Nonlinear coupling of flow harmonics in heavy-ion collisions

by

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A remarkable pattern of angular anisotropies. What do we understand?

$$P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \boxed{V_n} e^{-in\phi} \qquad V_{-n} = V_n^*$$

V2, **V3**: well-known response to geometry (ε2, ε3)[Teaney and Yan, **arXiv 1010:1876**]

What if we measure harmonics of order **n>3**?

To a given harmonic contribute all vectors that share the same symmetry under azimuthal rotation. [Gardim, Grassi, Luzum, Ollitrault, **arXiv 1111:6538**]

In other words, since V2 and V3 are large, we will have

$$V_4 \propto V_2^2 \qquad V_5 \propto V_2 \, V_3$$

...and combinations up to any order (potentially, even with a V1)

What should we do in hydro?

Great ideas from [Teaney and Yan, **arXiv 1206:1905**]: a formalism of nonlinear hydrodynamic response. The focus is on **nonlinear response coefficients**.

$$v_4 e^{-i4\Psi_4} = w_4 e^{-i4\Phi_4} + w_{4(22)} e^{-i4\Phi_2}$$

Yan and Ollitrault introduced a framework for nonlinear response which deals only with the coupling of final state anisotropies.

[Yan and Ollitrault, **arXiv 1502:02502**]

Simple procedure for V4:

$$V_4 = \chi_{42}(V_2)^2 + U_4$$

where U4 is the vector uncorrelated with V2^2,

$$\langle U_4 (V_2^*)^2 \rangle = 0$$

So that the coefficient is uniquely defined

$$\chi_{42} = \frac{\langle V_4 (V_2^*)^2 \rangle}{\langle |V_2|^4 \rangle}$$

In other words, if one knows V4 and V2^2 in a bunch of events, the coefficient is readily obtained.

$$v_n \equiv |V_n|$$
$$u_n \equiv |U_n|$$

So, writing down a few relevant harmonics:

$$V_4 = U_4 + \chi_{42} V_2^2$$
$$V_5 = U_5 + \chi_{523} V_2 V_3$$
$$V_6 = U_6 + \chi_{62} V_2^3 + \chi_{63} V_3^2 + \chi_{624} V_2 U_4$$
$$V_7 = U_7 + \chi_{723} V_2^2 V_3 + \chi_{725} V_2 U_5 + \chi_{734} V_3 U_4$$

[Qian, Heinz, Liu, arXiv 1602:02813]

Nonlinear response coefficients were introduced...

$$\chi_{42} = \frac{\langle V_4 V_2^{2*} \rangle}{\langle v_2^4 \rangle} \qquad \chi_{523} = \frac{\langle V_5 V_2^* V_3^* \rangle}{\langle v_2^2 v_3^2 \rangle}$$

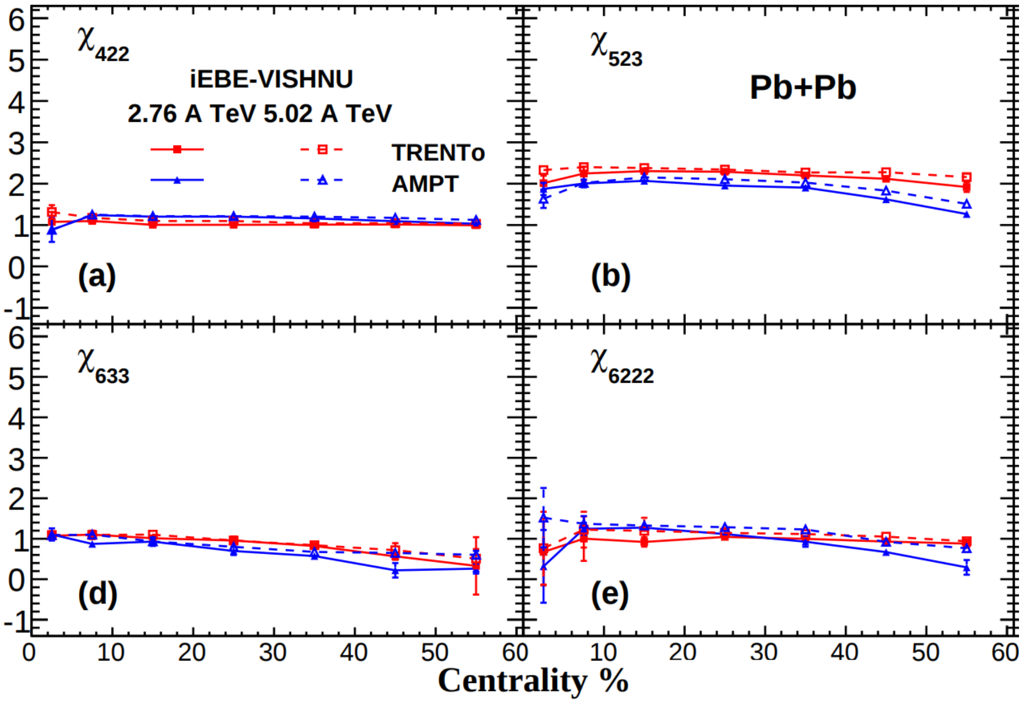
keep in mind, more later...

$$\chi_{62} = \frac{\langle V_6 V_2^{3*} \rangle}{\langle v_2^6 \rangle} \qquad \chi_{63} = \frac{\langle V_6 V_3^{2*} \rangle}{\langle v_3^4 \rangle} \qquad \chi_{624} = \frac{\langle V_6 V_2^* U_4^* \rangle}{\langle v_2^2 u_4^2 \rangle}$$

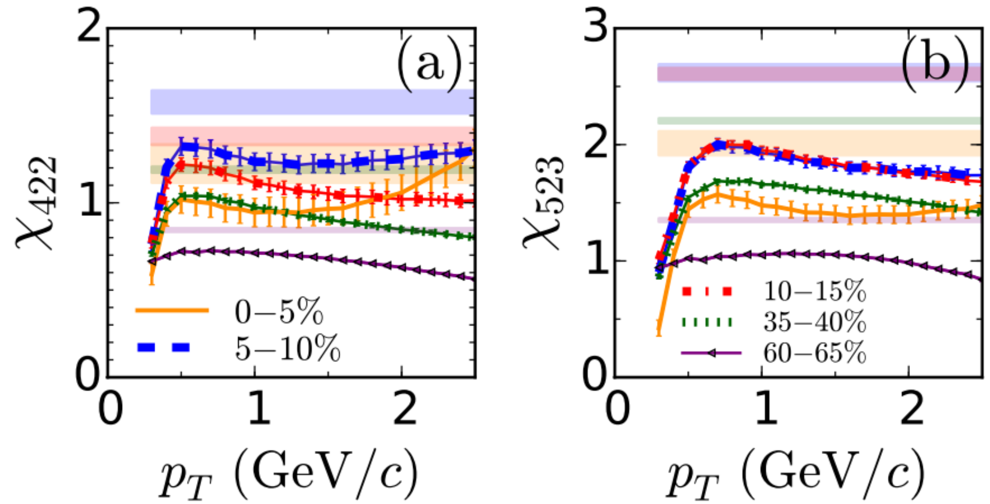
$$\chi_{723} = \frac{\langle V_7 V_3^* V_2^{2*} \rangle}{\langle v_2^4 v_3^2 \rangle} \qquad \chi_{725} = \frac{\langle V_7 V_2^* U_5^* \rangle}{\langle v_2^2 u_5^2 \rangle} \qquad \chi_{734} = \frac{\langle V_7 V_3^* U_4^* \rangle}{\langle v_3^2 u_4^2 \rangle}$$

...calculated in e-by-e hydro...

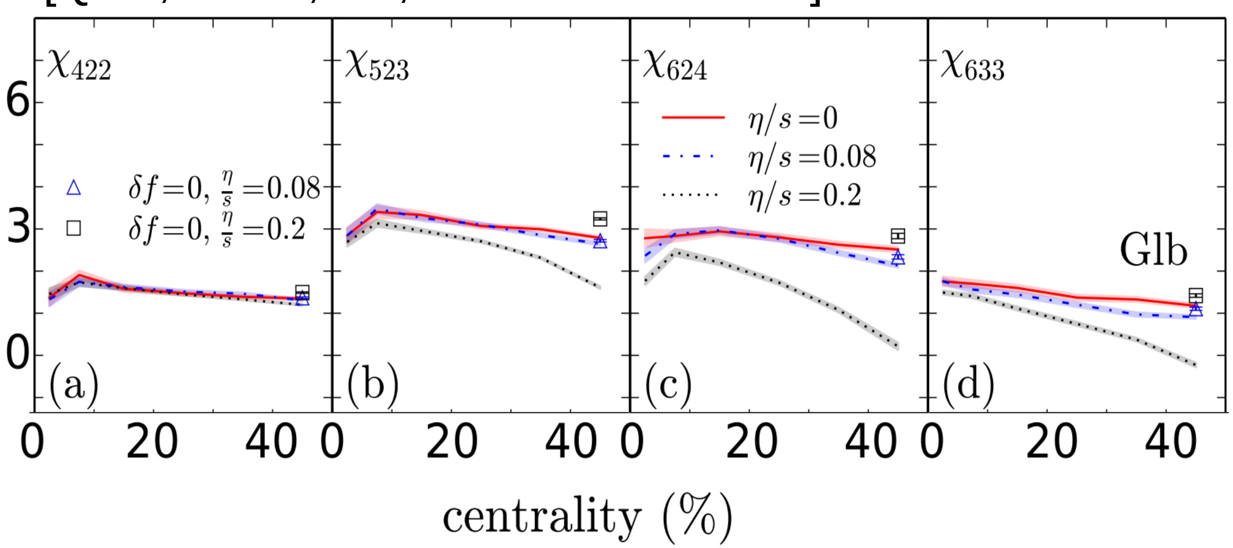
[Zhao, Xu, Song, **arXiv 1703:10792**]



[Qian, Heinz, He, Huo, **arXiv 1703:04077**]



[Qian, Heinz, Liu, **arXiv 1602:02813**]



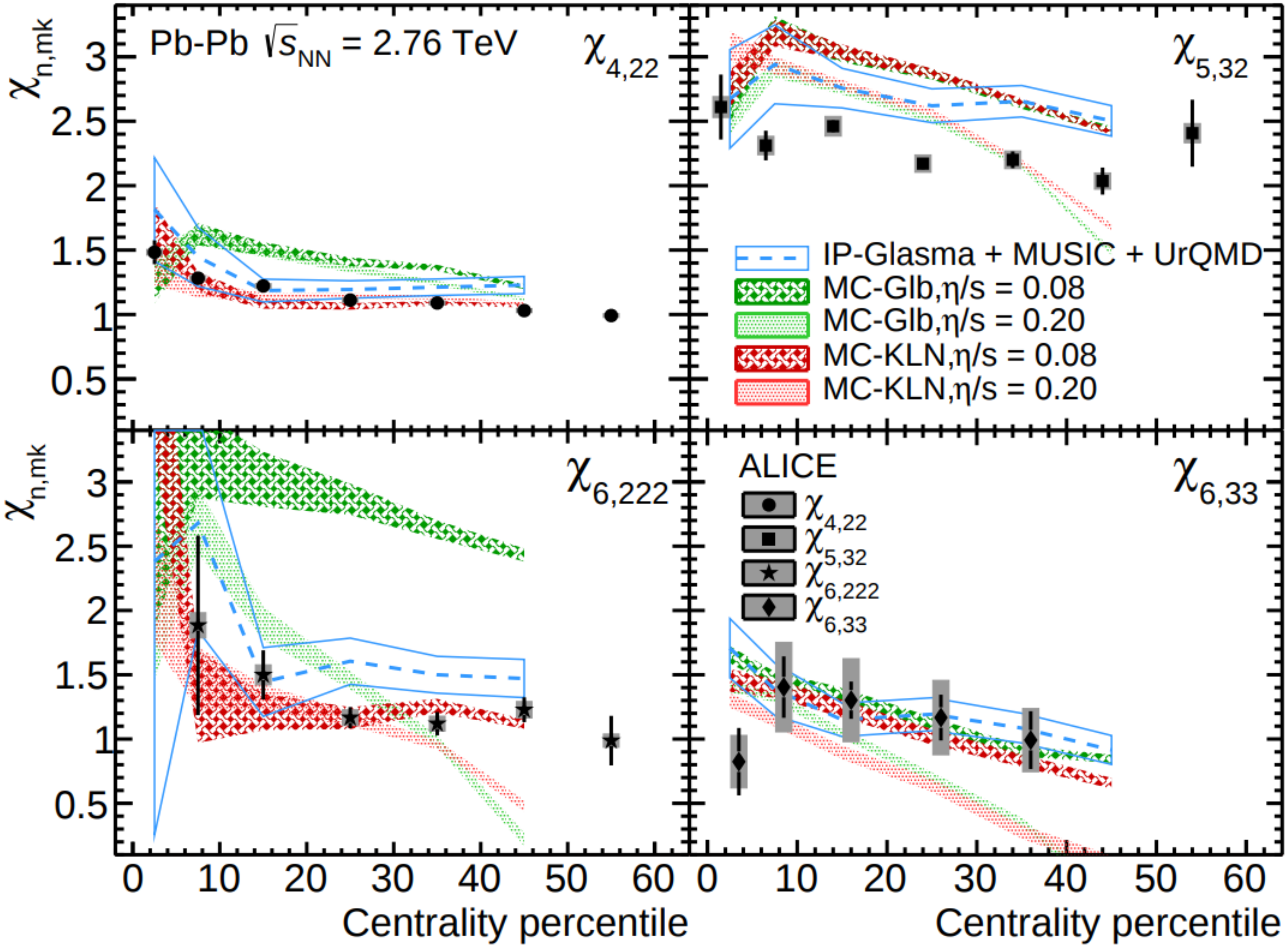
Little dependence on:

- centrality
- ICs
- viscosity
- sqrt(s)
- transverse momentum

Very robust probes!
No fine-tuning!

...and measured in experiment.

[ALICE collaboration, **arXiv 1705:04377**]



...the end of the story ? **NO**!

The formalism is not fully consistent yet! Let us fix it.

[Giacalone, Yan, Ollitrault, arXiv 1803:00253]

In full generality, start with:

$$\boxed{V = \sum_{k=1}^{p} \chi_k W_k + U} \qquad \text{and} \qquad \langle W_k^* U \rangle = 0$$

Now, multiply V by W_j^* and average over events. We obtain

$$\langle W_j^* V \rangle = \sum_{k=1}^{p} \chi_k \langle W_j^* W_k \rangle$$

This is a linear system of p equations for p coupling constants.
Define the following p x p symmetric matrix

$$\boxed{\Sigma_{jk} \equiv \langle W_j^* W_k \rangle}$$

And the following vectors:

$\boxed{M}$ ⟶ a p-vector whose components are the moments $\langle W_j^* V \rangle$

X ⟶ a p-vector whose components are the \chi coefficients

Eventually,

$$M = \Sigma X \quad \Longrightarrow \quad \boxed{X = \Sigma^{-1} M}$$

So, what were we missing before?
Let us have a look at V6:

$$V_6 = \chi_{62}(V_2)^3 + \chi_{63}(V_3)^2 + \chi_{624} V_2 U_4 + U_6$$

$$\Sigma^{(6)} = \begin{pmatrix} \langle v_2^6 \rangle & \langle (V_2^*)^3 (V_3)^2 \rangle & \langle v_2^2 U_4 (V_2^*)^2 \rangle \\ \langle (V_2)^3 (V_3^*)^2 \rangle & \langle v_3^4 \rangle & \langle (V_3^*)^2 U_4 V_2 \rangle \\ \langle v_2^2 U_4^* V_2^2 \rangle & \langle V_3^2 U_4^* V_2^* \rangle & \langle v_4^2 v_2^2 \rangle \end{pmatrix} \quad M = \begin{pmatrix} \langle (V_2^*)^3 V_6 \rangle \\ \langle (V_3^*)^2 V_6 \rangle \\ \langle V_2^* U_4^* V_6 \rangle \end{pmatrix}$$

then

$$X = \Sigma^{-1} M$$

yields exactly the coefficients that I showed previously if the matrix is diagonal!

$$\langle V_2^3 V_3^{2*} \rangle = 0 \qquad \langle v_2^2 V_2^2 U_4^* \rangle = 0 \qquad \langle V_3^2 V_2^* U_4^* \rangle = 0$$

We have always been assuming that mutual correlations between nonlinear terms (off-diagonal terms) are negligible!

good enough? Let us check directly using the experimental data.

Indeed, recent ALICE measurements allow to extract the whole $\Sigma^{(6)}$ and M from data.
In short:

$\langle v_2^6 \rangle$

$\langle v_3^4 \rangle$

From chi coeffs. + e.p. correlations measured by ALICE
[ALICE collaboration, **arXiv 1705:04377**]

$\langle v_4^2 v_2^2 \rangle$

From the symmetric cumulants SC(4,2) and NSC(4,2)
[ALICE collaboration, **arXiv 1604:07663**]

$\langle v_2^2 V_4 (V_2^*)^2 \rangle$

Higher-order moments in
[ALICE collaboration, **arXiv 1705:04377**]

$\langle (V_2^*)^3 (V_3)^2 \rangle$

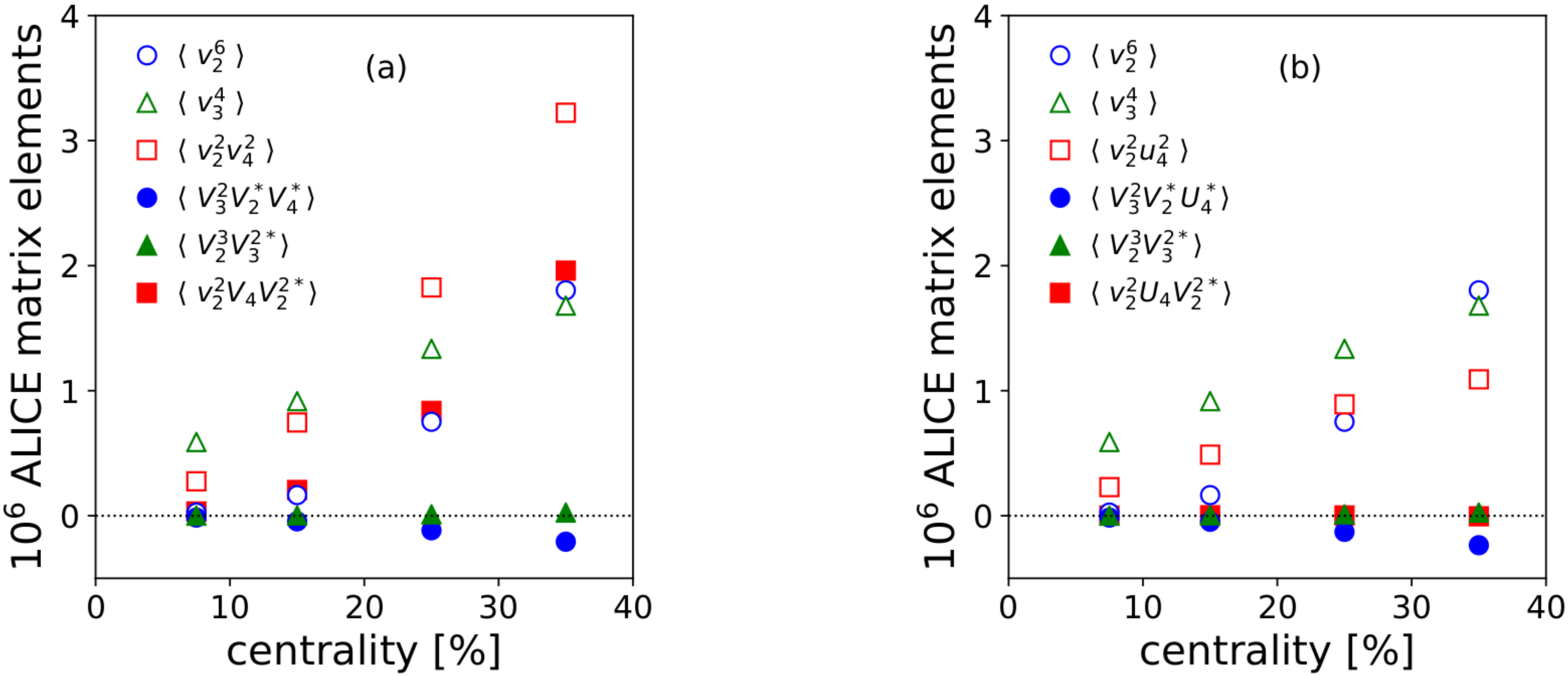
$\langle V_3^2 V_4^* V_2^* \rangle$

Need ATLAS data on e.p. correlations
[ATLAS collaboration, **arXiv 1403:0489**]

$$M = \begin{pmatrix} \langle (V_2^*)^3 V_6 \rangle \\ \langle (V_3^*)^2 V_6 \rangle \\ \langle V_2^* V_4^* V_6 \rangle \end{pmatrix}$$

Chi coeffs. from ALICE + $\langle v_2^6 \rangle$ and $\langle v_3^4 \rangle$

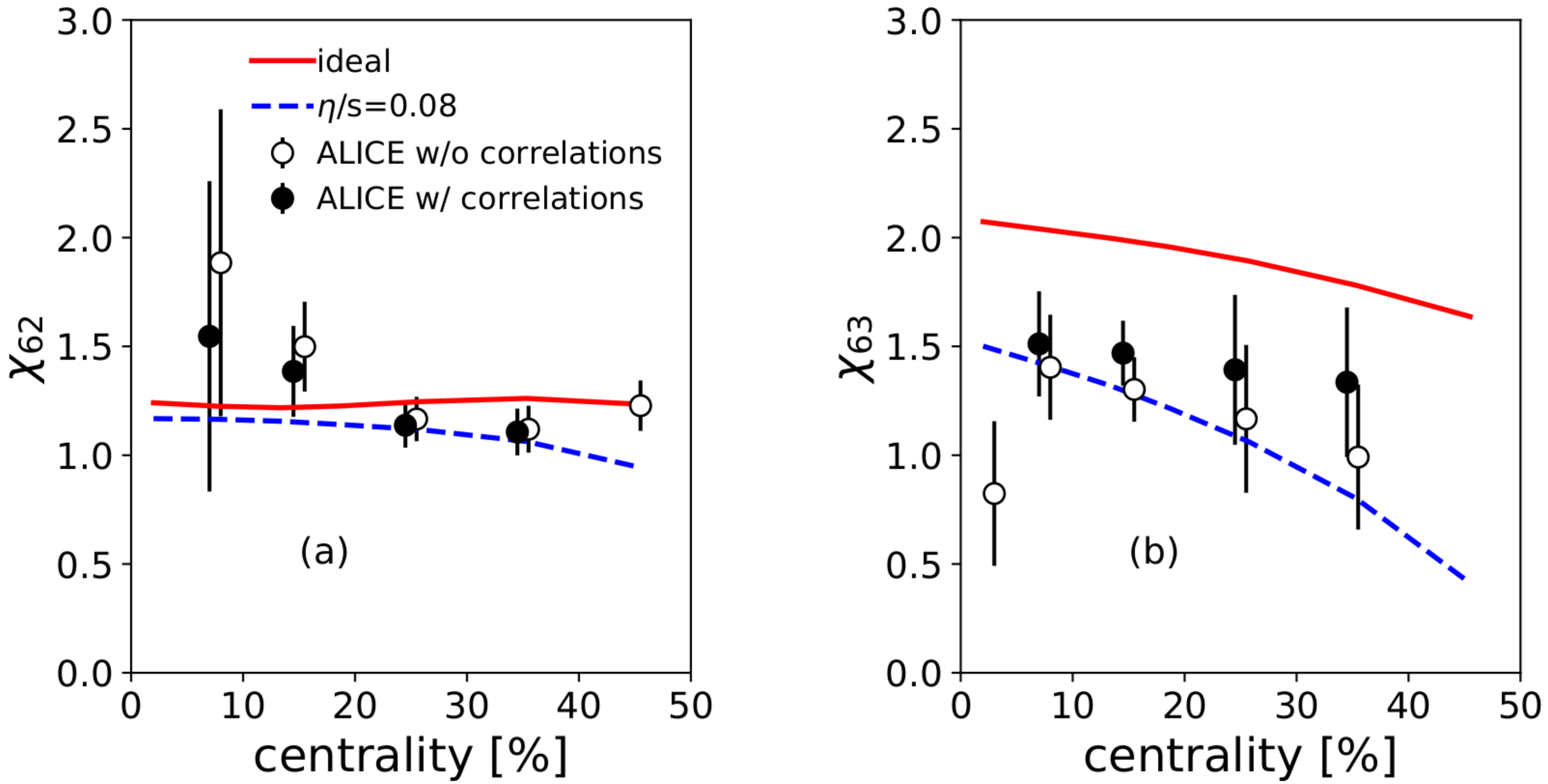
Need ATLAS data on e.p. correlations
[ATLAS collaboration, **arXiv 1403:0489**]



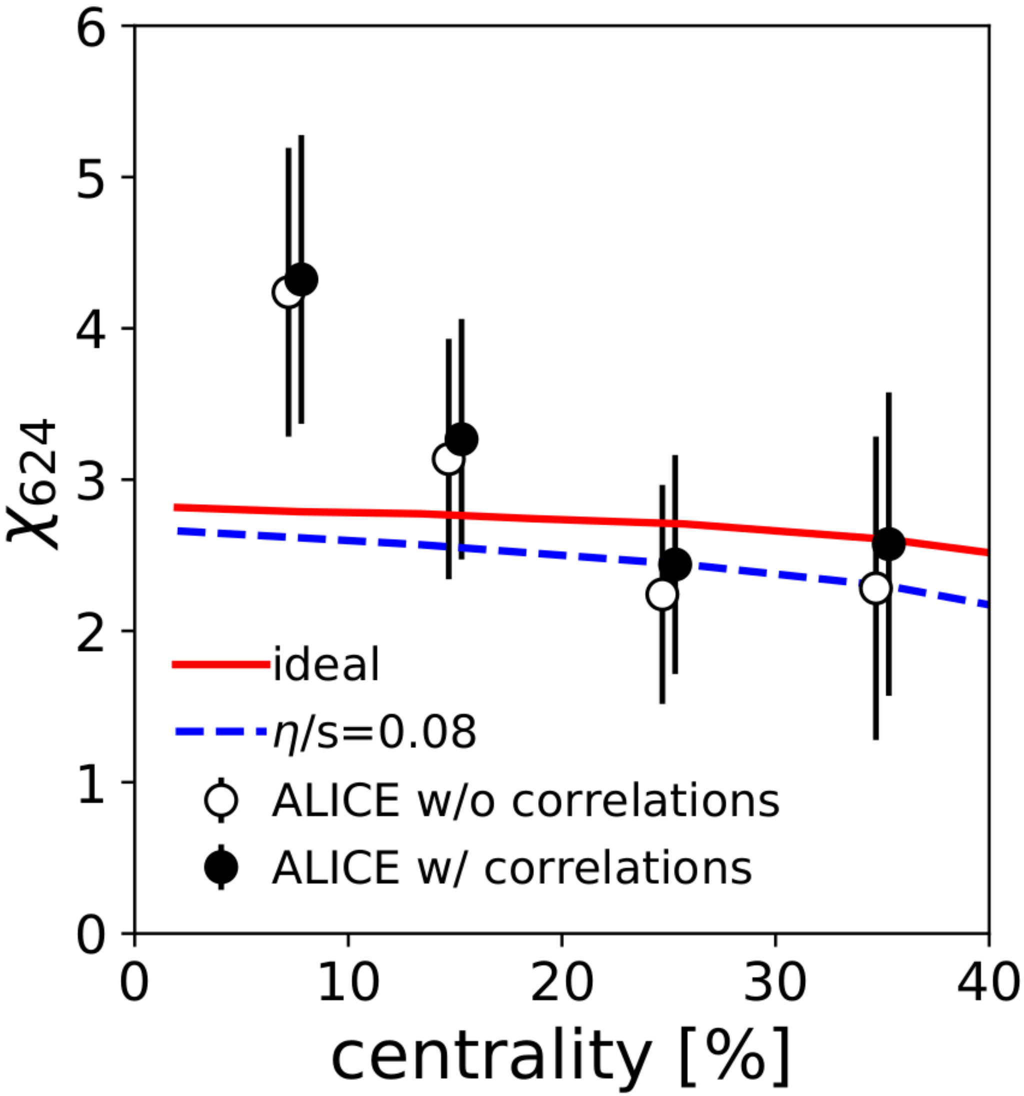
- Very much diagonal if we use U4 instead of V4.
- Interestingly, $\langle V_3^2 V_2^* U_4^* \rangle$ does not vanish.
- It is trivial to move from one figure to the other:

(i.e. in experiment, just measure the one which is more convenient)

$$
\begin{aligned}
\Sigma_{13} &\to \Sigma_{13} - \chi_{42}\Sigma_{11}, \\
\Sigma_{23} &\to \Sigma_{23} - \chi_{42}\Sigma_{21}, \\
\Sigma_{33} &\to \Sigma_{33} - 2\chi_{42}\Sigma_{31} + \chi_{42}^2\Sigma_{11}
\end{aligned}
$$



- No big effects, <u>good news</u>, old calculations and measurements are OK.
- Interestingly, the off-diagonal term seems to make the coefficients flatter, <u>good news</u> for hydro.
- Errors still large, not sure about the rise at low centrality. Likely measurements from other collaborations are needed.



- First extraction of this coefficient from data!
- Here the effect of correlations is not sizable, errors too large.
- Looks compatible with hydro. But hydro is just flat all the way to 0% (e-by-e as well). The rise at low centrality needs more investigation in experiment before anything can be claimed.

Before I conclude:
The limit of the formalism for applications to future data (run2, run3,..) is essentially our fantasy:

$$\begin{aligned} V_8 =&\big(\chi_{82} - \chi_{824}\chi_{42} - \chi_{826}\chi_{62} + \\ &+ \chi_{826}\chi_{624}\chi_{42} + \chi_{84}\chi_{42}^2\big) V_2^4 \\ &+ \big(\chi_{823} - \chi_{826}\chi_{63} - \chi_{835}\chi_{523}\big) V_2 V_3^2 \\ &+ \big(\chi_{824} - \chi_{826}\chi_{624} - 2\chi_{84}\chi_{42}\big) V_2^2 V_4 \\ &+ \chi_{826} V_2 V_6 + \chi_{835} V_3 V_5 + \chi_{84} V_4^2 + U_8 \end{aligned}$$

$$\Sigma^{(8)} = \begin{pmatrix} \langle v_2^8\rangle & \langle v_2^2 V_3^2 V_2^{3*}\rangle & \langle v_2^4 V_4 V_2^{2*}\rangle & \langle v_2^2 V_6 V_2^{3*}\rangle & \langle V_5 V_3 V_2^{4*}\rangle & \langle V_4^2 V_2^{4*}\rangle \\ \text{c.c.} & \langle v_2^2 v_3^4\rangle & \langle v_2^2 V_2 V_4 V_3^{2*}\rangle & \langle v_2^2 V_6 V_3^{2*}\rangle & \langle v_3^2 V_5 V_2^* V_3^*\rangle & \langle V_4^2 V_2^* V_3^{2*}\rangle \\ \text{c.c.} & \text{c.c.} & \langle v_2^4 v_4^2\rangle & \langle v_2^2 V_6 V_2^* V_4^*\rangle & \langle V_5 V_3 V_2^{2*} V_4^*\rangle & \langle v_4^2 V_4 V_2^{2*}\rangle \\ \text{c.c.} & \text{c.c.} & \text{c.c.} & \langle v_2^2 v_6^2\rangle & \langle V_5 V_3 V_2^* V_6^*\rangle & \langle V_4^2 V_2^* V_6^*\rangle \\ \text{c.c.} & \text{c.c.} & \text{c.c.} & \text{c.c.} & \langle v_5^2 v_3^2\rangle & \langle V_4^2 V_3^* V_5^*\rangle \\ \text{c.c.} & \text{c.c.} & \text{c.c.} & \text{c.c.} & \text{c.c.} & \langle v_4^4\rangle \end{pmatrix}$$

- I strongly doubt this is diagonal.
- Four-plane correlators off the diagonal, interesting new patterns (already for V7).
- Presumably possible in the near future. Keep in mind that everything we do to get the coefficients is **linear in V8**, therefore, simpler than anything involving v8^2, e.g., v8{2}.

- Conclusive remarks.
- Chi coefficients are very good observables, they are robust, i.e., just **numbers** with no specific centrality/viscosity/.. dependence;
- Correlations between mutual terms do not play an important role for the coefficients of V6, but some effects are sizable, in particular, a small off-diagonal term makes them flatter with centrality;
- The formalism is now fully consistent. **USE IT!** Analysis shown here performed with a bunch of ALICE run1 data… run2 data would improve it by orders of magnitudes (especially if ATLAS and CMS get involved)
- Insight about the physics: what are we exactly probing with these coefficients? The guess: the very late dynamics of the system. More theory work to do in view of future data.
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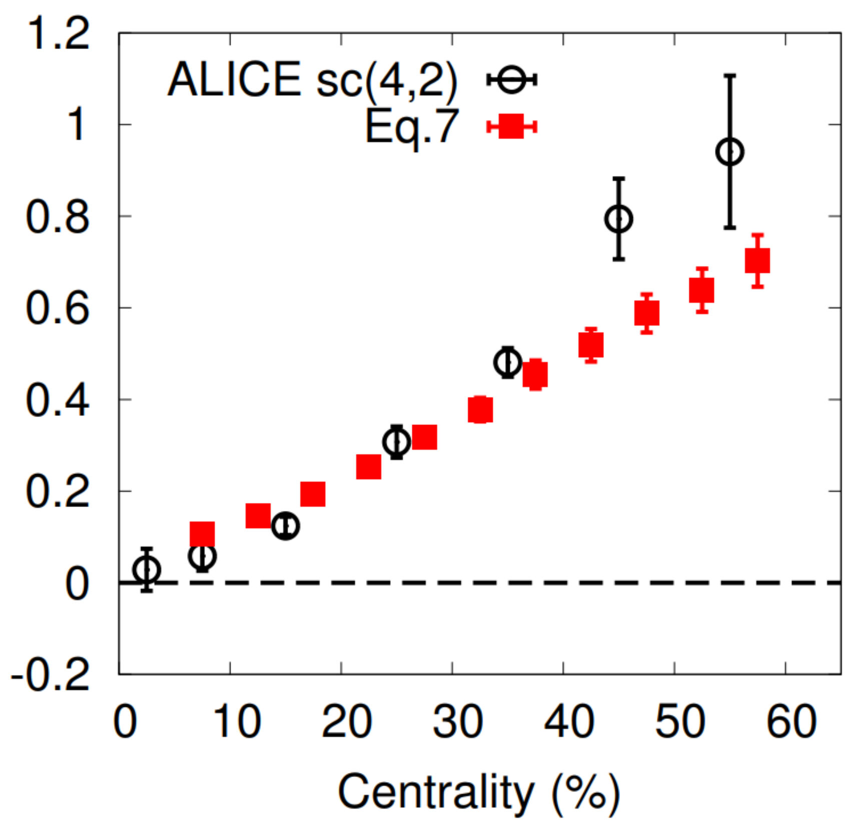
- Thank you all!

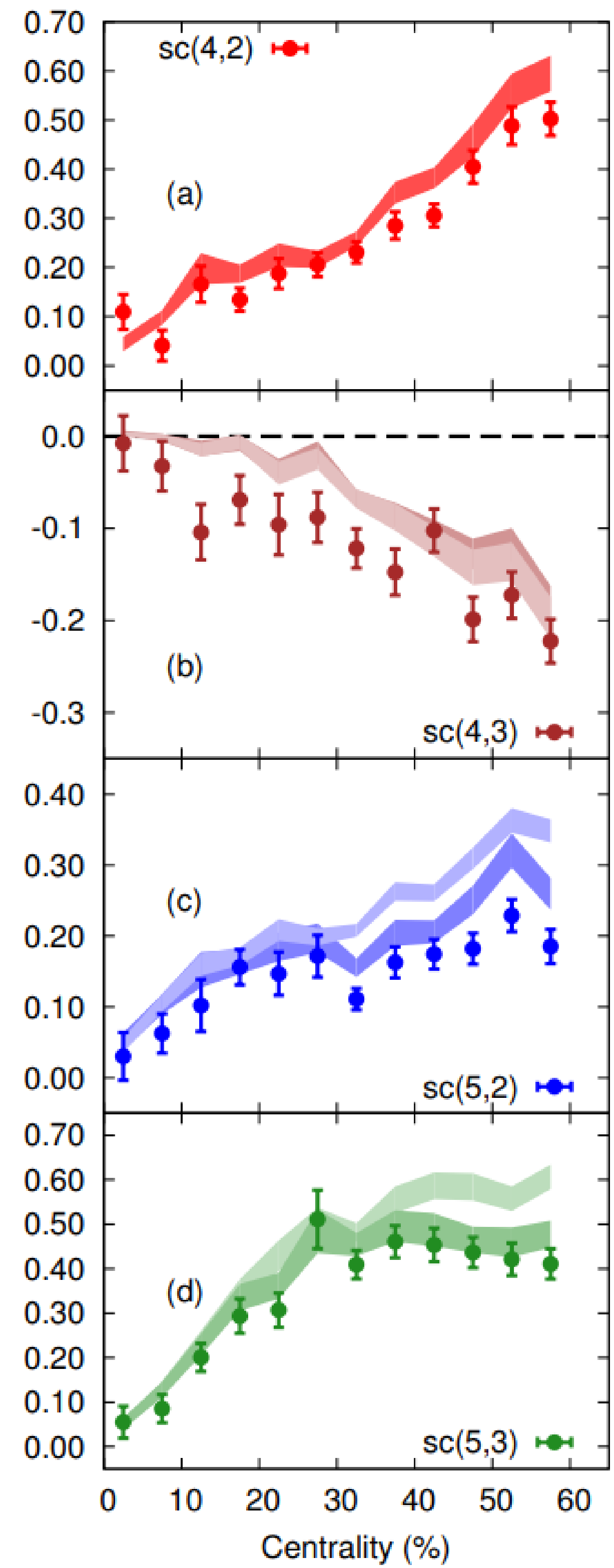
BACKUP

Phenomenology beyond the chi coefficients?

Any observable involving higher-order harmonics,
e.g., symmetric cumulants.

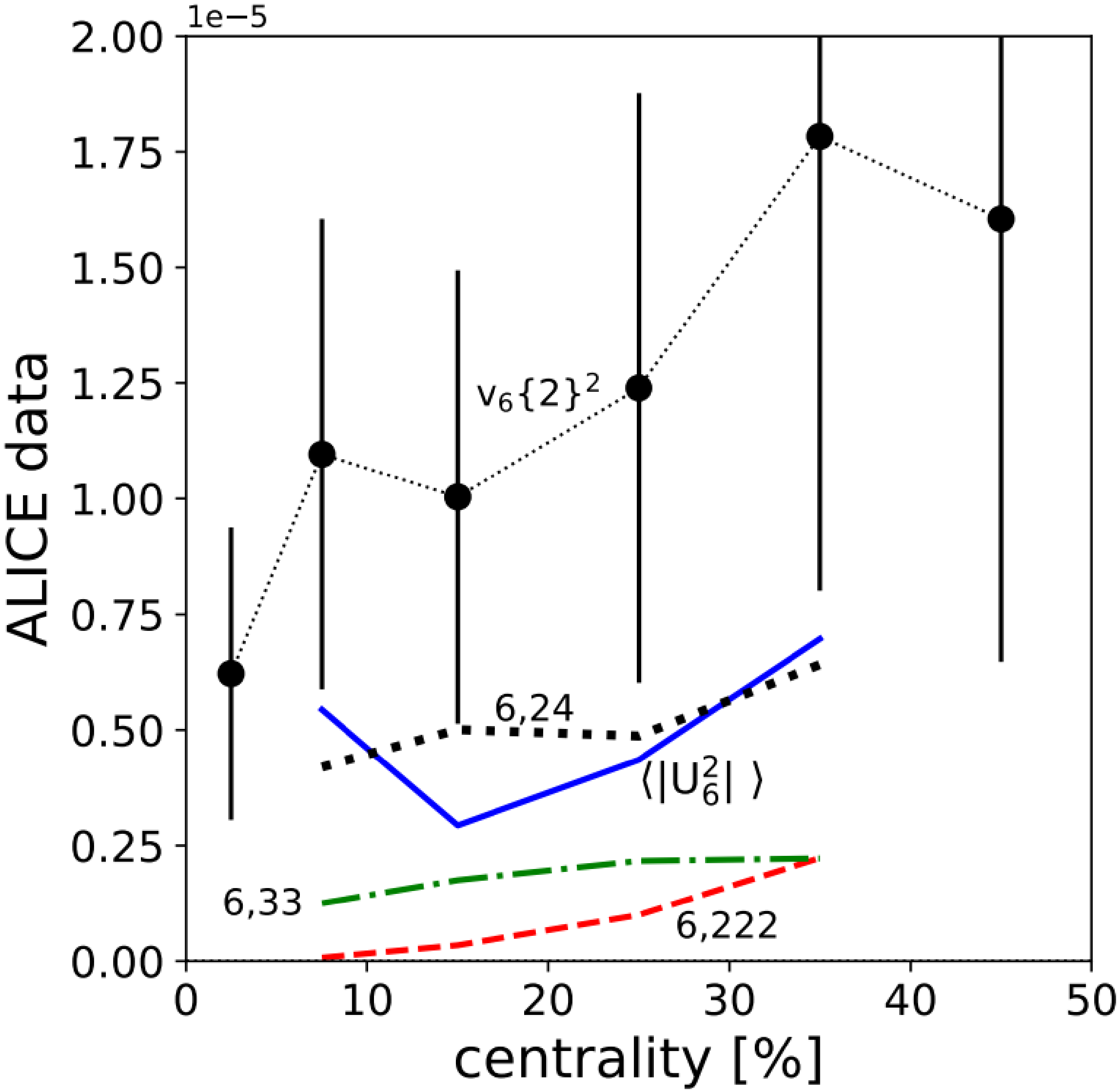
First attempt in:
[Giacalone, Yan, Noronha-Hostler, Ollitrault, arXiv 1605:08303]





One can go much beyond this.

$$\langle |V|^2 \rangle = \sum_{k=1}^{p} \chi_k \langle V^* W_k \rangle + \langle |U|^2 \rangle$$



Contribution to V6 proportional to V2V4 is the dominant one.