collisions at 2.76TeV with 20-25% centrality and second component from CMS data of Pb+Pb at 2.76TeV with 20-30% centrality.

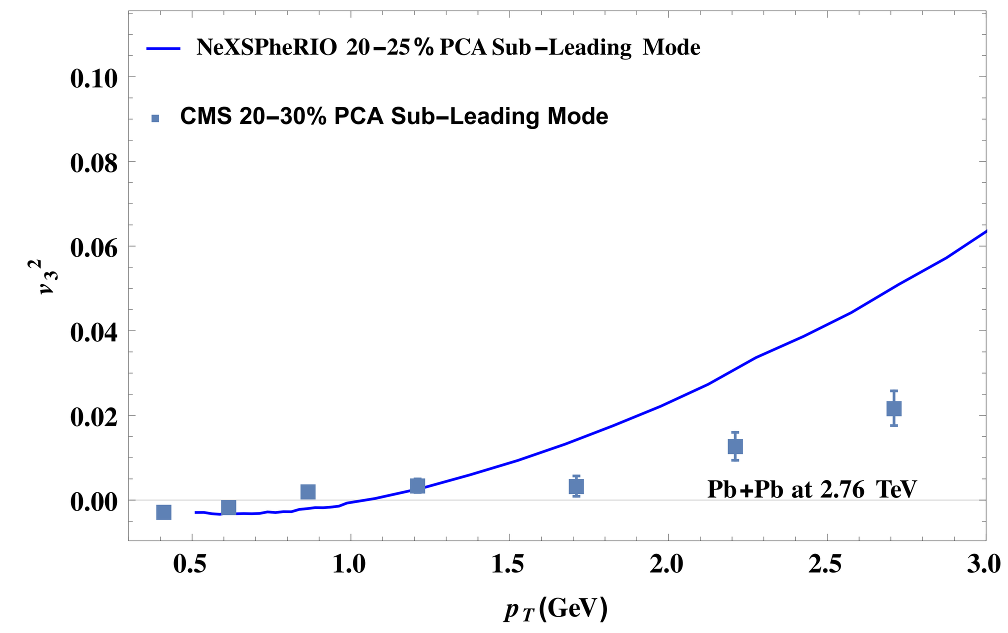

Figure 5: Second component of v_3 from simulations of Pb+Pb collisions at 2.76TeV with 20-25% centrality and second component from CMS data of Pb+Pb at 2.76TeV with 20-30% centrality.

Conclusions

Self-consistency

Our PCA is in agreement with standard calculations of v_n and r_n and can give more information with other components.

Comparison with data

NeXSPheRIO is in agreement with CMS data. The differences at high transverse momentum may be due to the ideal fluid approximation.

Next steps

Studies in the current centrality will continue and new ones will be made.

Error bars calculated with the Jackknife method will be added.

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Acknowledgements

P. I. is supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES). F. G. acknowledges support from Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). F. G. G. is supported by CNPq and Fundação de Amparo à Pesquisa do Estado de Minas Gerais (FAPEMIG).