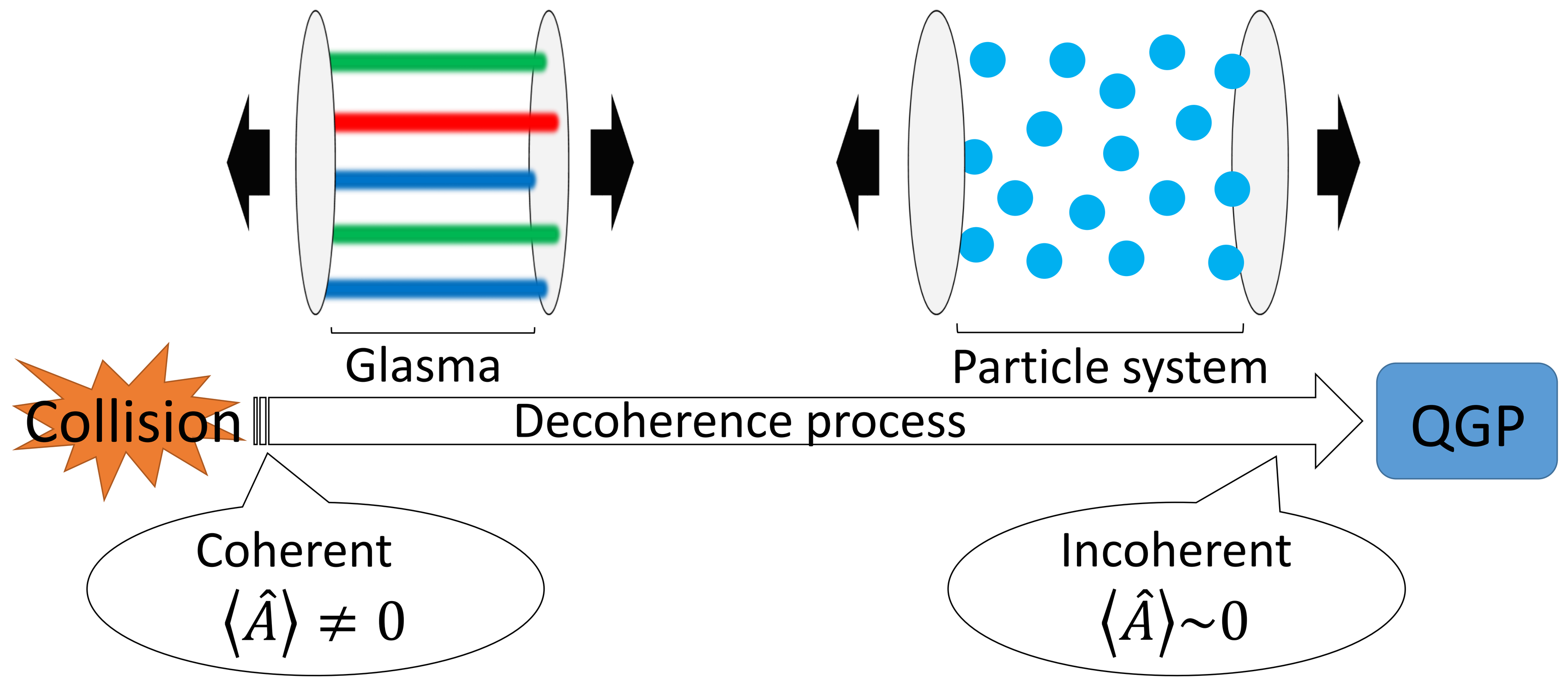
Decoherence and von Neumann entropy production of classical Yang-Mills fields in relativistic heavy ion collisions

Hidefumi Matsuda(Kyoto U.), Teiji Kunihiro(YITP), Akira Ohnishi(YITP)
Toru. T. Takahashi(GC), Hideaki Iida(FEFU), Hidekazu Tsukiji(Ashahikasei)

① Intoroduction

Thermalization process in heave ion collisions



Glasma

$\simeq$ Classical Yang-Mills(CYM) field — Glasma initial condition, EOM for CYM field

- Weibel instability — Weibel (1959), Romatschake and Venugopalan (2006)
- Nielsen-Olesen instability — Neilsen and Olsen (1978) , Fujii *et al.* (2009)
- parametric instability — Tsutsui *et al.* (2015)

→ Pressure isotropization

Romatschake and Venugopalan (2006), Epelbaum and Gelis (2013)

Content

We study the thermalization of CYM fields by calculating

- entropy production
- relaxation to equipartition

+pressure isotropization

② Method

EOM for CYM field

We calculate $\partial_0 A_i^a(x) = \frac{\partial H}{\partial E_i^a(x)}$, $\partial_0 E_i^a(x) = -\frac{\partial H}{\partial A_i^a(x)}$

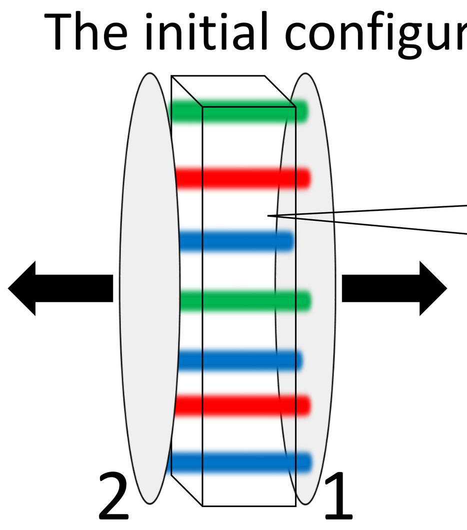
in (t,x,y,z) coordinate static box(=Non-expanding geometry).

MV like initial condition and fluctuations

McLerran and Venugopalan (1994)

The initial configuration mimicking MV model in the static box

Tsukiji, Kunihiro, Ohnishi *et al.* (2017)



$$A_{\mathrm{MV},\mu} = \delta_{\mu 1} A_{\mathrm{MV},1} + \delta_{\mu 2} A_{\mathrm{MV},2}$$
$$E_{\mathrm{MV},i} = \delta_{i3} E_{\mathrm{MV}}$$
$$B_{\mathrm{MV},i} = \delta_{i3} B_{\mathrm{MV}} \quad \text{at } t = 0+$$

temporal gauge $A^0 = 0$

+fluctuation respecting gauss law $\quad A_{\mathrm{MV}}, E_{\mathrm{MV}} \longrightarrow A_{\mathrm{MV}}, E_{\mathrm{MV}} + \delta E$

A trigger of instability — Romatschke and Venugopalan (2006)

We can control the size of fluctuations by changing the size of Δ

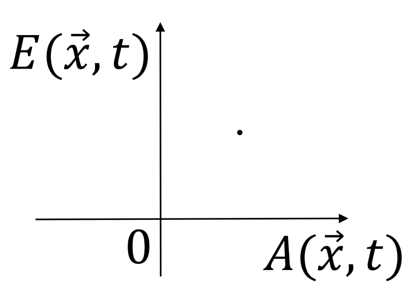
$\Delta = 0 \quad \Longrightarrow \quad \delta E = 0$

Δ =large $\quad \Longrightarrow \quad \delta E$ =large

Method for estimating entropy, assuming docoherence

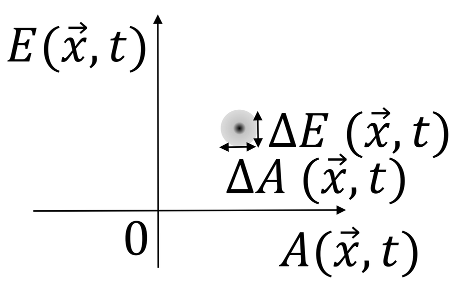
- Regard the classical state as the coherent state

$$(\cdots, A_i^a(\vec{x},t), \cdots, E^{ai}(\vec{x},t), \cdots)$$



$$|\alpha_{\mathrm{CYM}}(t)\rangle = \prod_{a,i,\vec{k}} \left[\sum_n C_n^{a,i,\vec{k}} |n(a,i,\vec{k})\rangle \right]$$

$$\boxed{\begin{aligned} A_i^a(\vec{x},t) &= \langle \alpha_{\mathrm{CYM}}(t) | \hat{A}_i^a(\vec{x}) | \alpha_{\mathrm{CYM}}(t) \rangle \\ E_i^a(\vec{x},t) &= \langle \alpha_{\mathrm{CYM}}(t) | \hat{E}_i^a(\vec{x}) | \alpha_{\mathrm{CYM}}(t) \rangle \end{aligned}}$$



- Assume that the phase coherence became lost in $|\alpha_{\mathrm{CYM}}(t)\rangle$(decoherence)

$$\rho_{\mathrm{CYM}} = |\alpha_{\mathrm{CYM}}(t)\rangle\langle\alpha_{\mathrm{CYM}}(t)| = \prod_{a,i,\vec{k}} \left[\sum_{m,n} C_m^{a,i,\vec{k}} \left(C_n^{a,i,\vec{k}} \right)^* |m(a,i,\vec{k})\rangle\langle n(a,i,\vec{k})| \right]$$

$$\rho_{\mathrm{dec}} = \prod_{a,i,\vec{k}} \left[\sum_n \left| C_n^{a,i,\vec{k}} \right|^2 |n(a,i,\vec{k})\rangle\langle n(a,i,\vec{k})| \right]$$

$\left| C_n^{a,i,\vec{k}} \right|^2$: Particle number distributon

- Define the von-Neumann entropy by using ρ_{dec}

$S_{\mathrm{dec}} \equiv -\mathrm{Tr}(\rho_{\mathrm{dec}} \ln \rho_{\mathrm{dec}})$ (Decoherence entropy)

③ Numerical result

Setup

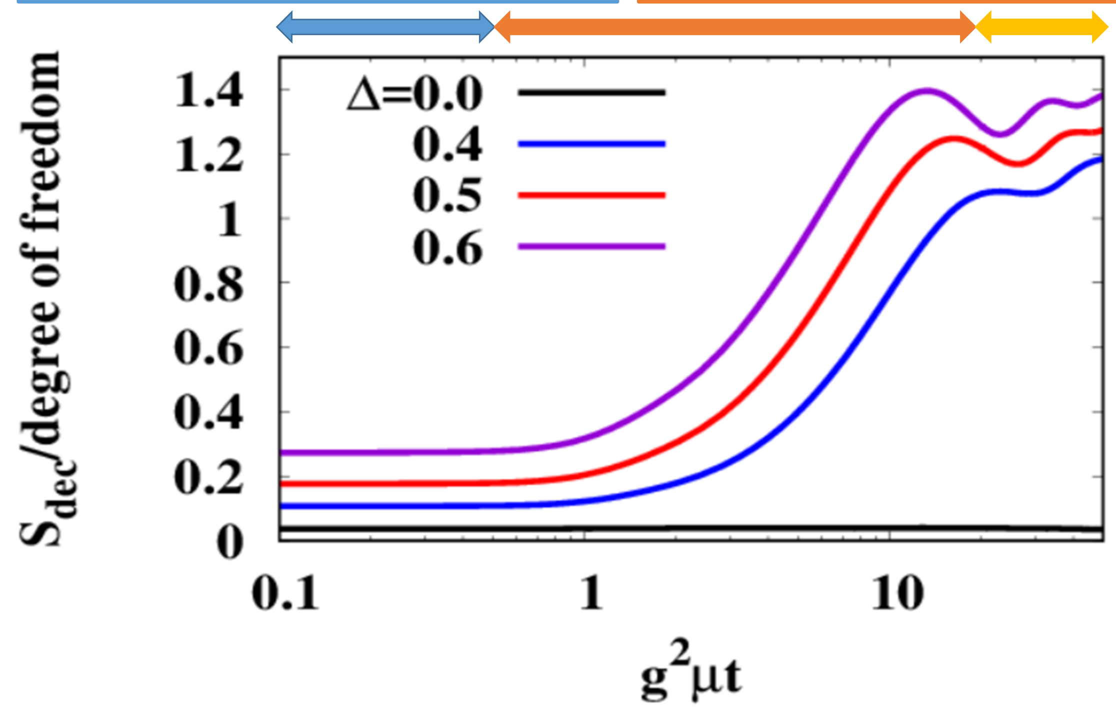
- number of sites : $64 \times 64 \times 64$
- SU(2) Yang-Mills theory
- $g^2 \mu a = 2$

Δ	ε	Increment(ε)[%]
0.0	3.3	0.0
0.4	3.6	10
0.5	4.0	23
0.6	4.8	48

ε = energy density

Ⅰ. Decohrence entropy

Entropy doesn't increase | Entropy increases rapidly | Entropy increases slowly

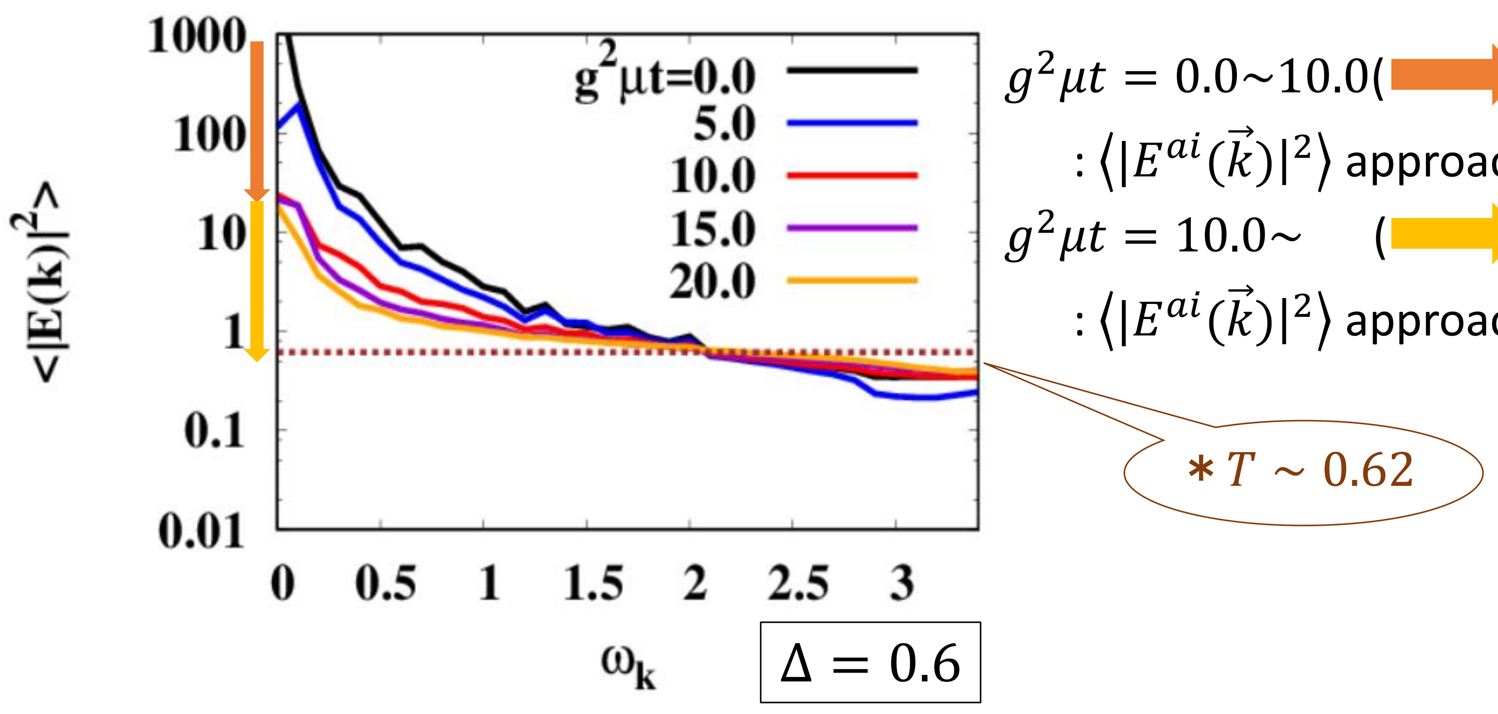


Entroy never increases($\Delta = 0$)

Δ	(blue arrow)	(orange arrow)	(yellow arrow)
0.4	$g^2\mu t = -0.5$	$g^2\mu t = 0.5 - 20.0$	$g^2\mu t = 20.0-$
0.5	-0.5	$0.5 - 15.0$	$15.0-$
0.6	-0.5	$0.5 - 10.0$	$10.0-$

Ⅱ. Equipartition

In classical equilibrium system, $\langle |E^{ai}(\vec{k})|^2 \rangle = T$. (equipartition theorem)



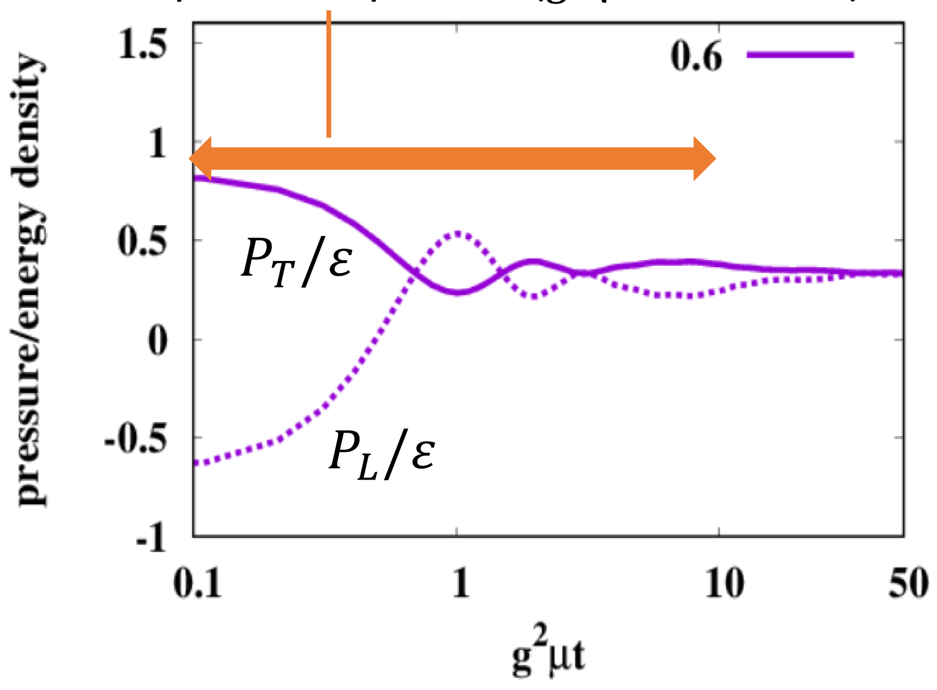
$g^2\mu t = 0.0\sim10.0$(→)
: $\langle |E^{ai}(\vec{k})|^2 \rangle$ approach "T" rapidly

$g^2\mu t = 10.0\sim$ (→)
: $\langle |E^{ai}(\vec{k})|^2 \rangle$ approach "T" slowly

Ⅲ. Pressure

From Ⅰ and Ⅱ, entropy production and relaxation to equipartition have same time scale.
And we find that pressure isotropization has same time scale, too.

Rapid isotropization($g^2\mu t = -10.0$)



④ Summery

- We have studied the thermalization process , focusing on decoherence.
- We have considered coherent state that correspond to CYM fields .
- We discuss the thermalization by using "entropy production", "relaxation to equipartition", "pressure isotropization".

→ two time scale

- Shorter time scale($g^2\mu t = 10.0 - 20.0$)
We fine that entropy production, relaxation to equipartition and pressure isotropization proceed rapidly.
→ Hydronization?(Incomplete thermalization)

- Longer time scale
We fine that entropy production and relaxation to equipartition proceed slowly.
→ Complete thermalization