# Towards an improved determination of thermal correlation functions

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## Physics motivation

- **Thermal physics:** transport of conserved charges, in-medium modification of heavy bound states, e.g. quarkonium.
- Access to **dynamical** information from **real-time** correlation functions.
- Transport coefficients such as the **shear viscosity** from spectral function of the energy-momentum tensor correlator

$$\eta = \lim_{\omega\to 0} \frac{1}{20}\frac{\rho_{\pi\pi}(\omega,0)}{\omega}\,, \qquad \rho_{\pi\pi}(\omega,0) = \int \frac{d^4x}{(2\pi)^4} e^{-i\omega x_0 + i\vec{p}\cdot\vec{x}} \,\langle [\pi_{ij}(x), \pi_{ij}(0)]\rangle\ .$$

- **Finite density** QCD and the sign problem.

## Motivation for lattice QCD

- **Spectral reconstruction** from numerical analytical continuation of imaginary time correlator data $\to$ ill-posed inverse problem.

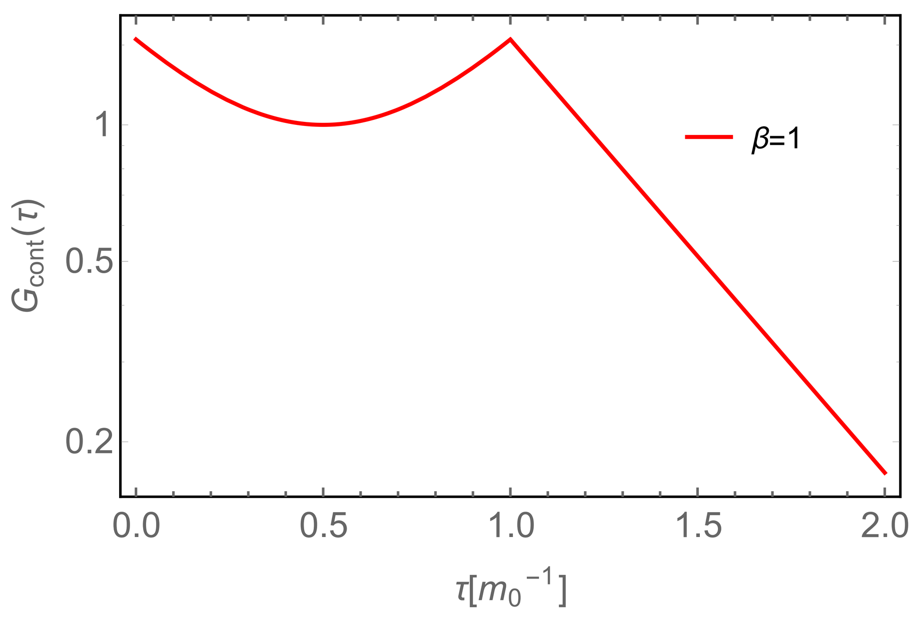
$$G(\tau) = \int_0^\infty d\mu\, \frac{\cosh(\mu(\tau-\beta/2))}{\sinh(\mu\beta/2)}\,\rho(\mu)\,, \qquad \beta = T^{-1}\,.$$

- **Problem:** Finite time extent $\to$ limited resolution of Matsubara frequencies $\omega_n = 2\pi T n$. Increasing $N_\tau$ not helpful.
- **Relevant thermal physics** encoded between $\omega_0$ and $\omega_1$. At larger $\omega_n$ thermal signal decays into vacuum.
- **Goal:** Higher resolution in imaginary frequencies in the physically interesting region $\to$ improve spectral reconstruction.

## Investigating analytic continuations

- **Proposal:** Simulate thermal real-time correlation functions on non-compact Wick-rotated time path similarly to the proposal in PLB 778 (2018) 221-226.
- **Requirements for proposed analytic continuation**

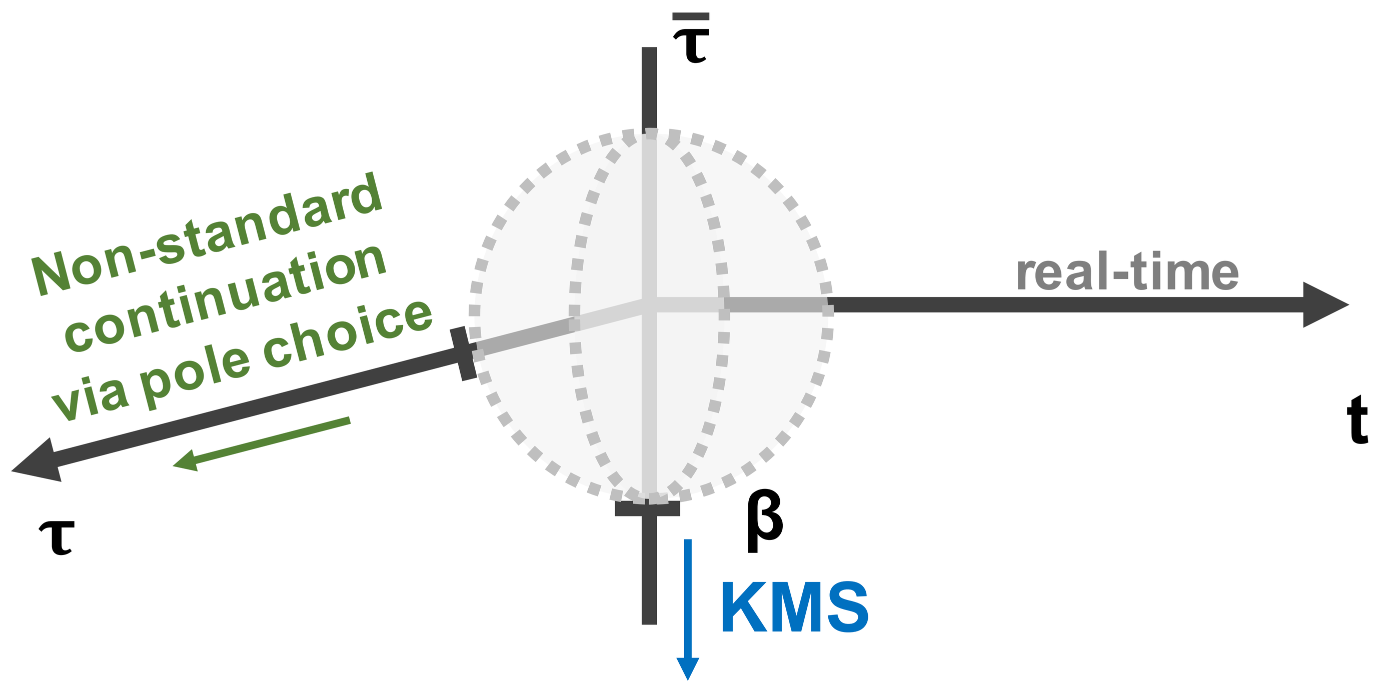
1. For $0 \le \tau \le \beta$ the contour correlator coincides with the Matsubara correlator
$$G_{\rm cont}(0,\tau) = \langle \phi_E(0)\,\phi_E(\tau)\rangle\ .$$
2. Postulating
$$\lim_{\tau\to\infty} G_{\rm cont}(0,\tau) = 0$$
defines a unique analytic continuation $\Rightarrow$ no KMS condition and no time translation invariance on the non-compact path.




- (1.) and (2.) define a **unique path integral representation**

$$G_{\rm cont}(0,\tau) = \frac{\displaystyle\int \mathcal{D}\phi_E e^{-S_E[\phi_E]} \left( \frac{\displaystyle\int_{\phi(0)=\phi_E(0)} \mathcal{D}\phi e^{-S_E[\phi]}\,\phi(0)\phi(\tau)}{\displaystyle\int_{\phi(0)=\phi_E(0)} \mathcal{D}\phi e^{-S_E[\phi]}} \right)}{\displaystyle\int \mathcal{D}\phi_E e^{-S_E[\phi_E]}}$$

- $G_{\rm cont}(\tau,\tau')$ has a Wigner transformation representation for $\tau,\tau' \ge \beta$.
- Non-standard **approximation** to the conventional imaginary time correlator having a **Källén-Lehmann spectral representation**.



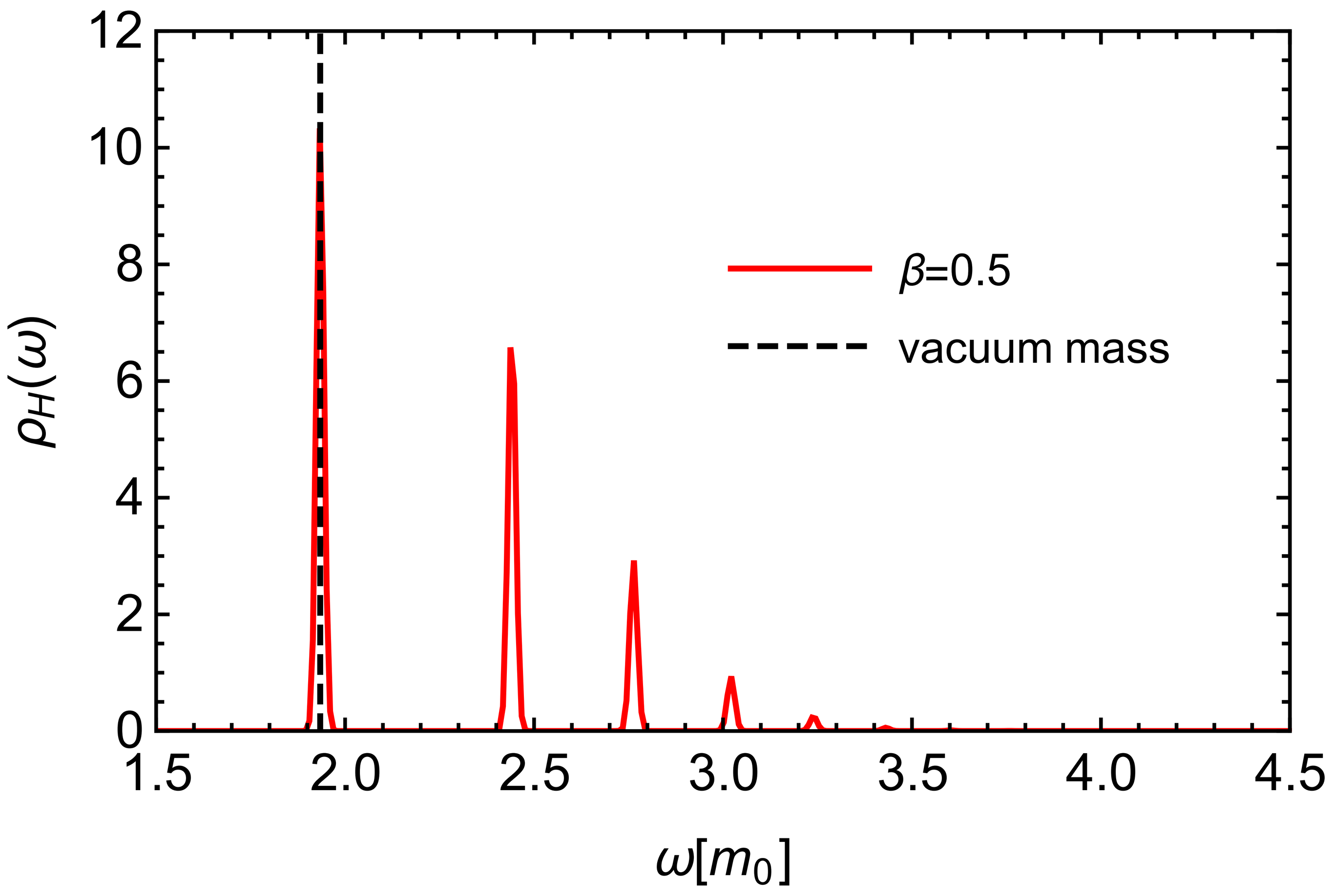

## Applications

- Test the above proposal on QM anharmonic oscillator with action

$$S_E[\phi] = \int d\tau \left[\frac{1}{2}(\partial_\tau\phi)^2 + \frac{1}{2}m_0^2\phi^2 + \frac{\lambda_0}{4!}\phi^4\right]\,, \qquad m_0 = 1, \lambda_0 = 24\,.$$

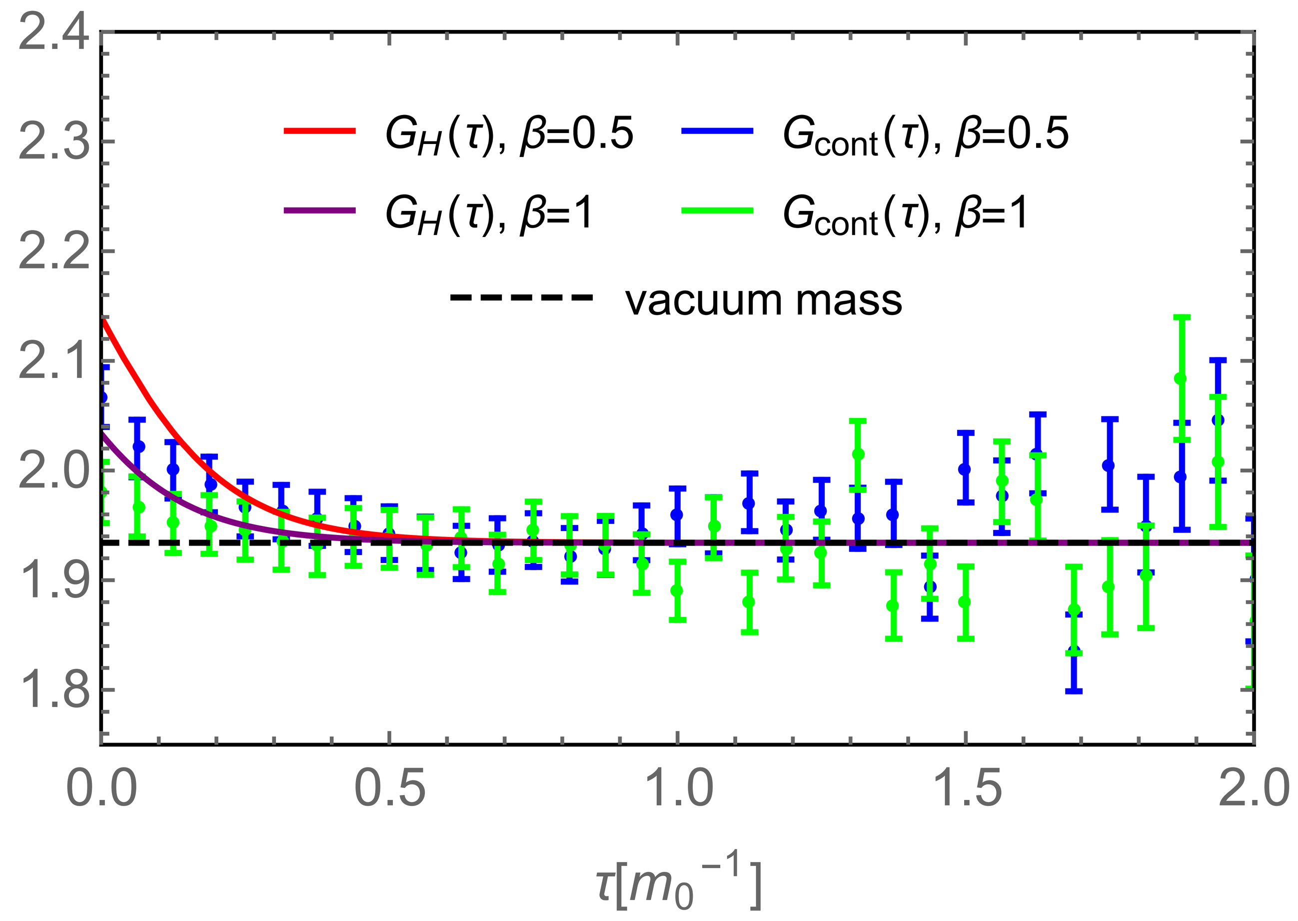
- **Cross-check** with exact thermal real-time correlation function $G_H(t)$ obtained by diagonalizing the Hamiltonian, i.e. solving

$$G_H(t) = -i\,{\rm tr}\left(e^{-\beta\hat{H}}[e^{-i\hat{H}t}\hat{\phi}(0)e^{i\hat{H}t}, \hat{\phi}(0)]\right)\ .$$




- From exact QM **spectral function** $\rho_H(\omega)$ we extract the thermally excited masses for comparisons with the contour correlator.

- **Conclusions and outlook**
  - Use current approximation for spectral reconstruction.
  - Extend this by applying the correct Wigner transformation.
  - Find the spectral representation for $G_{\rm cont}(0,\tau)$.



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