Effects of composite pions on the chiral condensate

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Abstract

We investigate the effect of composite pions on the behaviour of the chiral condensate at finite temperature within the Polyakov-loop improved NJL model. To this end we treat quark-antiquark correlations in the pion channel (bound states and scattering continuum) within a Beth-Uhlenbeck approach that uses medium-dependent phase shifts. A striking medium effect is the Mott transition which occurs when the binding energy vanishes and the discrete pion bound state merges the continuum. This transition is triggered by the lowering of the continuum edge due to the chiral restoration transition. This in turn also entails a modification of the Polyakov-loop so that the SU(3) center symmetry gets broken at finite temperature and dynamical quarks (and gluons) appear in the system, taking over the role of the dominant degrees of freedom from the pions. At low temperatures our model reproduces the chiral perturbation theory result for the chiral condensate while at high temperatures the PNJL model result is recovered. The new aspect of the current work is a consistent treatment of the chiral restoration transition region within the Beth-Uhlenbeck approach on the basis of mesonic phase shifts for the treatment of the correlations.

The model

We use the decomposition of the thermodynamical potential into a quark-gluon sector which is treated in the meanfield approximation to the SU(2) PNJL model and a hadronic part which is described by a Mott-hadron resonance gas, here restricted to just the pion channel for simplicity

$$\Omega(T) = \Omega_{PNJL}(T) + \Omega_\pi(T).$$

The quark-gluon thermodynamic potential at vanishing chemical potential $\mu = 0$ is given as

$$\Omega_{PNJL}(T) = -4\int \frac{d^3p}{(2\pi)^3}\{N_c E(p) + 2T\ln[N_\Phi(E(p))]\} + \frac{\sigma^2}{4G} + \mathcal{U}(\Phi;T) ,$$

$$\frac{\mathcal{U}(\Phi;T)}{T^4} = -\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{3}\Phi^3 + \frac{b_4}{4}\Phi^4$$

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

The contribution of composite meson to the thermodynamics we describe within the Beth-Uhlenbeck approach [1,2]

$$p_M(T) = d_M\int_0^\infty \frac{d\omega}{\pi}\int\frac{d^3q}{(2\pi)^3}\delta_M(\omega,\vec{q};T)[1+2g(\omega)] ,$$

using phase shift (for a more elaborate ansatz see Ref.[3])

$$\delta_M(\omega,\vec{q};T) = \pi\left[\Theta(\omega - E_M(q)) - \Theta(\omega - E_{\rm thr}(q))\right]\Theta(T_{\rm Mott} - T)\Theta(|\vec{q}| - \Lambda_M) ,$$

we can obtain meson pressure

$$p_M(T,\omega) = -\int_{\Lambda_\pi}\frac{d^3q}{(2\pi)^3}\left[\frac{\omega}{2} + T\ln\left(1 - e^{-\beta\omega}\right)\right] .$$

In this model the pion contribution to the thermodynamics is

$$p_\pi(T) = \left[p_\pi(T, E_\pi^2(q)) - p_\pi(T, E_{\rm thr}(q))\right]\Theta(T_{\rm Mott} - T) ,$$

where $E_\pi(q) = \sqrt{q^2 + m_\pi^2}$ and $E_{\rm thr}(q) = \sqrt{q^2 + m_{\rm thr}^2}$ with $m_{\rm thr} = 2m$.

In the following we shall check what is the effect on the chiral condensate. To this end we recall the standard formula for the chiral quark condensate

$$\langle\bar{q}q\rangle = \frac{\partial\Omega(T)}{\partial m_0} = \langle\bar{q}q\rangle_{PNJL} + \langle\bar{q}q\rangle_\pi , \qquad (1)$$

which receives contributions from the PNJL model in mean-field approximation

$$\langle\bar{q}q\rangle_{PNJL} = -4N_c\int^\Lambda \frac{dp\, p^2}{2\pi^2}\frac{m}{E(p)}\left[1 - 2f_\Phi(E(p))\right] , \qquad (2)$$

where $f_\Phi(E(p))$ is the Polyakov-loop generalized Fermi distribution function [4], and from the pion contribution

$$\langle\bar{q}q\rangle_\pi = \frac{d_\pi}{2}\int^{\Lambda_\pi}\frac{dq\, q^2}{2\pi^2}\left\{\frac{m_\pi^2}{2m_0E_\pi(q)}\left[1 + 2g(E_\pi(q))\right] - \frac{2m_{\rm thr}}{E_{\rm thr}(q)}\frac{\partial m}{\partial m_0}\left[1 + 2g(E_{\rm thr}(q))\right]\right\}\Theta(T_{\rm Mott} - T).$$

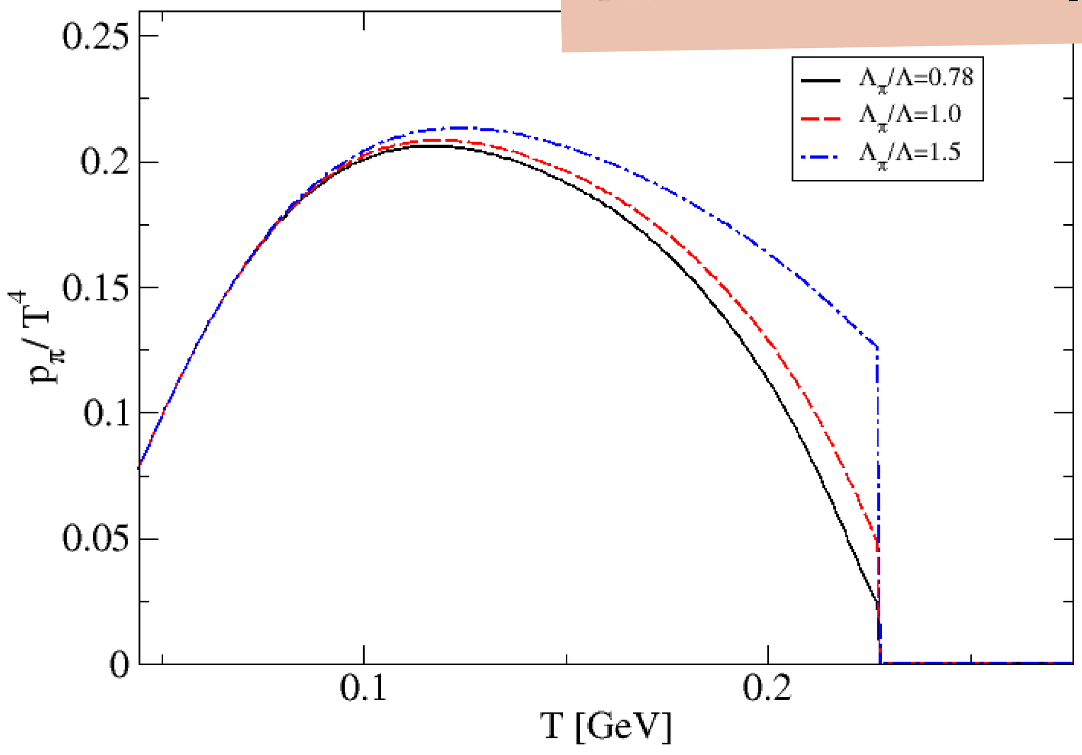
For the derivatives $\partial m_\pi^2/\partial m_0$ and $\partial m_\pi^2/\partial\sigma$ in equations above we employ the Gell-Mann–Oakes–Renner (GMOR) relation $m_\pi^2 f_\pi^2 = -m_0\langle\bar{q}q\rangle$.

Conclusion: A simple model ansatz for medium-dependent phase shifts was introduced for dealing with the quark-antiquark correlations in the pion channel (bound states and scattering continuum).

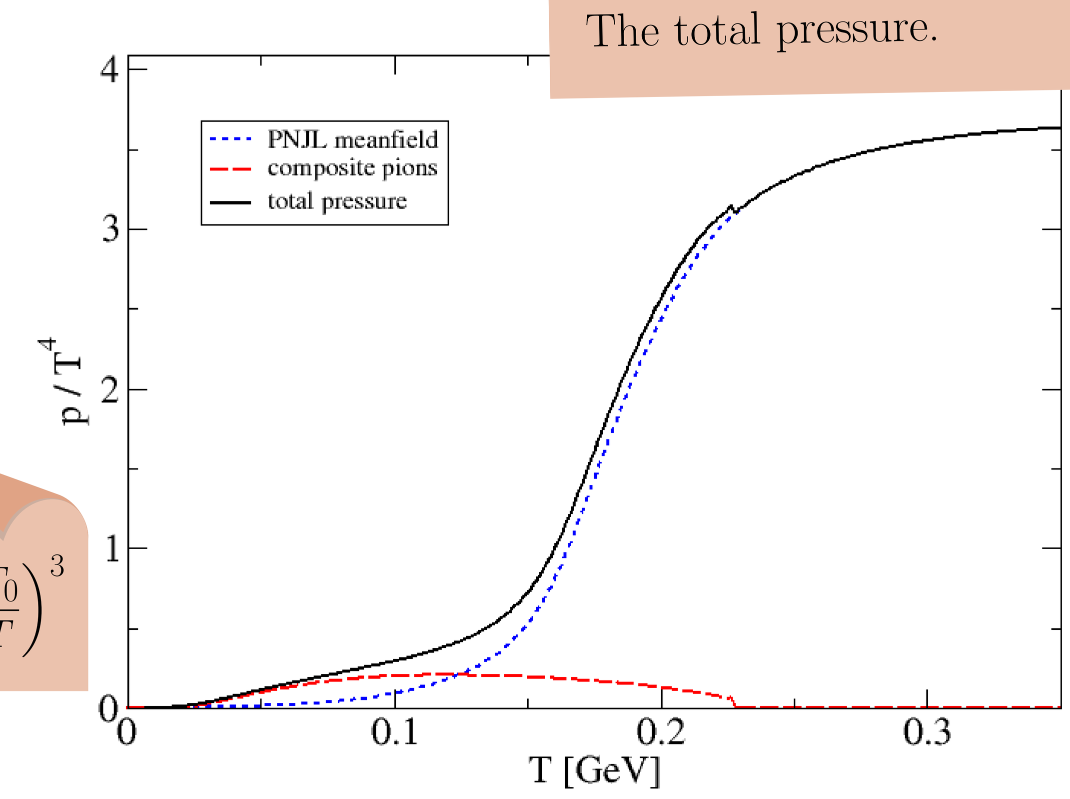
- this model describes a medium effect, the Mott transition for the pion bound state
- this ansatz is in accordance with the Levinson theorem generalized to medium-dependent phase shifts
- this model reproduces the chiral perturbation theory result for the chiral condensate at low temperatures
- But! While the pressure of the quark-gluon-meson system behaves continuous at the Mott transition, a discontinuous behaviour of the pressure derivatives, the chiral condensate and the quark mass gap results.

Results

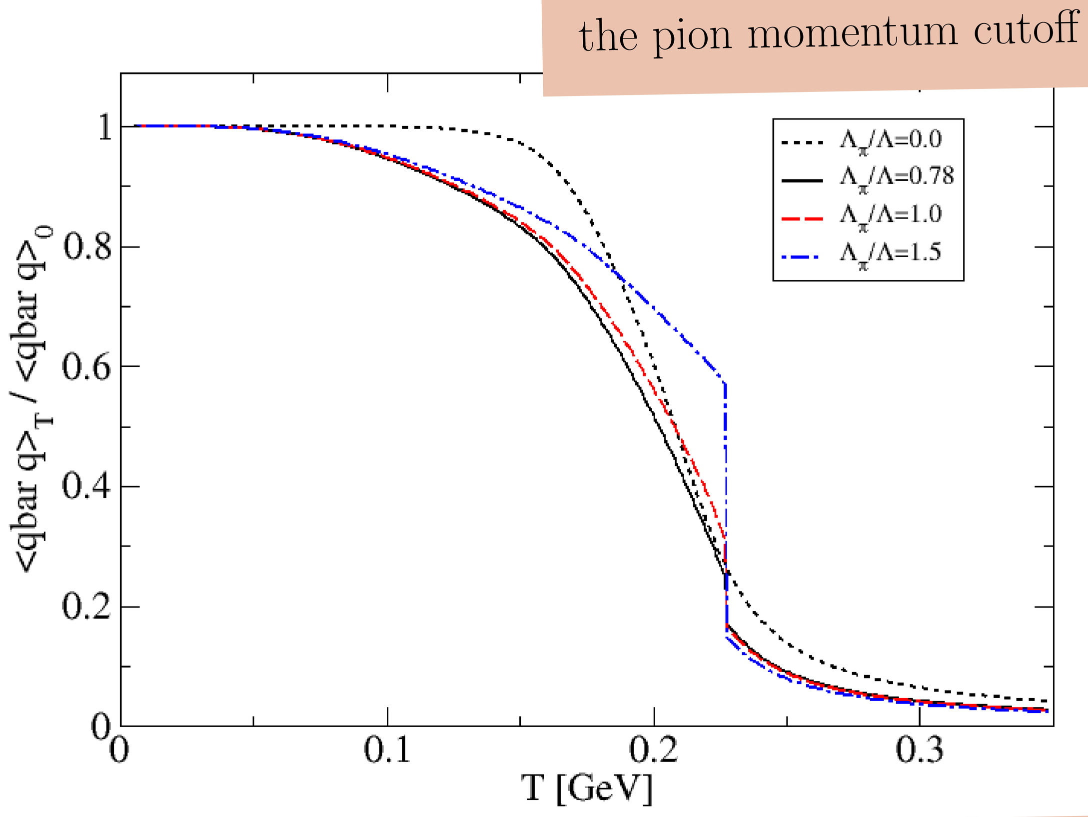
Pion contribution to the pressure.



The total pressure.



The chiral condensate for different choices of the pion momentum cutoff Λ_π.



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Challenges:

- to employ the ansatz for the density of states also for all other hadronic resonances, with appropriate threshold masses for the given hadronic channel to discuss the effect of hadronic excitations on the pseudocritical temperature of the chiral transition
- to construct a more realistic ansatz for the continuum
- to evaluate the quark Pauli blocking effect in a pion gas with respect to the backreaction of pions on the quark dynamics (as an example, using the Φ-derivable formulation of the cluster virial expansion for quark-meson matter [5]).

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