Collectivity from interference

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Observation of flow-like signatures in pp and pA motivates to revisit fluid dynamical paradigm.

- Fluid dynamics and transport theory invoke final state interactions.

- But final state interactions imply jet quenching, which is not seen in pp and pA.

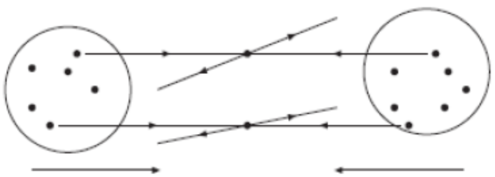
$\Rightarrow$ Either, jet quenching exists in pp & pA but effects are too small to be observed so far,

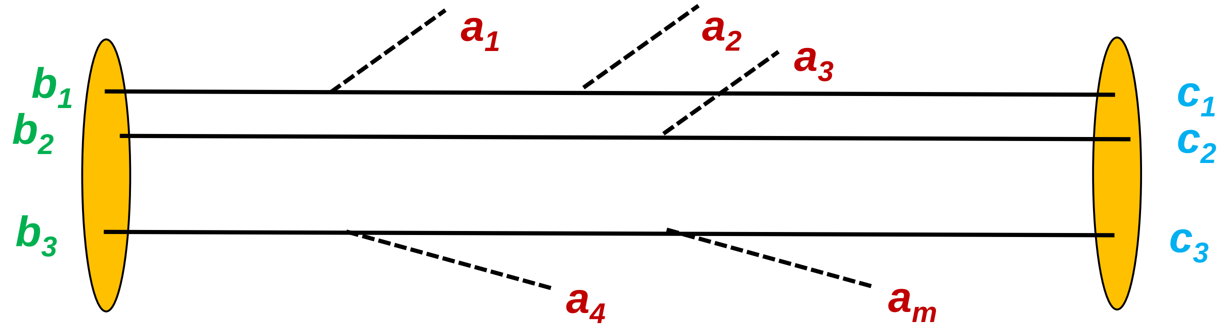
$\Rightarrow$ Or collectivity can build up without final state interactions. How? By quantum interference?

This question motivates the present study.

A simplified model of multi-parton production

- *Schematic picture: pp collision = multiple parton-parton interactions at positions y_i.*

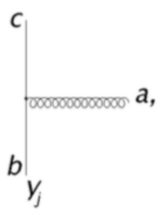




Source lines start (end) with colors b_i (c_j) at rapidity of 1st (2nd) hadron.

- *Diagrammatic rules: gluon emission keeps track of **color** and **phases** exactly.*

(basis for understanding QCD interference effects)

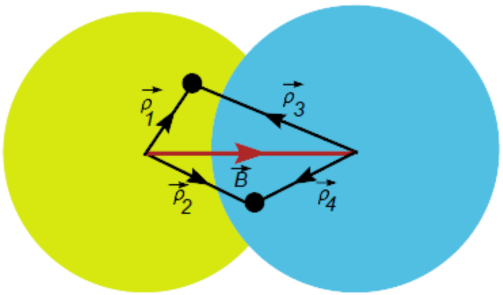


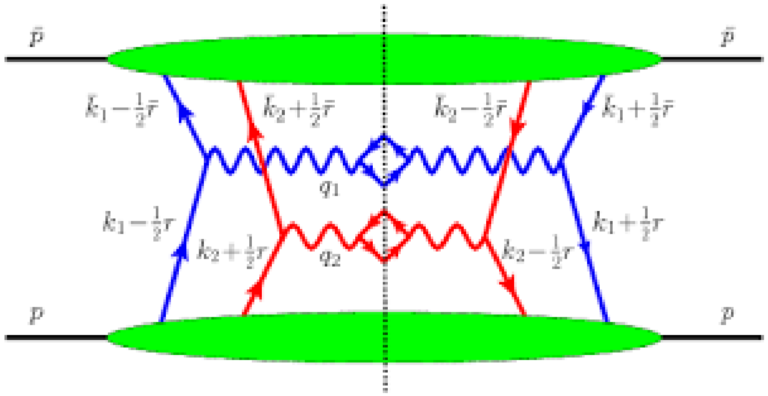
$$= \boxed{T^a_{b_i c_i}} \int d\mathbf{x}\, \vec{f}(\mathbf{x}-\mathbf{y}) \boxed{e^{i\,\mathbf{k}.\mathbf{x}}} \equiv T^a_{b_i c_i} \boxed{\vec{f}(\mathbf{k})} \exp\left[i\,\mathbf{y}.\mathbf{k}\right]$$

- *Simplifications (to make calculation of m-particle emission possible)*
  - *Don't specify **kinematics**.*
  - *Flat rapidity dependence of f(k).*
  - *Gluons do not cross.*

Set-up without final state interactions and without initial state density effects allows for calculation of m-particle interference and higher order cumulants.

Basic ideas about MPI geometry





$$\sigma_{2\,MPIs} = \frac{\sigma_1\,\sigma_2}{\sigma_{\rm eff}}$$

$$\sigma_{N\,\rm MPI} = \frac{\sigma_1 \ldots . \sigma_N}{K_N}\,.$$

- *Generalized parton distribution functions (GPDs) carry geometrical information*

$$\frac{1}{K_N} = \int \left(\prod_{i=1}^{N} \frac{d\mathbf{\Delta}_i}{(2\pi)^2} \right) \frac{G_N(\{x_i\},\{Q_i^2\},\{\mathbf{\Delta}_i\})\, G_N(\{x_i'\},\{Q_i^2\},\{\mathbf{\Delta}_i\})}{\prod_{i=1}^{N} (f(x_i,Q_i^2)\, f(x_i',Q_i^2))} \delta^{(2)} \left(\sum_{i=1}^{N} \mathbf{\Delta}_i \right)$$

- *Probabilistic interpretation of GPDs in a mean-field approximation* **Blok, Dokshitzer, Frankfurt, Strikman, PRD 83, 071501 (2011)**

$$G_N(\{x_i\},\{Q_i^2\},\{\mathbf{\Delta}_i\}) = \prod_{i=1}^{N} G_1(x_i,Q_i,\mathbf{\Delta}_i) = \prod_{i=1}^{N} f(x_i,Q_i) F_{2g}(\mathbf{\Delta}_i)$$

- *Only purpose for the following:* $F_{2g}^2(\Delta) = \exp(-B\Delta_i^2)$

$$\boxed{B = 2\,\mathrm{GeV}^{-2} \quad \longleftrightarrow \quad \sigma_{\rm eff} \approx 20\,\mathrm{mb}}$$

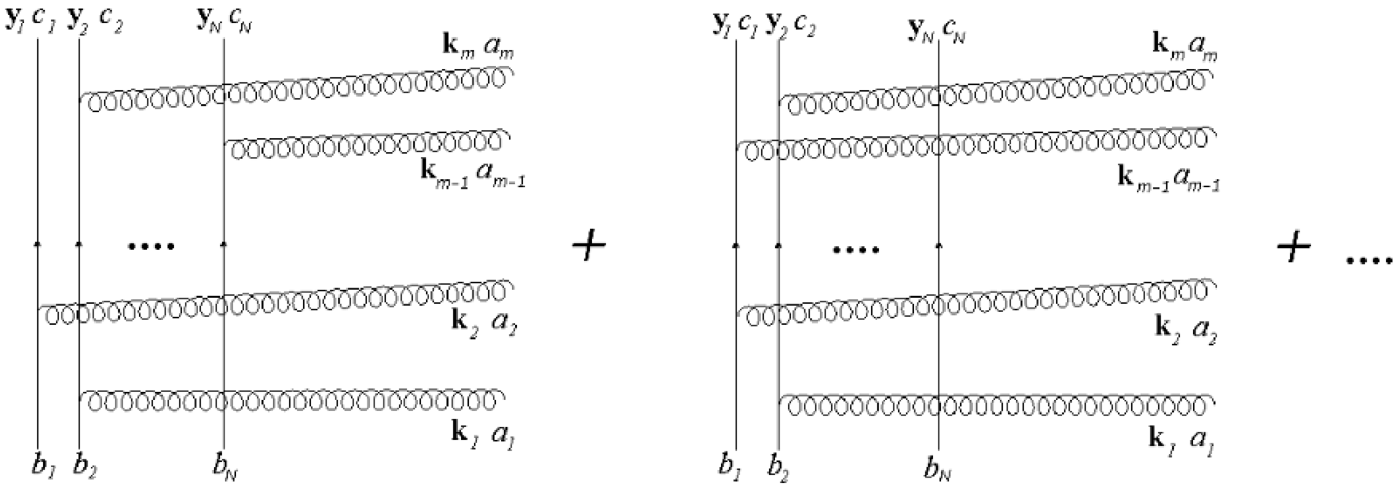
LHC data set scale of parameter B

- *Density distribution of colliding partons in pp*

$$\rho\left(\{\mathbf{y}_i\},\mathbf{b}\right) = \prod_j \frac{1}{(4\pi B)^2} \exp\left[-\frac{\mathbf{y}_j^2}{4B} \right] \exp\left[-\frac{(\mathbf{y}_j - \mathbf{b})^2}{4B} \right]$$

m-gluon emission from N sources: interference effects

- *This model has N^m different m-particle emission amplitudes:*



- *Summing up and squaring these emission amplitudes returns a* ***gluon spectrum for a fixed set of transverse positions y_i.*** *Averaging over transverse positions with a* ***classical weight****, one finds the spectrum*

$$\frac{d\Sigma}{d\mathbf{k}_1 \ldots d\mathbf{k}_m} = \int \left(\prod_{i=1}^{N} d\mathbf{y}_i \right) \boxed{\rho(\{y_i\})}\, \boxed{\hat{\sigma}\left(\{\mathbf{k}_j\}, \{y_i\}\right)}$$

We want to calculate this spectrum and its azimuthal anisotropies $v_n\{2k\}$ for arbitrary m and N.

Inclusive m-gluon cross section from N sources

- *Complete result in large N limit: (contains sums over source doublets, triplets, quadruplets, pairs of doublets, …)*

B. Blok, C.Jäkel, M. Strikman, UAW, JHEP 1712 (2017)

$$
\begin{aligned}
\hat{\sigma} \propto\; & N_c^m \left(N_c^2-1\right)^N \left(\prod_{i=1}^{m} \left|\vec{f}(\mathbf{k}_i)\right|^2\right) N^{m-4} \\
\times \Big\{ & N^4 + F_{\rm corr}^{(2)}(N,m)\frac{N^2}{(N_c^2-1)} \sum_{(ab)}\sum_{(lm)} 2^2 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{lm}) \\
& + F_{\rm corr}^{(3i)}(N,m)\frac{N}{(N_c^2-1)^2} \sum_{(abc)} \sum_{(lm)(mn)(nl)} 2^3 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm}) \\
& \qquad \times \cos(\mathbf{k}_b.\Delta\mathbf{y}_{mn})\cos(\mathbf{k}_c.\Delta\mathbf{y}_{nl}) \\
& + F_{\rm corr}^{(4i)}(N,m)\frac{1}{(N_c^2-1)^2} \sum_{(lm),(no)} \sum_{(ab)(cd)} 2^4 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{lm}) \\
& \qquad \times \cos(\mathbf{k}_c.\Delta\mathbf{y}_{no})\cos(\mathbf{k}_d.\Delta\mathbf{y}_{no}) \\
& + F_{\rm corr}^{(4ii)}(N,m)\frac{1}{(N_c^2-1)^3} \sum_{(lm)(mn)(no)(ol)} \sum_{(abcd)} 2^4 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{mn}) \\
& \qquad \times \cos(\mathbf{k}_c.\Delta\mathbf{y}_{no})\cos(\mathbf{k}_d.\Delta\mathbf{y}_{ol}) \\
& + F_{\rm corr}^{(5)}(N,m)\frac{N^{-1}}{(N_c^2-1)^3} \sum_{[(lm)(mn)(nl)](op)} \sum_{(abc)(de)} 2^2 \cos(\mathbf{k}_d.\Delta\mathbf{y}_{op})\cos(\mathbf{k}_e.\Delta\mathbf{y}_{op}) \\
& \qquad \times 2^3 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{mn})\cos(\mathbf{k}_c.\Delta\mathbf{y}_{nl}) \\
& + F_{\rm corr}^{(6)}(N,m)\frac{N^{-2}}{(N_c^2-1)^3} \sum_{(lm)(no)(pq)} \sum_{(ab)(cd)(ef)} 2^2 \cos(\mathbf{k}_a.\Delta\mathbf{y}_{lm})\cos(\mathbf{k}_b.\Delta\mathbf{y}_{lm}) \\
& \qquad \times 2^2 \cos(\mathbf{k}_c.\Delta\mathbf{y}_{no})\cos(\mathbf{k}_d.\Delta\mathbf{y}_{no})\, 2^2 \cos(\mathbf{k}_e.\Delta\mathbf{y}_{pq})\cos(\mathbf{k}_f.\Delta\mathbf{y}_{pq}) \\
& + O\left(\frac{1}{N}\right) + O\left(\frac{1}{(N_c^2-1)^4}\right) \Big\},
\end{aligned}
$$

From the spectra to v_n's and higher order cumulants

- *Once spectrum is known, azimuthal phase space averages can be formed*

$$T_n(k_1,k_2) = \binom{m}{2} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2\, \exp\left[in(\phi_1-\phi_2)\right] \left(\int \prod_{b=3}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

- *Suitably normalized, these define v_n's (2[nd] order cumulants)*

$$\overline{T}(k_1,k_2) = \binom{m}{2} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2 \left(\int \prod_{b=3}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

$$\boxed{v_n^2\{2\}(k_1,k_2) \equiv \langle\langle e^{in(\phi_1-\phi_2)} \rangle\rangle(k_1,k_2) \equiv \frac{T_n(k_1,k_2)}{\overline{T}(k_1,k_2)}}$$

- *Higher order cumulants obtained in close similarity*

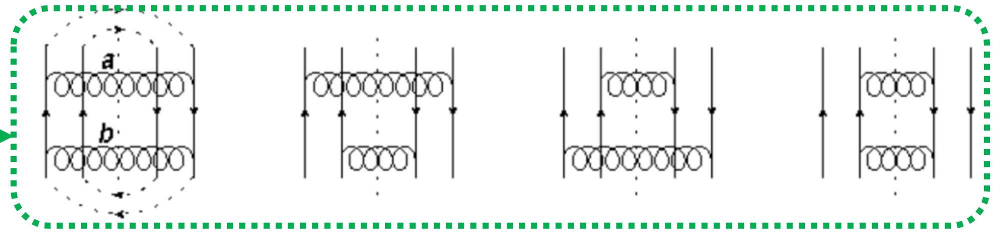
$$S(k_1,k_2,k_3,k_4) = \binom{m}{4} \int_\rho \int_0^{2\pi} d\phi_1\, d\phi_2\, d\phi_3\, d\phi_4\, e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \left(\int \prod_{b=5}^{m} k_b\, dk_b\, d\phi_b \right) \hat{\sigma}$$

$$\begin{aligned}\langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle_c = {} & \langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)} \rangle\rangle \\ & - \langle\langle e^{in(\phi_1-\phi_3)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_4)} \rangle\rangle - \langle\langle e^{in(\phi_1-\phi_4)} \rangle\rangle \langle\langle e^{in(\phi_2-\phi_3)} \rangle\rangle\end{aligned}$$

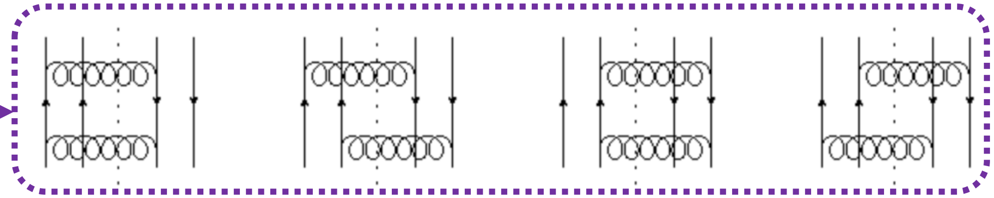
Simplest case: emitting m=2 gluons from N=2 sources

- *Color can be read easily from diagrams*

$$\boxed{\mathrm{Tr}\left[T^a T^b T^b T^a\right]\,\mathrm{Tr}\left[\mathbb{1}\right] = N_c^2\,\left(N_c^2-1\right)^2}$$

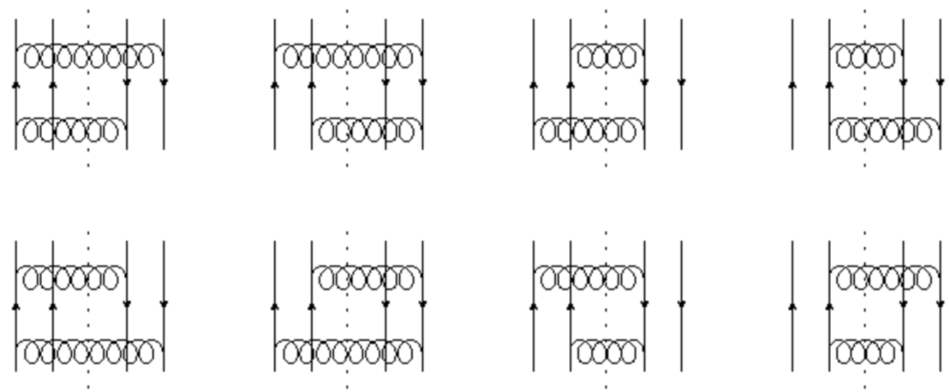


$$\boxed{\mathrm{Tr}\left[T^a T^b\right]\,\mathrm{Tr}\left[T^b T^a\right] = N_c^2\,\left(N_c^2-1\right)}$$



$$\text{phases} \propto e^{i\mathbf{k}.\mathbf{y}_i}\ \left(\propto e^{-i\mathbf{k}.\mathbf{y}_i}\right)$$

in amplitude (complex conj. amplitude)



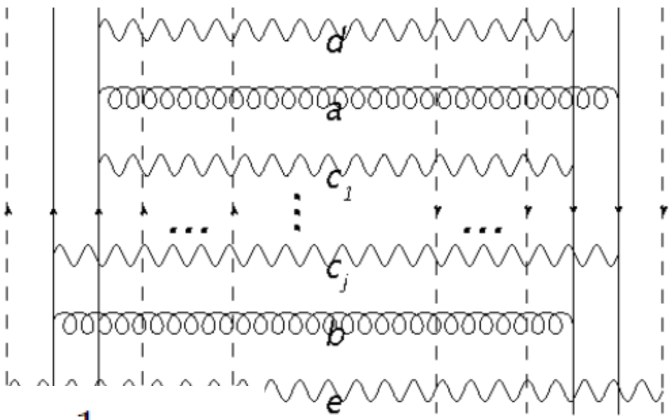
$$\boxed{\frac{d\Sigma}{d\mathbf{k}_1 d\mathbf{k}_2} \propto \left|\vec{f}(\mathbf{k}_1)\right|^2\,\left|\vec{f}(\mathbf{k}_2)\right|^2\left[1+\frac{\left(e^{-B(\mathbf{k}_1+\mathbf{k}_2)^2}+e^{-B(\mathbf{k}_1-\mathbf{k}_2)^2}\right)}{(N_c^2-1)}\right]}$$

- *For $B = 1/Q_s^2$, this QCD agrees with CGC calculations,*

Altinoluk et al, PLB 751 (2015) 448; PLB 752 (2016) 113
Lappi, Schenke, Schlichting, Venugopalan JHEP 1601 (2016) 061

but it does not invoke saturation effects.

Analytical control over effects of diagonal gluons



Using, $T^{c_j}T^aT^{c_j} = \frac{1}{2}N_c T^a$ we can resum contributions from arbitrarily many diagonal gluons in color correction factors

$$\hat{\sigma} \propto (N_c^2-1)^N N_c^m \left(\prod_{i=1}^{m} \left|\vec{f}(\mathbf{k}_i)\right|^2\right)$$
$$\times\left\{N^m + \boxed{F_{\rm corr}^{(2)}(N,m)}\frac{N^{m-2}}{(N_c^2-1)}\sum_{(ab)}\sum_{(ij)} 4\cos\left(\mathbf{k}_a.\Delta\mathbf{y}_{ij}\right)\cos\left(\mathbf{k}_b.\Delta\mathbf{y}_{ij}\right)\right.$$
$$\left. +O\left(\frac{1}{N}\frac{1}{(N_c^2-1)}\right)+O\left(\frac{1}{(N_c^2-1)^2}\right)\right\}.$$

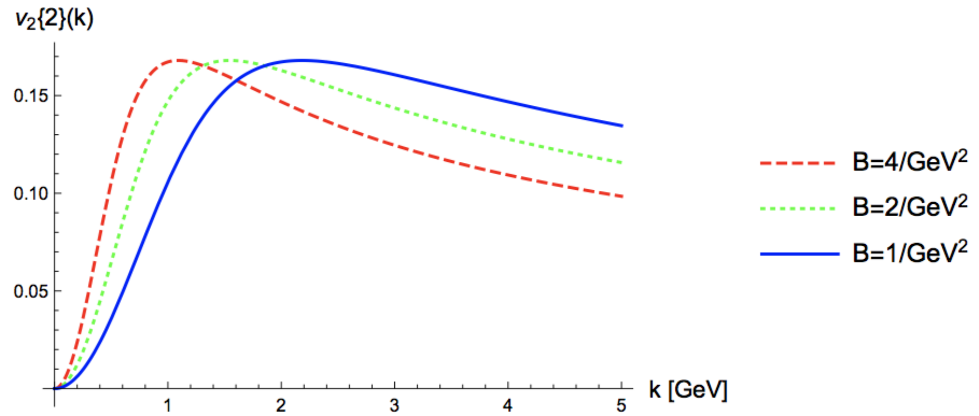
$$\boxed{\begin{aligned} F_{\rm corr}^{(2)}(N,m) &= \frac{1}{\mathcal{N}_{\rm incoh}}\sum_{j=0}^{m-2} N^{m-2-j}(m-1-j)\left(\sum_{l=0}^{j}\binom{j}{l}2^l(N-2)^{j-l}\frac{1}{2^l}\right) \\ &= \frac{2}{m(m-1)}N^{1-m}\left(N(N-1)^m + mN^m - N^{1+m}\right). \end{aligned}}$$

2nd order cumulant: v_2

$$v_2^2\{2\}(k_1,k_2) \equiv \langle\langle e^{i2(\phi_1-\phi_2)}\rangle\rangle(k_1,k_2)$$

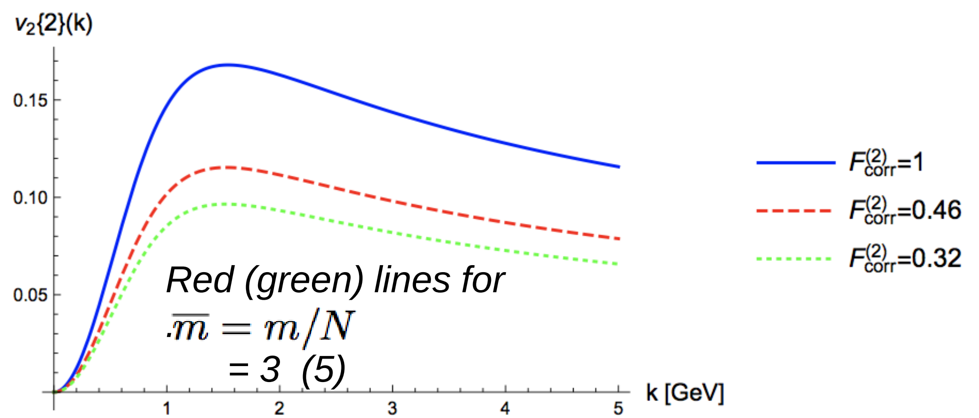
$$\equiv \frac{F^{(2)}_{\rm corr}(N,m)\int_\rho \frac{1}{N^2}\sum_{(ij)} 2^2 J_2\left(k_1\Delta y_{ij}\right)\, J_2\left(k_2\Delta y_{ij}\right)}{(N_c^2-1)+F^{(2)}_{\rm corr}(N,m)\int_\rho \frac{1}{N^2}\sum_{(ij)} 2^2 J_0\left(k_1\Delta y_{ij}\right)\, J_0\left(k_2\Delta y_{ij}\right)} + O\left(\frac{1}{(N_c^2-1)^2}\right)$$

- *Partonic v_2, may be modified by hadronization*
- *Signal persists to multi-GeV region*



- *For fixed average multiplicity per source, $\overline{m} = m/N$*

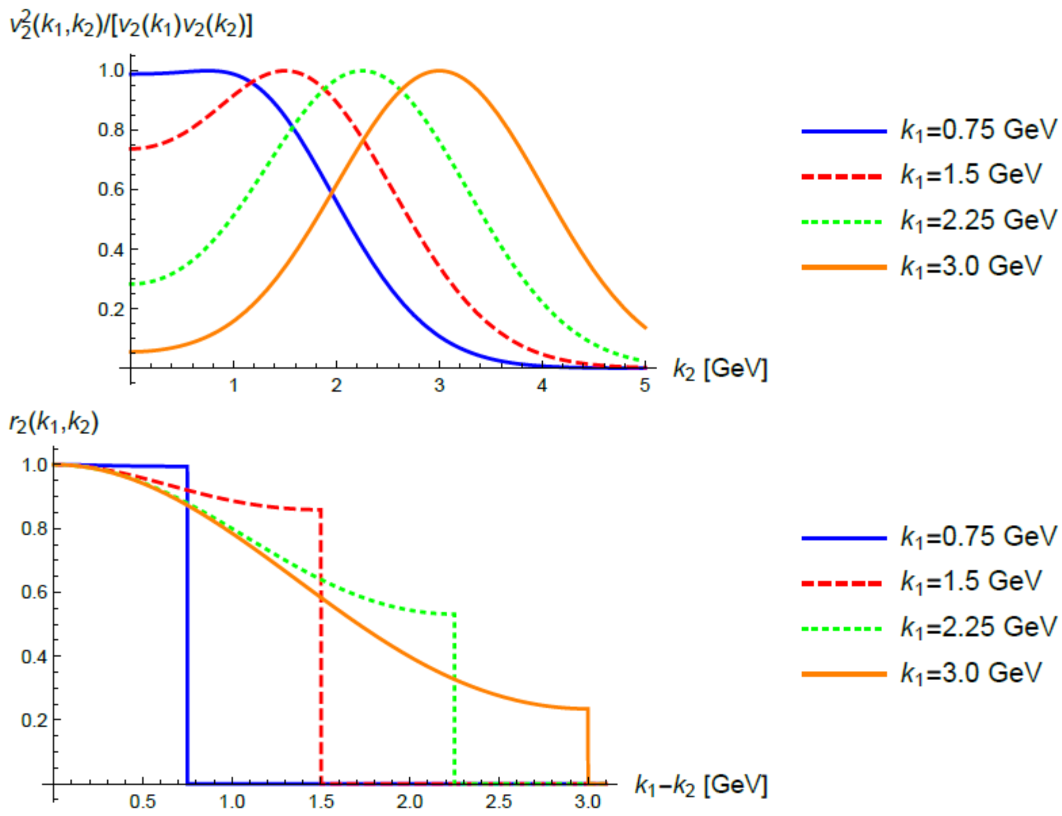
$$\lim_{m\to\infty} F^{(2)}_{\rm corr}(m/\overline{m}, m) = \frac{2\overline{m}+2e^{-\overline{m}}-2}{\overline{m}^2}\,.$$



For any multiplicity m, $v_2^2\{2\}$ is finite in the limit $N \to \infty$ of a large number of sources.

$v_2^2\{2\}(k_1, k_2)$ does not factorize except for small transverse momentum.

Compatible with hydrodynamics



4^{th} order cumulant: v_2

$$\langle\langle e^{i2(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c = (Bk_1^2)\,(Bk_2^2)\,(Bk_3^2)\,(Bk_4^2)$$

$$\Big\{\frac{1}{(N_c^2-1)^2}\left(\boxed{2\,F_{\rm corr}^{(4i)} - 2F_{\rm corr}^{(2)}F_{\rm corr}^{(2)}} + O\left(N^{-1}\right)\right)$$

$$+\frac{1}{(N_c^2-1)^3}\Big(2\,F_{\rm corr}^{(6)}(m-4)(m-5) - 4\,F_{\rm corr}^{(2)}\,F_{\rm corr}^{(4i)}(m-2)(m-3)$$

$$+4\,F_{\rm corr}^{(5)}(m-4) - 4\,F_{\rm corr}^{(2)}F_{\rm corr}^{(3)}(m-2)$$

$$+2\,F_{\rm corr}^{(4ii)} + 8\left(F_{\rm corr}^{(2)}\right)^3 - 4\,F_{\rm corr}^{(4i)}F_{\rm corr}^{(2)}\Big) + O\left(N^{-1}\right)\Big\}$$

$$+O\left(1/(N_c^2-1)^4\right)\,.$$

The leading order ~ $\frac{1}{(N_c^2-1)^2}$ *vanishes but the next order ~* $1/(N_c^2-1)^3$ *is always negative!*

$$v_n\{4\} \equiv \sqrt[4]{-\langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c}$$

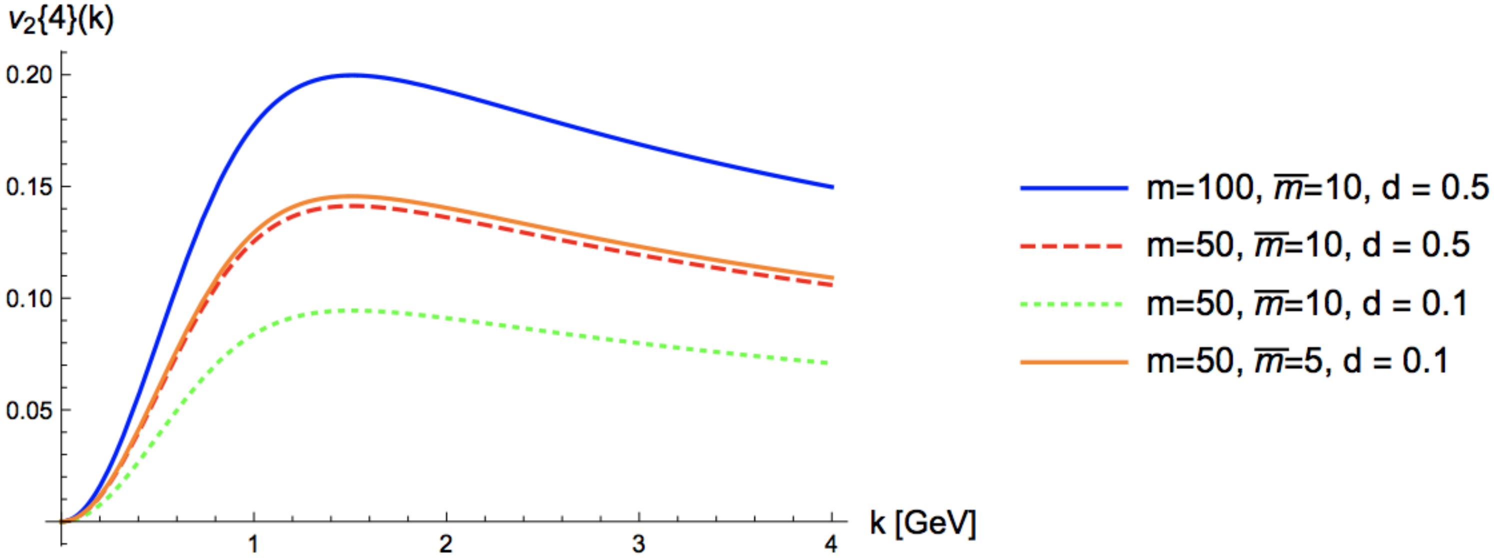
$$\boxed{v_2\{4\}(k) \simeq \frac{1}{(N_c^2-1)^{3/4}}2^{1/4}\sqrt{m}\,B\,k^2}$$

…4th order cumulant, cont'd …

CGC result for $v_2\{4\}$ has same N_c but different m- (N?)-dependencies,

$$v_n\{4\} \equiv \sqrt[4]{-\langle\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\rangle\rangle_c} \,.$$

$$\boxed{v_2\{4\}(k) \simeq \frac{1}{(N_c^2-1)^{3/4}} 2^{1/4}\sqrt{m}\, B\, k^2}$$



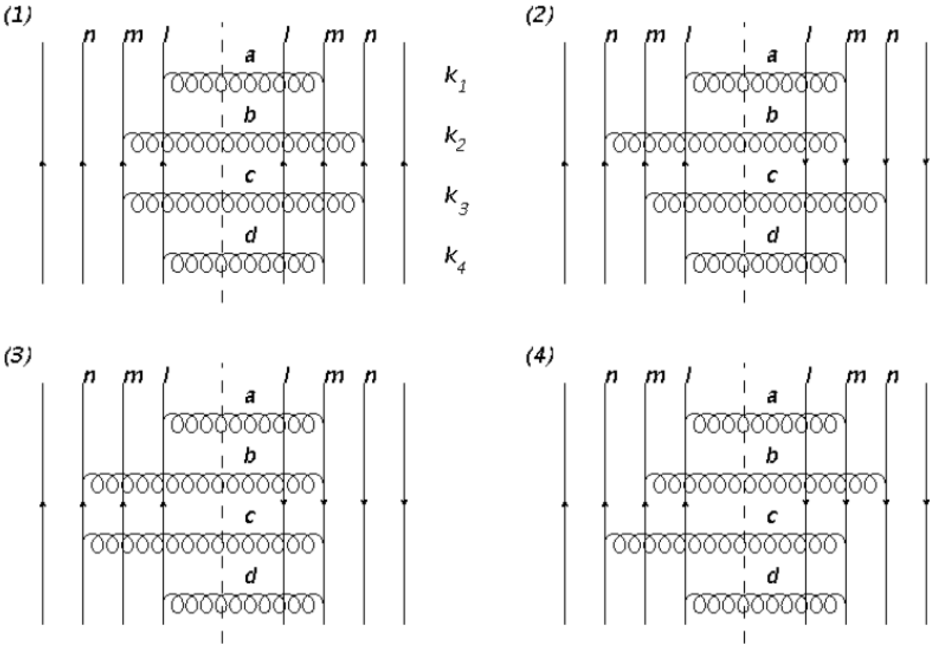
Odd harmonics

$$\mathrm{Tr}\,[\mathbb{1}]\ \mathrm{Tr}\left[T^c T^b\right]\ \mathrm{Tr}\left[T^b T^c T^d T^a\right]\ \mathrm{Tr}\left[T^a T^d\right] = N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\,[\mathbb{1}]\ \mathrm{Tr}\left[T^b T^c\right]\ \mathrm{Tr}\left[T^c T^d T^b T^a\right]\ \mathrm{Tr}\left[T^a T^d\right] = \frac{1}{2}N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\,[\mathbb{1}]\ \mathrm{Tr}\left[T^b T^c\right]\ \mathrm{Tr}\left[T^d T^c T^b T^a\right]\ \mathrm{Tr}\left[T^a T^d\right] = N_c^4\ (N_c^2-1)^2$$

$$\mathrm{Tr}\,[\mathbb{1}]\ \mathrm{Tr}\left[T^c T^b\right]\ \mathrm{Tr}\left[T^b T^d T^c T^a\right]\ \mathrm{Tr}\left[T^a T^d\right] = \frac{1}{2}N_c^4\ (N_c^2-1)^2$$



To order 1/N, differences in color factors break the k to –k symmetry

$$e^{i\,\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}}\left(e^{i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}+\frac{1}{2}e^{-i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}\right)+e^{-i\,\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}}\left(\frac{1}{2}e^{i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}+e^{-i\,\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}}\right)$$
$$=3\cos\left(\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)\,\cos\left(\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)-\sin\left(\mathbf{k}_2.\boldsymbol{\Delta}\mathbf{y}_{mn}\right)\,\sin\left(\mathbf{k}_3.\boldsymbol{\Delta}\mathbf{y}_{mn}\right). \qquad (5.$$

Odd harmonics occur!

Conclusion

- The zero-final state interaction baseline for v_2 (and v_n) is not vanishing, but it is given by quantum interference and it may take sizes and p_T-shapes comparable to those observed in pp, pA

- Work is in progress (Boris Blok & Urs Wiedemann)
  - to resume effects to all orders in $m^2/(N_c^2 -1)$
  - to push this line of investigation to higher order cumulants
  - to understand relation to CGC formalism

Stay tuned!