Dirac-mode expansion for quark-number holonomy in lattice QCD

Speaker: Takahiro Doi

Takahiro M. Doi (RIKEN, Japan)

In collaboration with Kouji Kashiwa (Fukuoka Institute of Technology, Japan)

Abstract

We consider the Dirac-mode expansion for quark number holonomy in both large and small quark mass regimes on the lattice. Quark number holonomy is a topological order parameter for the deconfinement transition. Dirac-mode is strongly related to chiral symmetry breaking. We derive the analytical Dirac-mode representation of the quark number holonomy and our results indicate that the low-lying Dirac modes are irrelevant for the quark-confinement.

Introduction

QCD at imaginary chemical potential

- Roberge Weiss (RW) periodicity at imaginary chemical potential:

$$Z_{\rm QCD}(T,\theta) = Z_{\rm QCD}(T,\theta+2\pi k/3)$$

$\mu = i\mu_{\rm I}$: Imaginary chemical potential

$\theta = \mu_{\rm I}/T$: dimensionless imaginary chemical potential

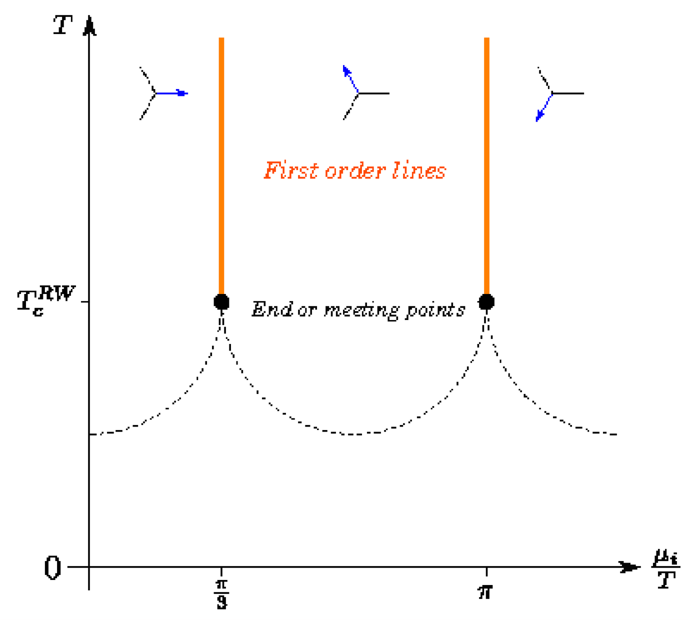
Quark confinement at imaginary chemical potential

- The RW periodicity has deep relations with the free-energy degeneracy.

Free energy is

$T \gg T_{\rm RW}$: degenerated

$T \ll T_{\rm RW}$: not degenerated



Motivation

Quark number holonomy

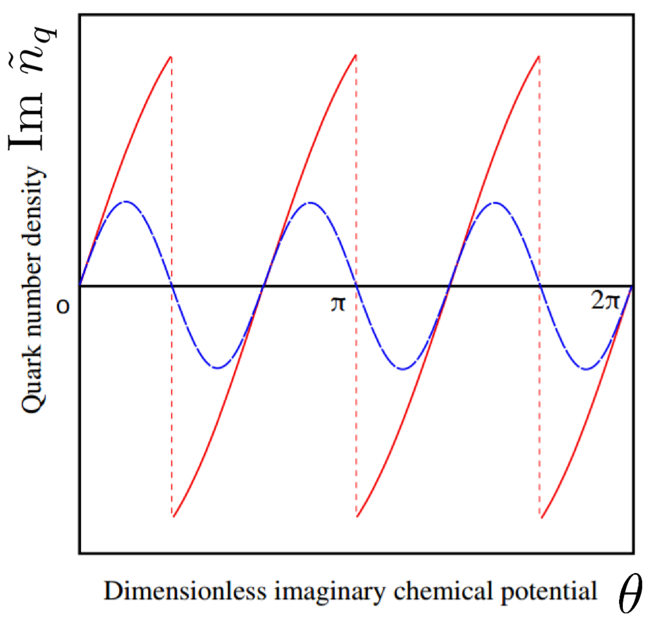
K. Kashiwa and A. Ohnishi, Phys. Lett. B750 (2015).
K. Kashiwa and A. Ohnishi, Phys. Rev. D93 (2016) 116002.

- Quark number holnomy $\Psi(T)$ is defined as a integral of the dimensionless quark number $\tilde{n}_q$ on a loop for the direction of the dimensionless imaginary chemical potential θ.

$$\Psi(T) = \left[\oint_0^{2\pi}\left\{ \mathrm{Im}\left(\frac{d\tilde{n}_q}{d\theta}\Big|_T\right)\right\} d\theta\right]$$

$\tilde{n}_q = Cn_q$: dimensionless quark number

- It counts gapped points of the quark number density along θ direction.
- Quark number holonomy is a topological order parameter for the confinement-deconfinement transition.



Red solid line: high temperature
Blue dotted line: low temperature

Dirac mode expansion

- Low-lying Dirac eigenmodes: the important modes for chiral symmetry breaking

$\langle\bar{q}q\rangle = -\lim_{m\to 0}\lim_{V\to\infty}\pi\langle\rho(0)\rangle$: Banks-Casher relation

$D|n\rangle = \lambda_n|n\rangle$: Dirac eigenvalue equation

D : Wilson-Dirac operator

$\rho(\lambda) = \frac{1}{V}\sum_n \delta(\lambda-\lambda_n)$: Dirac eigenvalue density

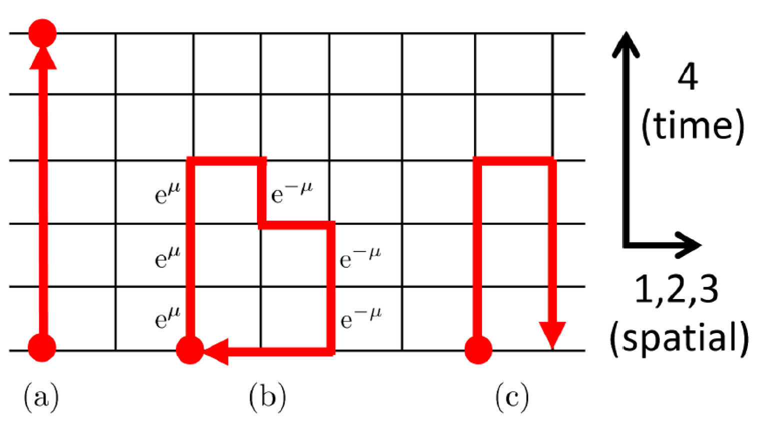
Dirac-mode expansion for quark number density

TMD and K. kashiwa, arXiv:1706.00614.

Large quark mass

- We expand the quark number density by 1/M. (Hopping parameter expansion)

$$n_q = \frac{1}{V}\left\langle \mathrm{Tr}_{\gamma,c}\left[\frac{\partial D}{\partial\mu}\frac{1}{D+m}\right]\right\rangle$$
$$= \frac{1}{MV}\left\langle \mathrm{Tr}_{\gamma,c}\left[\frac{\partial D}{\partial\mu}\sum_{n=0}^{\infty}\left(-\frac{\hat{D}}{M}\right)^n\right]\right\rangle$$
$$= \sum(\text{temporally widing loops})$$



(a): temporally winding loop

(b): spatially loop ⇒ μ-independent term ⇒ no contribution to n_q

(c): gauge-variant ⇒ no contribution to n_q

- **The quark number density shows the similar behavior with the Polyakov loop.**

$$n_q \sim L$$

small quark mass

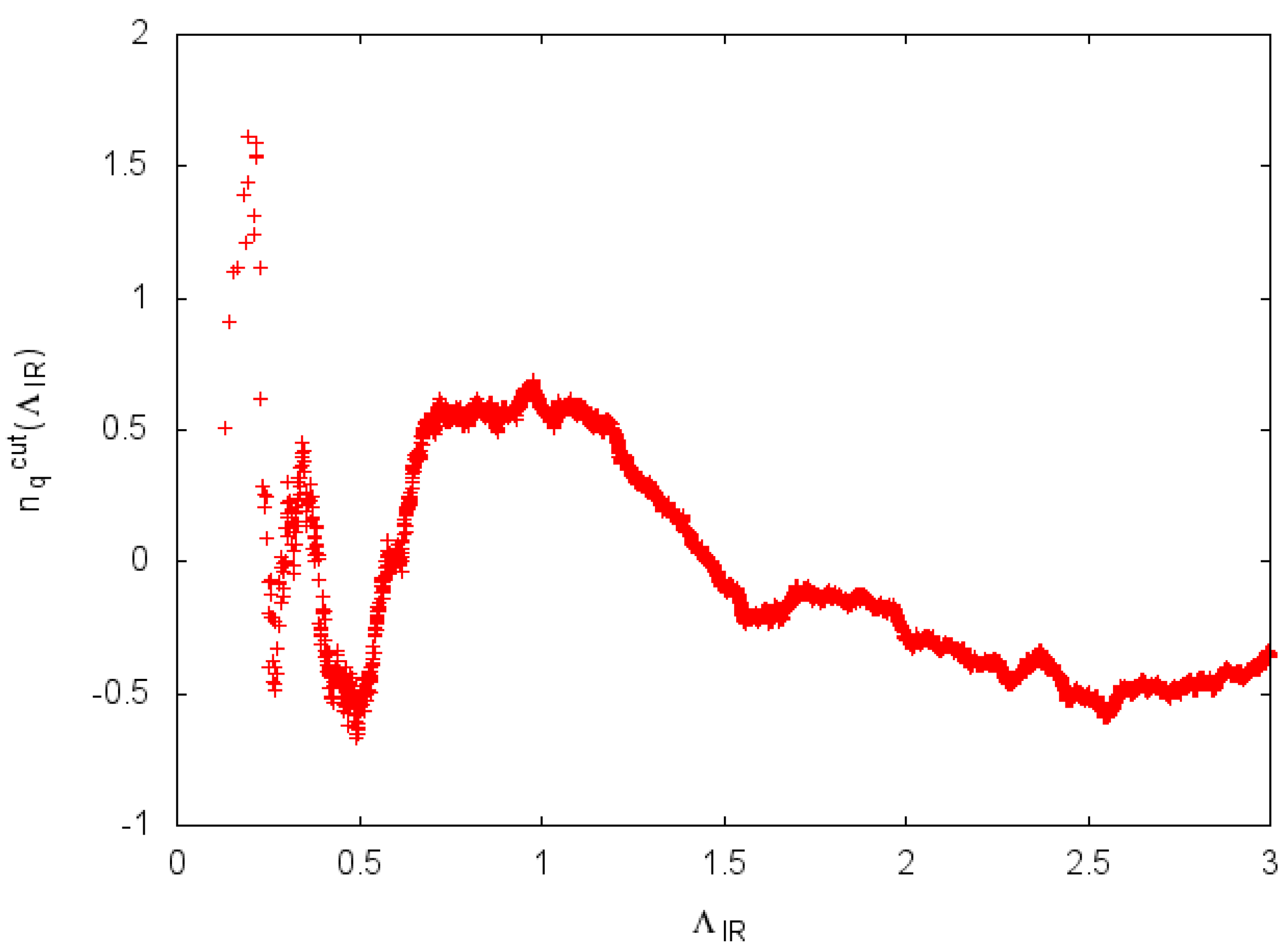
- We consider the IR-cutoff dependence of quark number density $n_q(\theta=\pi/3)$.

$$n_{\rm q}^{\rm cut}(\Lambda_{\rm IR}) = \frac{1}{n_{\rm q}}\sum_{|\Lambda_n|>\Lambda_{\rm IR}} n_{\rm q}^n$$

$$n_q \equiv \frac{1}{V}\langle n|\gamma_5\frac{\partial D}{\partial\mu}|n\rangle\frac{1}{\Lambda_n}$$

lattice setup:

- Configuration: quenched QCD
  - standard plaquette action
  - gauge coupling: $\beta = \frac{2N_c}{g^2} = 6.0$
  - lattice size: $N_\sigma^3\times N_\tau = 6^3\times 5$
  - temperature: $T \simeq 400$ MeV
- Wilson-Dirac operator
  - periodic boundary condition for link-variables and Dirac operator
  - hopping parameter: $\kappa = 0.151515$
  - chemical potential: $\mu a = 0.2094i$ ⇒ $\theta = \pi/3$



- The absolute value and sign of the quark number density is drastically changed by removal of the low-lying Dirac eigenmodes.
- However, the quark number holonomy is unchanged by removing the Dirac-modes because of the property of the quark number near $\theta=\pi/3$:

$$n_q(\theta=\pi/3-\epsilon) = -n_q(\theta=\pi/3+\epsilon)$$

- This result suggests **that the important modes for chiral symmetry breaking are not important for confinement-deconfinement transition defined by the quark number holonomy.**

Summary

1. In the large-quark mass regime, we analytically found that the quark number density shows the same behavior as the Polyakov loop, an usual order parameter for the confinement-deconfinemnet transition.
2. In the small-quark mass regime, we numerically found that the quark number holonomy, the topological order parameter for the deconfinement transition, is insensitive to the low-lying Dirac eigenmodes.
3. Our results suggest that low-lying Dirac modes do not play important roles in deconfinement transition as the topological phase transition defined by the quark number holonomy.

Reference

A. Roberge and N. Weiss, Nucl. Phys. B275, 734 (1986).
C. Czaban, et al., Phys.Rev. D93 (2016) 054507.

M. Sato, Phys.Rev. D77, 045013 (2008).

M. D'Elia and M.-P. Lombardo, Phys. Rev. D67, 014505 (2003).
H. Shimizu and K. Yonekura(2017), arXiv:1706.06104.

K. Kashiwa and A. Ohnishi, Phys. Lett. B750 (2015).
K. Kashiwa and A. Ohnishi, Phys. Rev. D93 (2016) 116002.

TMD and K. kashiwa, arXiv:1706.00614.