Extension of the Identity Method to Measurements of Differential Correlation Functions

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Introduction/Summary

Integral/differential correlation functions (CF) of particles produced in A-A collisions provide invaluable information on the particle production dynamics, the system evolution, and the determination of fundamental properties of the quark matter produced. CFs measured for specific species (e.g., π, K, p, etc.) are of particular interest as they probe processes determined by conservation laws. Measurements based on traditional particle identification (PID) methods require large datasets given severe particle rejection may be experimentally incurred to achieve high species purity and low contamination. The **identity method (IM)** [1] provides a technique to essentially recover the full statistics, extend the kinematic range of measurements while providing reliable disambiguation of particle species [2-4]. The method is here extended towards measurements of differential correlation functions, e.g., normalized cumulants R_2, and is developed for an arbitrary number of particle species, p_T dependent particle losses, as well as an arbitrary number of particle identification signals.

References

[1] M. Gazdzicki, EPJ C8 (1999) 131,
[2] A. Rustamov, et al. PRC 86 (2012) 044906,
[3] M. I. Gorenstein, PRC 84 (2011) 024902,
[4] C. Pruneau, PRC 96 (2017) 054902.

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Definitions:

Discretize particle momenta: $\left(y,\phi,p_T\right)\rightarrow\vec{\alpha}\equiv\left(\alpha_1,\alpha_2,\alpha_3\right)$ Dimensions: $n_\eta \times n_\phi \times n_{pT}$ Projections: $\left(y,\phi\right)\rightarrow\vec{\alpha}^{(2)}=\left(\alpha_1,\alpha_2\right)$ $\left(\Delta y,\Delta\phi\right)\rightarrow\Delta\vec{\alpha}^{(2)}=\left(\Delta\alpha_1,\Delta\alpha_2\right)$

Discretized single particle density (Species p): $\rho_1^p(\vec{\alpha})\rightarrow\left\langle N_p(\vec{\alpha})\right\rangle$ Discretized particle pair density (Species p,q): $\rho_2^{p,q}(\vec{\alpha},\vec{\beta})\rightarrow\left\langle N_p(\vec{\alpha})\left(N_q(\vec{\beta})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle$

Integration over pT: $\rho_1^p(\vec{\alpha}^{(2)})\rightarrow\left\langle N_p(\vec{\alpha}^{(2)})\right\rangle=\sum_{\alpha_3=1}^{m_p}N_p(\alpha_1,\alpha_2,\alpha_3)$ $\rho_2^{p,q}(\vec{\alpha}^{(2)},\vec{\beta}^{(2)})\rightarrow\left\langle N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle=\sum_{\alpha_3,\beta_3=1}^{m_p}\left\langle N_p(\vec{\alpha})\left(N_q(\vec{\beta})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle$

Normalized cumulant (4D): $R_2^{p,q}(\vec{\alpha}^{(2)},\vec{\beta}^{(2)})=\dfrac{\left\langle N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle}{\left\langle N_p(\vec{\alpha}^{(2)})\right\rangle\left\langle N_q(\vec{\beta}^{(2)})\right\rangle}-1$ Acceptance averaged cumulant: $R_2^{p,q}(\Delta\vec{\alpha}^{(2)})=\dfrac{1}{\Omega(\Delta\alpha_1)}\sum_{\alpha_1,\alpha_2,\beta_1,\beta_2=1}R_2^{p,q}(\vec{\alpha}^{(2)},\vec{\beta}^{(2)})\delta(\Delta\alpha_1-\alpha_1+\beta_1)\delta(\Delta\alpha_2-\alpha_2+\beta_2)$

Accounting for detection efficiencies:

True joint probability distribution of K species: $P_T\left(\vec{N}_1,\vec{N}_2,\ldots,\vec{N}_K\right)$

Measured distribution of K species: $P_M\left(\vec{n}_1,\vec{n}_2,\ldots,\vec{n}_K\right)=\sum_{\vec{N}}P_T\left(\vec{N}_1,\vec{N}_2,\ldots,\vec{N}_K\right)\prod_{\vec{\alpha}_1}B\left(n_1(\vec{\alpha}_1)|N_1(\vec{\alpha}_1),\varepsilon_1(\vec{\alpha}_1)\right)\times\prod_{\vec{\alpha}_2}B\left(n_2(\vec{\alpha}_2)|N_2(\vec{\alpha}_2),\varepsilon_2(\vec{\alpha}_2)\right)\times\cdots\times\prod_{\vec{\alpha}_K}B\left(n_K(\vec{\alpha}_K)|N_K(\vec{\alpha}_K),\varepsilon_K(\vec{\alpha}_K)\right)$

Binomial losses
$B(n|N,\varepsilon)=\dfrac{N!}{n!(N-n)!}\varepsilon^n(1-\varepsilon)^{N-n}$

Expectation values:

True expectation values:

$$\left\langle N_p(\vec{\alpha}^{(2)})\right\rangle=\sum_{N}N_p(\vec{\alpha}^{(2)})P_T(\vec{N}_1,\ldots,\vec{N}_K)$$

$$\left\langle N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle=\sum_{N}N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)P_T(\vec{N}_1,\ldots,\vec{N}_K)$$

Measured expectation values:

$$\left\langle n_p(\vec{\alpha}^{(2)})\right\rangle=\sum_{N}n_p(\vec{\alpha}^{(2)})P_M(\vec{n}_1,\ldots,\vec{n}_K)$$

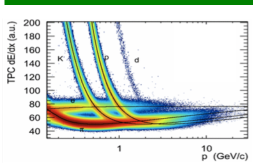
$$\left\langle n_p(\vec{\alpha}^{(2)})\left(n_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle=\sum_{N}n_p(\vec{\alpha}^{(2)})\left(n_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)P_M(\vec{n}_1,\ldots,\vec{n}_K)$$

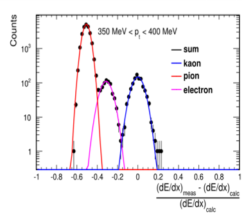
Measured multiplicities corrected for efficiencies:

$$\left\langle N_p(\vec{\alpha}^{(2)})\right\rangle=\sum_{\alpha_3=1}^{m_p}\frac{\left\langle n_p(\vec{\alpha})\right\rangle}{\varepsilon_p(\vec{\alpha})}$$

$$\left\langle N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle=\sum_{\alpha_3=1}^{m_p}\frac{\left\langle n_p(\vec{\alpha}^{(2)})\left(n_q(\vec{\beta}^{(2)})-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle}{\varepsilon_p(\vec{\alpha})\varepsilon_p(\vec{\beta})}$$

Identity Method – Variable Definitions





Probability signal "m" corresponds to species k:

$$\omega_k(m|\vec{\alpha})=\frac{P(m|d,k,\vec{\alpha})\varepsilon_k(\vec{\alpha})P(k,\vec{\alpha})}{\sum_{k'=1}^{K}P(m|d,k',\vec{\alpha})\varepsilon_{k'}(\vec{\alpha})P(k',\vec{\alpha})}$$

Probability of species k given PID signal "m":

$$P(k|m,d,\vec{\alpha})=\frac{P(m|d,k,\vec{\alpha})\varepsilon_k(\vec{\alpha})P(k,\vec{\alpha})}{P(m|d,\vec{\alpha})P(d,\vec{\alpha})}.$$

Identity variable for species "p" :

$$W_p(\vec{\alpha})\equiv\sum_{i=1}^{M}\omega_p(m_i|\vec{\alpha}).$$

Response functions:

$$u_{pq}(\vec{\alpha})=\int\omega_p(m|\vec{\alpha})P(m|d,q,\vec{\alpha})dm,$$

$$u_{pk}^{(2)}(\vec{\alpha})=\int\omega_p(m|\vec{\alpha})^2P(m|d,k,\vec{\alpha})dm,$$

$$u_{pqk}(\vec{\alpha})=\int\omega_p(m|\vec{\alpha})\omega_q(m|\vec{\alpha})P(m|d,k,\vec{\alpha})dm,$$

First Moments:

$$\left\langle W_p(\vec{\alpha}^{(2)})\right\rangle=\sum_{k=1}^{K}u_{pk}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\right\rangle\varepsilon_p(\vec{\alpha})$$

Second & Cross-Moments:

$$\left\langle\left(W_p(\vec{\alpha})\right)^2\right\rangle=\sum_{k,k'=1}^{K}u_{pk}^{(2)}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\right\rangle\varepsilon_p(\vec{\alpha})+\sum_{k,k'=1}^{K}u_{pk}(\vec{\alpha})u_{pk'}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\left(N_{k'}(\vec{\alpha})-\delta_{kk'}\right)\right\rangle\left(\varepsilon_p(\vec{\alpha})\right)^2$$

$$\left\langle W_p(\vec{\alpha})W_q(\vec{\beta})\right\rangle=\delta_{\vec{\alpha}\vec{\beta}}\sum_{k,k'=1}^{K}u_{pqk}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\right\rangle\varepsilon_p(\vec{\alpha})+\sum_{k,k'=1}^{K}u_{pk}(\vec{\alpha})u_{qk'}(\vec{\beta})\left\langle N_k(\vec{\alpha})\left(N_{k'}(\vec{\beta})-\delta_{kk'}\delta_{\vec{\alpha}\vec{\beta}}\right)\right\rangle\varepsilon_p(\vec{\alpha})\varepsilon_q(\vec{\beta})$$

Identity Method – Matrix formulation & Solution

$$V_{pq}(\vec{\alpha},\vec{\beta})=\left\langle W_p(\vec{\alpha})W_q(\vec{\beta})\right\rangle-\delta_{\vec{\alpha}\vec{\beta}}\delta_{pq}\sum_{k=1}^{K}u_{pk}^{(2)}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\right\rangle\varepsilon_k(\vec{\alpha})-\delta_{\vec{\alpha}\vec{\beta}}\sum_{k=1}^{K}u_{pqk}(\vec{\alpha})\left\langle N_k(\vec{\alpha})\right\rangle\varepsilon_k(\vec{\alpha})$$

$$\mathbb{V}_{pq}(\vec{\alpha},\vec{\beta})=\begin{bmatrix}V_{11}(\vec{\alpha},\vec{\beta}) & V_{12}(\vec{\alpha},\vec{\beta}) & \cdots & V_{1K}(\vec{\alpha},\vec{\beta})\\ V_{21}(\vec{\alpha},\vec{\beta}) & V_{22}(\vec{\alpha},\vec{\beta}) & \cdots & V_{1K}(\vec{\alpha},\vec{\beta})\\ \cdots & \cdots & \ddots & \vdots\\ V_{K1}(\vec{\alpha},\vec{\beta}) & \cdots & \cdots & V_{KK}(\vec{\alpha},\vec{\beta})\end{bmatrix}$$

$$\mathbb{N}^{(2)}(\vec{\alpha},\vec{\beta})=\begin{bmatrix}\left\langle N_1(\vec{\alpha})\left(N_1(\vec{\beta})\right)-\delta_{\vec{\alpha}\vec{\beta}}\right\rangle & \left\langle N_1(\vec{\alpha})N_2(\vec{\beta})\right\rangle & \cdots & \left\langle N_1(\vec{\alpha})N_K(\vec{\beta})\right\rangle\\ \left\langle N_2(\vec{\alpha})N_1(\vec{\beta})\right\rangle & \left\langle N_2(\vec{\alpha})\left(N_2(\vec{\beta})\right)-\delta_{\vec{\alpha}\vec{\beta}}\right\rangle & \cdots & \left\langle N_2(\vec{\alpha})N_K(\vec{\beta})\right\rangle\\ \cdots & \cdots & \ddots & \vdots\\ \left\langle N_K(\vec{\alpha})N_1(\vec{\beta})\right\rangle & \cdots & \cdots & \left\langle N_K(\vec{\alpha})\left(N_K(\vec{\beta})\right)-\delta_{\vec{\alpha}\vec{\beta}}\right\rangle\end{bmatrix}$$

$$\mathbb{N}^{(2)}(\vec{\alpha},\vec{\beta})=\mathbb{U}(\vec{\alpha})^{-1}\mathbb{V}(\vec{\alpha},\vec{\beta})\left(\mathbb{U}(\vec{\beta})^T\right)^{-1}\Rightarrow\left\langle N_p(\vec{\alpha})\left(N_q(\vec{\beta})\right)-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right\rangle=\frac{\left\langle n_p(\vec{\alpha})\left(n_q(\vec{\beta})\right)-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right\rangle}{\varepsilon_p(\vec{\alpha})\varepsilon_q(\vec{\beta})}\Rightarrow R_2^{p,q}(\vec{\alpha}^{(2)},\vec{\beta}^{(2)})=\frac{\left\langle N_p(\vec{\alpha}^{(2)})\left(N_q(\vec{\beta}^{(2)})\right)-\delta_{pq}\delta_{\vec{\alpha}\vec{\beta}}\right\rangle}{N_p(\vec{\alpha}^{(2)})N_q(\vec{\beta}^{(2)})}-1$$

Summary:

The formalism presented enables measurements of differential correlation functions for pairs of species, e.g., ππ, πK, πp, KK, Kp, pp, that essentially recover the full statistics, extend the kinematic range of measurements while providing reliable disambiguation of particle species [2-3]. The method was developed towards measurements of differential correlation functions, e.g., R_2, for an arbitrary number of particle species, pT dependent particle losses, as well as an arbitrary number of particle identification signals (not discussed here) but can be extended to other correlation observables.