Computation of the Berry curvature in lattice QCD

Arata Yamamoto (U. of Tokyo)

[1] A. Yamamoto, PRL 117, 052001 (2016)

[2] S. Pu and A. Yamamoto, arXiv:1712.02218

Berry curvature

quark ground state $\Phi(p)$

momentum space		coordinate space
p	$\leftrightarrow$	x
Berry connection		gauge connection
$\tilde{A}_i(p) = -i\Phi^\dagger(p)\frac{\partial}{\partial p_i}\Phi(p)$	$\leftrightarrow$	$A_i(x)$
Berry curvature		gauge curvature
$\tilde{F}_{ij}(p)$	$\leftrightarrow$	$F_{ij}(x)$

purpose:

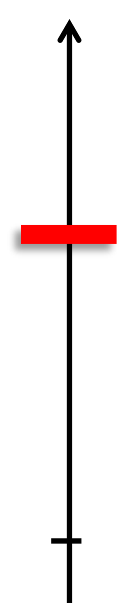
to compute the Berry curvature of interacting massive quarks in lattice QCD

applications:

- ✓ chiral kinetic theory in QCD
- ✓ topological materials in condensed matter physics

Free quarks

quark helicity is conserved



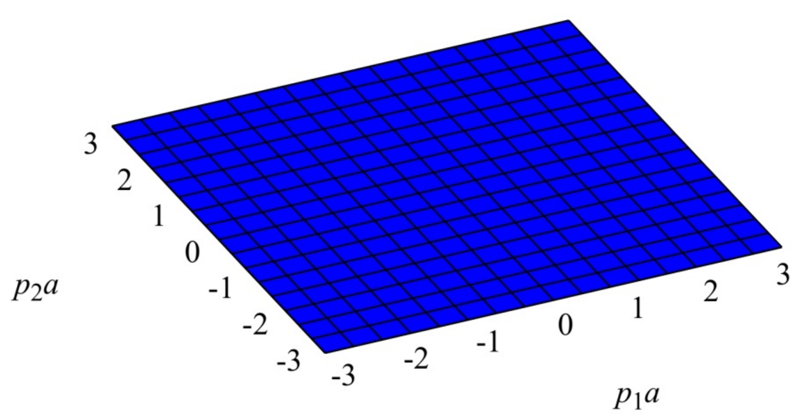
2 helicity modes

degenerate ground state

$\longrightarrow$ U(2) Berry curvature

E = 0

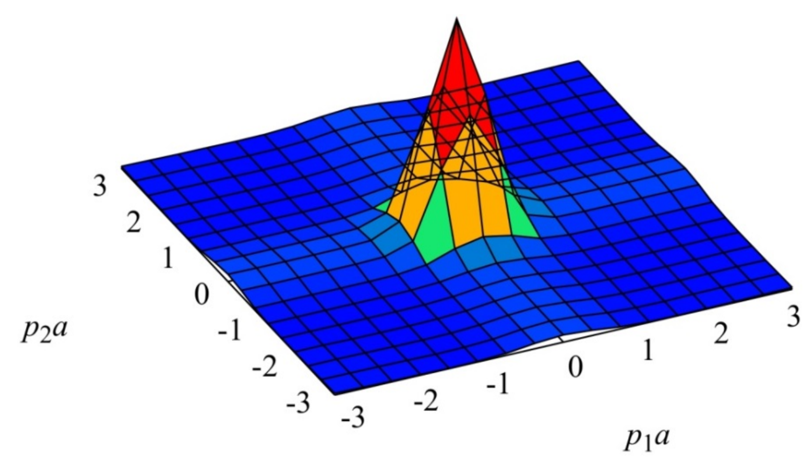
U(1) part $\frac{1}{4}\tilde{F}^0_{ij}(p)\tilde{F}^0_{ij}(p)$



cancellation between 2 helicity modes

at zero chiral chemical potential

SU(2) part $\sum_{a=1,2,3}\frac{1}{4}\tilde{F}^a_{ij}(p)\tilde{F}^a_{ij}(p)$

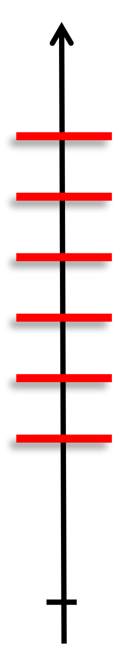


peak $\sim 1/m^4$

Lattice QCD

quark helicity is not conserved

(total helicity is conserved)



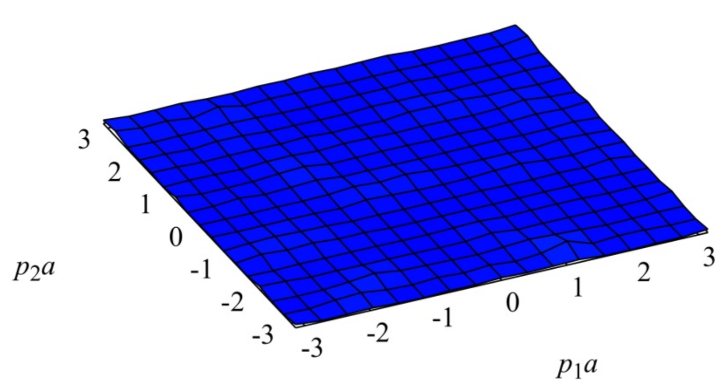
2 helicity × 3 color modes

non-degenerate ground state

$\longrightarrow$ U(1) Berry curvature

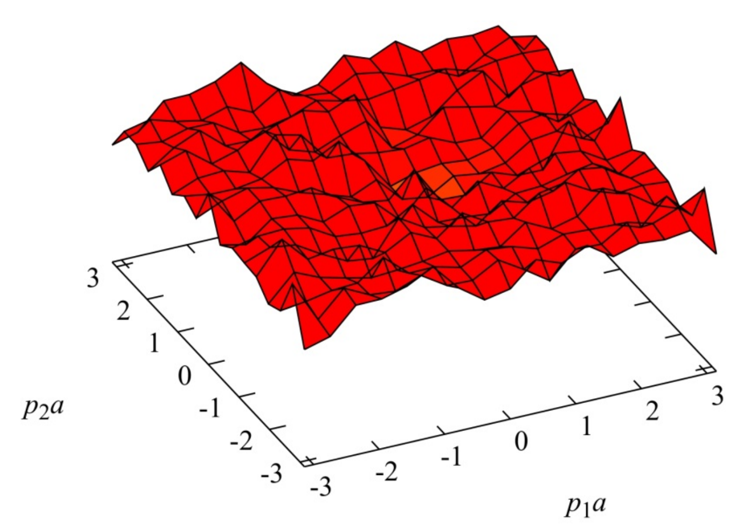
E = 0

"average" $\left\langle \frac{1}{2}\tilde{F}_{ij}(p)\right\rangle^2$



consistent with free quarks

"fluctuation" $\left\langle \frac{1}{4}\tilde{F}_{ij}(p)\tilde{F}_{ij}(p)\right\rangle$



quantum fluctuation ~ (0.5 GeV)$^{-4}$

at zero temperature