

Fluid dynamics of out of equilibrium boost invariant plasmas

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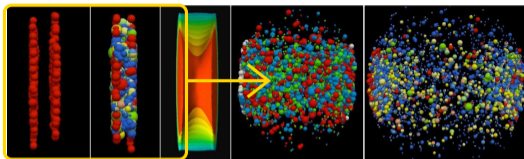


Quark Matter 2018, Venice, Italy

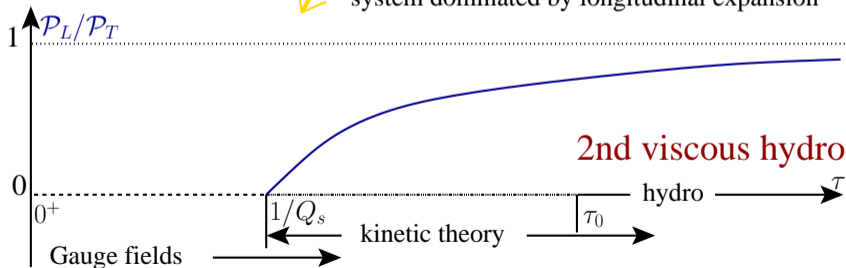


Out-of-equilibrium QGP system in heavy-ion collisions

\mathcal{P}_T : transverse pressure
 \mathcal{P}_L : longitudinal pressure



system dominated by longitudinal expansion



Beyond pressure anisotropy: \mathcal{L} -moment

p^2 -moment weighted with Legendre Polynomial P_{2n} :

$$\mathcal{L}_n = \int \frac{d^3p}{(2\pi)^3 p^0} p^2 P_{2n}(p_z/p_\perp) f(\tau, \vec{p}_\perp, p_z),$$

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- Mass dimension of \mathcal{L}_n is same as $T^{\mu\nu}$.
- $n = 0 \Leftrightarrow$ energy density: $\mathcal{L}_0 = \mathcal{E}$.
- $n = 1 \Leftrightarrow$ pressure anisotropy: $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$.
- $n \geq 2 \Leftrightarrow$ finer structure of f (or δf).

Equation of motion for \mathcal{L}_n

Transport equation with relaxation time approximation :

$$\left[\partial_\tau - \frac{p_z}{\tau} \partial_{p_z} \right] f(\mathbf{p}, \tau) = - \frac{f(\mathbf{p}, \tau) - f_{\text{eq}}(p/T)}{\tau_R}, \quad \tau_R = \tau_R(T) \sim \frac{\eta}{s}$$

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which leads to

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = - \frac{1}{\tau} [a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}] - \frac{\mathcal{L}_n}{\tau_R} (1 - \delta_{n0}), \quad n = 0, 1, \dots$$

- a_n , b_n and c_n are constant coefficients.

$$a_0 = \frac{4}{3}, \quad a_1 = \frac{38}{21}, \quad \dots$$

- τ_R/τ defines Knudsen number \Leftrightarrow How far a system is away from equilibrium

Truncation of the coupled equations

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- at $n = 0$

$$\frac{\partial \mathcal{E}}{\partial \tau} + \frac{4}{3} \frac{\mathcal{E}}{\tau} = 0 \quad \rightarrow \quad \mathcal{E} \sim \tau^{-4/3} \quad \text{ideal hydro}$$

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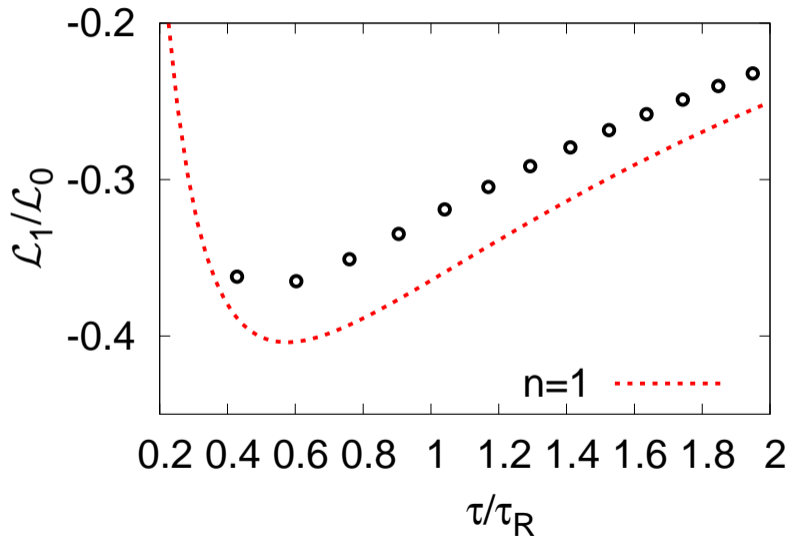
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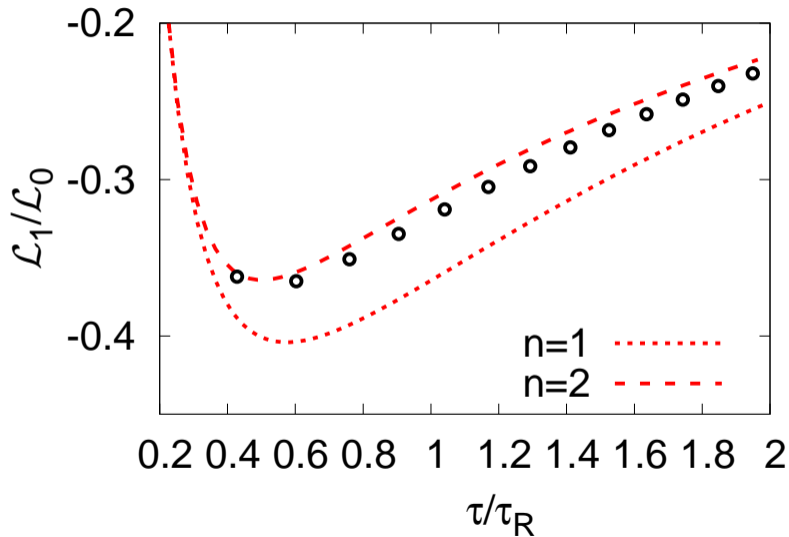
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- at higher orders ...

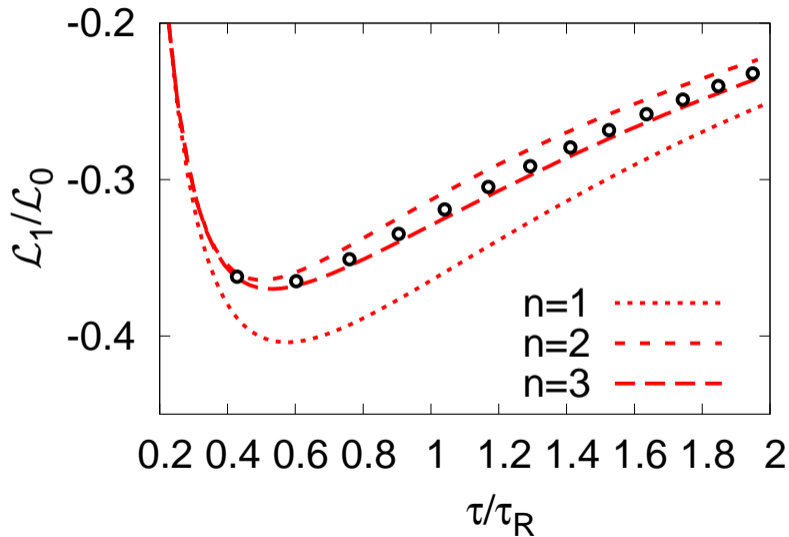
Convergence of truncation



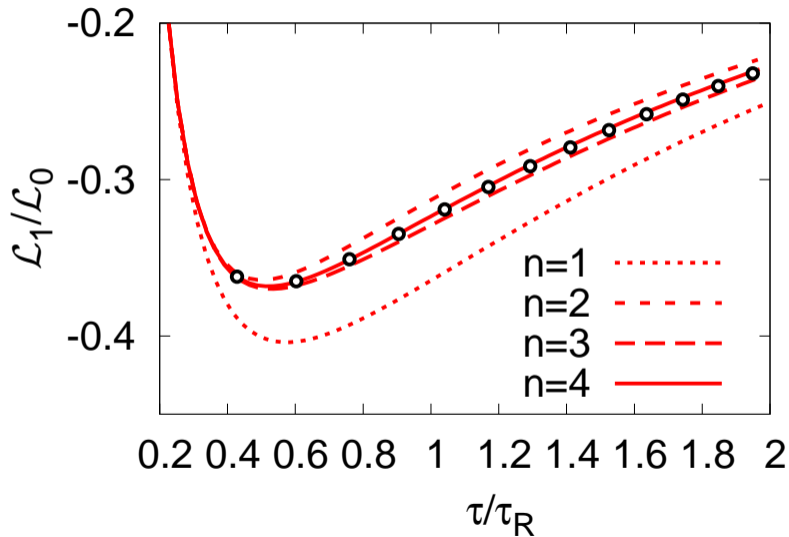
Convergence of truncation



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Convergence of truncation



The free-streaming fixed points: $\tau/\tau_R \rightarrow 0$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau} [a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}]$$

For infinite n :

- $\mathcal{L}_0 = \mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \dots$

$$\Rightarrow \mathcal{L}_n = \mathcal{L}_n(\tau_0) \left(\frac{\tau_0}{\tau}\right)^2 \quad \rightarrow \tau \partial_\tau \ln \mathcal{L}_n = -2$$

- $\mathcal{L}_n(\tau) = P_{2n}(0) \mathcal{L}_0(\tau),$

$$\Rightarrow \mathcal{L}_n(\tau) = \mathcal{L}_n(\tau_0) \left(\frac{\tau_0}{\tau}\right) \quad \rightarrow \tau \partial_\tau \ln \mathcal{L}_n = -1$$

For finite n ,

$$\begin{pmatrix} a_0 & c_0 & 0 & 0 & \dots \\ b_1 & a_1 & c_1 & 0 & \dots \\ 0 & b_2 & a_2 & c_2 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \Rightarrow \quad \approx -2(\text{unstable}) \text{ and } \approx -1(\text{stable})$$

The hydro fixed points: $\tau/\tau_R \rightarrow \infty$

Hydro EoM via truncations

- Truncation at $n = 1$ gives 2nd order viscous hydro: ($c_0 \mathcal{L}_1 = \Pi = \Pi_\xi^\xi$)

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0 \mathcal{L}_0 + c_1 \mathcal{L}_1) \quad \rightarrow \quad \partial_\tau \mathcal{E} + \frac{\mathcal{E} + \mathcal{P}_L}{\tau} = 0$$

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1 \mathcal{L}_1 + b_1 \mathcal{L}_0) - \frac{1}{\tau_R} \mathcal{L}_1 \quad \rightarrow \quad \Pi = -\eta\sigma - a_1 \frac{\tau_R}{\tau} \Pi - \tau_R \partial_\tau \Pi.$$

- * Note $a_1 = 38/21 = \beta_{\pi\pi}$ is 2nd transport coefficient in DNMR.

Denicol, Niemi, Molnar and Rischke

- * Also compatible with BRSSS with respect to conformal symmetry:

Baier, Romatschke, Son, Starinets and Stephanov

$$\Pi = -\eta\sigma - \underbrace{\tau_R}_{\tau_\pi} \left[\partial_\tau \Pi + \frac{4}{3} \Pi \nabla \cdot u \right] + \tau_R \underbrace{\left(a_1 - \frac{4}{3} \right)}_{\frac{\lambda_1}{\eta\tau_\pi}} \frac{3}{4} \frac{\Pi^2}{\eta},$$

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- A systematic way to derive viscous hydro of any order (*Bjorken flow*) !

The hydro fixed points: $\tau/\tau_R \rightarrow \infty$

Ansatz form of gradient expansion

$$\mathcal{L}_n = \sum_{m=0} \frac{\alpha_m^{(n)}}{\tau^m}$$

asymptotic decay rate determined by the leading term: $\mathcal{L}_n \sim \alpha_n^{(n)}/\tau$

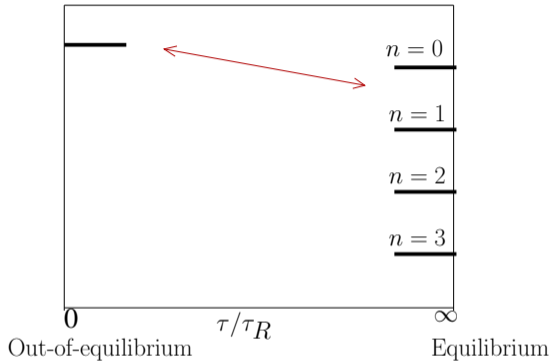
$$\Rightarrow \tau \partial_\tau \ln \mathcal{L}_n = -\frac{4+2n}{3} \quad (\tau_R \propto 1/T)$$

$$\Rightarrow \tau \partial_\tau \ln \mathcal{L}_n = -\frac{4+3n}{3} \quad (\tau_R \text{ constant})$$

These are stable fixed points in the hydro regime.

Fixed points and hydro attractor

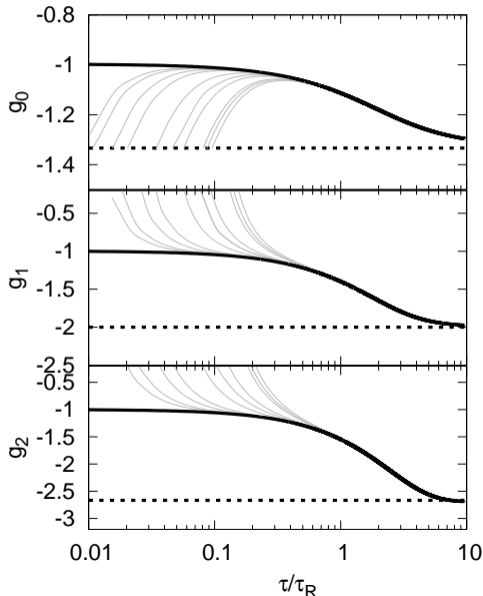
Define: $g_n = \tau \partial_\tau \ln \mathcal{L}_n$



- Free-streaming limit: fixed points of all g_n degenerate ≈ -1 .
- Hydro limit: fixed points of g_n split according to n .
- System evolves between these two types of fixed points \Rightarrow *attractor*.

e.g., ideal hydro is a trivial attractor solution: $g_0 = \text{const.} = -4/3$

Remarks on attractor solution



- Attractor solution exists, with or without conformal symmetry, beyond Bjorken symmetry.

P. Romatschke, M. Martinez, M. Strickland, G. Denicol, ...

- Non-hydro modes decay exponentially, w.r.t. attractor solutions. Quasi-normal modes.

M. Heller, M. Spalinski, P. Romatschke, A. Kurkela, U. Wiedemann, ...

- Attractor corresponds to Borel-summation of hydro gradient expansion.

M.Heller, M. Spalinski, R. Janik, P. Witaszczyk, G. Basar, G. Dunne, ...

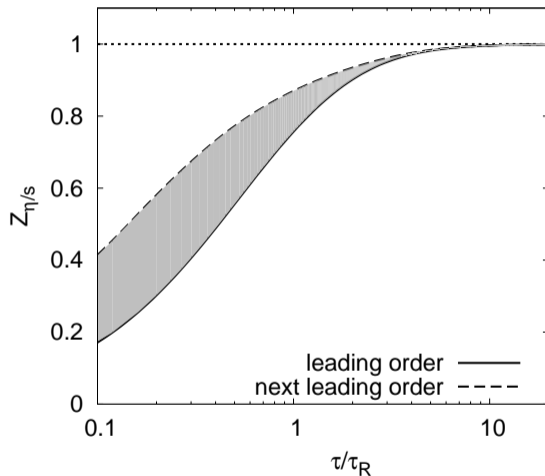
Renormalization of η/s

Effects from higher order moments/viscous hydro (leading order):

$$\begin{aligned}\partial_\tau \mathcal{L}_0 &= -\frac{1}{\tau}(a_0 \mathcal{L}_0 + c_0 \mathcal{L}_1), \\ \partial_\tau \mathcal{L}_1 &= -\frac{1}{\tau}(a_1 \mathcal{L}_1 + b_0 \mathcal{L}_0) - \underbrace{\left[1 + \frac{c_1 \tau_R}{\tau} \frac{\mathcal{L}_2}{\mathcal{L}_1}\right]}_{Z_{\eta/s}^{-1}} \frac{\mathcal{L}_1}{\tau_R} \quad (\text{2nd hydro}), \\ g_2(\tau/\tau_R) &= -a_2 - b_2 \frac{\mathcal{L}_2}{\mathcal{L}_1} - \frac{\tau}{\tau_R}.\end{aligned}$$

- Taking attractor solution for g_2 : Borel-resummed gradients.
- Off-equilibrium effects w.r.t. 2nd order hydro \Leftrightarrow renormalized η/s !

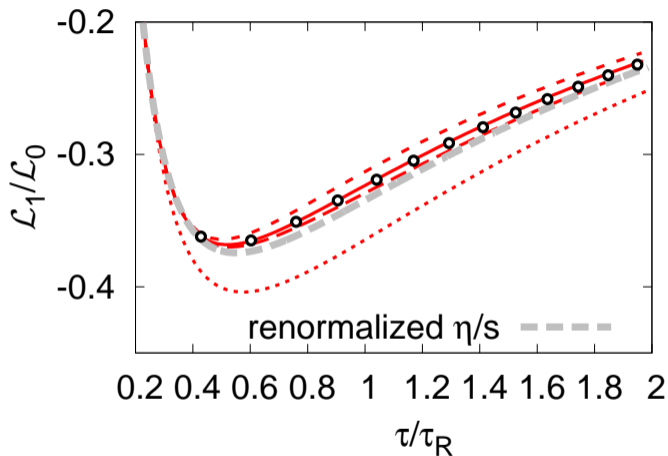
Renormalization of η/s



Out-of-equilibrium physics can be effectively absorbed into a reduced η/s .

E. Shuryak, M. Lublinsky, P. Romatschke

Numerical test of η/s renormalization



2nd order viscous hydro using renormalized η/s

Summary

- \mathcal{L} -moments are proposed to quantify system thermalization.
- Coupled equations for \mathcal{L}_n are derived, with valid truncations.
 \Rightarrow fluid dynamics for out-of-equilibrium system
- Attractor solutions smoothly connect fixed points of \mathcal{L}_n in two limits.
- Hydro can be (or has been) used in out-of-equilibrium systems with renormalized η/s :
 - * Hydro starts much earlier in heavy-ion collisions.
 - * Physical value of $\eta/s >$ phenomenological expectations $\sim O(1/4\pi)$.

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\Rightarrow strongly coupled QGP?

Back-up slides

- Truncation at $n = 2$ gives 3rd order viscous hydro: ($c_0 \mathcal{L}_2 = \Sigma = \Sigma_\xi^\xi$)

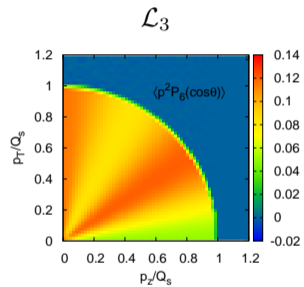
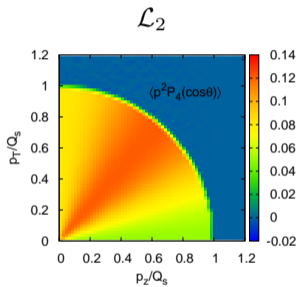
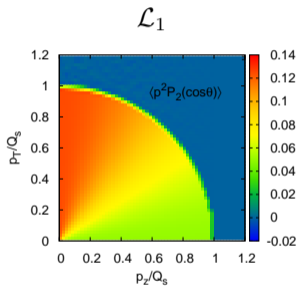
$$\Pi = -\eta\sigma - \tau_\pi \left[\partial_\tau \Pi + \frac{4}{3} \Pi \nabla \cdot u \right] + \underbrace{\tau_\pi \left(a_1 - \frac{4}{3} \right) \frac{3}{4} \frac{\Pi^2}{\eta}}_{\frac{\lambda_1}{\eta\tau_\pi}}$$

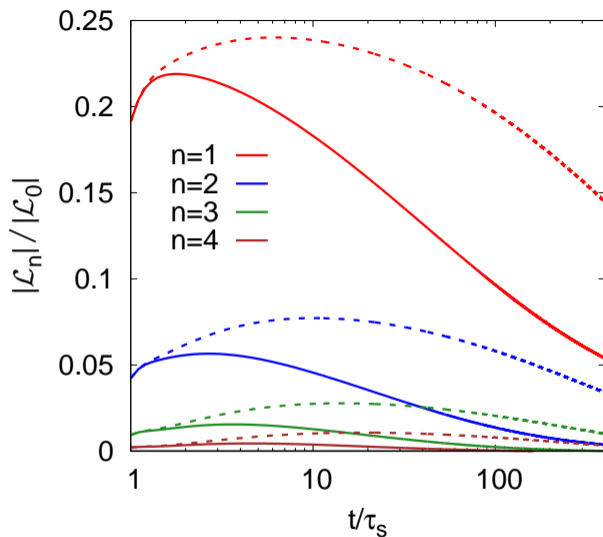
$$+ \underbrace{\frac{3c_1\tau_\pi}{4}}_{\sim \text{3rd order transport}} \frac{\Sigma\Pi}{\eta}$$

A. Jaiswal (2013)

$$\Sigma = \frac{\lambda_1 + \eta\tau_\pi}{2\eta^2} \Pi^2 - \tau_\pi \left(\partial_\tau \Sigma + \frac{4}{3} \Sigma \nabla \cdot u \right) + \underbrace{\tau_\pi \left(a_2 - \frac{4}{3} \right) \frac{3}{4} \frac{\Sigma\Pi}{\eta}}_{\sim \text{3rd order transport}}$$

- ...





Solved by kinetic theory with respect to QCD 2-to-2 scattering.