Hybrid Model of Heavy-Ion Collisions at BES Energies with Dynamical Sources

Lipei Du

The Ohio State University

In collaboration with Gojko Vujanovic and Ulrich Heinz

Department of Physics, The Ohio State University, USA

May 14, 2018
QCD phase diagram and heavy-ion collisions

2007 NSAC Long Range Plan

Hybrid models


<table>
<thead>
<tr>
<th>Model</th>
<th>Initial condition</th>
<th>Hydro</th>
<th>Switching criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>UrQMD hybrid [17]</td>
<td>UrQMD cascade</td>
<td>ideal 3+1D, SHASTA</td>
<td>$t_{CM} = \max(2R\sqrt{\frac{E_{ab}}{2m_N}}, 1.0) \text{ fm/c}$</td>
</tr>
<tr>
<td>Skokov-Toneev hybrid [18]</td>
<td>Quark-Gluon-String Model</td>
<td>ideal 3+1D, SHASTA</td>
<td>$t_{CM}$ such that $S/Q_B = \text{const}$</td>
</tr>
<tr>
<td>EPOS [19]</td>
<td>Strings (Regge-Gribov model)</td>
<td>ideal 3+1D</td>
<td>$\tau$</td>
</tr>
<tr>
<td>NeXSPheRIO hybrid [20,27]</td>
<td>Strings (Regge-Gribov model)</td>
<td>ideal 3+1D, SPH</td>
<td>$\tau = 1 \text{ fm [28]}$</td>
</tr>
<tr>
<td>Gale et al. [21]</td>
<td>IP glasma</td>
<td>viscous 3+1D, MUSIC</td>
<td>$\tau = 0.2 \text{ fm/c (}\sqrt{s_{NN}} = 2.76 \text{ TeV)}$</td>
</tr>
<tr>
<td>Karpenko hybrid [22]</td>
<td>UrQMD cascade</td>
<td>viscous 3+1D</td>
<td>$\tau_{\text{geom}}$</td>
</tr>
<tr>
<td>Pang et al. hybrid [23]</td>
<td>AMPT</td>
<td>ideal 3+1D, SHASTA</td>
<td>$\tau$</td>
</tr>
<tr>
<td>Bhalerao et al. hybrid [24]</td>
<td>AMPT</td>
<td>viscous 2+1D, VISH2+1</td>
<td>$\tau = 0.4 \text{ fm/c (}\sqrt{s_{NN}} = 2.76 \text{ TeV)}$</td>
</tr>
</tbody>
</table>

- The switching criteria: either constant proper time $\tau$ or constant time in the center-of-mass frame $t_{CM}$, determined by geometrical criterion.
Chun Shen, *Dynamical initialization and hydrodynamic modeling of relativistic heavy-ion collisions*

16 May 2018, 11:30


The conservation laws are
\[ d_\mu T^{\mu\nu} = d_\mu \left[ T_{\text{fluid}}^{\mu\nu} + T_{\text{particle}}^{\mu\nu} \right] = 0, \]
\[ d_\mu N^\mu = d_\mu \left[ N_{\text{fluid}}^\mu + N_{\text{particle}}^\mu \right] = 0, \]
from which we can get the source terms for the fluid as
\[ d_\mu T^{\mu\nu}_{\text{fluid}} = J^\nu_{\text{source}}(x) \equiv -d_\mu T_{\text{particle}}^{\mu\nu}(x), \]
\[ d_\mu N_{\text{fluid}}^\mu = \rho_{\text{Bsource}}(x) \equiv -d_\mu N_{\text{particle}}^\mu(x). \]

What is needed to get source terms:
- **Pre-equilibrium dynamics** to give information of particles;
- **Energy-momentum tensor** and net baryon current constructed from particles.
Pre-equilibrium dynamics
Our model: Dynamical initialization based on UrQMD

Instead of rescattering after production, particles get thermalized after formation time.

$t - z$ plot of particles at production time

$\tau - \eta_s$ plot of particles at formation time
Energy deposition

Our model:

\[ \eta \left( \frac{\tau}{\tau_0} \right) \]


Lipei Du (The Ohio State University)
Baryon distribution

Our model (produced mesons not shown)


Lipei Du (The Ohio State University) Hybrid Model with Dynamical Sources May 14, 2018 9 / 26
Baryon distribution

Particle formation time strongly affects the baryon distribution in space-time rapidity

Interesting question: how will this difference affect the final momentum distributions?
Baryon distribution in space-time rapidity (central AuAu @ 200 GeV)

The initial spacetime distribution of net baryon number can be strongly model dependent, within our model on the value of the formation time.

- Similar rapidity distributions can correspond to very different space-time rapidity distributions, and different space-time rapidity distributions may lead to different hydrodynamic evolution.

Baryon distribution in space-time rapidity (central AuAu @ 19.6 GeV)

\[
\Delta \tau = \frac{\tau_{\text{form}}}{\tau_{\text{formation}}} = \frac{\tau_{\text{formation}}}{\tau_{\text{formation}}} = \frac{\tau_{\text{formation}}}{\tau_{\text{formation}}}
\]

This model

- The initial spacetime distribution of net baryon number can be strongly model dependent, within our model on the value of the formation time.
- Similar rapidity distributions can correspond to very different space-time rapidity distributions, and different space-time rapidity distributions may lead to different hydrodynamic evolution.

Source terms
Constructing the source terms

Baryon current and energy-momentum tensor of the particles are given by [D. Oliinychenko and H. Petersen, Phys.Rev. C93 (2016) 034905]

\[
T_{\text{particle}}^{\mu\nu}(t, \mathbf{r}) = \sum_i \frac{p_i^\mu p_i^\nu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i(t), p_i),
\]

\[
N_{\text{particle}}^\mu(t, \mathbf{r}) = \sum_i b_i \frac{p_i^\mu}{p_i^0} K(\mathbf{r} - \mathbf{r}_i(t), p_i).
\]

The smearing kernel in the rest frame is given by

\[
K_i(t_{\text{rf}}, \mathbf{x}_{\text{rf}}, p_i) = \gamma_i \frac{1}{(2\pi\sigma^2)^{3/2}} \exp \left[ \frac{-(\mathbf{x}_{\text{rf}} - \mathbf{r}_{\text{rf},i})^2}{2\sigma^2} \right] \Theta(t_{\text{form},i} - t_{\text{rf}}),
\]

\[
\equiv \gamma_i \frac{1}{(2\pi\sigma^2)^{3/2}} \exp \left[ \frac{(x - x_i(t))^2 - (u_i \cdot (x - x_i(t)))^2}{2\sigma^2} \right] \Theta(t_{\text{form},i} - t_{\text{rf}}).
\]
\[ \Theta(t_{\text{form}} - t) = \frac{1}{2} \left[ \tanh \left( \frac{t_{\text{form}} - t}{\Delta \tau_{\text{th}}} \right) + 1 \right] \xrightarrow{\Delta \tau_{\text{th}} \to 0} \theta(t_{\text{form}} - t). \]
Dynamical sources (central AuAu @ 200 GeV)

Energy-momentum source $J_{\text{source}}^{\tau}$

Baryon number source $\rho_{\text{source}}^{B}$

$\tau = 0.5 \text{ fm/c}$
Dynamical sources (central AuAu @ 200 GeV)

Energy-momentum source $J^\tau_{\text{source}}$

Baryon number source $\rho^B_{\text{source}}$

$\tau = 0.7 \text{ fm/c}$
Dynamical sources (central AuAu @ 200 GeV)

Energy-momentum source $J_{\text{source}}^\tau$

Baryon number source $\rho_{\text{source}}^B$

$\tau = 0.9 \text{ fm/c}$
Dynamical sources (central AuAu @ 200 GeV)

Energy-momentum source $J^\tau_{\text{source}}$

Baryon number source $\rho^B_{\text{source}}$

$\tau = 1.1 \text{ fm/c}$
In our model, bumpiness of $\rho_B$ is not correlated with that of $e$. 
Hydrodynamics with dynamical sources
Dissipative hydrodynamics

The conservation law of energy momentum is

\[ d_\mu T_{\text{fluid}}^{\mu\nu} = d_\mu (e u^\mu u^\nu - (p_0 + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}) = J_\nu^{\text{source}}. \]

The dissipative transport equations are

\[
\begin{align*}
\tau_\Pi \dot{\Pi} + \Pi &= -\zeta \theta - \delta \Pi \Pi \theta + \lambda_\Pi \pi^{\mu\nu} \sigma_{\mu\nu} - \ell_\Pi n \cdot n - \tau_\Pi n \cdot \nabla p_0 - \lambda_\Pi n \cdot \nabla \left( \frac{\mu B}{T} \right), \\
\tau_\pi \dot{\pi}^{(\mu\nu)} + \pi^{\mu\nu} &= 2\eta \sigma^{\mu\nu} + 2\tau_\pi \pi^{(\mu \chi} \omega^{\nu)\lambda} - \delta_\pi \pi^{\mu\nu} \theta - \tau_\pi \pi^{\lambda (\mu} \sigma_{\nu)} + \lambda_\pi \Pi \sigma^{\mu\nu} \\
&= -\tau_\pi n^{(\mu \nabla \nu)} p_0 + \ell_\pi n \nabla^{(\mu n \nu)} + \lambda_\pi n^{(\mu \nabla \nu)} \left( \frac{\mu B}{T} \right),
\end{align*}
\]

The conservation law of the net baryon number is

\[ d_\mu N_{\text{fluid}}^\mu = d_\mu (\rho_B u^\mu + n^\mu) = \rho_{B\text{source}}. \]

The dissipative transport equation for the baryon diffusion current is

\[ \tau_n \dot{n}^{\langle \mu \rangle} + n^\mu = \kappa_B \nabla^\mu \left( \frac{\mu_B}{T} \right) - \tau_n n_\nu \omega^{\nu\mu} - \delta_{nn} n^\mu \theta - \lambda_{nn} n_\nu \sigma^{\mu\nu}. \]

In the Navier-Stokes limit, the baryon diffusion current is

\[ n^\mu \sim \kappa_B \nabla^\mu \left( \frac{\mu_B}{T} \right). \]

- \( \rho_B u^\mu \) and \( n^\mu \) together control transportation of the net baryon number when there is no source.

Baryon diffusion coefficient

Relaxation time approximation [G. S. Denicol et al, arXiv:1804.10557 [nucl-th]]

\[ \kappa_B = \frac{C_B}{T} \rho_B \left( \frac{1}{3} \coth \left( \frac{\mu_B}{T} \right) - \frac{\rho_B T}{e + P} \right) \]


- Holographic model smoothes out spatial fluctuations of \( \kappa_B \).
Initial longitudinal profiles and driving forces

Net baryon number is transported to midrapidity by diffusion, and to large rapidities by longitudinal expansion; energy density is insensitive to baryon diffusion.

In spite of the expansion of the fireball, net baryon number at large rapidity increases due to continued feeding by baryon number sources (dynamical sources cease at 4 fm/c).

Net baryon number doesn’t decrease strongly in midrapidity because the baryon diffusion is against the flow velocity.
Conclusions

- Based on modified UrQMD, a dynamical source model for both the energy-momentum and net baryon current is constructed;

- A (3+1)D hydrodynamical code with baryon evolution has been developed which can handle dynamical sources;

- The initial space-time rapidity distribution of net baryon number is strongly model dependent even within a single model, on the value of the formation time in our model; its influence on the final momentum distributions is an interesting question under investigation.

- The baryon evolution in the longitudinal direction is studied and is consistent with previous studies when the same initial condition is used; how the evolution will be changed with the quite different initial conditions got from our dynamical source model needs to be studied.

- Study of final spectra from full (3+1)D hydrodynamics with dynamical sources for both the energy-momentum and net baryon current is upcoming.
Thank you very much!