

General equilibrium second-order hydrodynamic coefficients for quantum fields

Matteo Buzzegoli



Dipartimento di Fisica e Astronomia & INFN, Firenze

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Based on previous work with **E. Grossi** and **F. Becattini**: arXiv:1704.02808
and on a work in preparation

Motivations

Relativistic hydrodynamic

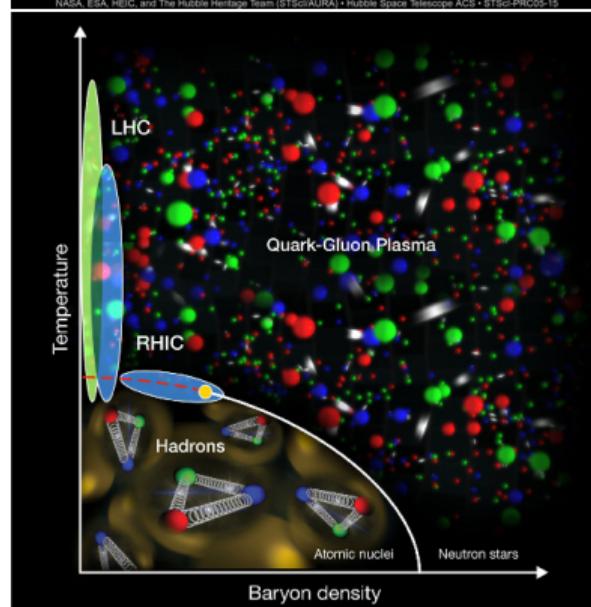
- Astrophysics
- Cosmology
- Heavy ions collisions

General equilibrium

- Quantum-relativistic formulation
- Vorticity effects

Chiral matter

- Anomalous transport
- Chiral Vortical Effect



Introduction

Quantum theory

$$\hat{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^a)} \partial^\nu \psi^a - g^{\mu\nu} \mathcal{L}$$

Statistical operator $\hat{\rho}$

Corrispondence principle

$$T^{\mu\nu} = \text{tr} \left[\hat{\rho} \hat{T}^{\mu\nu} \right] = \langle \hat{T}^{\mu\nu} \rangle \quad j^\mu = \text{tr} \left[\hat{\rho} \hat{j}^\mu \right] = \langle \hat{j}^\mu \rangle$$

Hydro equations

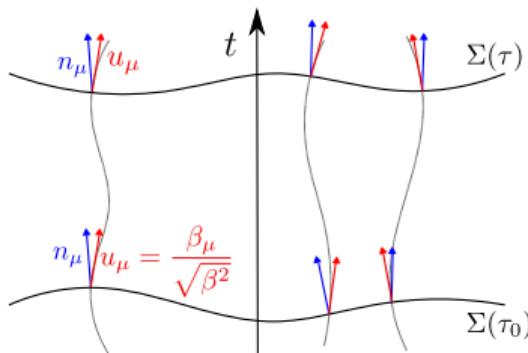
$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu j^\mu = 0$$

Global Generalized Equilibrium

General covariant density operator for any quantum theory [Zubarev et al. (1979); Van Weert (1982)]

$$\hat{\rho}(t) = \frac{1}{\mathcal{Z}} \exp \left[- \int_{\Sigma} d\Sigma_{\mu} \left(\hat{T}^{\mu\nu}(x) \beta_{\nu}(x) - \hat{j}_i^{\mu}(x) \zeta_i(x) \right) \right]$$

$d\Sigma_{\mu}$ is an element of a space-like, 3-dimensional hypersurface Σ_{μ} ,



$$\nabla_{\mu} \hat{T}^{\mu\nu}(x) = 0 \quad \nabla_{\mu} \hat{j}_i^{\mu}(x) = 0$$

In global equilibrium $\hat{\rho}(t)$ should be independent of time

$$\nabla_{\mu} \left(\hat{T}^{\mu\nu}(x) \beta_{\nu}(x) - \hat{j}_i^{\mu}(x) \zeta_i(x) \right) = \hat{T}^{\mu\nu}(x) \nabla_{\mu} \beta_{\nu}(x) - \hat{j}_i^{\mu}(x) \nabla_{\mu} \zeta_i(x) = 0$$

Killing equation: $\nabla_{\mu} \beta_{\nu} + \nabla_{\nu} \beta_{\mu} = 0$ and $\nabla_{\mu} \zeta_i = 0$

also used by [Hayata et al. (2015), Hongo (2016), Florkowski et al. (2017), Harutyunyan et al. (2018)]

Global Generalized Equilibrium (GE)

Solution in Minkowski space:

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

$\zeta_i, b_\mu, \varpi_{\mu\nu}$ are const.

Thermal Vorticity: $\varpi_{\mu\nu} = -\varpi_{\nu\mu} = \partial_\nu \beta_\mu$

Then, the global generalized equilib. density operator in Minkowski space is [Becattini et al 2014]

$$\hat{\rho}_{\text{GE}} = \frac{1}{Z_{\text{GE}}} \exp \left[-b_\mu \hat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \hat{\mathcal{J}}^{\mu\nu} + \zeta_i \hat{Q}_i \right]$$

Features

Lorentz Generators:

$$\hat{\mathcal{J}}^{\mu\nu} = \int_\Sigma d\Sigma_\lambda (x^\mu \hat{T}^{\lambda\nu} - x^\nu \hat{T}^{\lambda\mu})$$

- 10 parameters (as Poincaré generators)
- is a local equilibrium with a specific $\beta(x)$ form
- is an actual global equilibrium (time independent)
- contains all known form of global equilibrium
- $\varpi \neq 0 \Rightarrow$ non-vanishing acceleration and angular velocity

Global Generalized Equilibrium, Special cases

$$\beta_\mu = b_\mu + \varpi_{\mu\nu} x^\nu$$

Homogeneous equilibrium

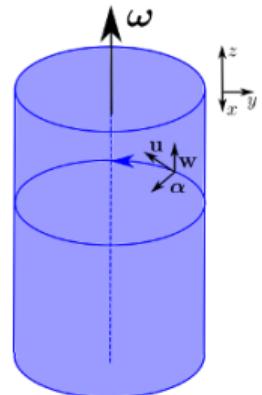
$$b_\mu = \text{const.}, \quad \varpi = 0 \quad \hat{\rho}_\beta = \frac{1}{\mathcal{Z}_\beta} \exp \left(-\beta \cdot \hat{P} + \zeta_i \hat{Q}_i \right)$$

$$T_{\text{ideal}}^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu}$$

Equilibrium with rotation

$$b_\mu = (1/T, \mathbf{0}), \quad \varpi_{\mu\nu} = \omega/T (\delta_\mu^1 \delta_\nu^2 - \delta_\mu^2 \delta_\nu^1)$$

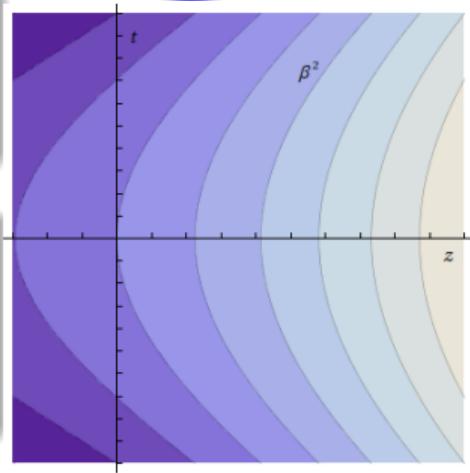
$$\hat{\rho}_\omega = \frac{1}{\mathcal{Z}_\omega} \exp \left(-\hat{H}/T + \omega \hat{J}_z/T \right)$$



Equilibrium with acceleration

$$b_\mu = (1/T, 0, 0, 0), \quad \varpi_{\mu\nu} = (a/T)(g_{0\mu}g_{3\nu} - g_{3\mu}g_{0\nu})$$

$$\hat{\rho}_a = \frac{1}{\mathcal{Z}_a} \exp \left(-\hat{H}/T + a \hat{K}_z/T \right)$$



Stress energy tensor mean value

$$\beta_\mu(x) = b_\mu + \varpi_{\mu\nu} x^\nu$$

$$\hat{\rho}_{\text{GE}} = \frac{1}{\mathcal{Z}_{\text{GE}}} e^{-b_\mu \hat{P}^\mu + \zeta_i \hat{Q}_i + \frac{\varpi_{\mu\nu}}{2} \hat{J}^{\mu\nu}} = \frac{1}{\mathcal{Z}_{\text{GE}}} \exp \left[-\beta_\mu(x) \hat{P}^\mu + \zeta_i \hat{Q}_i + \frac{1}{2} \varpi_{\mu\nu} \hat{J}_x^{\mu\nu} \right]$$

$$\hat{J}_x^{\mu\nu} \equiv \hat{J}^{\mu\nu} - x^\mu \hat{P}^\nu + x^\nu \hat{P}^\mu$$

How does thermal vorticity affects hydrodynamic equations?

$$\langle \hat{T}^{\mu\nu}(x) \rangle = \text{tr} \left[\hat{\rho}_{\text{GE}} \hat{T}^{\mu\nu}(x) \right] = T^{\mu\nu}$$

$$\partial_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = 0$$

$$u^\mu = \frac{\beta^\mu}{\sqrt{\beta^2}}$$

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu} + \delta T^{\mu\nu}(\partial\beta)$$

Cumulant expansion

$$\hat{\rho}_{\text{GE}} = \frac{1}{\mathcal{Z}_{\text{GE}}} \exp \left[-\beta_\mu(x) \hat{P}^\mu + \zeta_i \hat{Q}_i + \frac{1}{2} \varpi_{\mu\nu} \hat{J}_x^{\mu\nu} \right]$$

Expansion for $\varpi \ll 1$ (it is adimensional)

path ordered expansion [van Weert (1982)]

$$\begin{aligned} \langle \hat{O}(x) \rangle &= \text{tr} \left[\hat{\rho}_{\text{GE}} \hat{O}(x) \right] \\ &= \langle \hat{O}(0) \rangle_{\beta(x)} + \frac{\varpi_{\mu\nu}}{2} \langle \langle \hat{J}^{\mu\nu} \hat{O} \rangle \rangle + \frac{\varpi_{\mu\nu} \varpi_{\rho\sigma}}{8} \langle \langle \hat{J}^{\mu\nu} \hat{J}^{\rho\sigma} \hat{O} \rangle \rangle + \mathcal{O}(\varpi^3) \end{aligned}$$

with $\langle \langle \hat{J}^{\mu\nu} \dots \hat{J}^{\rho\sigma} \hat{O} \rangle \rangle \equiv \int_0^{|\beta|} \frac{d\tau_1}{|\beta|} \dots \frac{d\tau_n}{|\beta|} \text{tr} \left[T_\tau \left(\hat{\rho}_{\text{h}} \hat{J}_{-\text{i}\tau_1 u}^{\mu\nu} \dots \hat{J}_{-\text{i}\tau_n u}^{\rho\sigma} \hat{O}(0) \right) \right]_{\text{connected}}$ and

$$\hat{\rho}_{\text{h}} = \frac{1}{\mathcal{Z}_{\text{h}}} \exp \left[-\beta_\mu(x) \hat{P}^\mu + \zeta_i \hat{Q}_i \right]$$

Acceleration and vorticity decomposition

Tetrad: $\{u, \alpha, w, \gamma\}$

$$\partial_\nu \beta_\mu = \varpi_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} w^\rho u^\sigma + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

thermal acceleration:

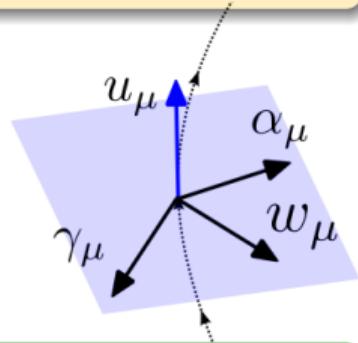
$$\alpha^\mu = \frac{1}{T} a^\mu \quad |\alpha| = \frac{\hbar |\mathbf{a}|}{c k_B T}$$

thermal angular velocity:

$$w^\mu = \frac{1}{T} \omega^\mu \quad |w| = \frac{\hbar |\boldsymbol{\omega}|}{k_B T}$$

transverse vector:

$$\gamma^\mu = \epsilon^{\mu\nu\rho\sigma} w_\nu \alpha_\rho u_\sigma$$



Numerical estimate in heavy ions collisions

[STAR, Nature (2017)]

$$T = 200 \text{ MeV}, |\mathbf{a}| \approx c|\boldsymbol{\omega}| \approx 0.05 c^2/\text{fm} \rightarrow |\varpi| \sim 10^{-2}$$

$$\hat{J}^{\mu\nu} = u^\mu \hat{K}^\nu - u^\nu \hat{K}^\mu - u_\rho \epsilon^{\rho\mu\nu\sigma} \hat{J}_\sigma$$

$$\begin{aligned} \langle \hat{O}(x) \rangle &= \langle \hat{O}(x) \rangle_{\beta(x)} - \alpha_\rho \langle \langle \hat{K}^\rho \hat{O} \rangle \rangle - w_\rho \langle \langle \hat{J}^\rho \hat{O} \rangle \rangle + \frac{\alpha_\rho \alpha_\sigma}{2} \langle \langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle \rangle \\ &\quad + \frac{w_\rho w_\sigma}{2} \langle \langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle \rangle + \frac{\alpha_\rho w_\sigma}{2} \langle \langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle \rangle + \mathcal{O}(\varpi^3) \end{aligned}$$

Acceleration and vorticity decomposition

Tetrad: $\{u, \alpha, w, \gamma\}$

$$\partial_\nu \beta_\mu = \varpi_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} w^\rho u^\sigma + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

thermal acceleration:

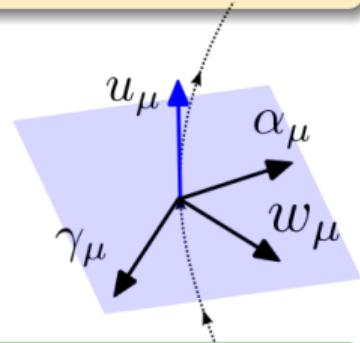
$$\alpha^\mu = \frac{1}{T} a^\mu \quad |\alpha| = \frac{\hbar |\mathbf{a}|}{c k_B T}$$

thermal angular velocity:

$$w^\mu = \frac{1}{T} \omega^\mu \quad |w| = \frac{\hbar |\boldsymbol{\omega}|}{k_B T}$$

transverse vector:

$$\gamma^\mu = \epsilon^{\mu\nu\rho\sigma} w_\nu \alpha_\rho u_\sigma$$



Numerical estimate in heavy ions collisions

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$$\hat{J}^{\mu\nu} = u^\mu \hat{K}^\nu - u^\nu \hat{K}^\mu - u_\rho \epsilon^{\rho\mu\nu\sigma} \hat{J}_\sigma$$

$$\begin{aligned} \langle \hat{O}(x) \rangle &= \langle \hat{O}(x) \rangle_{\beta(x)} - \alpha_\rho \langle \langle \hat{K}^\rho \hat{O} \rangle \rangle - w_\rho \langle \langle \hat{J}^\rho \hat{O} \rangle \rangle + \frac{\alpha_\rho \alpha_\sigma}{2} \langle \langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle \rangle \\ &\quad + \frac{w_\rho w_\sigma}{2} \langle \langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle \rangle + \frac{\alpha_\rho w_\sigma}{2} \langle \langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle \rangle + \mathcal{O}(\varpi^3) \end{aligned}$$

Even and odd Parity fluid

$$\langle\langle \hat{K}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle\rangle$$

Even and odd Parity fluid

$$\langle\langle \hat{K}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle\rangle$$

$$\widehat{\rho}_h = \begin{cases} \widehat{\rho}_{\text{Even}} \propto \exp \left[-\beta_\mu(x) \widehat{P}^\mu + \zeta \widehat{Q} \right] \\ \widehat{\rho}_{\text{Odd}} \propto \exp \left[-\beta_\mu(x) \widehat{P}^\mu + \zeta \widehat{Q} + \zeta_5 \widehat{Q}_5 \right] \end{cases}$$

	\widehat{Q}	\widehat{Q}_5	$\widehat{\rho}_{\text{Even}}$	$\widehat{\rho}_{\text{Odd}}$
P	+	-	+	\pm
T	+	+	+	+
C	-	+	\pm	\pm

Even and odd Parity fluid

$$\langle\langle \hat{K}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle\rangle$$

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	\widehat{Q}	\widehat{Q}_5	$\widehat{\rho}_{\text{Even}}$	$\widehat{\rho}_{\text{Odd}}$
P	+	-	+	\pm
T	+	+	+	+
C	-	+	\pm	\pm

Observables

Lorentz generators

\widehat{T}^{00}			\widehat{T}^{0i}			\widehat{T}^{ij}			\widehat{j}_V^0			$\widehat{\mathbf{j}}_V$			\widehat{j}_A^0			$\widehat{\mathbf{j}}_A$			$\widehat{\phi}$			$\widehat{\phi}_A$			$\widehat{\mathbf{K}}$			$\widehat{\mathbf{J}}$			$(\widehat{K}\widehat{K}, \widehat{J}\widehat{J})$			$\widehat{K}\widehat{J}$		
P	+	-	+	+	-	-	+	+	+	-	-	+	-	-	+	+	-	+	-	+	-	+	-	+	-	+	-	-	-	-								
T	+	-	+	+	-	+	-	+	-	+	+	-	-	+	-	-	-	+	-	+	-	+	-	+	-	+	-	-	-	-								
C	+	+	+	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+								

Even and odd Parity fluid

$$\langle\langle \hat{K}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle\rangle$$

$$\widehat{\rho}_h = \begin{cases} \widehat{\rho}_{\text{Even}} \propto \exp \left[-\beta_\mu(x) \hat{P}^\mu + \zeta \hat{Q} \right] \\ \widehat{\rho}_{\text{Odd}} \propto \exp \left[-\beta_\mu(x) \hat{P}^\mu + \zeta \hat{Q} + \zeta_5 \hat{Q}_5 \right] \end{cases}$$

	\hat{Q}	\hat{Q}_5	$\widehat{\rho}_{\text{Even}}$	$\widehat{\rho}_{\text{Odd}}$
P	+	-	+	\pm
T	+	+	+	+
C	-	+	\pm	\pm

Observables

Lorentz generators

\widehat{T}^{00}			\widehat{T}^{0i}			\widehat{T}^{ij}			\widehat{j}_V^0	$\widehat{\mathbf{j}}_V$	\widehat{j}_A^0	$\widehat{\mathbf{j}}_A$	$\widehat{\phi}$	$\widehat{\phi}_A$	$\widehat{\mathbf{K}}$	$\widehat{\mathbf{J}}$	$(\widehat{K}\widehat{K}, \widehat{J}\widehat{J})$	$\widehat{K}\widehat{J}$
P	+	-	+	+	-	-	+	+	+	-	-	+	+	-	-	+	-	
T	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	-	
C	+	+	+	-	-	+	+	+	+	+	+	+	+	+	+	+	+	

Axial Vortical Effect

$$\langle\langle \hat{J}^3 \hat{j}_A^3 \rangle\rangle \longrightarrow \text{tr} \left[\widehat{\rho}_{\text{Even}} \hat{J}^3 \hat{j}_A^3 \right] \neq 0 \quad \text{tr} \left[\widehat{\rho}_{\text{Odd}} \hat{J}^3 \hat{j}_A^3 \right] \neq 0$$

Even and odd Parity fluid

$$\langle\langle \hat{K}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{O} \rangle\rangle \quad \langle\langle \hat{K}^\rho \hat{K}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \hat{J}^\rho \hat{J}^\sigma \hat{O} \rangle\rangle \quad \langle\langle \{\hat{K}^\rho, \hat{J}^\sigma\} \hat{O} \rangle\rangle$$

$$\widehat{\rho}_h = \begin{cases} \widehat{\rho}_{\text{Even}} \propto \exp \left[-\beta_\mu(x) \widehat{P}^\mu + \zeta \widehat{Q} \right] \\ \widehat{\rho}_{\text{Odd}} \propto \exp \left[-\beta_\mu(x) \widehat{P}^\mu + \zeta \widehat{Q} + \zeta_5 \widehat{Q}_5 \right] \end{cases}$$

	\widehat{Q}	\widehat{Q}_5	$\widehat{\rho}_{\text{Even}}$	$\widehat{\rho}_{\text{Odd}}$
P	+	-	+	\pm
T	+	+	+	+
C	-	+	\pm	\pm

Observables

Lorentz generators

	\widehat{T}^{00}	\widehat{T}^{0i}	\widehat{T}^{ij}	\widehat{j}_V^0	$\widehat{\mathbf{j}}_V$	\widehat{j}_A^0	$\widehat{\mathbf{j}}_A$	$\widehat{\phi}$	$\widehat{\phi}_A$	$\widehat{\mathbf{K}}$	$\widehat{\mathbf{J}}$	$(\widehat{K}\widehat{K}, \widehat{J}\widehat{J})$	$\widehat{K}\widehat{J}$
P	+	-	+	+	-	-	+	+	-	-	+	+	-
T	+	-	+	+	-	+	-	+	-	+	-	+	-
C	+	+	+	-	-	+	+	+	+	+	+	+	+

Chiral Vortical Effect

$$\langle\langle \widehat{J}^3 \widehat{j}_V^3 \rangle\rangle \longrightarrow \text{tr} \left[\widehat{\rho}_{\text{Even}} \widehat{J}^3 \widehat{j}_V^3 \right] = 0 \quad \text{tr} \left[\widehat{\rho}_{\text{Odd}} \widehat{J}^3 \widehat{j}_V^3 \right] \neq 0$$

Stress-energy tensor corrections

Thermodynamic coefficient definitions

Expansion up to second order on $\varpi_{\mu\nu} = \partial_\nu \beta_\mu$

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & \mathbb{A} \epsilon_{\mu\nu\kappa\lambda} \alpha^\kappa u^\lambda + \mathbb{W}_1 w_\mu u_\nu + \mathbb{W}_2 w_\nu u_\mu + (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu \\ & - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

$$\mathbb{A} = \langle\langle \hat{K}^3 \frac{\hat{T}^{12} - \hat{T}^{21}}{2} \rangle\rangle$$

$$\mathbb{W}_1 = \langle\langle \hat{J}^3 \hat{T}^{30} \rangle\rangle$$

$$\mathbb{W}_2 = \langle\langle \hat{J}^3 \hat{T}^{03} \rangle\rangle$$

$$U_\alpha = \frac{1}{2} \langle\langle \hat{K}^3 \hat{K}^3 \hat{T}^{00} \rangle\rangle$$

$$U_w = \frac{1}{2} \langle\langle \hat{J}^3 \hat{J}^3 \hat{T}^{00} \rangle\rangle$$

$$D_\alpha = \frac{1}{2} \langle\langle \hat{K}^3 \hat{K}^3 \hat{T}^{11} \rangle\rangle - \frac{1}{3} \langle\langle \hat{K}^1 \hat{K}^2 \frac{\hat{T}^{12} + \hat{T}^{21}}{2} \rangle\rangle$$

$$D_w = \frac{1}{2} \langle\langle \hat{J}^3 \hat{J}^3 \hat{T}^{11} \rangle\rangle - \frac{1}{3} \langle\langle \hat{J}^1 \hat{J}^2 \frac{\hat{T}^{12} + \hat{T}^{21}}{2} \rangle\rangle$$

$$A = \langle\langle \hat{K}^1 \hat{K}^2 \frac{\hat{T}^{12} + \hat{T}^{21}}{2} \rangle\rangle$$

$$W = \langle\langle \hat{J}^1 \hat{J}^2 \frac{\hat{T}^{12} + \hat{T}^{21}}{2} \rangle\rangle$$

$$G_1 = -\frac{1}{2} \langle\langle \{\hat{K}^1, \hat{J}^2\} \hat{T}^{03} \rangle\rangle$$

$$G_2 = -\frac{1}{2} \langle\langle \{\hat{K}^1, \hat{J}^2\} \hat{T}^{30} \rangle\rangle$$

Currents corrections

Vector (electric) current

$$\langle \hat{j}_V^\mu \rangle = n_V u^\mu + (\alpha^2 N_\alpha^V + w^2 N_\omega^V) u^\mu + W^V w^\mu + G^V \gamma^\mu + \mathcal{O}(\varpi^3)$$

$$N_\alpha^V = \frac{1}{2} \langle\langle \hat{K}^3 \hat{K}^3 \hat{j}_V^0 \rangle\rangle$$

$$N_w^V = \frac{1}{2} \langle\langle \hat{J}^3 \hat{J}^3 \hat{j}_V^0 \rangle\rangle$$

$$G^V = \frac{1}{2} \langle\langle \{\hat{K}^1, \hat{J}^2\} \hat{j}_V^3 \rangle\rangle$$

$$W^V = \langle\langle \hat{J}^3 \hat{j}_V^3 \rangle\rangle$$

Axial current

$$\langle \hat{j}_A^\mu \rangle = n_A u^\mu + (\alpha^2 N_\alpha^A + w^2 N_\omega^A) u^\mu + W^A w^\mu + G^A \gamma^\mu + \mathcal{O}(\varpi^3)$$

$$N_\alpha^A = \frac{1}{2} \langle\langle \hat{K}^3 \hat{K}^3 \hat{j}_A^0 \rangle\rangle$$

$$N_\omega^A = \frac{1}{2} \langle\langle \hat{J}^3 \hat{J}^3 \hat{j}_A^0 \rangle\rangle$$

$$G^A = \frac{1}{2} \langle\langle \{\hat{K}^1, \hat{J}^2\} \hat{j}_A^3 \rangle\rangle$$

$$W^A = \langle\langle \hat{J}^3 \hat{j}_A^3 \rangle\rangle$$

Chiral coefficients in red

Results for non-interacting fields

Non chiral coefficients evaluated in [M.B., E. Grossi and F. Becattini (2017)]
for massless and massive charged free Dirac and Scalar field;

Chiral coefficients evaluated in [M.B. in preparation]
for free massless Dirac field;

$$\mathcal{L} = \frac{i}{2} [\bar{\psi} \gamma^\mu (\partial_\mu \psi) - (\partial_\mu \bar{\psi}) \gamma^\mu \psi] \quad \hat{j}_V^\mu = \bar{\psi} \gamma^\mu \psi \quad \hat{j}_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi \quad \partial_\mu \hat{j}_A^\mu = 0$$

$$\langle \hat{T}^{\mu\nu}(x) \rangle = \mathbb{A} \epsilon^{\mu\nu\kappa\lambda} \alpha_\kappa u_\lambda + \mathbb{W}_1 w^\mu u^\nu + \mathbb{W}_2 w^\nu u^\mu + \dots$$

$$\mathbb{W}_2 = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{6\pi^2 |\beta|^4}$$

agree with previous determinations

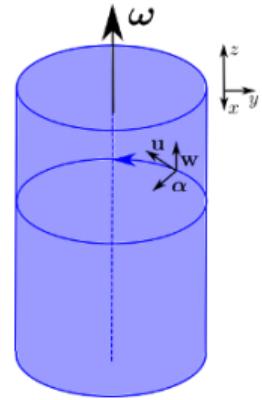
- Vilenkin (1979)
- Holography: Landsteiner et al. (2013)
- CKT: Chen et al. (2015), Hidaka et al. (2018)
- Kubo Formulae: Landsteiner et al. (2011), Chowdhury and David (2015)

Axial Vortical Effect

Massless field

$$\hat{j}_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi \quad j_A = \frac{W^A}{T} \omega$$

ω , j_A and angular momentum \mathbf{J} are axial vectors



Massless field

$$j_A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} + \frac{\mu_5^2}{2\pi^2} \right) \omega$$

$$\partial_\mu \hat{j}_A^\mu = 0 \quad \partial_\mu \langle \hat{j}_A^\mu \rangle = 0$$

Anomaly?

Axial Vortical Effect

Massive field

$$\partial_\mu \hat{j}_A^\mu = 2m i \bar{\psi} \gamma^5 \psi \quad \partial_\mu \langle \hat{j}_A^\mu \rangle = 2m i \langle \bar{\psi} \gamma^5 \psi \rangle \quad \mathbf{j}_A = \frac{W^A}{T} \boldsymbol{\omega}$$

$$W^A = \frac{1}{2\pi^2 |\beta|} \int_0^\infty \frac{dp}{E_p} (n_F(E_p - \mu) + n_F(E_p + \mu)) (2p^2 + m^2)$$

At $T = 0$

$$\frac{W^A}{T} = \frac{\mu^2}{2\pi^2} \sqrt{\frac{\mu^2 - m^2}{\mu^2}}$$

For $T \ll m$

$$\frac{W^A}{T} = \frac{m^2}{2\sqrt{2}\pi^{3/2}} \sqrt{\frac{T}{m}} e^{-|\beta|(m-\mu)} \left[1 + \frac{15}{8} \frac{T}{m} + O\left(\frac{T^2}{m^2}\right) \right]$$

For $T \gg m$

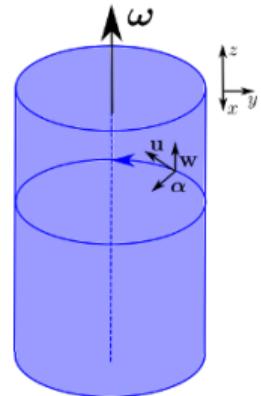
$$\frac{W^A}{T} = \frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{m^2}{4\pi^2} - \frac{7\zeta'(-2)}{8\pi^2} \frac{m^4}{T^2} + O\left(\frac{m^3}{T^3}\right)$$

Chiral Vortical Effect

Massless field

$$\hat{j}_V^\mu = \bar{\psi} \gamma^\mu \psi \quad j_V = \frac{W^V}{T} \omega$$

ω and angular momentum \mathbf{J} are axial vectors
 j_V is a vector, \hat{Q}_5 provides the Parity breaking



Massless field

$$j_V = \frac{\mu\mu_5}{\pi^2} \omega$$

$$\partial_\mu \hat{j}_V^\mu = 0 \quad \partial_\mu \langle \hat{j}_V^\mu \rangle = 0$$

Anomaly?

Summary

We have studied quantum relativistic free fields of spin 0 and 1/2 at general thermodynamic equilibrium with non-vanishing **acceleration** and **vorticity**.

- Expressed as correlators of conserved quantities ($T_{\mu\nu}$ and Poincaré groups generators)
- Such corrections may be phenomenologically relevant for system with very high acceleration, such as in the early stage of relativistic heavy ion collisions
- These corrections are pure **quantum** effects
- Chiral coefficients by odd Parity of Axial charge
- Axial/Chiral vortical effects **without anomaly**

Thanks for the attention!

Results, massless free Dirac [Odd Parity]

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & \mathbb{A} \epsilon^{\mu\nu\kappa\lambda} \alpha_\kappa u_\lambda + \mathbb{W}_1 w^\mu u^\nu + \mathbb{W}_2 w^\nu u^\mu \\ & + (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} \\ & + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

A generic coefficient can be written as

$$K \int_0^\infty P_N(p) \left[n_F(p - \mu_R) + \eta \eta_5 n_F(p + \mu_R) + \eta_5 n_F(p - \mu_L) + \eta n_F(p + \mu_L) \right] dp$$

Fermi-Dirac distribution function $n_F(x) = \frac{1}{\exp(|\beta|x)+1}$

$$\mathbb{A}^{\text{Sym}} = 0, \quad \mathbb{A}^{\text{Can}} = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{6\pi^2 |\beta|^4},$$

$$\mathbb{W}_1^{\text{Sym}} = \mathbb{W}_2^{\text{Sym}} = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{3\pi^2 |\beta|^4},$$

$$\mathbb{W}_1^{\text{Can}} = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{2\pi^2 |\beta|^4}, \quad \mathbb{W}_2^{\text{Can}} = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{6\pi^2 |\beta|^4}.$$

Results, massless free Dirac [Odd Parity]

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & \mathbb{A} \epsilon^{\mu\nu\kappa\lambda} \alpha_\kappa u_\lambda + \mathbb{W}_1 w^\mu u^\nu + \mathbb{W}_2 w^\nu u^\mu \\ & + (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} \\ & + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

$$\rho = 3p = \frac{30\pi^2 (\zeta^2 + \zeta_5^2) + 15 (\zeta^4 + 6\zeta^2\zeta_5^2 + \zeta_5^4) + 7\pi^4}{60\pi^2 |\beta|^4},$$

$$U_\alpha = 3D_\alpha = \frac{3 (\zeta^2 + \zeta_5^2) + \pi^2}{24\pi^2 |\beta|^4},$$

$$G_1^{\text{Sym}} = G_2^{\text{Sym}} = \frac{3 (\zeta^2 + \zeta_5^2) + \pi^2}{18\pi^2 |\beta|^4},$$

$$G_1^{\text{Can}} = \frac{2 [3 (\zeta^2 + \zeta_5^2) + \pi^2]}{9\pi^2 |\beta|^4}, \quad G_2^{\text{Can}} = -\frac{3 (\zeta^2 + \zeta_5^2) + \pi^2}{9\pi^2 |\beta|^4}.$$

Results, massless free Dirac [Odd Parity]

Vector (electric) current

$$\langle \hat{j}_V^\mu \rangle = n_V u^\mu + (\alpha^2 N_\alpha^V + w^2 N_\omega^V) u^\mu + W^V w^\mu + G^V \gamma^\mu + \mathcal{O}(\varpi^3)$$

$$n_V = \frac{\zeta (\zeta^2 + 3\zeta_5^2 + \pi^2)}{3\pi^2 |\beta|^3}, \quad N_\alpha^V = \frac{\zeta}{4\pi^2 |\beta|^3}, \quad W^V = \frac{\zeta \zeta_5}{\pi^2 |\beta|^3}$$

$$N_w^V = \frac{\zeta}{4\pi^2 |\beta|^3}, \quad G^V = \frac{\zeta}{6\pi^2 |\beta|^3},$$

Axial current

$$\langle \hat{j}_A^\mu \rangle = n_A u^\mu + (\alpha^2 N_\alpha^A + w^2 N_\omega^A) u^\mu + W^A w^\mu + G^A \gamma^\mu + \mathcal{O}(\varpi^3)$$

$$n_A = \frac{\zeta_5 (\pi^2 + 3\zeta^2 + \zeta_5^2)}{3\pi^2 |\beta|^3}, \quad N_\alpha^A = \frac{\zeta_5}{4\pi^2 |\beta|^3}, \quad W^A = \frac{3 (\zeta^2 + \zeta_5^2) + \pi^2}{6\pi^2 |\beta|^3}$$

$$N_\omega^A = \frac{\zeta_5}{4\pi^2 |\beta|^3}, \quad G^A = \frac{\zeta_5}{6\pi^2 |\beta|^3}.$$

Stress-energy tensor corrections

Features

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & \mathbb{A} \epsilon_{\mu\nu\kappa\lambda} \alpha^\kappa u^\lambda + \mathbb{W}_1 w_\mu u_\nu + \mathbb{W}_2 w_\nu u_\mu + (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu \\ & - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

- Because are evaluated at global equilibrium they are **non-dissipative**

$\partial^\mu \langle \hat{T}_{\mu\nu} \rangle = 0$ implies relations between coefficients

$$-2\mathbb{A} = |\beta| \frac{\partial \mathbb{W}_1}{\partial |\beta|} + 3\mathbb{W}_1 + \mathbb{W}_2$$

$$U_\alpha = -|\beta| \frac{\partial}{\partial |\beta|} (D_\alpha + A) - (D_\alpha + A)$$

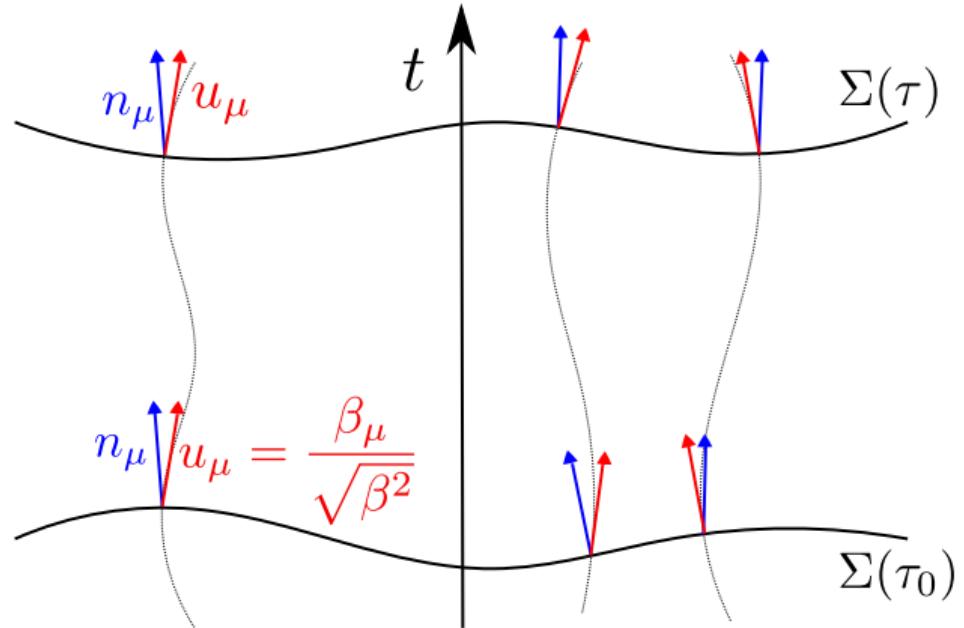
$$U_w = -|\beta| \frac{\partial}{\partial |\beta|} (D_w + W) - D_w + 2A - 3W$$

$$G_1 + G_2 = 2(D_\alpha + D_w) + A + |\beta| \frac{\partial}{\partial |\beta|} W + 3W$$

Local Thermal Equilibrium

Foliation with 3D spatial hypersurfaces Σ of normal vector n

$$n_\mu \langle \hat{T}^{\mu\nu}(x) \rangle = n_\mu T^{\mu\nu}(x)$$
$$n_\mu \langle \hat{j}^\mu(x) \rangle = n_\mu j^\mu(x)$$



$$\hat{\rho} = \frac{1}{\mathcal{Z}} \exp \left[- \int_{\Sigma(\tau)} d\Sigma n_\mu(x) \left(\hat{T}^{\mu\nu}(x) \beta_\nu(x) - \hat{j}^\mu(x) \zeta(x) \right) \right]$$

Stevin law

Consider a non-relativistic fluid with $\mu = 0$ in a gravitational field $\mathbf{g} = -g\hat{z}$

$$\beta^\mu = \frac{1}{T_0}(1 + gz, 0, 0, gt) \quad \frac{1}{T(z)} = \sqrt{\beta^2} \simeq \frac{1}{T_0}(1 + gz) + \mathcal{O}(g^2)$$

The pressure p and the numerical density n are given by

$$m n = \frac{m^4}{\sqrt{2}\pi^{3/2}} e^{-m/T(z)} \left(\frac{T(z)}{m}\right)^{3/2} \quad p = \frac{m^4}{\sqrt{2}\pi^{3/2}} e^{-m/T(z)} \left(\frac{T(z)}{m}\right)^{5/2}$$

The coordinate dependence are completely contained on $T(z)$

$$\frac{\partial p}{\partial z} \simeq -m^2 n \frac{T(z)}{m} \frac{1}{T_0} \frac{\partial}{\partial z}(1 + gz) + \mathcal{O}(g^2) = -m n g + \mathcal{O}(g^2)$$

$$p = p_0 + d g h \quad d \text{ mass density } h \text{ depth}$$

Dirac Bilinears

Scalar

$$\langle \hat{\bar{\psi}} \hat{\psi}(x) \rangle_{\text{GE}} = \langle \hat{\bar{\psi}} \hat{\psi}(x) \rangle_T + \alpha^2 k_s + w^2 j_s$$

Pseudoscalar

$$\langle \hat{\bar{\psi}} \gamma_5 \hat{\psi}(x) \rangle_{\text{GE}} = \alpha \cdot w l^{\text{PS}}$$

Tensor

$$\langle \hat{\sigma}_{\alpha\beta}(x) \rangle_{\text{GE}} = (u_\alpha \alpha_\beta - u_\beta \alpha_\alpha) A^T + \epsilon_{\alpha\beta\rho\lambda} w^\rho u^\lambda W^T$$

Free fields

Charged free scalar field

$$\hat{T}_{\mu\nu} = \partial_\mu \hat{\phi}^\dagger \partial_\nu \hat{\phi} + \partial_\nu \hat{\phi}^\dagger \partial_\mu \hat{\phi} - g_{\mu\nu} (\partial \hat{\phi}^\dagger \cdot \partial \hat{\phi} - m^2 \hat{\phi}^\dagger \hat{\phi}) - 2\xi (\square - \partial_\mu \partial_\nu) \hat{\phi}^\dagger \hat{\phi}$$

Free Dirac field

$$\hat{T}_{\text{can}}^{\mu\nu} = \frac{i}{2} \left[\hat{\bar{\psi}} \gamma^\mu (\partial^\nu \hat{\psi}) - \gamma^\mu (\partial^\nu \hat{\bar{\psi}}) \hat{\psi} \right]$$

$$\hat{T}_{\text{sym}}^{\mu\nu} = \frac{i}{4} \left[\hat{\bar{\psi}} \gamma^\mu (\partial^\nu \hat{\psi}) + \hat{\bar{\psi}} \gamma^\nu (\partial^\mu \hat{\psi}) - (\partial^\nu \hat{\bar{\psi}}) \gamma^\mu \hat{\psi} - (\partial^\mu \hat{\bar{\psi}}) \gamma^\nu \hat{\psi} \right]$$

Example coefficient

$$U_\alpha = \int_0^{|\beta|} \frac{d\tau_1}{2} \int_0^{|\beta|} d\tau_2 \int d^3x \int d^3y \langle \hat{T}_{00}^{\text{sym}}(X) \hat{T}_{00}^{\text{sym}}(Y) \hat{T}_{00}(0) \rangle_{T,c} x_3 y_3$$

Results [Even Parity]

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} \\ & + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

A generic coefficient can be written as

$$\frac{1}{\pi^2 \beta^2} \int_0^\infty \frac{dp}{E_p} \left(n''_{\text{B/F}}(E_p - \mu) + n''_{\text{B/F}}(E_p + \mu) \right) (Ap^4 + Bm^2 p^2 + Cm^4)$$

Bose-Einstein/Fermi-Dirac distribution function $n_{\text{B/F}}(x) = \frac{1}{\exp(|\beta|x) - \eta}$

Bose Field

$$G_1^{\text{sym}} = G_2^{\text{sym}}$$

Coeff. depends on ξ

$W = [\text{Moore and Sohrabi 2011}]$

Dirac Field

$$A = W = 0$$

$$G_1^{\text{sym}} = G_2^{\text{sym}}$$

$$G_1^{\text{can}} = -2G_2^{\text{can}}$$

$W \neq [\text{Moore and Sohrabi 2011}]$

A and $W = [\text{Megias and Valle 2014}]$

Results [Even Parity]

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} \\ & + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

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Bose-Einstein/Fermi-Dirac distribution function $n_{\text{B/F}}(x) = \frac{1}{\exp(|\beta|x) - \eta}$

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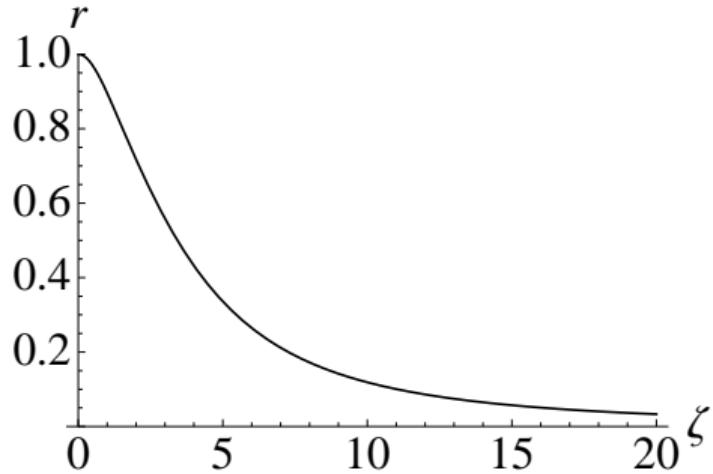
$$G_1^{\text{can}} = -2G_2^{\text{can}}$$

$W \neq [\text{Moore and Sohrabi 2011}]$

A and $W = [\text{Megias and Valle 2014}]$

Massless fields

	Bose $\mu = 0$	Dirac
U_α	$\frac{1-6\xi}{6}T^4$	$\frac{1}{24} \left(1 + \frac{3\beta^2\mu^2}{\pi^2}\right) T^4$
D_α	$\frac{6\xi-1}{9}T^4$	$\frac{1}{72} \left(1 + \frac{3\beta^2\mu^2}{\pi^2}\right) T^4$
A	$\frac{1-6\xi}{6}T^4$	0
U_w	$\frac{1-4\xi}{6}T^4$	$\frac{1}{8} \left(1 + \frac{3\beta^2\mu^2}{\pi^2}\right) T^4$
D_w	$\frac{\xi}{3}T^4$	$\frac{1}{24} \left(1 + \frac{3\beta^2\mu^2}{\pi^2}\right) T^4$
W	$\frac{2\xi-1}{6}T^4$	0
G	$\frac{1+\xi}{18}T^4$	$\frac{1}{18} \left(1 + \frac{3\beta^2\mu^2}{\pi^2}\right) T^4$



$$\rho_{\text{eff}} \equiv \langle \hat{T}_{\mu\nu} \rangle u^\mu u^\nu = \rho - \alpha^2 U_\alpha - w^2 U_w,$$

$$p_{\text{eff}} \equiv -\frac{1}{3} \langle \hat{T}_{\mu\nu} \rangle \Delta^{\mu\nu} = p - \alpha^2 D_\alpha - w^2 D_w$$

Dirac case:

$$\rho_{\text{eff}} = \rho \left[1 - \frac{5}{14\pi^2} (\alpha^2 + 3w^2) r(\zeta) \right] = 3p_{\text{eff}}$$

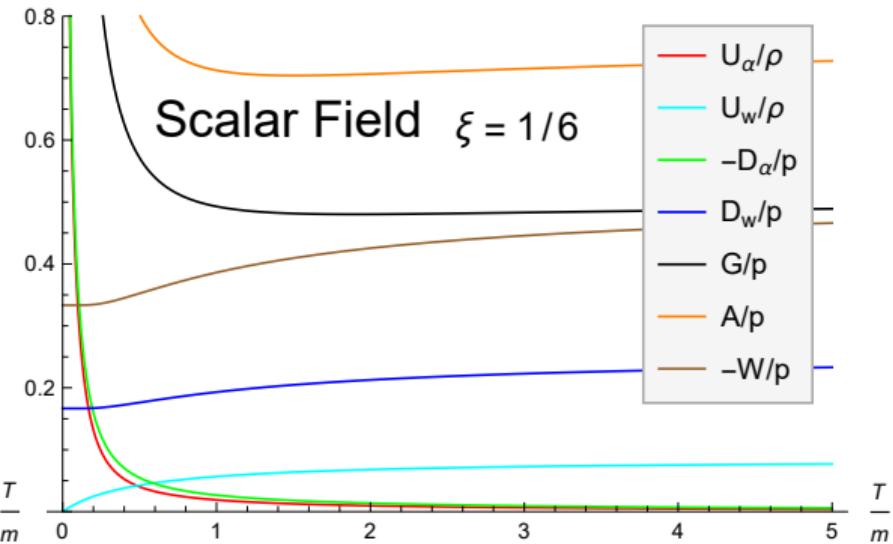
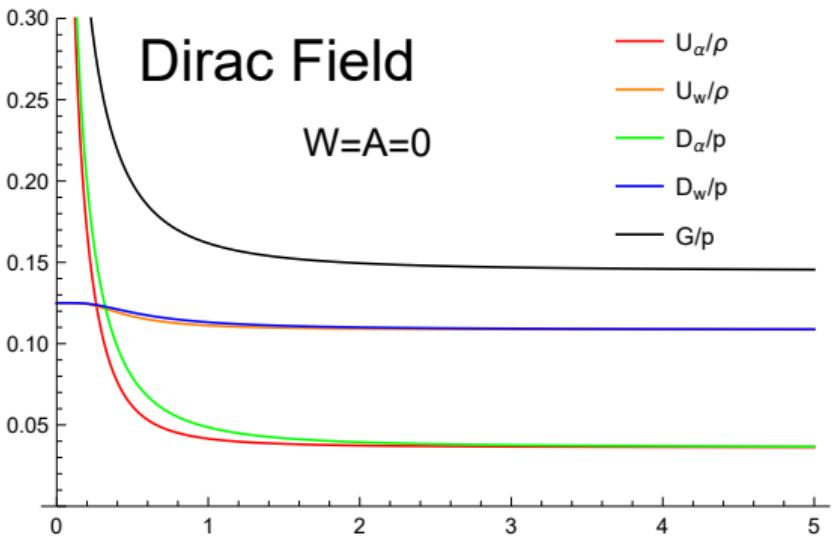


Figure: The coefficients divided by the energy ρ or the pressure p with $m > 0, \mu = 0$

Quark gluon plasma

Values: $k_B T = 300$ MeV, $a = 10^{30} \frac{\text{m}}{\text{s}^2} \rightarrow \frac{\alpha^2 U_\alpha}{\rho} \sim 10^{-3}$

Non relativistic limit

$$\begin{aligned}\langle \hat{T}_{\mu\nu}(x) \rangle \simeq & (\rho - \alpha^2 U_\alpha - w^2 U_w) u_\mu u_\nu - (p - \alpha^2 D_\alpha - w^2 D_w) \Delta_{\mu\nu} \\ & + A \alpha_\mu \alpha_\nu + W w_\mu w_\nu + G_1 u_\mu \gamma_\nu + G_2 u_\nu \gamma_\mu\end{aligned}$$

Every coefficient

$$\frac{1}{\pi^2 \beta^2} \int_0^\infty \frac{dp}{E_p} \left(n''_{\text{B/F}}(E_p - \mu) + n''_{\text{B/F}}(E_p + \mu) \right) (Ap^4 + Bm^2 p^2 + Cm^4)$$

in the limit $m/T \gg 1$ becomes $m \frac{dN}{dx^3} f(m/T)$

where f remains finite and dN/dx^3 is the classical expression of particle density in Boltzmann limit

$$|\alpha| = \frac{\hbar |\mathbf{a}|}{c k_B T}, \quad |w| = \frac{\hbar |\boldsymbol{\omega}|}{k_B T}$$

Quantum effects

Massive Dirac field at zero temperature

$$\rho_{\text{eff}} = \rho - \alpha^2 U_\alpha - w^2 U_w, \quad p_{\text{eff}} = p - \alpha^2 D_\alpha - w^2 D_w$$

At $T = 0$ we have $n_F(E - \mu) = \theta(\mu - E)$

Ultra relativistic limit

$$\frac{\alpha^2 U_\alpha}{\rho} = -\frac{1}{2} \left(\frac{|a|}{\mu} \right)^2$$

$$\frac{w^2 U_w}{\rho} = -\frac{3}{2} \left(\frac{|\omega|}{\mu} \right)^2$$

$$\frac{\alpha^2 D_\alpha}{p} = -\frac{1}{2} \left(\frac{|a|}{\mu} \right)^2$$

$$\frac{w^2 D_w}{p} = -\frac{3}{2} \left(\frac{|\omega|}{\mu} \right)^2$$

Non relativistic limit ($\mu^{\text{NR}} = \mu - m$)

$$\frac{\alpha^2 U_\alpha}{\rho} = \frac{1}{64} \left(\frac{|a|}{\mu^{\text{NR}}} \right)^2 \frac{m}{\mu^{\text{NR}}}$$

$$\frac{w^2 U_w}{\rho} = -\frac{3}{32} \left(\frac{|\omega|}{\mu^{\text{NR}}} \right)^2$$

$$\frac{\alpha^2 D_\alpha}{p} = -\frac{5}{64} \left(\frac{|a|}{\mu^{\text{NR}}} \right)^2 \frac{m}{\mu^{\text{NR}}}$$

$$\frac{w^2 D_w}{p} = -\frac{15}{32} \left(\frac{|\omega|}{\mu^{\text{NR}}} \right)^2$$

For white dwarfs and neutron stars these corrections are negligible: $10^{-40} \div 10^{-50}$

Axial Vortical Effects

ω , j_A and angular momentum \mathbf{J} are axial vectors.

$$\langle \hat{j}_A^\mu(x) \rangle = \frac{w^\mu}{|\beta|} \int_0^{|\beta|} d\tau \langle T_\tau \left(\hat{J}_{-\text{i}\tau}^3 \hat{j}^{3,A}(0) \right) \rangle_{T,c}$$

Massless field

$$\hat{j}_A^\mu = \bar{\hat{\psi}} \gamma^\mu \gamma^5 \hat{\psi} \quad \partial_\mu \hat{j}_A^\mu = 0 \quad \partial_\mu \langle \hat{j}_A^\mu \rangle = 0$$

Propagator with rotation [Vilenkin 1980]

$$\langle \hat{\psi}(\chi, \mathbf{p}) \hat{\bar{\psi}}(\chi', \mathbf{q}) \rangle_\omega = \bar{\delta}(\chi - \chi', \mathbf{p} - \mathbf{q}) e^{\omega \frac{1}{2} \Sigma \frac{\partial}{\partial \chi}} \frac{-i\gamma^0(\chi + \mu) - i\gamma \mathbf{p}}{(\chi + \mu)^2 + \mathbf{p}^2}$$

$$\Sigma^i = \epsilon^{ijk} \frac{i}{4} [\gamma_j, \gamma_k]$$

$$\langle \hat{j}_A^\mu \rangle_\omega \propto \text{tr} [\gamma^5 \gamma^\mu \gamma^\rho \gamma^\sigma \gamma^\nu] = 4i \epsilon^{\mu\rho\sigma\nu}$$