Series expansions are useful tools to study the phase diagram in the region which is not directly accessible to MonteCarlo simulations due to the sign problem.

Typical parameters for the expansion are the chemical potential $\mu$ and the fugacity $e^\mu/T$, while the observables to be expanded include the Grand Canonical Partition Function or the associated thermodynamic quantities, derivatives of the GCPF [3].

Which choice is most convenient depends on the thermodynamic region: at high temperatures the pressure in the massless limit is a closed polynomial, hence the expansion of any observable should be possible. In the hadronic phase the GCPF is well approximated by a simple trigonometric function, which suggests to use a virial expansion for the thermodynamics. Moreover, Cluster Model Expansions provide a phenomenological guidance for the value of coefficients, which constrain the position of the hypothesized critical point of QCD [3].

Series for thermodynamic quantities are limited by the position of the singularities, if any. This property has been exploited for instance in the search of a critical point by use of the Taylor expansion. On the other hand, series for the GCPF do not have this limitation: GCPF has zeros, not critical points!

Recently a strategy has been proposed which begins from the virial expansion at imaginary chemical potential - with a limited radius of convergence - and reconstruct the canonical partition functions, hence the thermodynamic quantities. Then, $Z(\xi)$ can be computed from $Z_0$ and $Z_0(\xi)\xi^n$ for various order of approx.

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MonteCarlo simulations due to the sign problem.

Series expansions are useful tools to study the phase diagram in the region which is not directly accessible to MonteCarlo simulations due to the sign problem. First, for $\mu_1$:

Then, for $\mu$:

Comments on QCD1d:
- The radius of convergence can be read off the virial coefficients
- A negative $Z_0$ may appear and signals an insufficient order of the virial expansion
- The fugacity series is a double series with two radii of convergence
- The virial-like expansion is the typical outcome of analytic continuation from imaginary chemical potential

QCD - preliminary results.

Virial expansion, radius of convergence, CEM

One dimensional QCD is an interesting, exactly solvable model. There is no spontaneous symmetry breaking, but there are hysteresis. Its partition function is formally the same as the one obtained in 4d QCD at strong coupling, once an explicit mass term is identified with the 4d dynamically generated mass. For any temperature $T$ and varying $\mu$ there is a crossover to a huygens-rich phase, which turns into a first order transition at zero temperature.

The partition function reads

$Z(\mu,T) = \sum_{n=0}^{\infty} \frac{(-\mu T)^n}{n!} d^n(\mu T)$

The virial expansion is the typical outcome of analytic continuation from imaginary $\mu$ and its radius of convergence depends on the zeros of the GCPF in complex fugacity plane.

Virial expansion, zeros of the GCPF and radius of convergence

The zeros for the GCPF determine the radius of convergence of the virial expansion of $\log(Z(\mu,T)) = \sum_{n=1}^{\infty} a_n (\mu/T)^n$

The GCPF in QCD 1d beyond the radius of convergence

First, for $\mu_1$:

Then, for $\mu$:

The GCPF can be reconstructed beyond the radius of convergence

Then calculate $Z$ in real regions.

Calculation in pure $\mu_1$ regions, where no sign problem.

$Z_n = \int d\theta \frac{e^{i n \theta}}{2\pi} Z(T, \theta \equiv \frac{Im \mu}{T})$

Then calculate $Z$ in real $\mu$ regions.

$Z(\xi, T) = \sum_{n=1}^{\infty} Z_n(T)^{n/2}$

We use pure field configuration prepared at $T = 1.035$ for both QCD with three fermions. When $\mu$ is changed the changed configuration is used for the calculation.

The simulations were made at $T = 0.1$ in lattice units $T/T_c = 1.035$ and $0.08$ and $0.01$ in the limit of massive quark and gluon degrees of freedom in the continuum phase along the line of constant physics with $\mu/T_c = 0.5$. The parameters of the action, including $\mu$, were tuned from the comparison of the results of the calculations with $\mu = 0$ and 0.5 with available analytic results in QCD [1].

We compare the quark number density $\rho_q$ at imaginary quark chemical potential $\mu_1$, which is defined as

$\rho_q = \frac{1}{2(\pi T)^3} \int d\mathbf{p} \delta(E - m) Z_0(\mathbf{p})$.