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Multiparticle femtoscopy with marginal distributions

Correlations in femtoscopy

- Classically, one defines correlation functions in femtoscopy as the ratios, for instance between the 2- and 3-particle coincidence cross sections and the products of independent single-particle cross sections:

$$C_2(\vec{p}_1,\vec{p}_2) \equiv \frac{N_2(\vec{p}_1,\vec{p}_2)}{N_1(\vec{p}_1)N_1(\vec{p}_2)}$$

$$C_3(\vec{p}_1,\vec{p}_2,\vec{p}_3) \equiv \frac{N_3(\vec{p}_1,\vec{p}_2,\vec{p}_3)}{N_1(\vec{p}_1)N_1(\vec{p}_2)N_1(\vec{p}_3)}$$

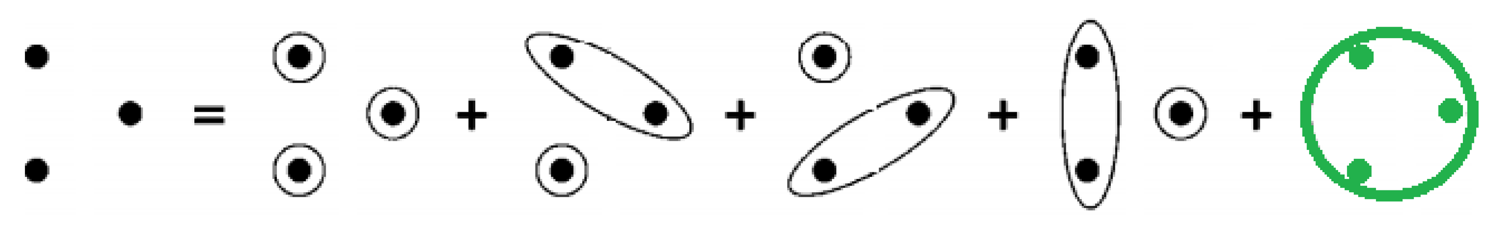
- Single-particle cross sections are typically obtained from mixed-event technique
- Due to large demand on statistics in the full differential forms above, correlations are projected onto Lorentz invariant quantities:

$$Q_{ij}^2 \equiv -(p_i - p_j)^2$$

$$Q_3^2 \equiv Q_{12}^2 + Q_{13}^2 + Q_{23}^2$$

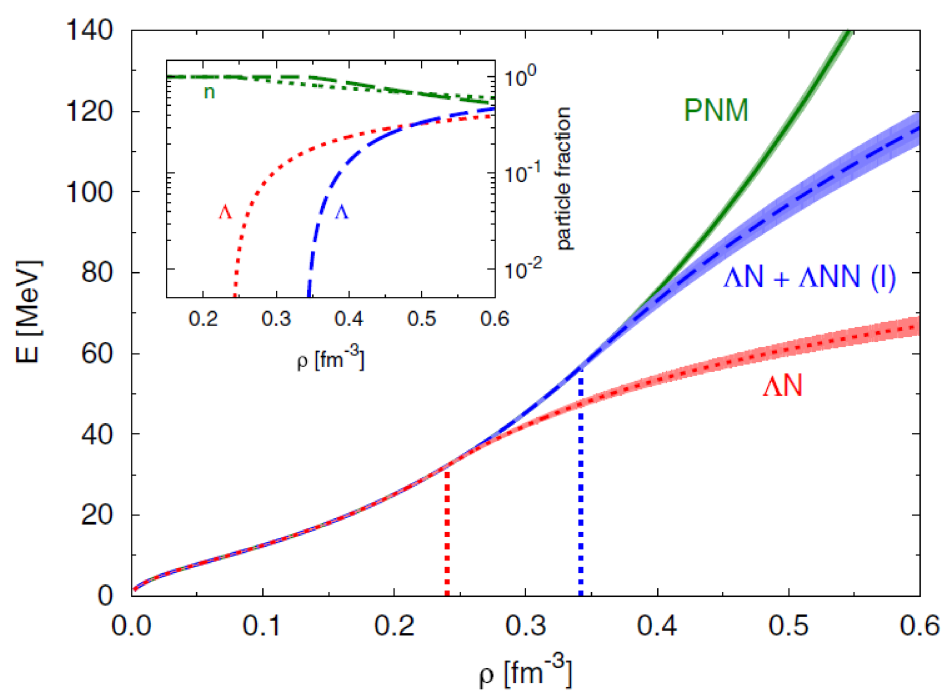
Genuine multi-body correlations

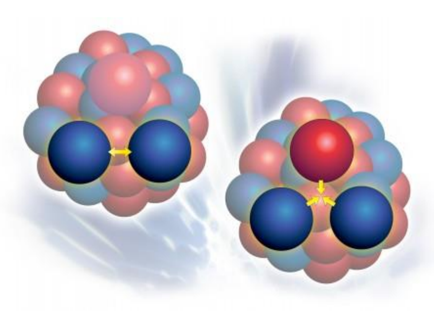
- From the measured correlations, one can isolate the genuine multi-body correlation by using the mathematical formalism of cumulants
- Genuine multi-body correlation can provide independent constraints if measured multiparticle correlation is not a trivial superposition of lower order correlations
- Example: 3-particle cumulant in terms of measured correlations:



$$\langle X_1X_2X_3\rangle_c = \langle X_1X_2X_3\rangle - \langle X_1X_2\rangle\langle X_3\rangle - \langle X_1X_3\rangle\langle X_2\rangle - \langle X_2X_3\rangle\langle X_1\rangle + 2\langle X_1\rangle\langle X_2\rangle\langle X_3\rangle$$

- Genuine 3-body interactions are relevant for instance in the description of EoS of neutron stars:





Lonardoni et al, Phys. Rev. Lett. 114, 092301 (2015)

Marginal distributions

- If the interaction of two particle species, labeled *a* and *b*, is described by joint 2-particle p.d.f., $f_{ab}(p_a,p_b)$, then the marginal distributions are:

$$f_a(p_a) = \int_B f_{ab}(p_a,p_b)\,dp_b$$

$$f_b(p_b) = \int_A f_{ab}(p_a,p_b)\,dp_a$$

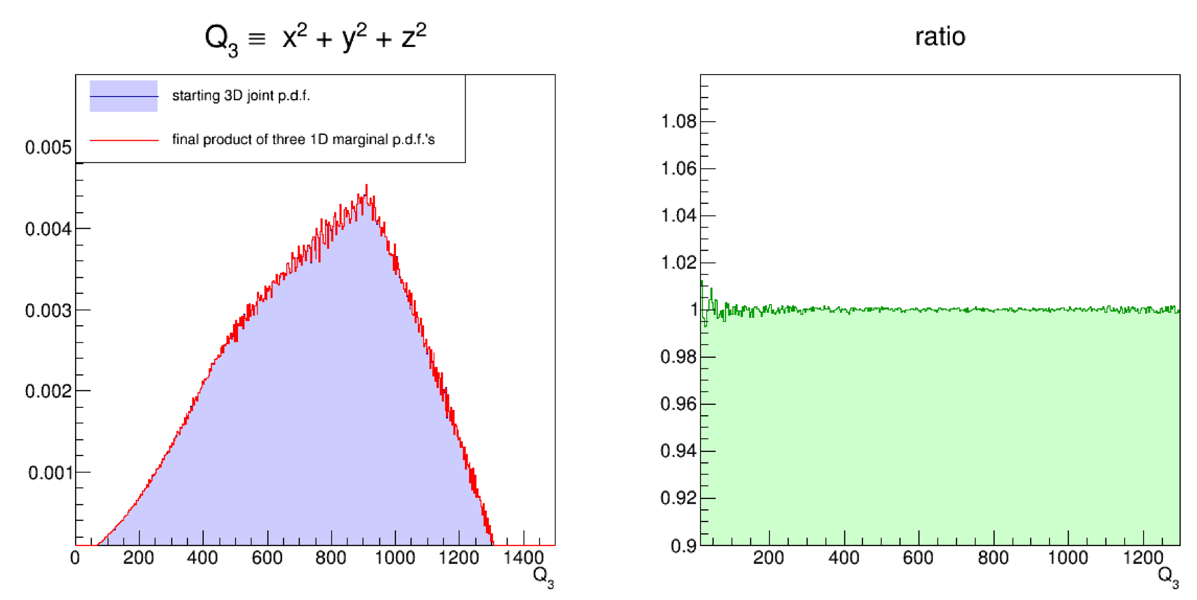
- If two particles are emitted independently, then:

$$f_{ab}(p_a,p_b) = f_a(p_a)f_b(p_b)$$

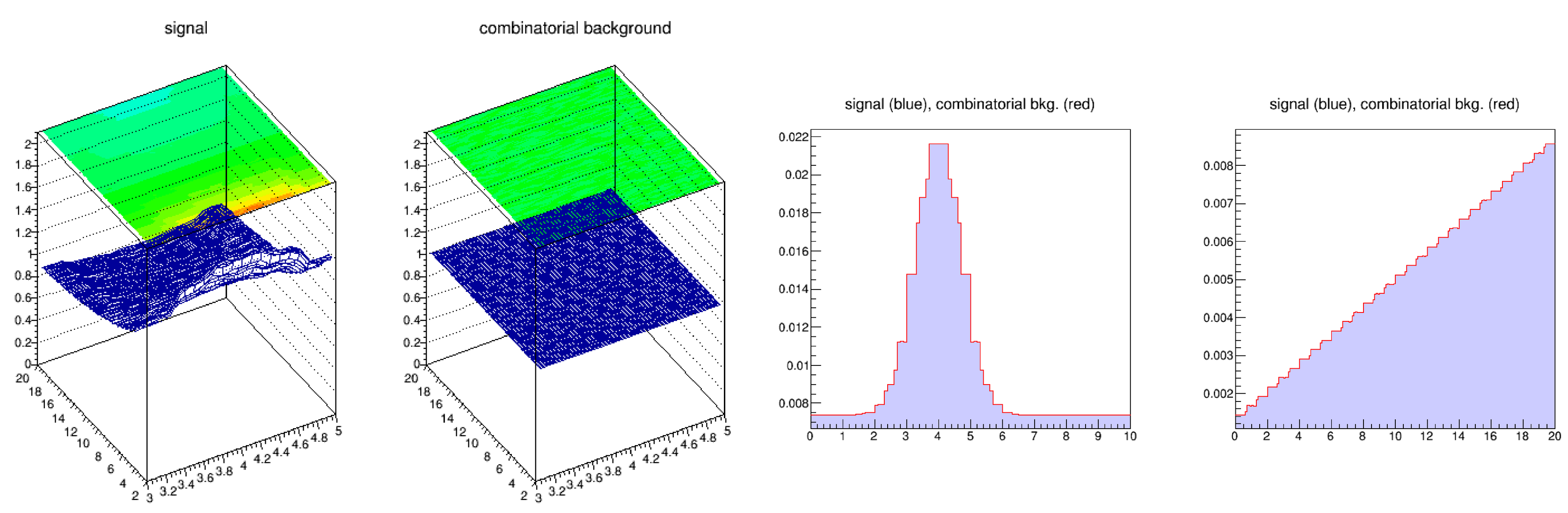
- Very important property of marginal distributions is that they preserve statistical independence also after projecting onto any lower dimensional variable (e.g. Q_3). This can be seen formally from the following general expression (where $a_1 = a_1(x_1,\ldots,x_n)$, etc.), since in both cases the Jacobian of transformation is the same:

$$g(a_1,\ldots,a_n) = f(x_1,\ldots,x_n)|J|$$

$$g'(a_1,\ldots,a_n) = f_{x_1}(x_1)\cdots f_{x_n}(x_n)|J|$$



- Another important property of marginal distributions is that they are the same for signal and background, for the given emission process:



- If the initial 3D distribution is normalized, then all 2D and 1D marginal distributions obtained from it are normalized by definition. This circumvents the notorious normalization problem when for instance 3-particle cumulants are projected onto lower dimensional observable (e.g. Q_3), and when the 2D and 1D distributions which appear in its decomposition are obtained from mixed-event technique
- Lorentz invariance in the approach with marginal distributions is ensured also by observing that energy prefactors E which make $d^3\vec{p}/E$ Lorentz invariant cancel in the ratios
- Remaining problems: Low sensitivity for small multiplicities
- Large computational demands to obtain marginal distributions