Fluid dynamics of out of equilibrium boost invariant plasmas

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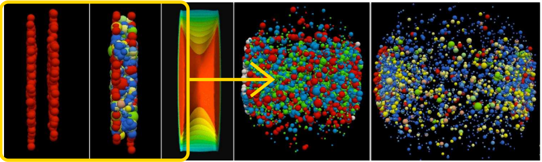
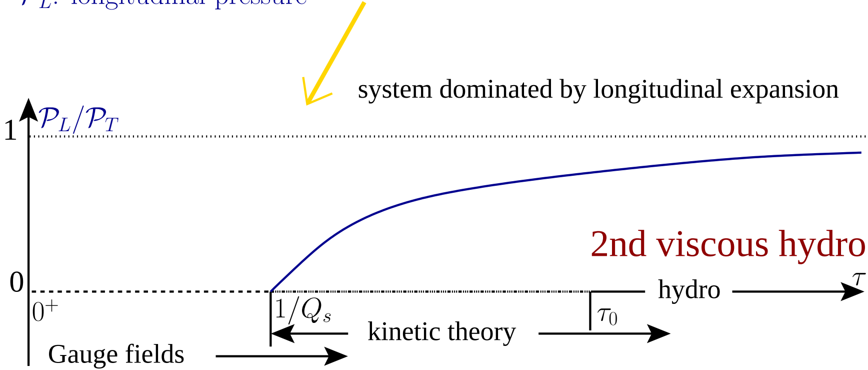
1712.03856, 1703.10694, work in progress, with Jean-Paul Blaizot

Out-of-equilibrium QGP system in heavy-ion collisions

$\mathcal{P}_T$: transverse pressure

$\mathcal{P}_L$: longitudinal pressure





- We consider Bjorken symmetry ($z-$boost and $\vec{x}_\perp-$translational)

Beyond pressure anisotropy: $\mathcal{L}$-moment

p^2-moment weighted with Legendre Polynomial P_{2n}:

$$\mathcal{L}_n = \int \frac{d^3p}{(2\pi)^3 p^0} p^2 P_{2n}(p_z/p_\perp) f(\tau, \vec{p}_\perp, p_z),$$

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- Mass dimension of $\mathcal{L}_n$ is same as $T^{\mu\nu}$.
- $n = 0 \quad \Leftrightarrow \quad$ energy density: $\mathcal{L}_0 = \mathcal{E}$.
- $n = 1 \quad \Leftrightarrow \quad$ pressure anisotropy: $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$.
- $n \geq 2 \quad \Leftrightarrow \quad$ finer structure of f (or δf).

Equation of motion for $\mathcal{L}_n$

Transport equation with relaxation time approximation :

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p},\tau) = -\frac{f(\mathbf{p},\tau) - f_{\mathrm{eq}}(p/T)}{\tau_R}, \qquad \tau_R = \tau_R(T)$$

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which is equivalent to

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}), \quad n = 0, 1, ...$$

- a_n, b_n and c_n are constant coefficients.

$$a_0 = \frac{4}{3}, \quad a_1 = \frac{38}{21}, \quad \ldots$$

- τ_R/τ defines Knudsen number $\Leftrightarrow$ How far a system is away from equilibrium

Truncation of the coupled equations

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R}(1-\delta_{n0}) \quad n = 0, 1, ...$$

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- at $n = 0$

$$\frac{\partial \mathcal{E}}{\partial \tau} + \frac{4}{3}\frac{\mathcal{E}}{\tau} = 0 \quad \rightarrow \quad \mathcal{E} \sim \tau^{-4/3} \qquad \text{ideal hydro}$$

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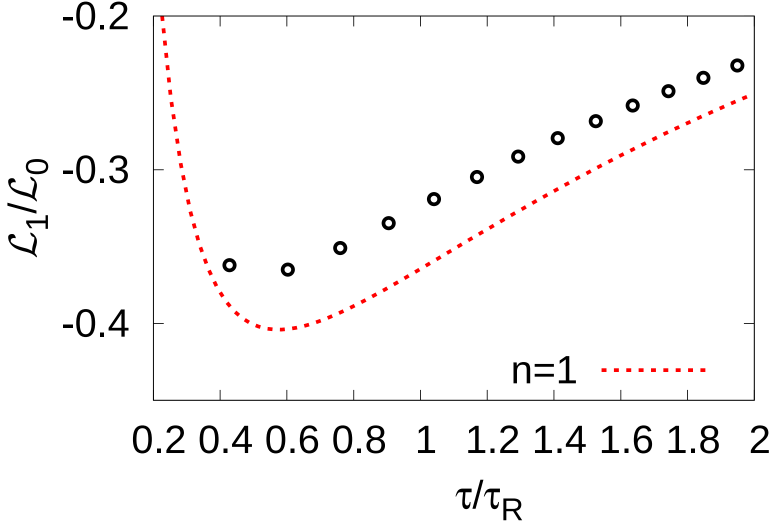
- at higher orders ...

Convergence of truncation



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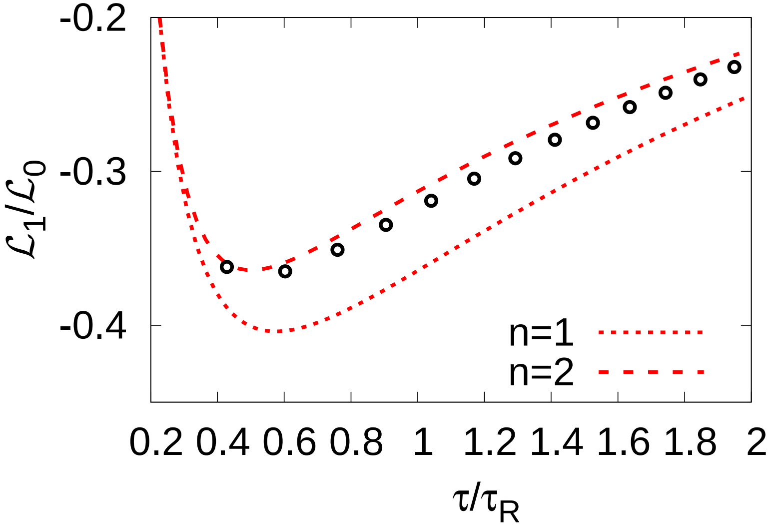

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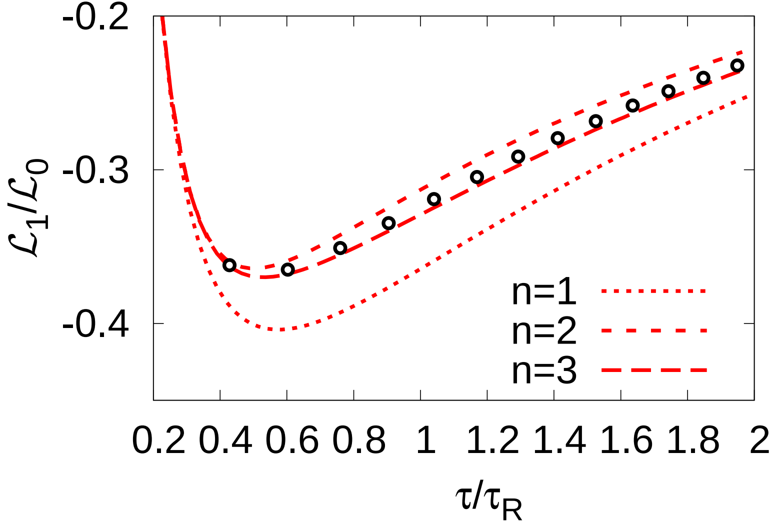

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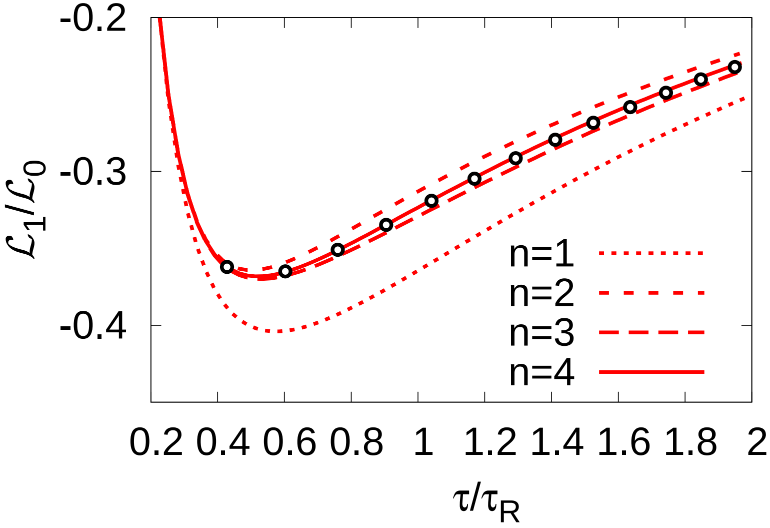

The free-streaming fixed points: $\tau/\tau_R \to 0$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right]$$

For infinite n:

- $\mathcal{L}_0 = \mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \ldots$

$$\Rightarrow \mathcal{L}_n = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right)^2 \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = -2$$

- $\mathcal{L}_n(\tau) = P_{2n}(0)\mathcal{L}_0(\tau)$,

$$\Rightarrow \mathcal{L}_n(\tau) = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right) \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = -1$$

For finite n,

$$\begin{pmatrix} a_0 & c_0 & 0 & 0 & \ldots \\ b_1 & a_1 & c_1 & 0 & \ldots \\ 0 & b_2 & a_2 & c_2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix} \Rightarrow \qquad \approx -2(\text{unstable}) \text{ and } \approx -1(\text{stable})$$

The hydro fixed points: $\tau/\tau_R \to \infty$

Hydro EoM via truncations

- Truncation at $n = 1$ gives 2nd order viscous hydro: $(c_0\mathcal{L}_1 = \Pi = \Pi^\xi_\xi)$

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_1\mathcal{L}_1) \quad \to \quad \partial_\tau \mathcal{E} + \frac{\mathcal{E} + \mathcal{P}_L}{\tau} = 0$$

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_1\mathcal{L}_0) - \frac{1}{\tau_R}\mathcal{L}_1 \quad \to \quad \Pi = -\eta\sigma - a_1\frac{\tau_R}{\tau}\Pi - \tau_R\partial_\tau\Pi\,.$$

* Note $a_1 = 38/21 = \beta_{\pi\pi}$ is 2nd transport coefficient in DNMR.
Denicol, Niemi, Molnar and Rischke

* Also compatible with BRSSS with respect to conformal symmetry:
Baier, Romatschke, Son, Starinets and Stephanov

$$\Pi = -\eta\sigma - \underbrace{\tau_R}_{\tau_\pi}\left[\partial_\tau\Pi + \frac{4}{3}\Pi\nabla\cdot u\right] + \tau_R\underbrace{\left(a_1 - \frac{4}{3}\right)\frac{3}{4}}_{\frac{\lambda_1}{\eta\tau_\pi}}\frac{\Pi^2}{\eta}\,,$$

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- A systematic sway to derive viscous hydro of any order (*Bjorken flow*) !

The hydro fixed points: $\tau/\tau_R \to \infty$

Ansatz form of gradient expansion

$$\mathcal{L}_n = \sum_{m=0} \frac{\alpha_m^{(n)}}{\tau^n}$$

asymptotic decay rate determined by the leading term: $\mathcal{L}_n \sim \alpha_n^{(n)}/\tau$

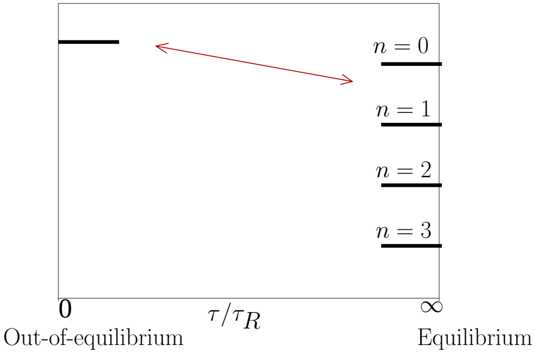
$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+2n}{3} \quad (\tau_R \propto 1/T)$$

$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+3n}{3} \quad (\tau_R \text{ constant})$$

These are stable fixed points in the hydro regime.

Fixed points and hydro attractor

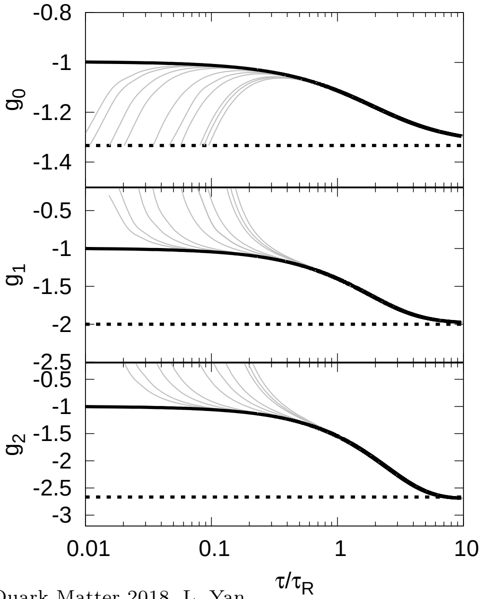
Define: $g_n = \tau \partial_\tau \ln \mathcal{L}_n$



- Free-streaming limit: fixed points of all g_n degenerate ≈ -1.
- Hydro limit: fixed points of g_n split according to n.
- System evolves between these two types of fixed points $\Rightarrow$ *attractor*.

e.g., ideal hydro is a trivial attractor solution: $g_0 = \text{const.} = -4/3$

Remarks on attractor solution



- Attractor solution exists, with or without conformal symmetry, beyond Bjorken symmetry.
  P. Romatschke, M. Martinez, M. Strickland, G. Denicol, ...
- Non-hydro modes decay exponentially, w.r.t. attractor solutions. Quasi-normal modes.
  M. Heller, M. Spalinski, P. Romatschke, A. Kurkela, U. Wiedemann, ...
- Attractor corresponds to Borel-summation of hydro gradient expansion.
  M.Heller, M. Spalinski, R. Janik, P. Witaszczky, G. Basar, G. Dunne, ...

Renormalization of η/s

Effects from higher order moments/viscous hydro (leading order):

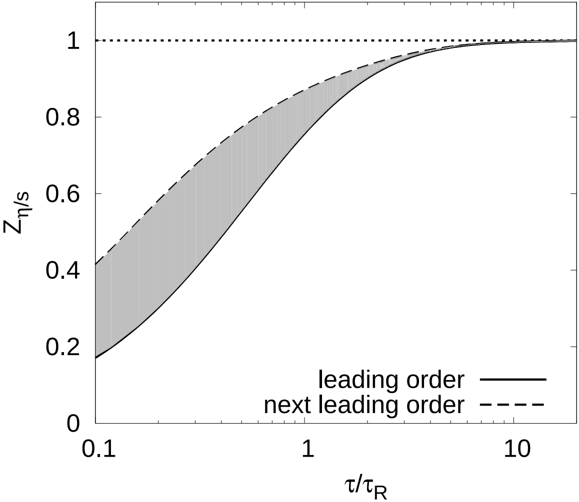
$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_0\mathcal{L}_1)\,,$$

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_0\mathcal{L}_0) - \underbrace{\left[1 + \frac{c_1\tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}\right]}_{Z^{-1}_{\eta/s}}\frac{\mathcal{L}_1}{\tau_R} \qquad \text{(2nd hydro)}\,,$$

$$g_2(\tau/\tau_R) = -a_2 - b_2\frac{\mathcal{L}_2}{\mathcal{L}_1} - \frac{\tau}{\tau_R}\,.$$

- Taking attractor solution for g_2: Borel-resummed gradients.
- Off-equilibrium effects w.r.t. 2nd order hydro $\Leftrightarrow$ renormalized η/s !

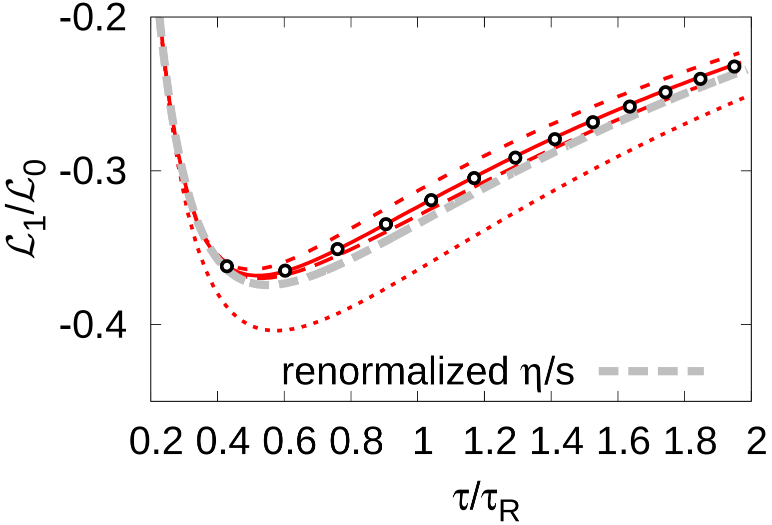
Renormalization of η/s



Out-of-equilibrium physics can be effectively absorbed into a reduced η/s.

E. Shuryak, M. Lublinsky, P. Romatschke

Numerical test of η/s renormalization



2nd order viscous hydro using renormalized η/s

Summary

- $\mathcal{L}$-moments are proposed to quantify system thermalization.
- Coupled equations for $\mathcal{L}_n$ are derived, with valid truncations.

$\Rightarrow$ fluid dynamics for out-of-equilibrium system

- Attractor solution smoothly connect fixed points of $\mathcal{L}_n$ in two limits.
- The system in heavy-ion collisions could be out-of-equilibrium:

Hydro could start much earlier in heavy-ion collisions.

Physical value of $\eta/s >$ phenomenological expectations $\sim O(1/4\pi)$.

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$\Rightarrow$ strongly coupled QGP?

Back-up slides

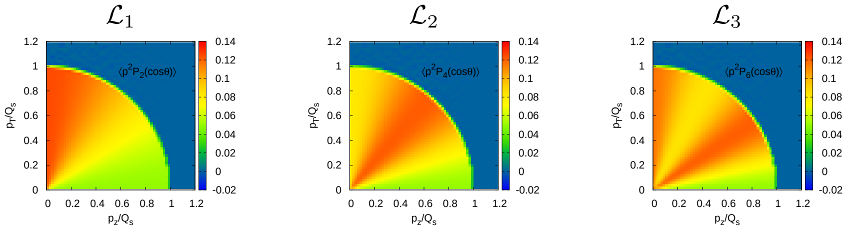
- Truncation at $n = 2$ gives 3rd order viscous hydro: $(c_0\mathcal{L}_2 = \Sigma = \Sigma^\xi_\xi)$

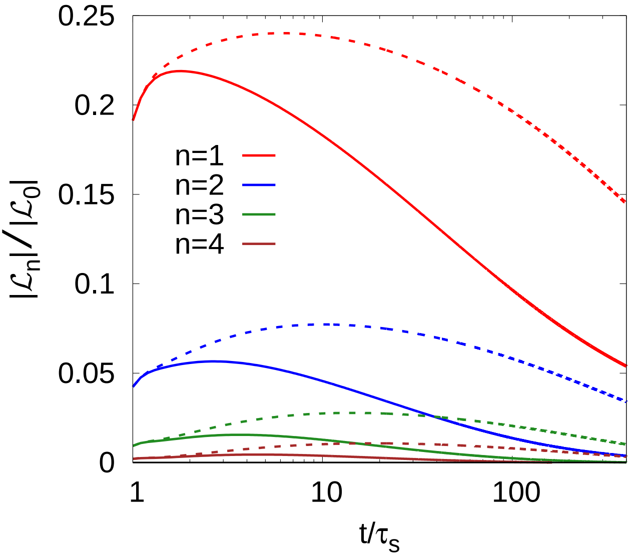
$$\Pi = -\eta\sigma - \tau_\pi\left[\partial_\tau\Pi + \frac{4}{3}\Pi\nabla\cdot u\right] + \underbrace{\tau_\pi\left(a_1 - \frac{4}{3}\right)\frac{3}{4}}_{\frac{\lambda_1}{\eta\tau_\pi}}\frac{\Pi^2}{\eta} + \underbrace{\frac{3c_1\tau_\pi}{4}}_{\sim\text{ 3rd order transport}}\frac{\Sigma\Pi}{\eta}$$

A. Jaiswal (2013)

$$\Sigma = \frac{\lambda_1 + \eta\tau_\pi}{2\eta^2}\Pi^2 - \tau_\pi\left(\partial_\tau\Sigma + \frac{4}{3}\Sigma\nabla\cdot u\right) + \underbrace{\tau_\pi\left(a_2 - \frac{4}{3}\right)\frac{3}{4}}_{\sim\text{ 3rd order transport}}\frac{\Sigma\Pi}{\eta}$$

- ...


$\mathcal{L}_1$
$\langle p^2 P_2(\cos\theta)\rangle$
p_T/Q_s
p_z/Q_s
$\mathcal{L}_2$
$\langle p^2 P_4(\cos\theta)\rangle$
p_T/Q_s
p_z/Q_s
$\mathcal{L}_3$
$\langle p^2 P_6(\cos\theta)\rangle$
p_T/Q_s
p_z/Q_s



Solved by kinetic theory with respect to QCD 2-to-2 scattering.