Global $Λ$ polarization in intermediate & high energy heavy ion collisions

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Introduction: Spin-orbit Coupling

Uncharged body will be spontaneously magnetized when rotated with fixed axis

- Coupling between macroscopic orbital angular momentum and microscopic spin angular momentum

- Another interpretation of this coupling: Equipartition of angular momentum degrees of freedom

Ferromagnet at rest, when put into an external H field, will start to rotate
[A. Einstein, W. J. de Haas, Koninklijke Akademie van Wetenschappen te Amsterdam, Proceedings, 18 I, 696711 (1915)]
Introduction: Angular momentum

- Tilted initial state (Fig. 1) carries the angular momentum from impact.
- The velocity shear (Fig. 2b) will further rotate this initial state, and even leads to Kelvin Helmholtz Instability (KHI).
Introduction: Vorticity and Polarization

Fig. 3 The vorticity calculated in the reaction (xz) plane at t = 0.17 fm/c after the start of fluid dynamical evolution.

• The rotation can lead to observable vorticity (Fig. 3), and polarization (Fig. 4).

• The initial angular momentum can be transfer to the polarization at final state, via spin-orbital coupling or equipartition of the freedom of angular momentum.

Fig. 4. The dominant y component of the observable polarization, $\Pi_y(p)$ in the $\Lambda$'s rest frame.

[ L. P. Csernai, et al, PRC 87, 034906 (2013)]

An initial state based Yang-Mills fields (flux tube) is formed after Lorentz contracted nuclei penetrate each other, then system evolves with a (3+1)D fluid dynamics: the high resolution Particle-In-Cell Relativistic (PICR) hydrodynamic model.

The major part of freeze out hypersurface is assumed to be time like here. And we use the ideal-gas post-FO distribution. The precise FO prescription in Hydro fluid dynamics was discussed in Ref. [Yu Cheng, 2011].

Polarization vector

Refs. [Becattini, 2008, 2013] revisited the relativistic thermodynamics with spin: adding a rotation term into the density operator for a rotating gas system in local equilibrium:

\[ \hat{\rho} = \frac{1}{Z} \exp \left[ -\frac{\hat{H}}{T} + \mu \hat{Q} + [\mathbf{\omega}] \right] \]

→ distribution function → Spin tensor → Pauli-Lubanski vector → Polarization 4-vector

→ The \( \Lambda \) polarization (3-)vector in CM frame:

\[ \Pi(p) = \frac{\hbar \varepsilon}{8m} \int dV n_F(x, p)(\nabla \times \beta) + \frac{\hbar p}{8m} \times \left( \frac{\partial_t \beta + \nabla \beta^0}{\int dV n_F(x, p)} \right) \]

where \( \beta^\mu(x) = [1 / T(x)] u^\mu(x) \) is the inverse temperature four-vector field. Then thermal vorticity is \( \omega = \nabla \times \beta \).

In experiments, the polarization is measured in particle’s rest frame---- Lorentz-boosting:

\[ \Pi_0(p) = \Pi(p) - \frac{p}{p^0(p^0 + m)} \Pi(p) \cdot p \; . \]

In experiments, the polarization is measured globally-----Integrating the y component of polarization \( \Pi_{0y}(p) \) over momentum space, to obtain the global polarization:

\[ < \Pi_{0y} >_p = \frac{\int dpdx \Pi_{0y}(p, x)n_F(x, p)}{\int dpdx n_F(x, p)} = \frac{\int dp \Pi_{0y}(p)n_F(p)}{\int dp n_F(p)} \]

Polarization vector

- Lambdas are “self-analyzing”: Reveal polarization by preferentially emitting daughter proton in the spin direction.
- In experiments, we measure the emitted protons’ angular distribution in Λ’s frame to obtain the Lambda’s polarization.

For an ensemble of Λs with polarization $\vec{P}$:

$$\frac{dW}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \vec{P} \cdot \hat{p}_p) = \frac{1}{4\pi}(1 + \alpha P \cos \theta^*)$$

$\alpha = 0.642$ [measured]

$\hat{p}_p^*$ is daughter proton momentum direction in Λ frame

$0 < |\vec{P}| < 1$: \[ \vec{P} = \frac{3}{\alpha} \hat{p}_p^* \]
Results: Y Component

- The y component is dominant, up to ~20%, as we can compare it with x and z components later.
- Opposite directions. Result into a relatively small value of global polarization.

Fig. 6  The first (left) and second (right) term of the dominant y component of the $\Lambda$ polarization for momentum vectors in the transverse plane at $p_z = 0$, for the FAIR U+U reaction at 8.0 GeV at time $t = 2.5+4.75$ fm/c.
Results: X and Z components

Fig. 7  The first (left) and second (right) terms of the x(up) and y(down) components of the $\Lambda$ polarization for momentum vectors in the transverse plane at $p_z = 0$, for the FAIR U+U reaction at 8.0 GeV

1. Small magnitude.
2. Anti-symmetric

ECHO-QGP numerical code, implementing relativistic dissipative hydrodynamics in the causal Israel-Stewart framework in 3+1 dimensions with an initial Bjorken flow profile

Results: Polarization at FAIR

Fig. 8 The y component (left) of polarization vector in center of mass frame and Λ’s rest frame. The right sub-figure are the modulus of the polarization in Λ’s rest frame. At FAIR 8.0 GeV at time 2.5+4.75 fm/c.

- The modulus of polarization is very similar with the y component of polarization, both in magnitude and the structure. I. e. the other x and z components do not contribute to the polarization, which is in line with previous observations in this work and other papers.

The similarity of the magnitude and structure between $\Pi_{0y}$ and $\Pi_0$ here is even more obvious than that in previous figures.

I.e. The contribution from x and z components is even smaller in earlier time.
Results: RHIC Au+Au Collisions

- Recently, RHIC BES program has measured the global polarization at energies $\sqrt{s} = 7.7, 11.5, 14.5, 19.6, 27.0, 39.0, 62.4, \text{ and } 200 \text{ GeV}$.

Thus we perform the simulation and calculation for these energies.

- Transverse Momentum Analysis: Similar behavior as before: The Structure and magnitude between $\Pi_{0y}$ and $|\Pi_0|$ are similar. $y$ directed component dominates the polarization.

$< \Pi_{0y} >_p = \frac{\int dp dxd\Pi_{0y}(p,x)n_F(x,p)}{\int dp dxdn_F(x,p)} = \frac{\int dp \Pi_{0y}(p)n_F(p)}{\int dp n_F(p)}$

Fig. 10 The $y$ component (left) and the modulus (right) of the polarization for momentum vectors in the transverse plane at $p_z = 0$, for the RHIC Au+Au collisions at 11.5 GeV, at time $t=2.5+4.75 \text{ fm/c}$. The figure is in the $\Lambda$’s rest frame.
Results: Centrality dependence

- Since the y component is dominant, we integrate the y component of polarization $\Pi_{0y}$ over momentum space, to get the global polarization: $\langle \Pi_{0y} \rangle_p = \frac{\int dp dx \Pi_{0y}(p,x) n_F(x,p)}{\int dp dx n_F(x,p)}$.

- Perform some dependence analysis for obtained global polarization: Centrality dependence, Energy dependence, Time evolution.

- This linear dependence of global polarization on impact parameter clearly shows that the polarization arises from initial angular momentum.

- At the higher energy, the central and semi-central collision have vanishing polarization. This is why in 200GeV, the polarization magnitude is closed to zero.
Results: Energy dependence

The polarization magnitude and the tendency of our results are very similar with the results from RHIC’s Beam Energy Scan program.

The polarization decreases with increasing collision energy. What is the reason?
Results: Energy dependence

Interpretations of the energy dependence behavior:

1. Drastic thermal motion of particles in higher temperature/energy.

\[ \frac{dP}{dt} \equiv \tau_q = -\frac{1}{\tau_q} \frac{(1 - P^2)\pi \mu_p}{E(E + m) - P\pi \mu_p} \]

Mean Free Path \( \propto \) Temperature

[ P. Huovinen, X.N. Wang et al., PRC 84,054910 (2011). ]

2. The vorticity decreases with the increased energy

3. Initial State: tilted more for low energy, Larger \( \omega_{xz} \)

4. Freeze out: Longer hydro evolution time for larger \( \sqrt{s} \)


Why is the $\Lambda$ polarization larger than anti-$\Lambda$'s polarization, in RHIC BES program?

1. Polarization induced by magnetic field might split the vorticity induced polarization?

2. Effect of baryon chemical potential: accounts for only 1%

$$n_F(x, p) = \frac{1}{e^{\beta(p\mu_+\mu^-)} + 1}$$

R.H. Fang, et al., PRC 94 (2016) 024904;
M.A. Lisa, invited talk in WPCF, Budapest 2017

3. Axial Anomaly Charge: The same for $\Lambda$s and anti-$\Lambda$s. But $N_\Lambda > N_{\bar{\Lambda}}$

A. Sorin, O. Teryaev, PRC 94 011902(R) (2017)
Outlook

• What is the relation between the final hyperon polarization and the quark’s Chiral Vortical Effect (CVE)? Can we predicate the other one when we know one of them?  A. Sorin, O. Teryaev, PRC 94 011902(R) (2017)

\[ \xi_5 = \frac{1}{6} T^2 + \frac{1}{2\pi^2} \left( \mu^2 + \mu_2^2 \right) \]

[X.N. Wang, et al., PRL 109, 232301 (2012)]

• What is the full relativistic fluid dynamics for particle system with spin? In Becattini’s deduction of polarization 4-vector, he used mainly the spin thermodynamics, but also a little fluid dynamics. Then what is the full spin hydrodynamics. Nice attempt: [W. Florkowski, et al., arxive:1705.00587.]
The y component of polarization dominates the global polarization.

The polarization in our model linearly depends on impact parameter, which clearly shows the polarization arises from initial angular momentum.

The polarization decreases very fast with increased energies, and this can be attributed to drastic thermal motion of particles in higher temperature/energy. Or other effects.

We hope the future RHIC BES II program can provide a higher centrality resolution, to determine precisely the centrality dependence of polarization.
Thank You
Results: Time evolution

At earlier time (0-5 fm/c), the polarization is increasing. However, it is not possible to have $\Lambda$ at this time.

From time 5 fm/c, the polarization is decreasing with the time. After 10 fm/c, the polarization even goes to negative value, which shows the limitation of hydro model in the later stage of collisions, due to the large surface to volume ratio.

Fig. 13  The time evolution of global polarization for energy $\sqrt{s} = 11.5$, 27, and 62.4 GeV.

New Initial State

- Nowadays, particle physicists are more and more concerning the initial state, because the final state is very sensitive to the initial state configuration.

Aim: Compact Initial State with Shear
Main Features: Steak by Steak collisions, Bjorken expansion, Matching, Propagation(mapping)

New Initial State
**Configuration**

Fig. 1. Sketch of peripheral heavy ion collisions at high energy. The $\Lambda$ polarization points essentially into the direction of the total angular momentum $(-y)$ of the interaction region, it is orthogonal to the reaction plane.

Fig. 2  Due to the azimuthal symmetry the rotation is in the reaction plane. The radial component of momentum is chosen to be $p_x$ only.

Fig.3  The Shear in transvers plane, and the collective flow viewed in different directions.
Equation of State

- Both in the initial state and subsequent CFD simulation, the frequently used bag model equation of state (EoS) was applied: \( P = c_0^2 e^2 - \frac{4}{3}B \), with constant \( c_0^2 = 1/3 \) and a fixed bag constant \( B \).

- The energy density takes the form \( e = \alpha T^4 + \beta T^2 + \gamma + B \), where \( \alpha, \beta, \gamma \) are constants arising from the degeneracy factors for (anti)quarks and gluons.
3 Results: Polarization at NICA

- Similarity between $y$ component and modulus of Polarization, in magnitude and structure.
- Similarity between NICA and FAIR’s polarization results.
- The net polarization is still negative, which means the first term is larger than the second term, at this time.

Fig. 9 The $y$ component (left) and the modulus (right) of the polarization for momentum vectors in the transverse plane at $p_z = 0$, for the NICA $Au+Au$ reaction at 9.3 GeV. The figure is in the $\Lambda$’s rest frame.