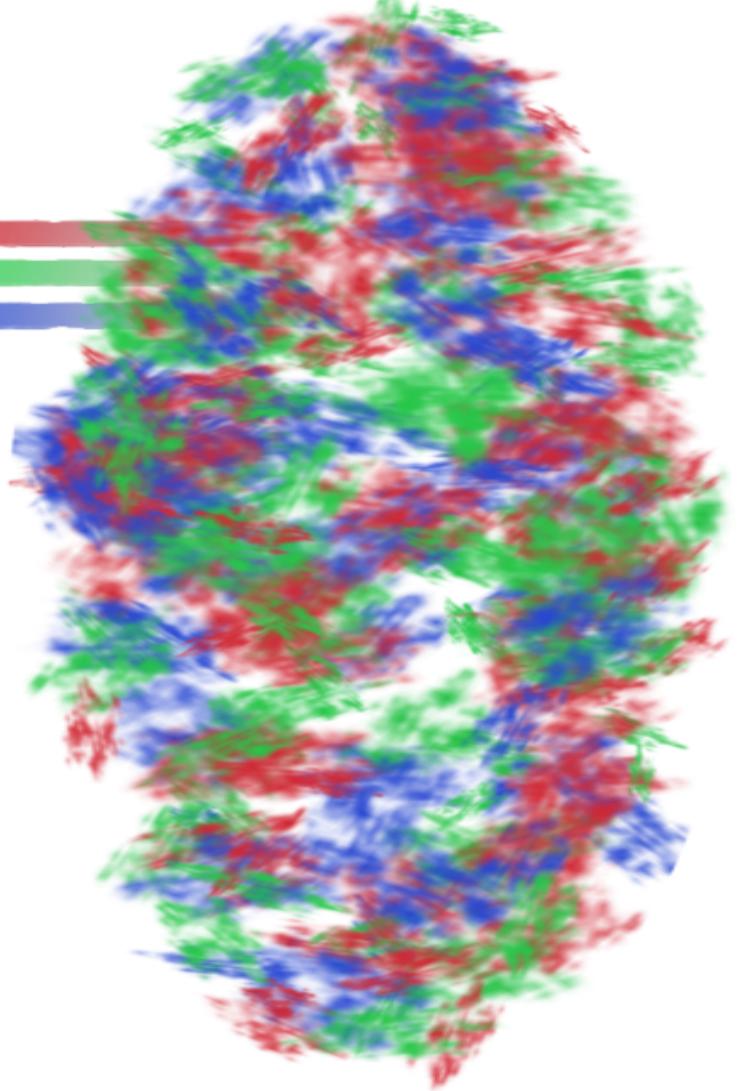


Multi-particle correlations and collectivity in small systems from the initial state



Mark Mace

Stony Brook University and Brookhaven National Lab

Outline

1. Demonstration of multi-particle collectivity with proof of principle parton model

K. Dusling, MM, R. Venugopalan PRL 120, 042002 (2018) [arXiv:1705.00745],
PRD 97, 016014 (2018) [arXiv:1706.06260]

2. Demonstration of hierarchy of v_2 and v_3 across small systems in CGC EFT

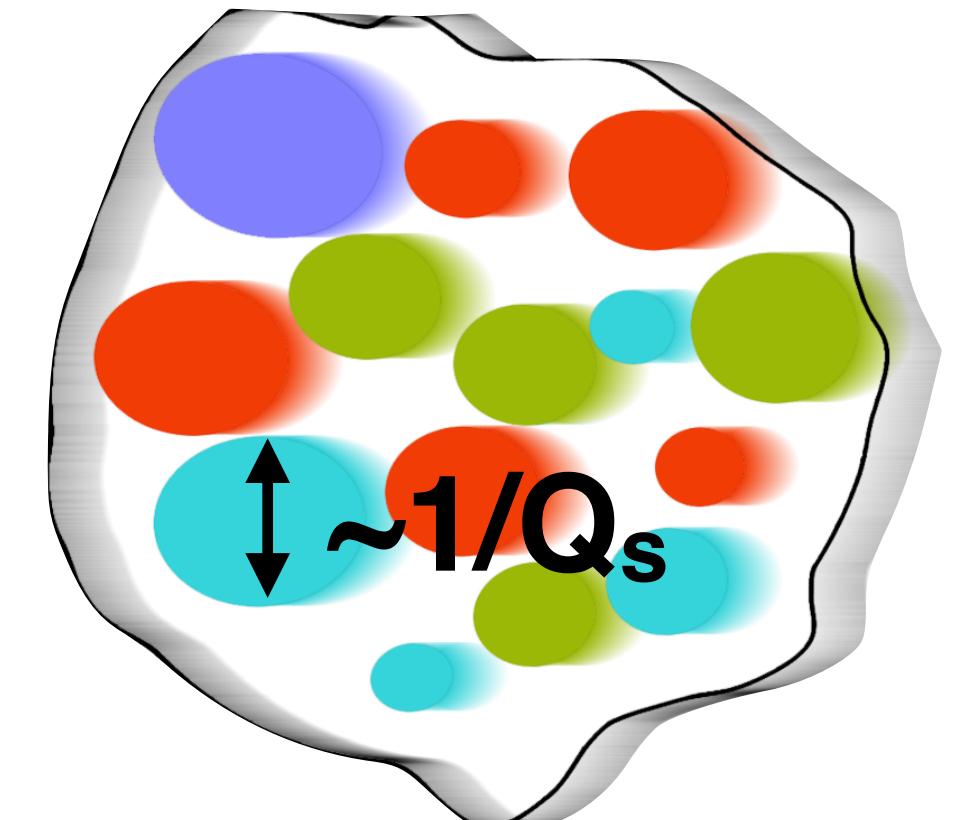
MM, V. Skokov, P. Tribedy, R. Venugopalan, in preparation

Initial State Flow

At high energy → high density gluon matter described by **Color Glass Condensate**

McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204

High gluon density in QCD generates dynamical saturation scale $Q_s^2 \sim A^{1/3} s^\lambda$



Intuitive picture of CGC:

Nucleus becomes saturated with high occupancy gluons for $k_T < Q_s$

For $k_T \gg Q_s$ smooth matching of framework to pQCD

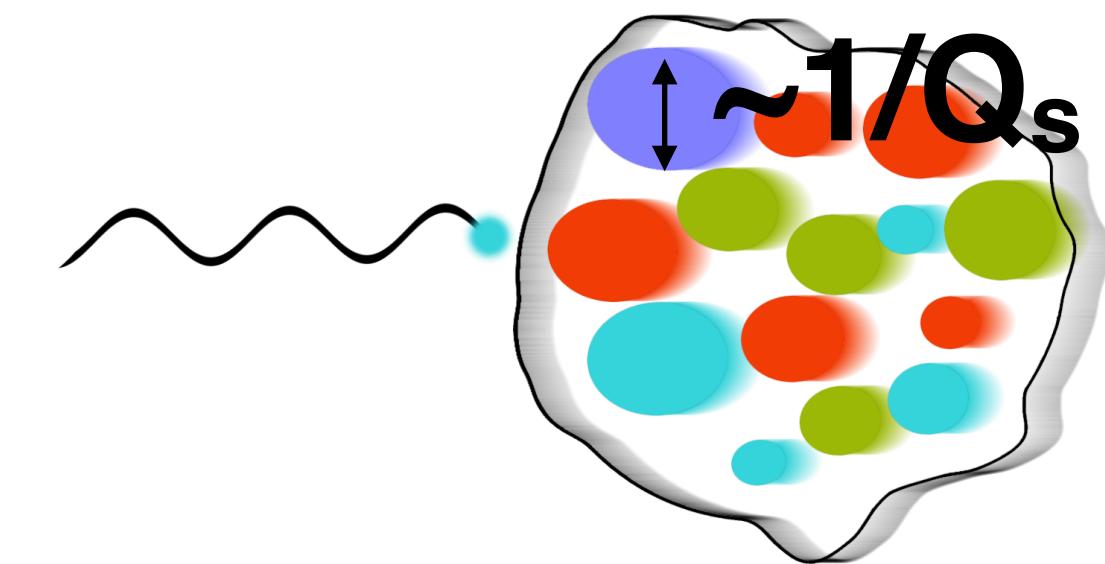
Note: Very strongly correlated system. Dependence on coupling drops out

This talk: CGC has “flow” in line with observations

A parton model

Eikonal quark scattering off dense nuclear target with color domains of size $\sim 1/Q_s$

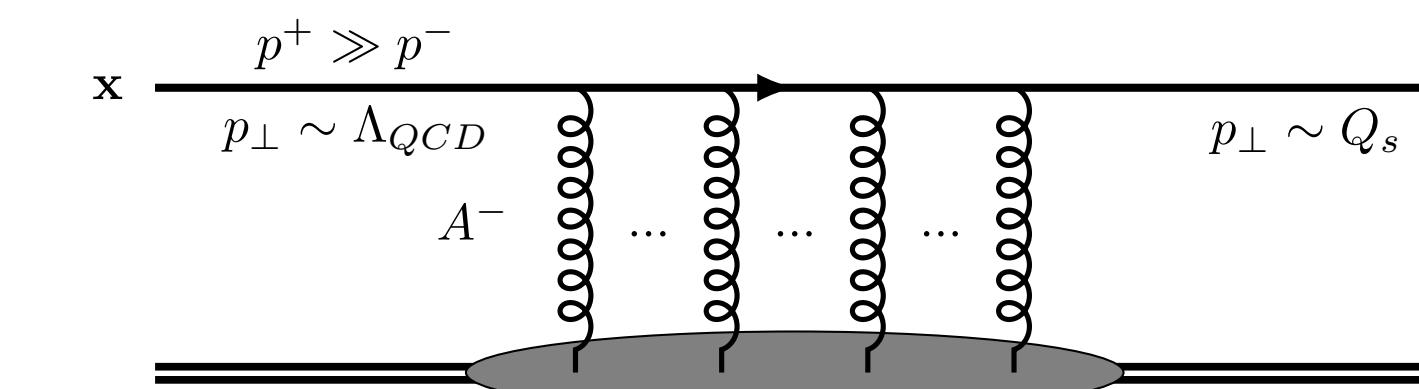
Lappi, PLB 744, 315 (2015); Lappi, Schenke, Schlichting, Venugopalan, JHEP 1601 (2016); Dusling, MM, Venugopalan PRL 120 (2018), PRD 97 (2018)



Quark coherent multiple scattering off target represented by Wilson line phase

Bjorken, Kogut, Soper, PRD (1971), Dumitru, Jalilian-Marian, PRL 89 (2002)

$$U(\mathbf{x}) = \mathcal{P}\exp\left(-ig \int dz^+ A^{a-}(\mathbf{x}, z^+) t^a\right)$$



Single inclusive quark distribution

$$\left\langle \frac{dN_q}{d^2\mathbf{p}} \right\rangle \simeq \int_{\mathbf{b}, \mathbf{r}, \mathbf{k}} e^{-|\mathbf{b}|^2/B_p} e^{-|\mathbf{k}|^2 B_p} e^{i(\mathbf{p}-\mathbf{k}) \cdot \mathbf{r}} \left\langle \frac{1}{N_c} \text{Tr} \left(U(\mathbf{b} + \frac{\mathbf{r}}{2}) U^\dagger(\mathbf{b} - \frac{\mathbf{r}}{2}) \right) \right\rangle$$

Projectile: Wigner function

*Single scale defines projectile $B_p = 4 \text{ GeV}^{-2}$

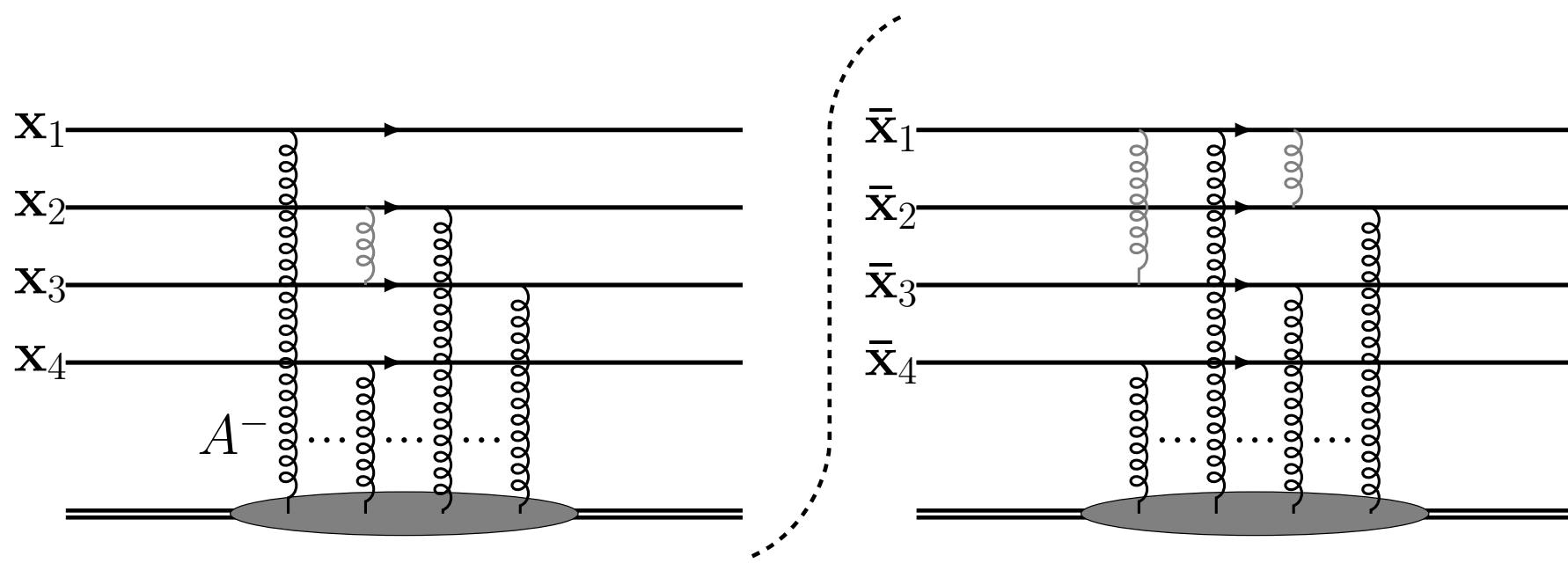
Target scattering:
Dipole operator $D(x, y)$

A parton model

Generalize to multiple particle correlations with a *simple* parton model

Introduced novel method to compute arbitrary
Wilson line correlators in MV - arXiv:1706.06260

$$\left\langle \frac{d^m N}{d^2 \mathbf{p}_1 \dots d^2 \mathbf{p}_m} \right\rangle = \left\langle \frac{dN}{d^2 \mathbf{p}_1} \dots \frac{dN}{d^2 \mathbf{p}_m} \right\rangle \sim \int \boxed{\langle D \dots D \rangle}$$



Average over color correlations in single event and all events

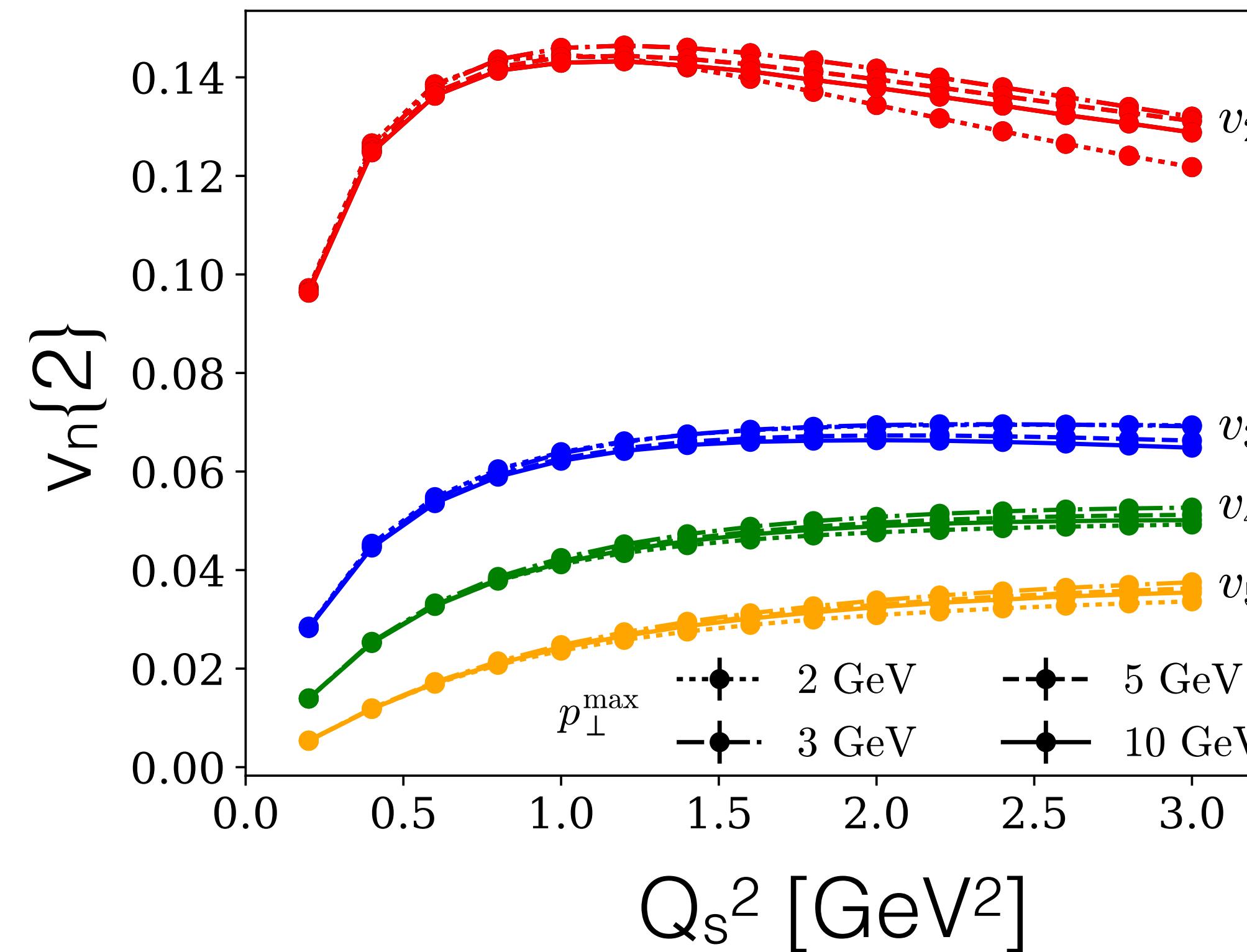
McLerran, Venugopalan, PRD 49, 3352, 2233 (1994)

Generate cumulants, integrate to scale p_{\perp}^{max}

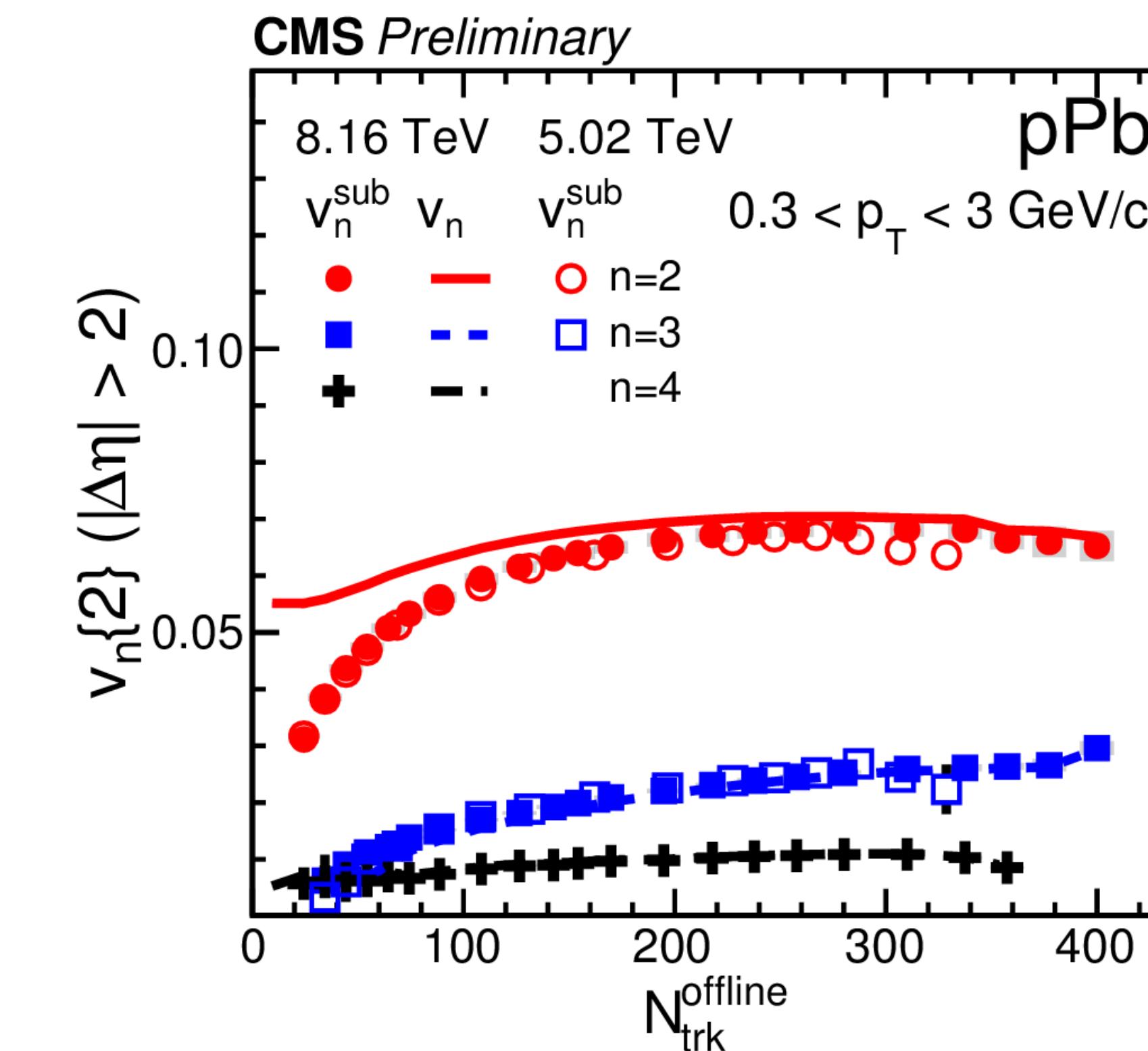
$$\kappa_n\{m\} = \int_{\mathbf{p}_1 \dots \mathbf{p}_m} \cos(n(\phi_1^p + \dots + \phi_m^p)) \left\langle \frac{d^m N}{d^2 \mathbf{p}_1 \dots d^2 \mathbf{p}_m} \right\rangle , \quad c_2\{2\} = \frac{\kappa_2\{2\}}{\kappa_0\{2\}} , \dots$$

Multi-particle quark correlations

Ordering in two particle Fourier harmonics similar to data



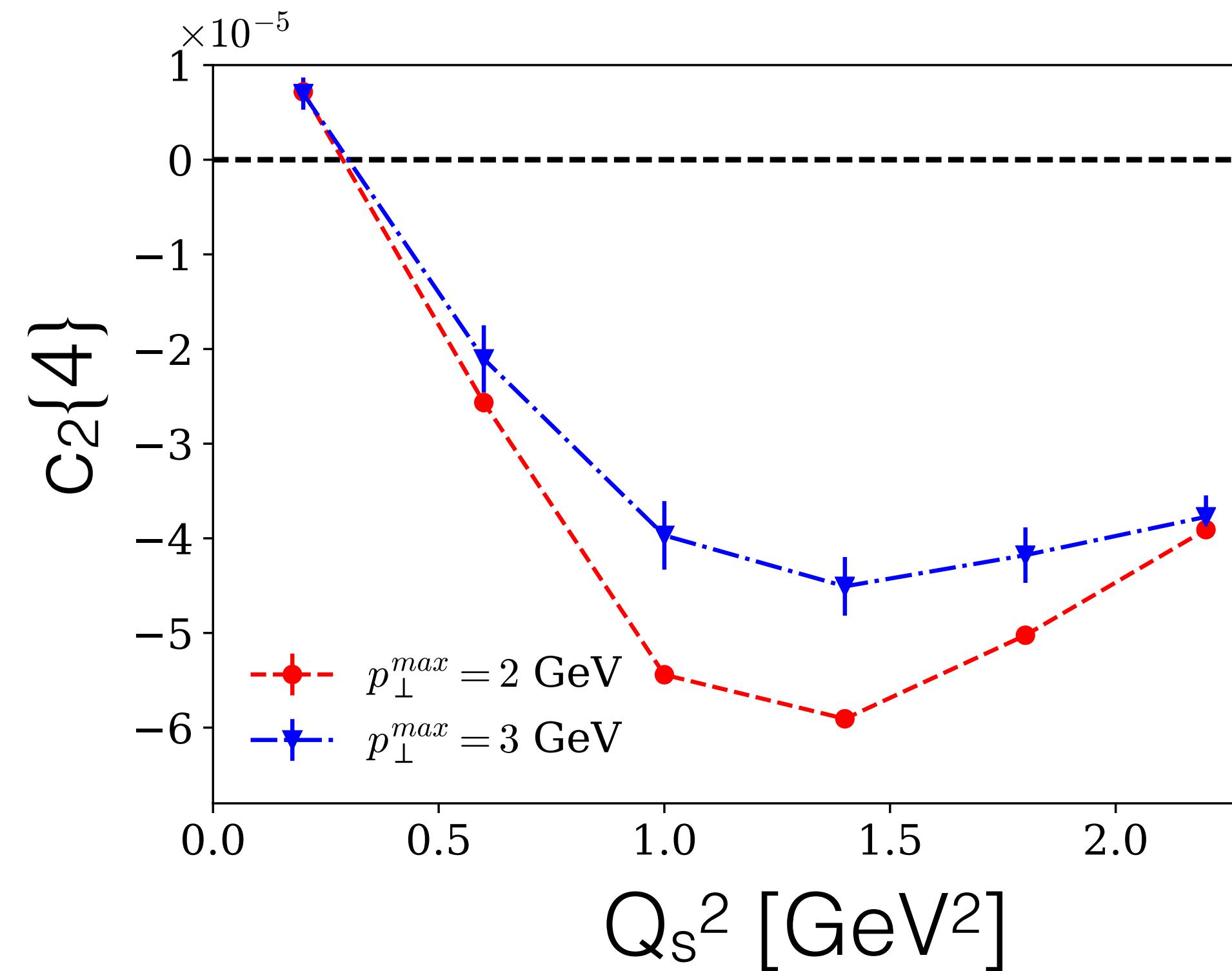
Dusling, MM, Venugopalan PRL 120 (2018)



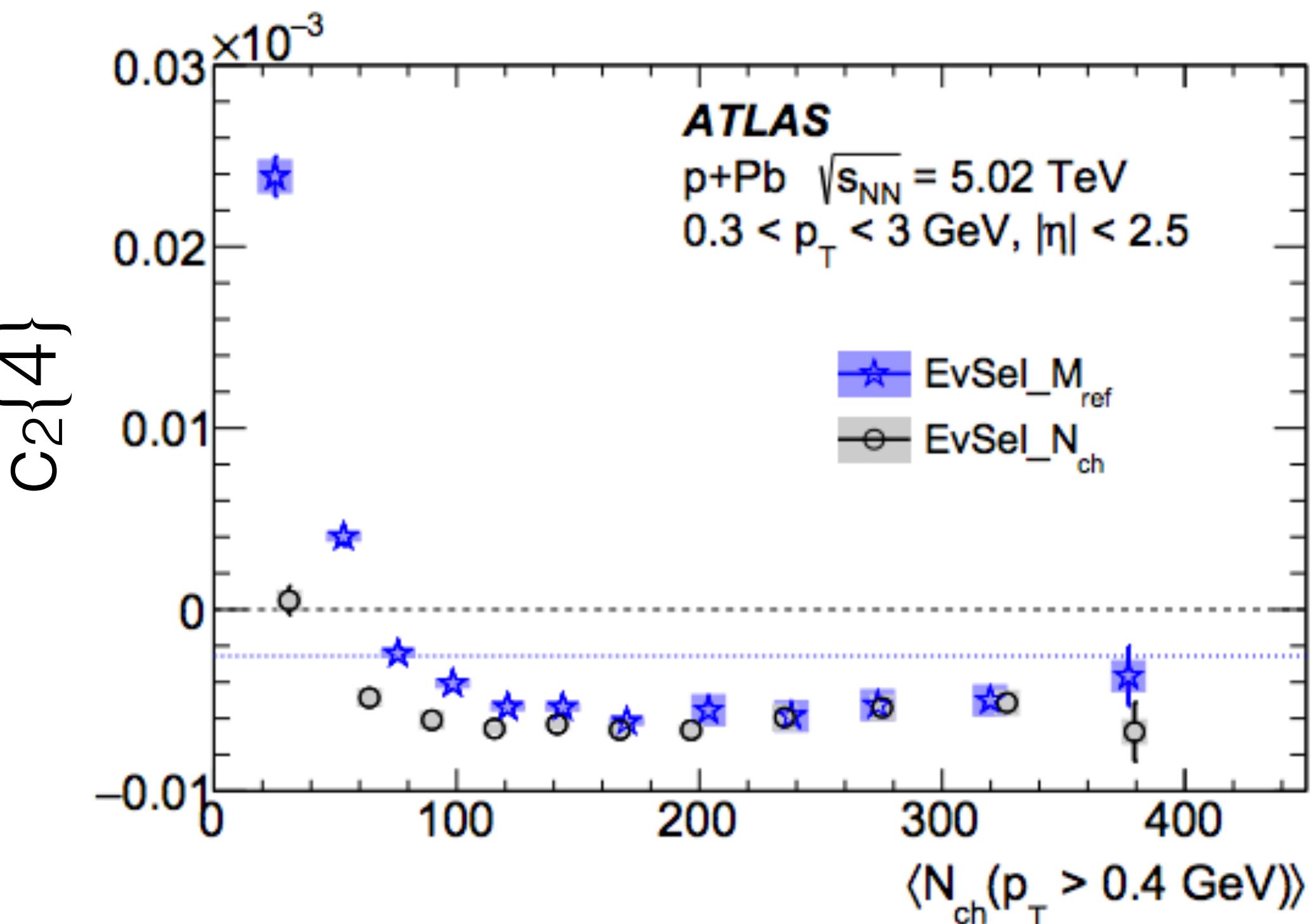
CMS-PAS-HIN-16-022

Multi-particle quark correlations

$C_2\{4\}$ becomes negative for increasing Q_s



Dusling, MM, Venugopalan PRD 97 (2018)

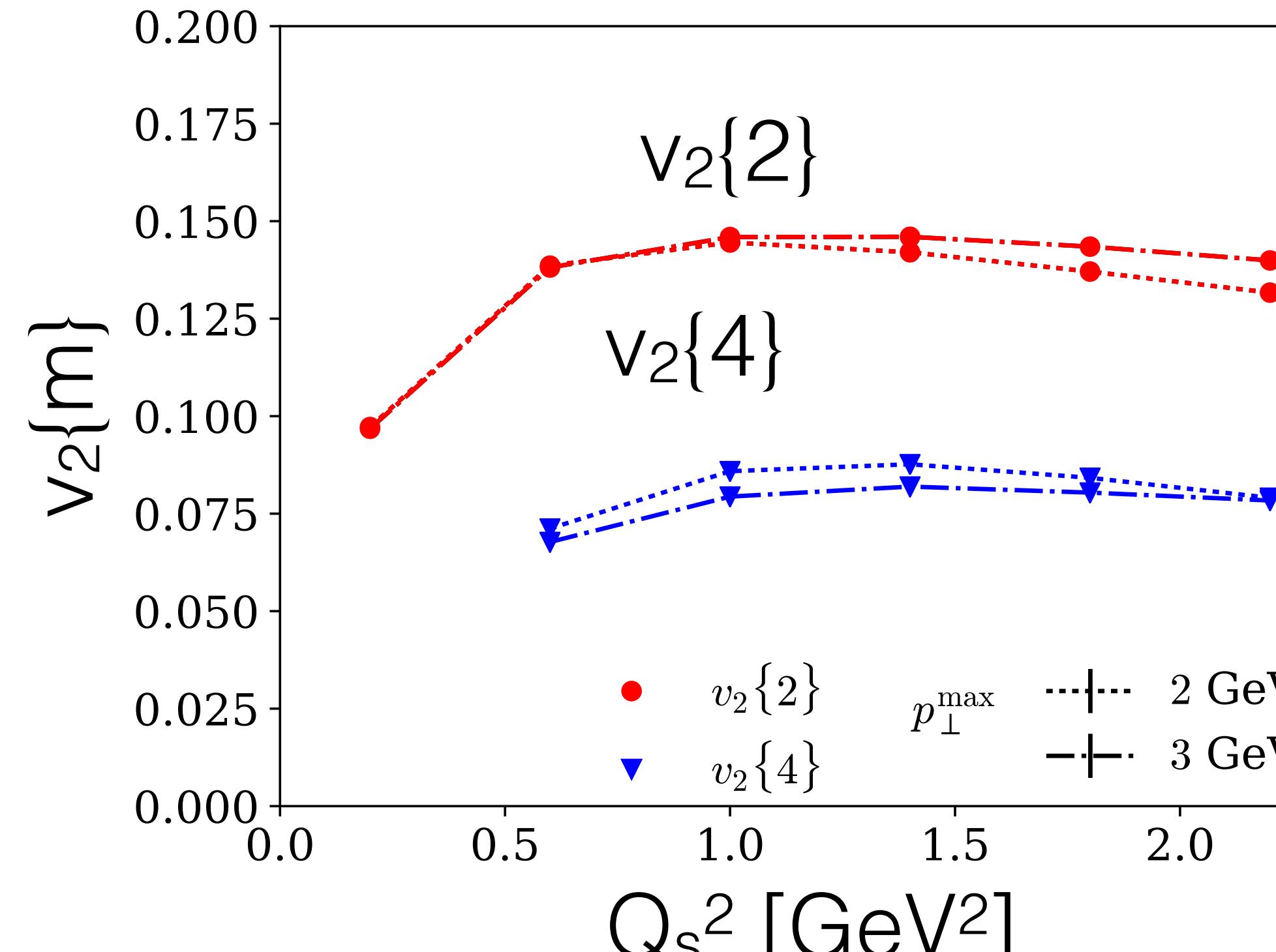


ATLAS EPJC 77 (2017)

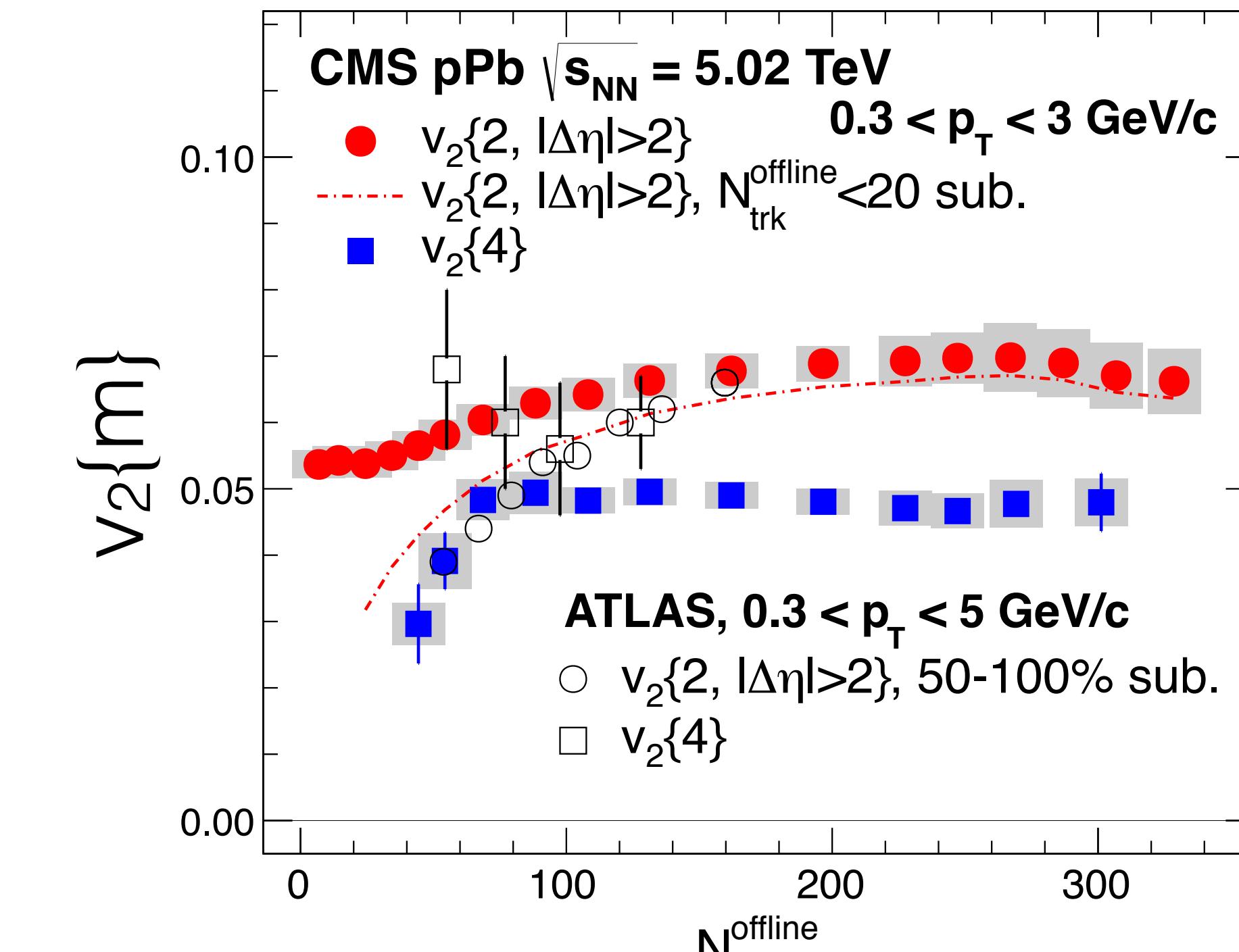
Mild dependence on maximum integrated p_{\perp}

Multi-particle quark correlations

Obtain real $v_2\{4\}$, fairly independent of ‘multiplicity’



Dusling, MM, Venugopalan PRL 120 (2018)



CMS PLB 724 (2013) 213

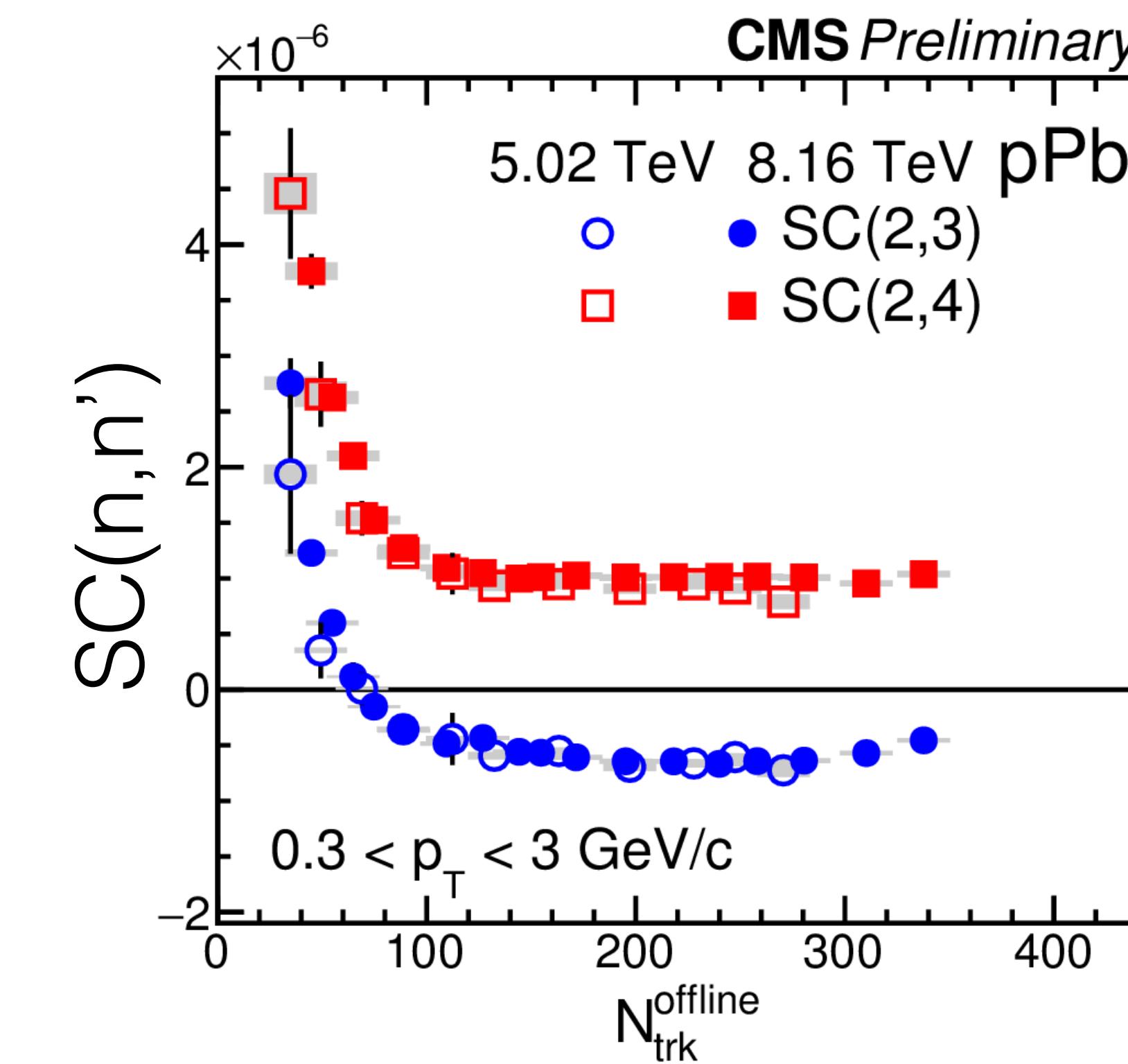
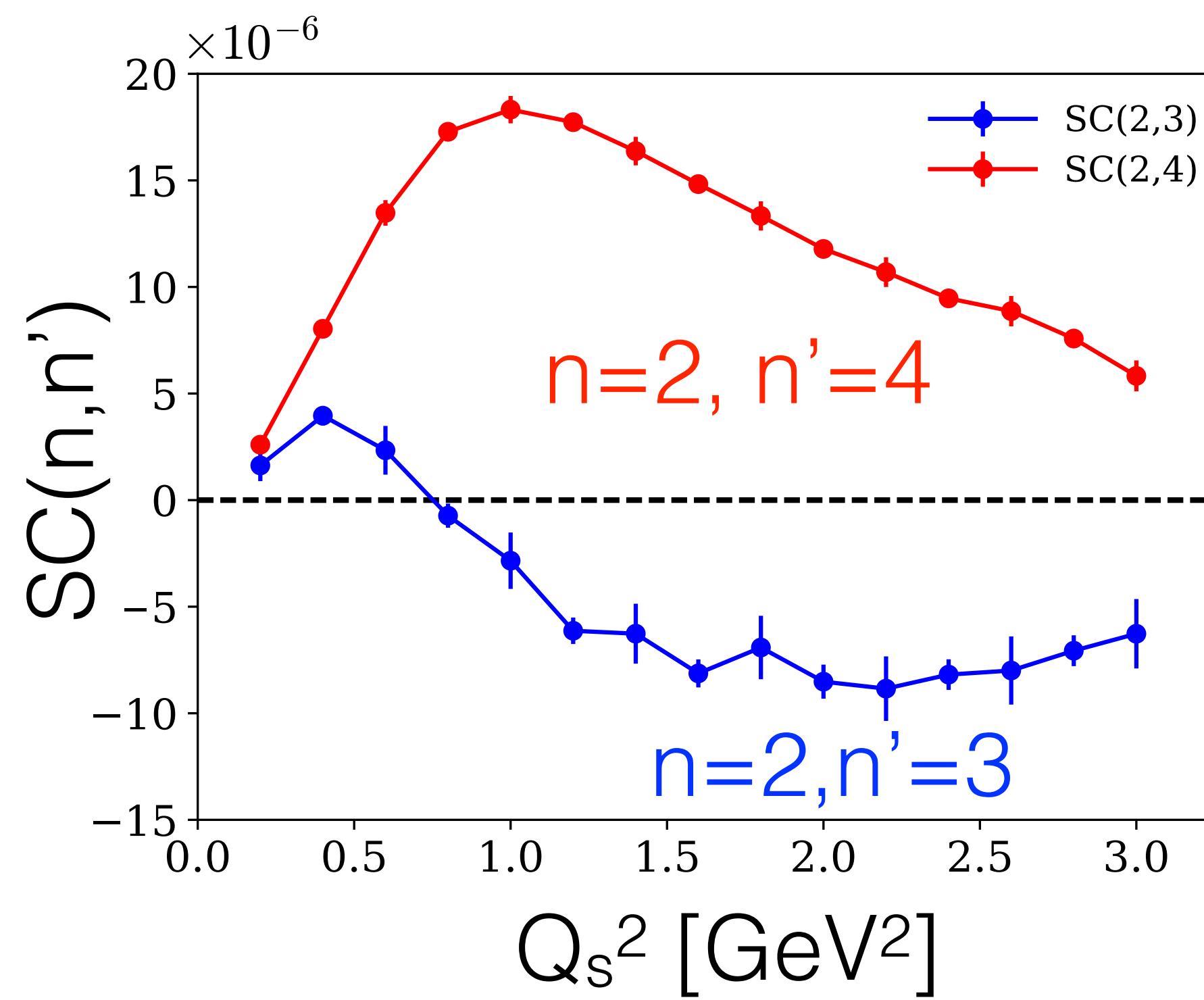
No inverse scaling by number of domains in CGC and data

Symmetric Quark Cumulants

Symmetric cumulants: mixed harmonic cumulants

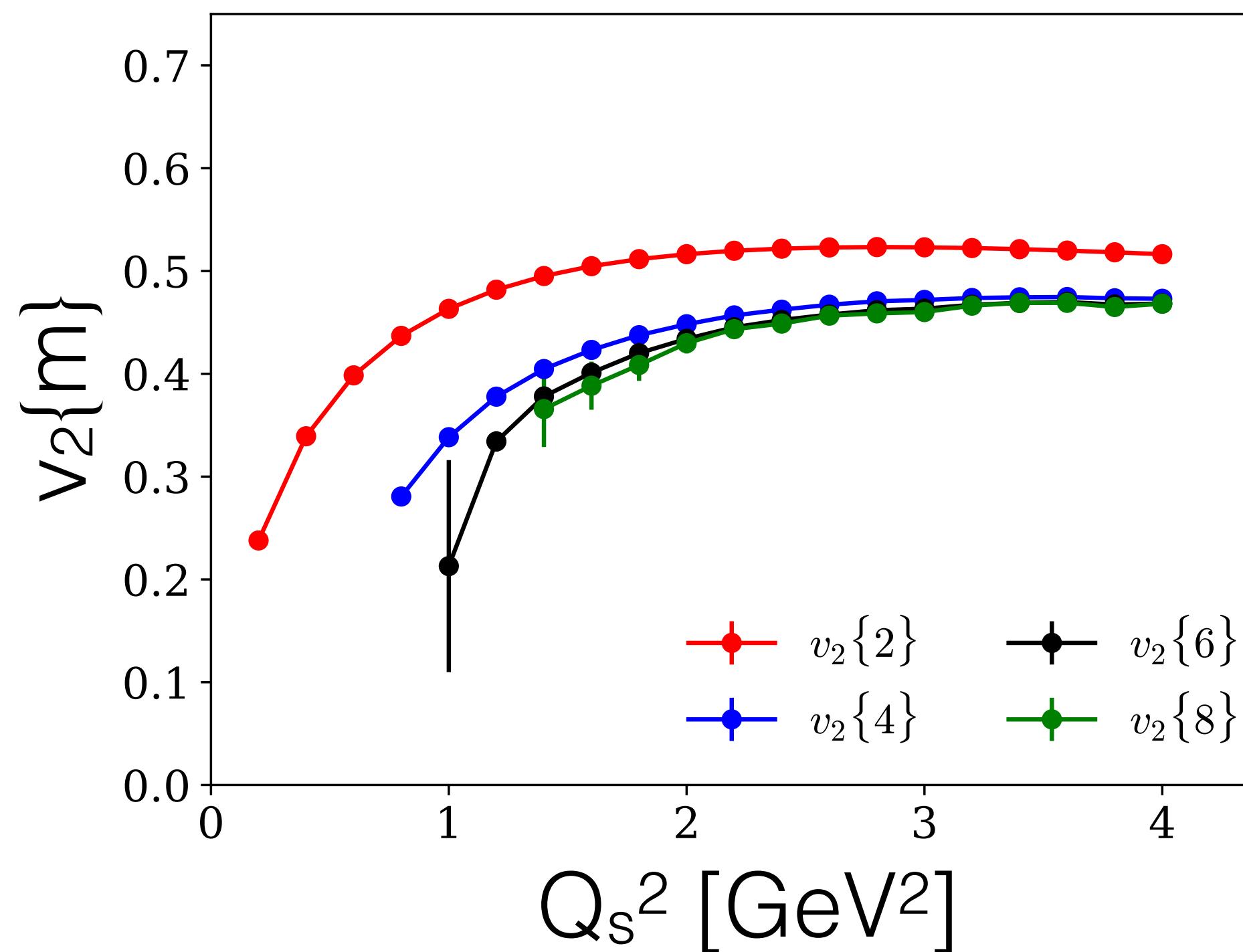
$$SC(n, n') = \langle e^{i(n(\phi_1 - \phi_3) - n'(\phi_2 - \phi_4))} \rangle - \langle e^{in(\phi_1 - \phi_3)} \rangle \langle e^{in'(\phi_2 - \phi_4)} \rangle$$

Bilandzic et al, PRC 89, no. 6, 064904 (2014)

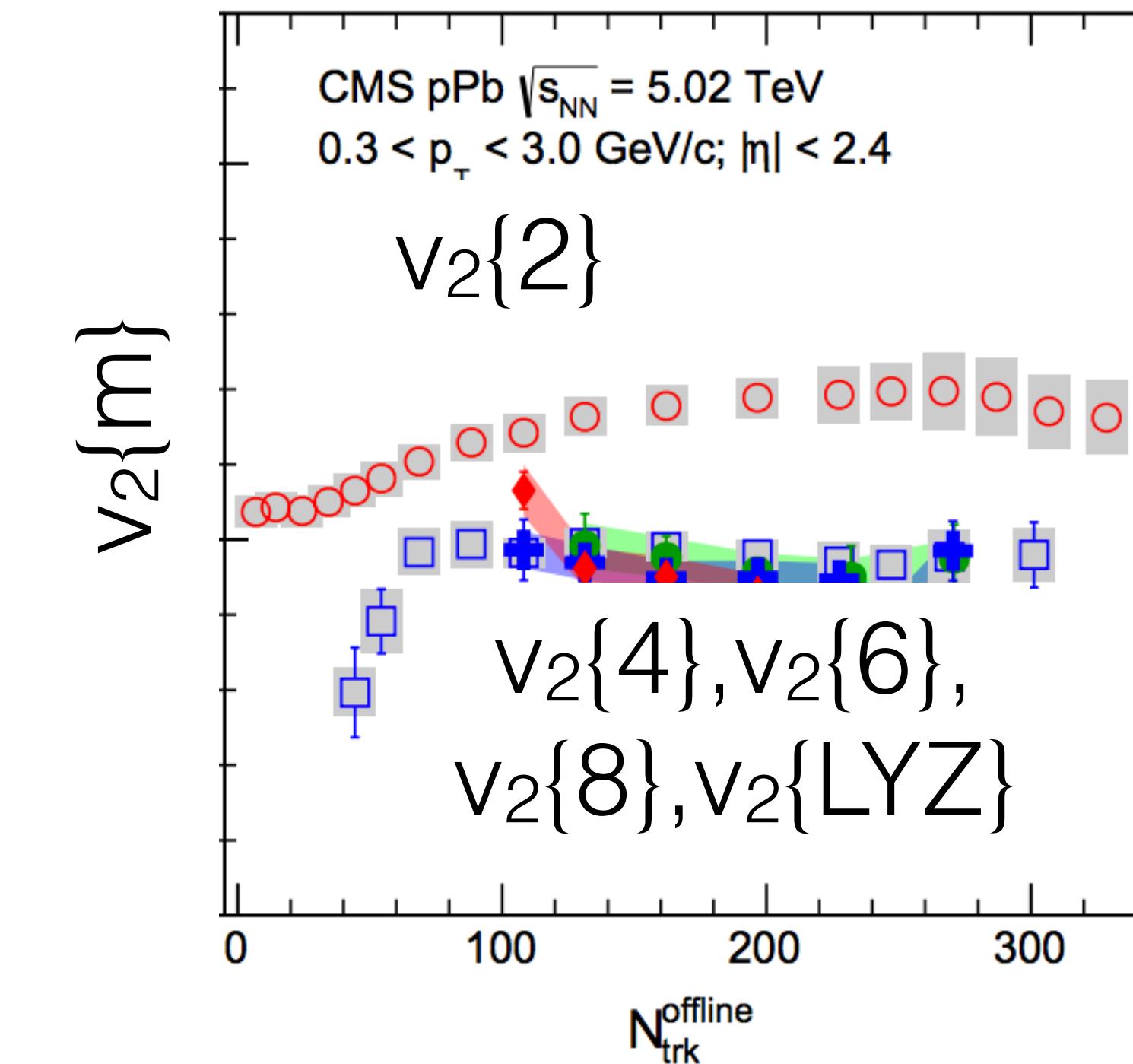


Collectivity from parton model

For computational reduction, consider Abelian version



Dusling, MM, Venugopalan PRL 120 (2018)



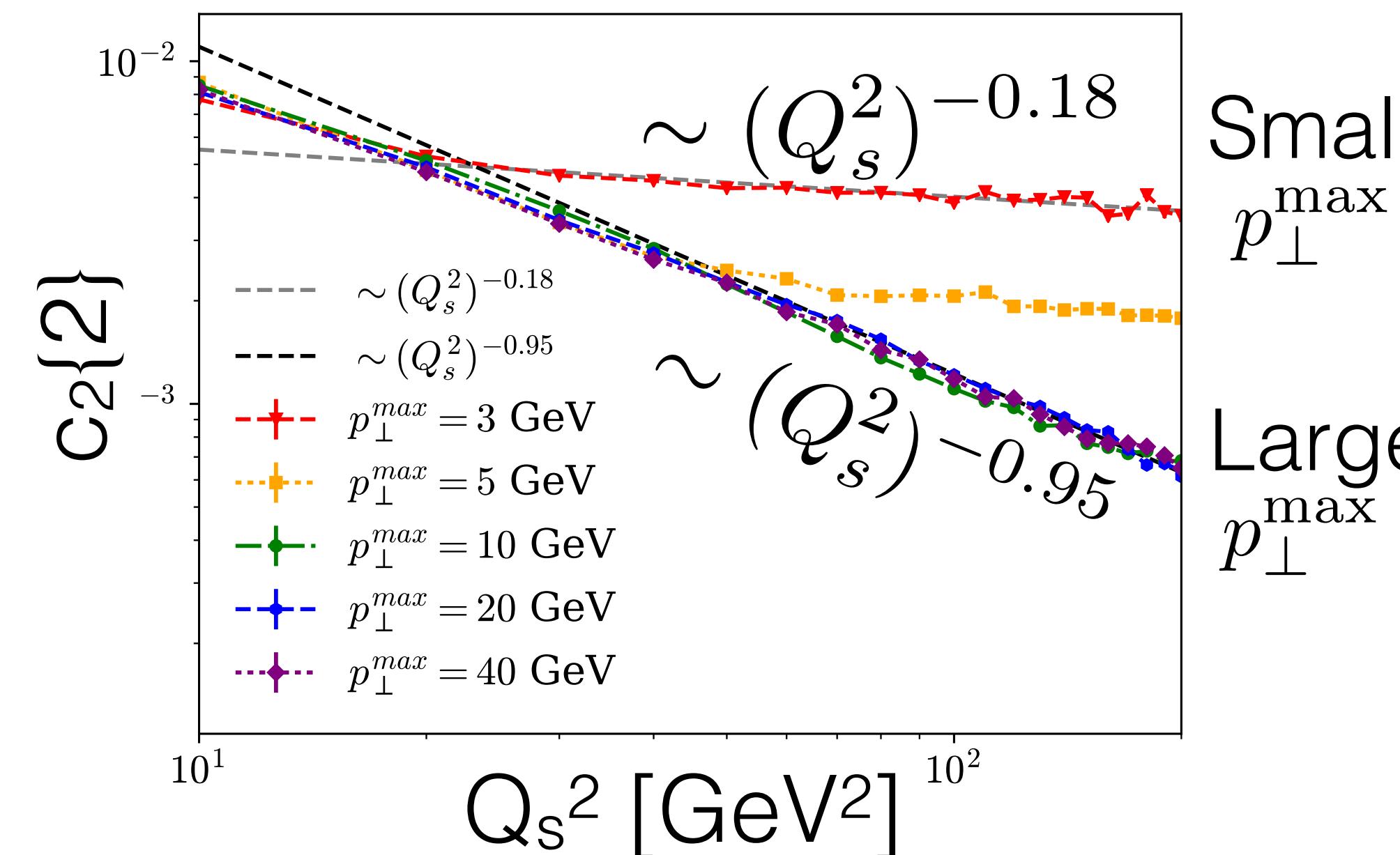
CMS PRL 115 (2015) 012301

Clear demonstration that $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$
collectivity not unique to hydrodynamics

Yan, Ollitrault PRL 112 (2014) 082301, Bzdak, Skokov NPA 943 (2015)

Scale dependence

Two dimensionless scales: $Q_s^2 B_p$, the number of domains, and the ratio of resolution scales, $Q_s^2 / (p_{\perp}^{\max})^2$



$(p_{\perp}^{\max})^2 \lesssim Q_s^2$: probe coarse graining over multiple domains

$(p_{\perp}^{\max})^2 \gtrsim Q_s^2$: probe resolves area less than domain size

Scaling with inverse number of domains seen only for large p_{\perp}^{\max}

Comparison to glasma graphs

Glasma graph approximation, valid only for
 $p_\perp > Q_s$, only considers single gluon
exchange

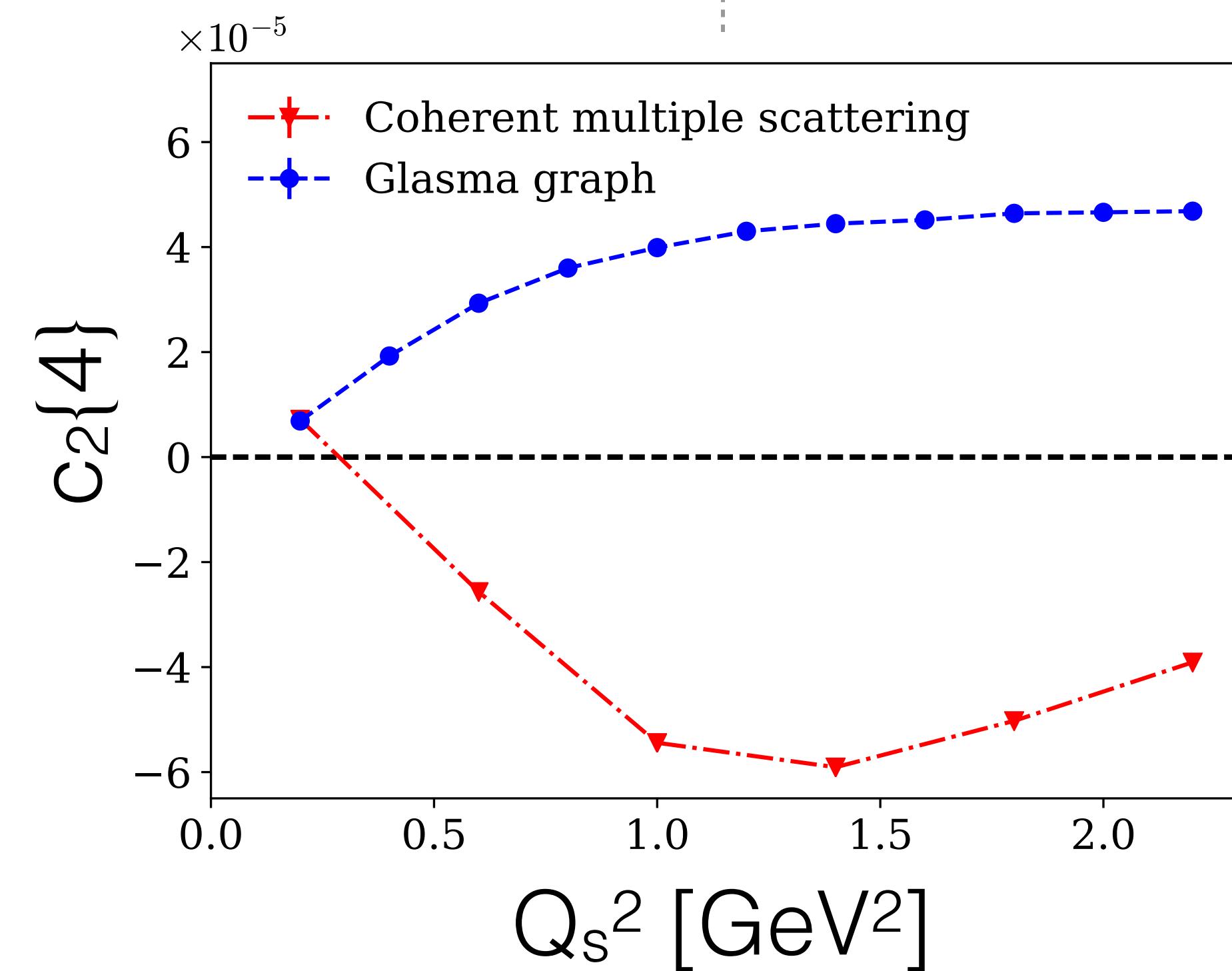
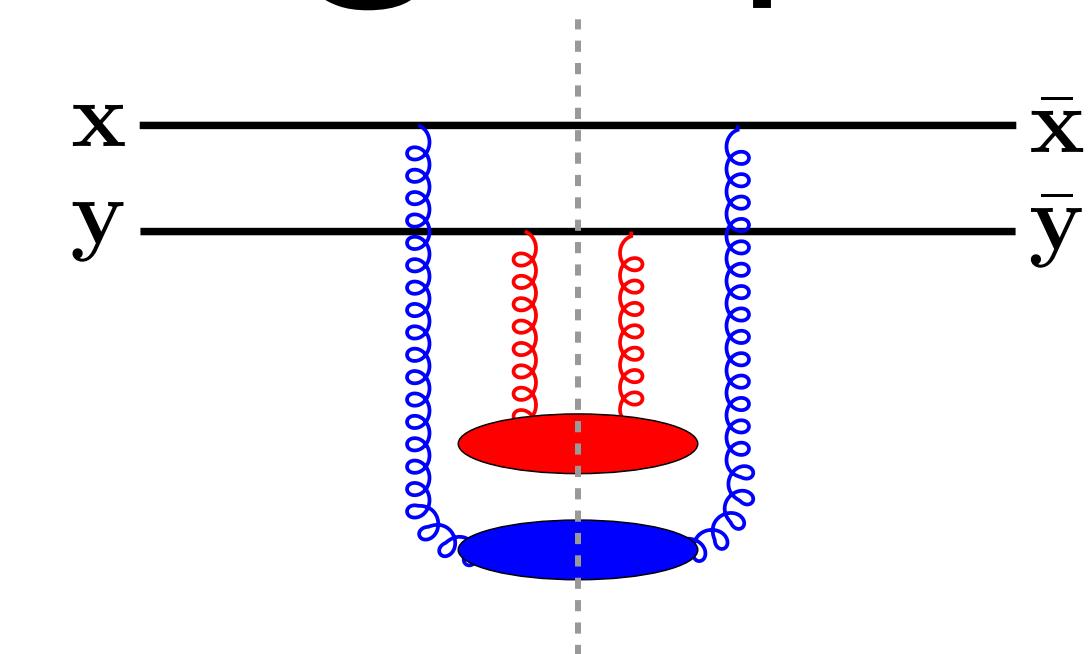
Dumitru, Gelis, McLerran, Venugopalan, NPA 810 (2008),
Dusling, Venugopalan PRL 108 (2012), PRD 87 (2013)

Glasma graphs have very strong
correlations, close to a Bose distribution
(as in a laser)

Gelis, Lappi, McLerran NPA 828 (2009)

For higher particle cumulants and
harmonics, Glasma graph gives $c_2\{4\} > 0$

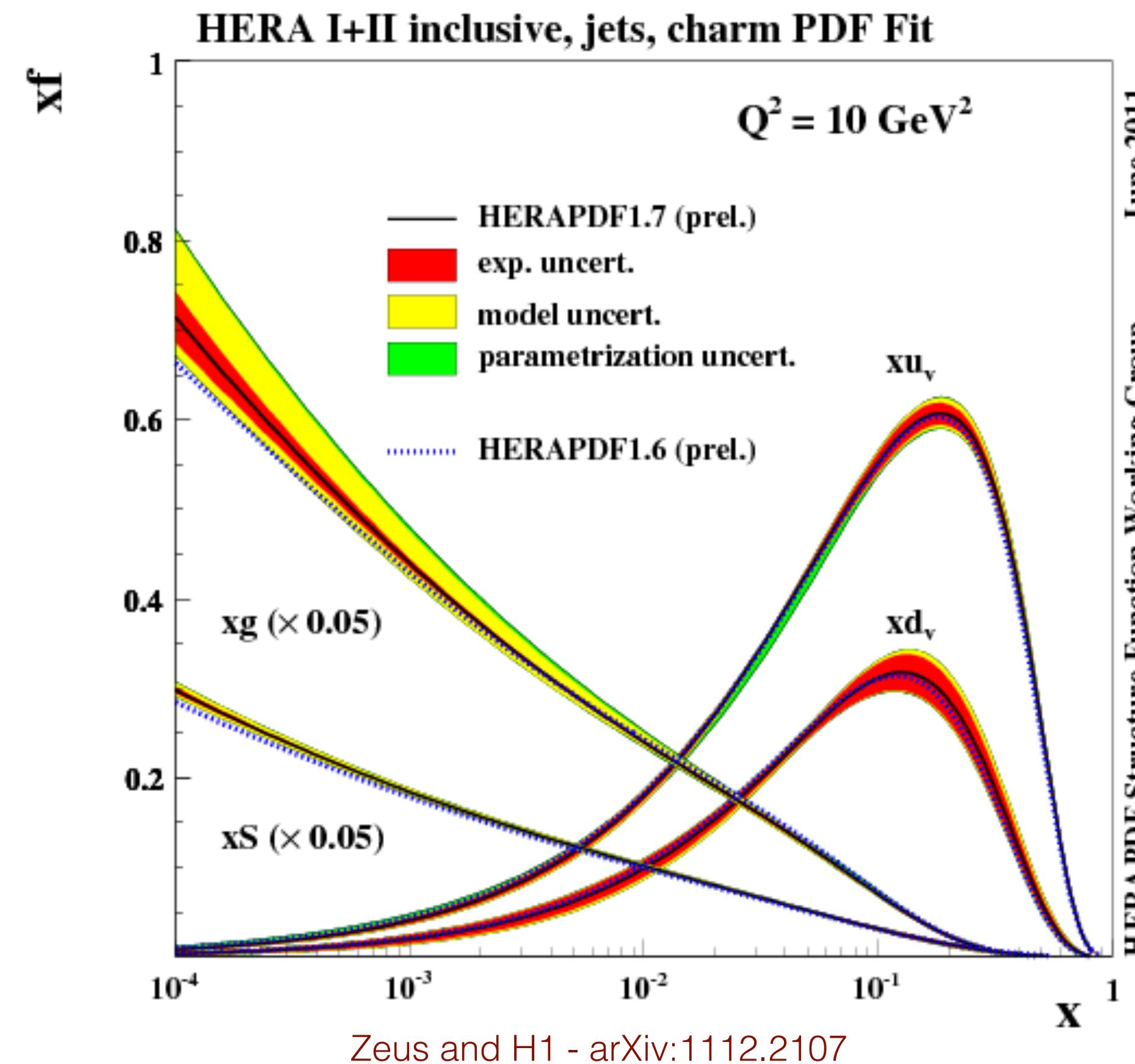
Dusling, MM, Venugopalan PRD 97 (2018)



Multiple scattering suppresses higher cumulants $\rightarrow c_2\{4\} < 0$

The role of glue?

Previous discussion only included quarks scattering off CGC...



What about gluons, which are dominant at small x or high energies?

Dilute-dense CGC EFT framework

Can compute scattering of gluons off saturated nuclear target in dilute-dense CGC

Kovner, Wiedemann PRD 64 (2001)
Dumitru, McLerran NPA 700 (2002),
Blaizot, Gelis, Venugopalan NPA 743 (2004)
McLerran, Skokov NPA 959 (2017)
Kovchegov, Skokov arXiv:1802.08166

Glauber+IP-Sat initial conditions

Kowalski, Teaney, Phys.Rev. D68 (2003),
Schenke, Tribedy, Venugopalan PRL 108 (2012)

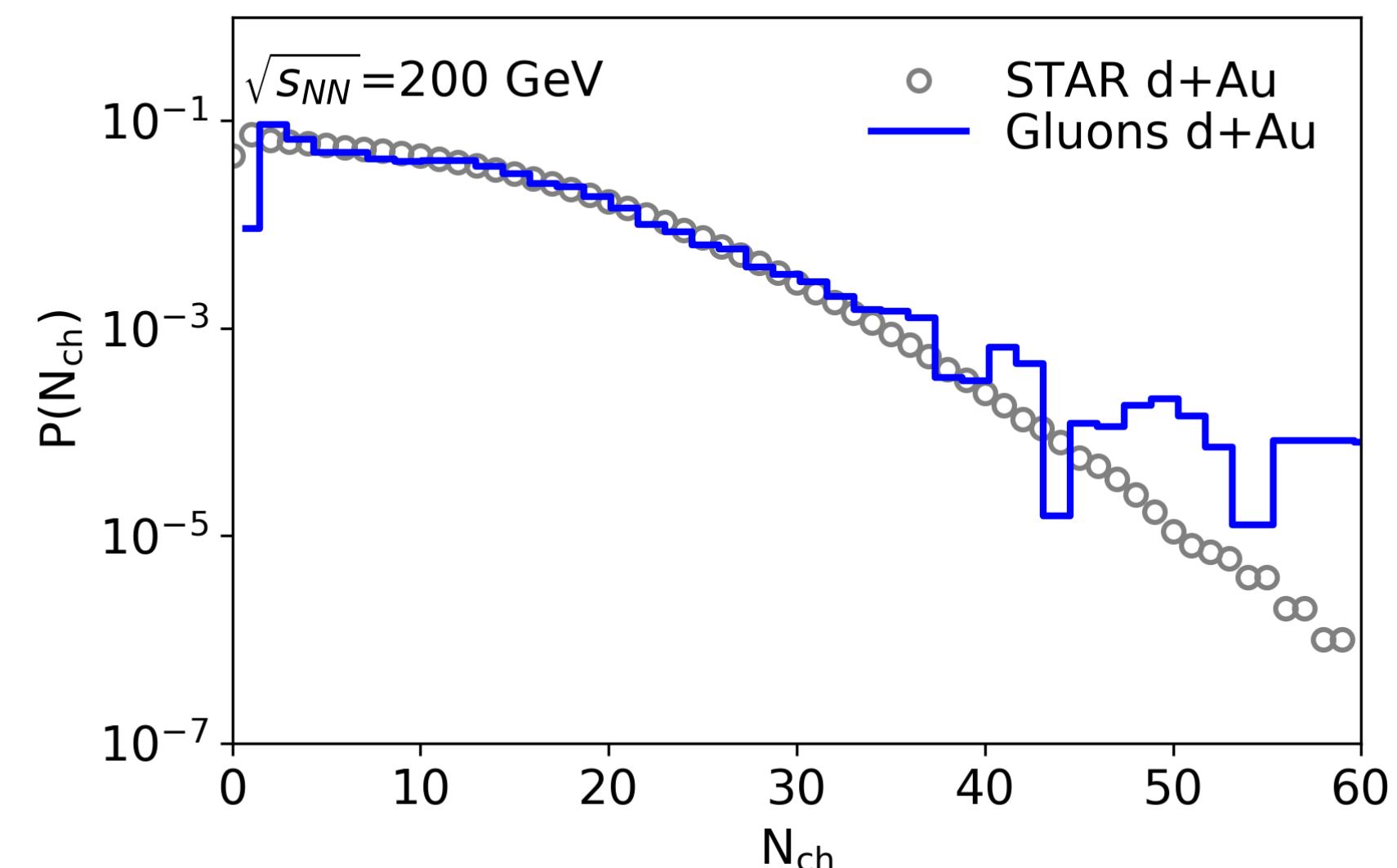
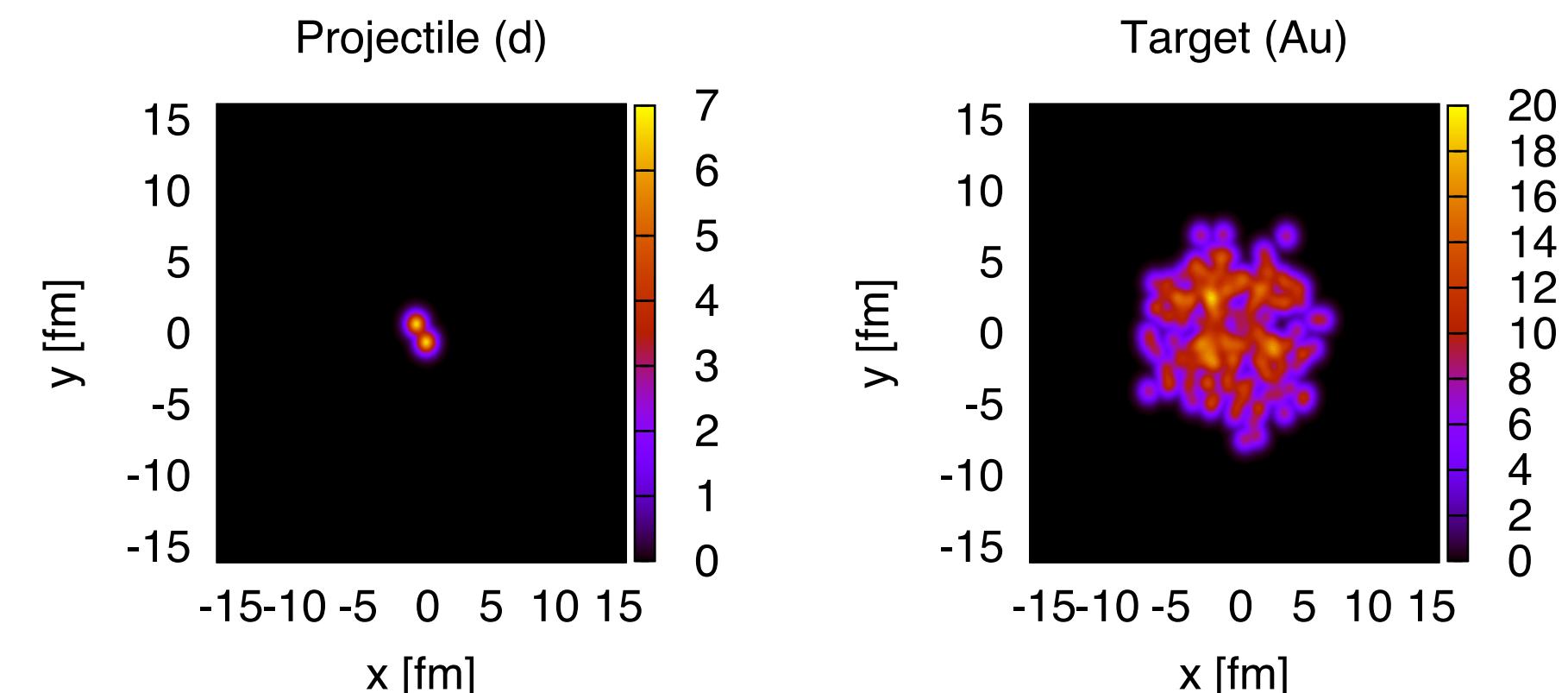
Generates negative binomial distributions from first principles, *not an input!*

Schenke, Tribedy, Venugopalan PRC 86 (2012)
McLerran, Tribedy NPA 945 (2016)

Good agreement found with STAR d+Au multiplicity distribution

See talk by P. Tribedy (Tuesday 12:10) - for more comparisons to multiplicity distributions in p+p at RHIC & LHC

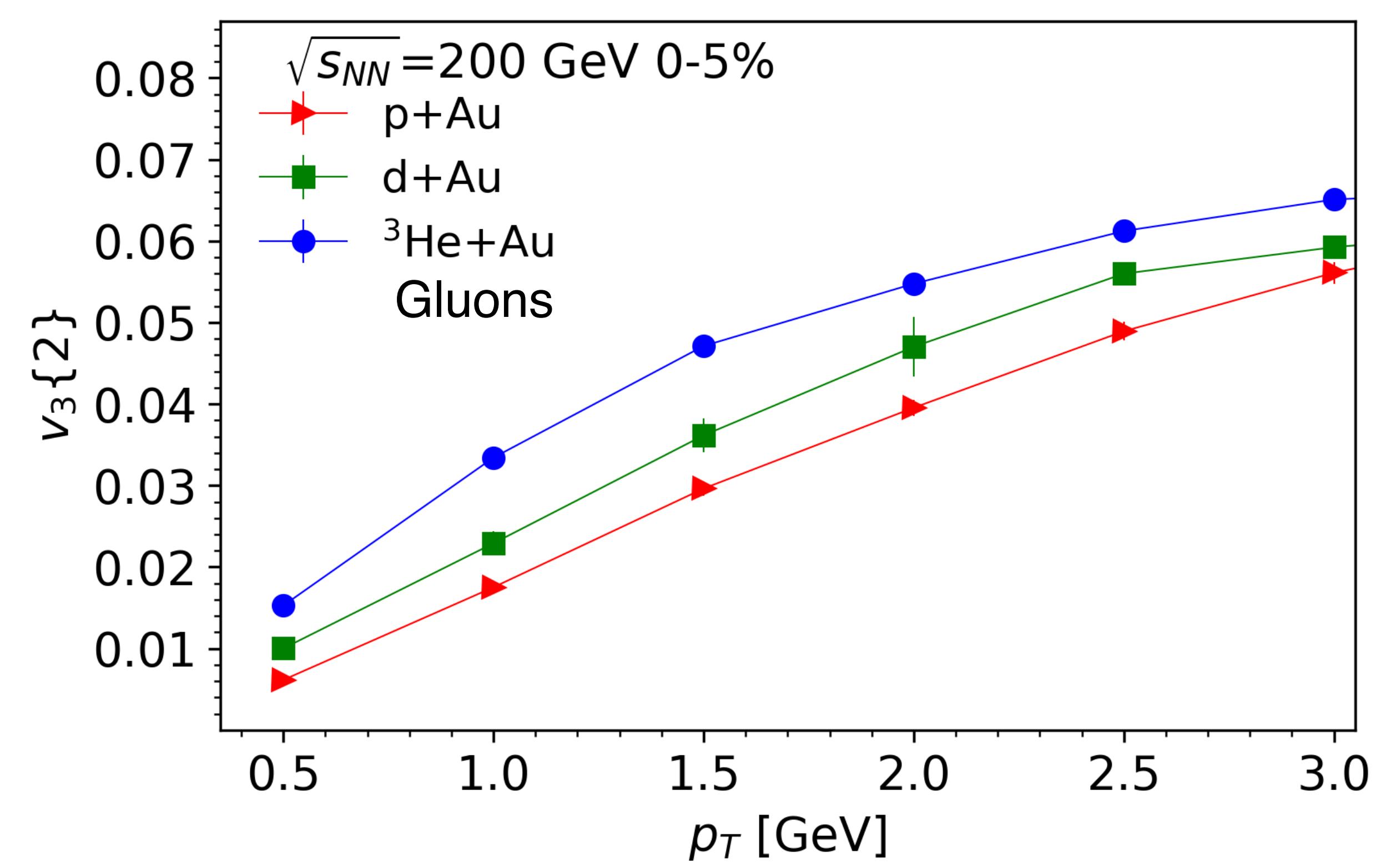
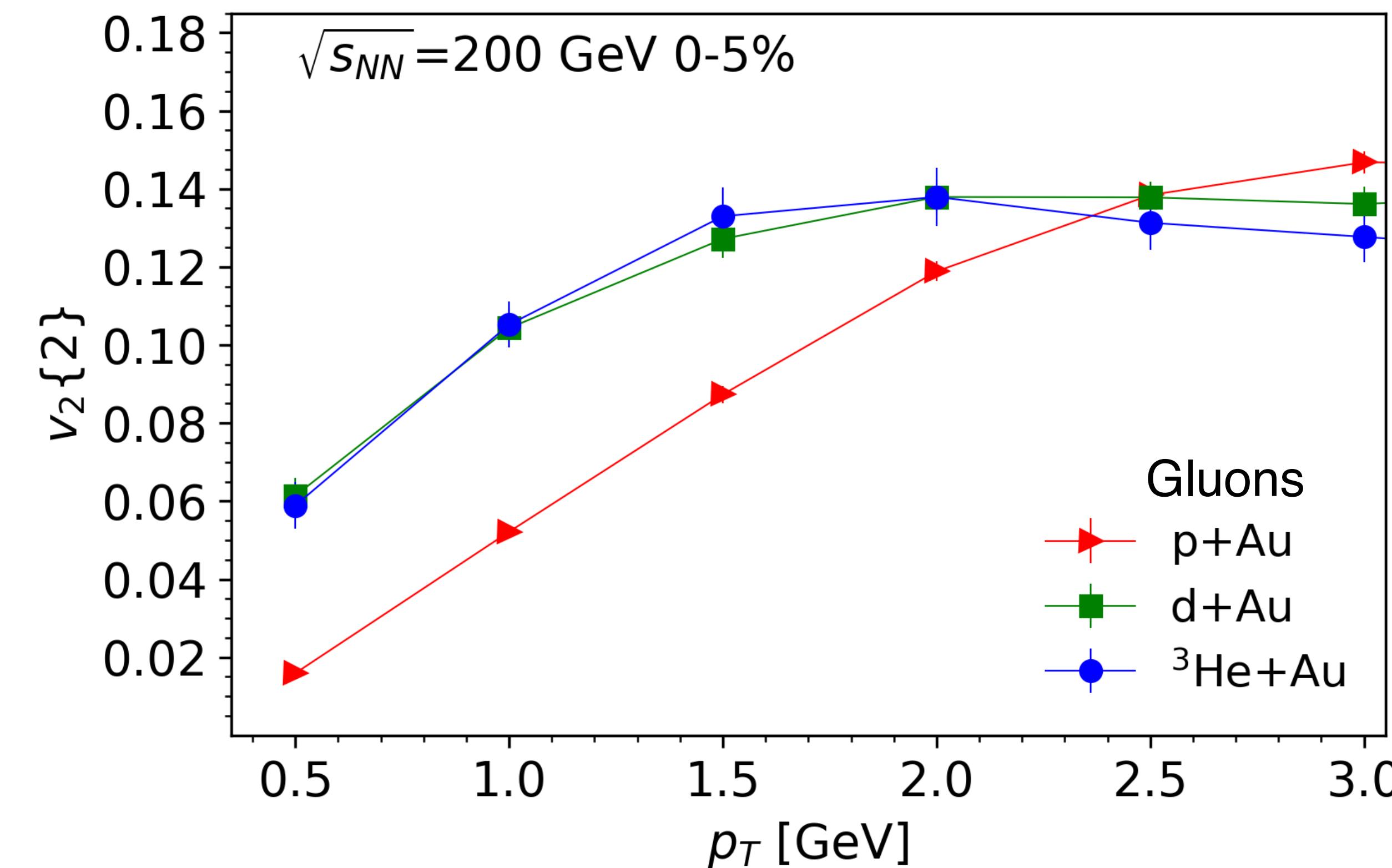
Color charge density



MM, Skokov, Tribedy, Venugopalan, in preparation
STAR PRC 79 (2009)

Hierarchy of anisotropies across systems

System size dependence at RHIC captured by CGC initial state gluon correlations



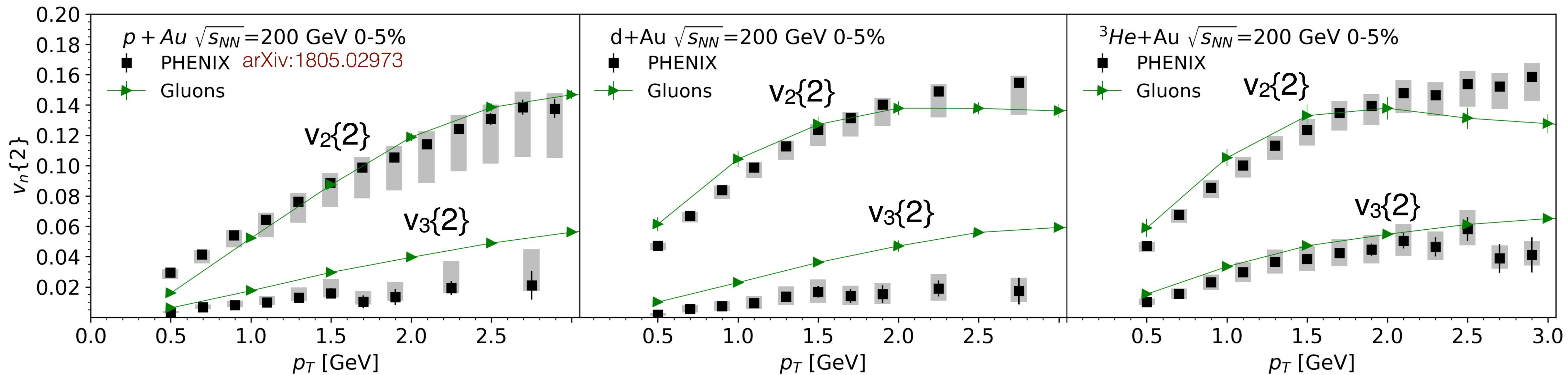
Quantifying systematic uncertainties

- Dilute-dense approximation: high density effects need to be quantified
- All parameters are fixed, even for p and ^3He , by fit to STAR d+Au multiplicity distribution. Would be useful to have p/ $^3\text{He}+\text{Au}$ multiplicity distributions
- Nuclear wave function: strong short-range correlations (measured at JLab). Exciting prospect; quantify influence on high multiplicity events in $^3\text{He}+\text{Au}$

c.f. Hen, Miller, Piasetzky, Weinstein Rev.Mod.Phys. 89 (2017);
Cruz-Torres, Schmidt, Miller, Weinstein, Barnea, Weiss, Piasetzky, Hen arXiv:1710.07966
Hen, MM, Schmidt, Venugopalan, in progress.
- Fragmentation: See talk by P. Tribedy (Tuesday 12:10)

e.g. Schenke, Schlichting, Tribedy, Venugopalan, PRL 117 (2016) no.16, 162301

Gluon correlations vs RHIC data for small systems



Key features of system dependence captured by initial state gluon correlations

Multi-particle gluons correlations

Finite $v_2\{4\}$ seen in high multiplicity d+Au collisions.

Due to high statistics needed, results will not be shown here.

Detailed comparison across system size and different energies is feasible
and will be presented elsewhere

Conclusions

Multiparticle collectivity demonstrated through purely initial state correlations with simple proof of principle parton model

Dusling, MM, Venugopalan PRL 120, 042002 (2018), PRD 97, 016014 (2018)

Full dilute-dense CGC framework able to describe system size hierarchy of v_2 and v_3 at RHIC — systematic uncertainties need to be quantified further

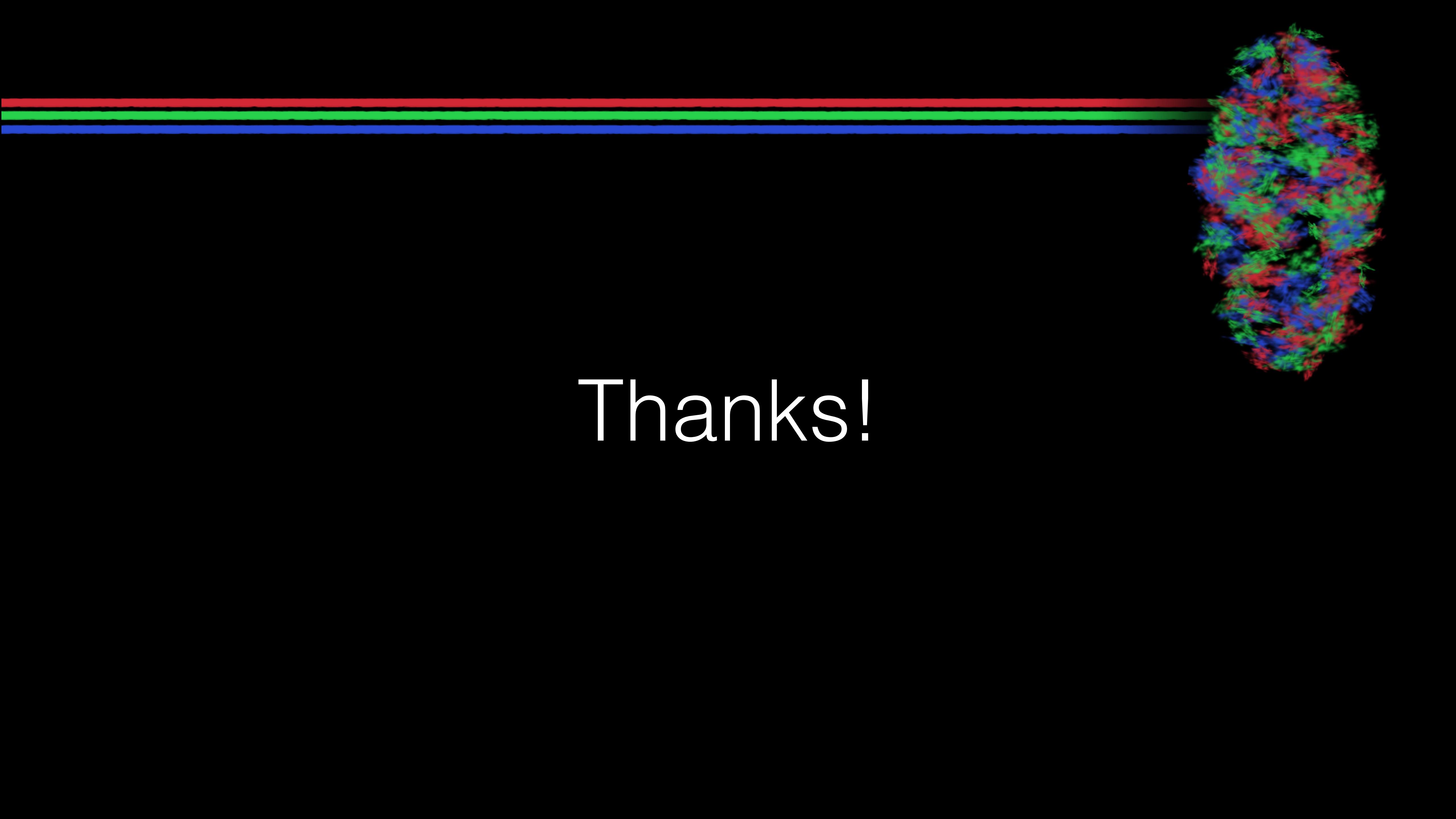
MM, Skokov, Tribedy, Venugopalan, in preparation, McLerran, Skokov NPA 959 (2017)

To distinguish between hydro and initial state explanation, important to have $p/{}^3He$ +Au multiplicity distributions and anisotropies in different event classes

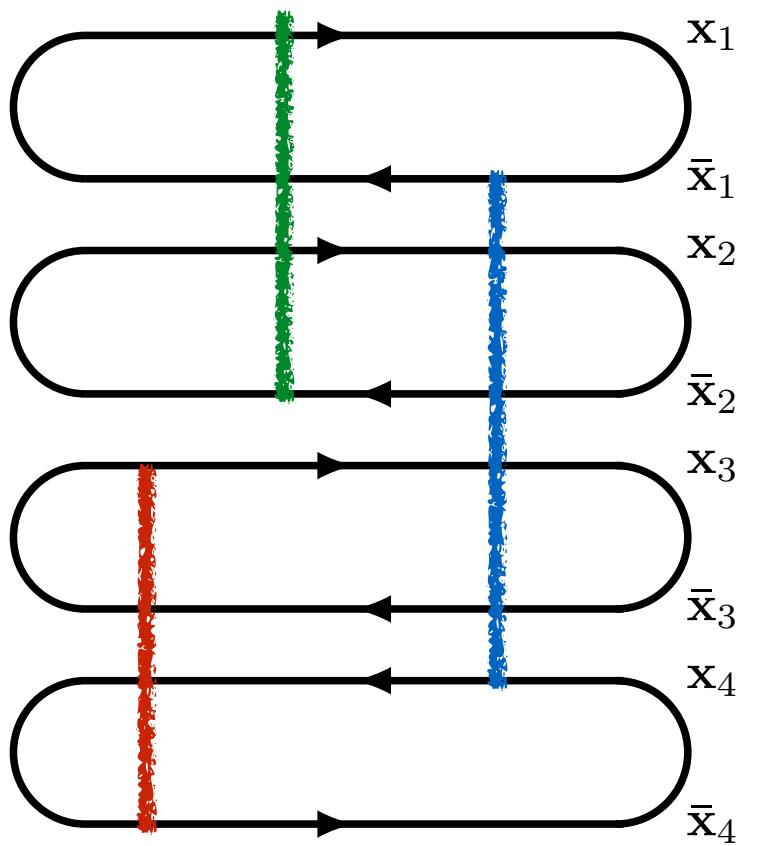
Can compute $v_n\{m\}$ in framework and compare to data, such comparison also important to do in hydrodynamical models for definitive conclusions

CGC EFT allows for computation of both flow and non-flow in same framework. Can quantify to understand their relative contributions

Fukushima, Hidaka NPA 813 (2008), Dusling, Venugopalan PRD 87 (2013)



Thanks!



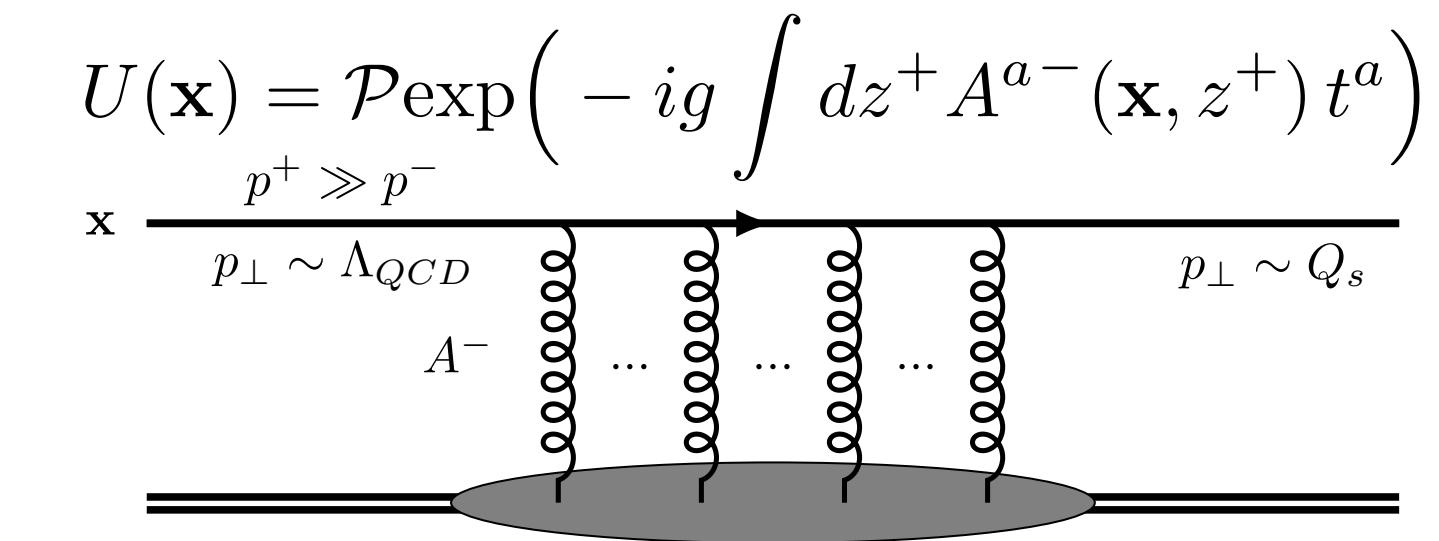
BACKUP

Dipole correlators

First, need to be able to compute correlation functions
expectation values of dipoles

Consider dipole scattering matrix

$$\langle D(\mathbf{x}, \mathbf{y}) \rangle_U = \left\langle \frac{1}{N_c} \text{tr}(U(\mathbf{x}) U^\dagger(\mathbf{y})) \right\rangle$$



Expand out Wilson line in slices in rapidity

$$U(\mathbf{x}) = \mathcal{P}\exp \left(-ig \int dx^+ A^{a-}(\mathbf{x}, x^+) t^a \right) \simeq V(\mathbf{x}) [1 - ig A^{a-}(\zeta, \mathbf{x}) t^a + \dots]$$

Then gluons emissions with MV model

$$g^2 \langle A_a^-(x^+, \mathbf{x}_\perp) A_b^-(y^+, \mathbf{y}_\perp) \rangle = \delta_{ab} \delta(x^+ - y^+) L_{\mathbf{xy}}$$

$$\text{where } L_{\mathbf{x}_\perp, \mathbf{y}_\perp} = -\frac{(g^2 \mu)^2}{16\pi^2} |\mathbf{x} - \mathbf{y}|^2 \log \left(\frac{1}{|\mathbf{x}_\perp - \mathbf{y}_\perp| \Lambda} + e \right)$$

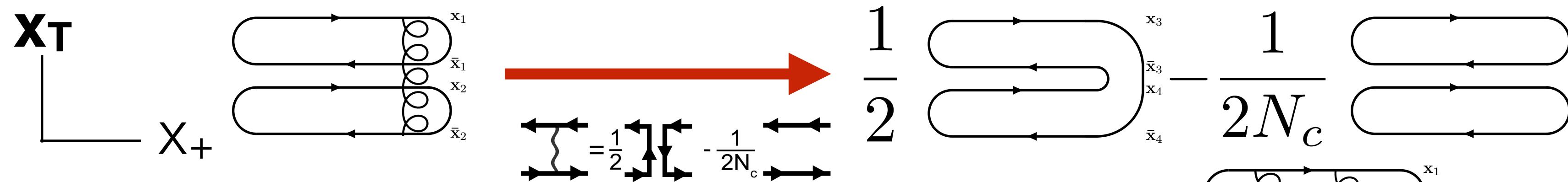
We can re-exponentiate

$$\langle D(\mathbf{x}, \mathbf{y}) \rangle_U = \exp(C_F L(\mathbf{x}, \mathbf{y}))$$

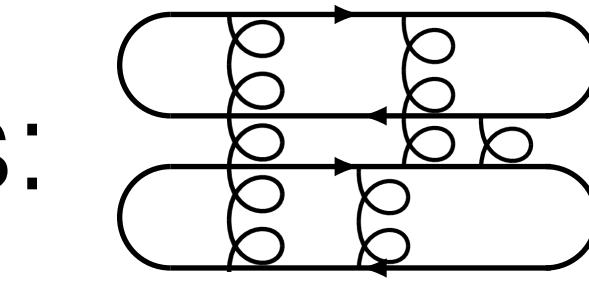
Dipole correlators

Multiple dipole correlation functions encode projectile nucleus scattering, depends on scale Q_s

Then we can obtain $\langle DD \rangle$ similarly, first considering single gluon exchange, given by Fierz identity



Doing this for all possible exchanges:



$$\begin{pmatrix} \langle D_{x_1 \bar{x}_1} D_{x_2 \bar{x}_2} \rangle \\ \langle Q_{x_1 \bar{x}_2 x_2 \bar{x}_1} \rangle \end{pmatrix}_U = \begin{pmatrix} \alpha_{x_1 \bar{x}_1 x_2 \bar{x}_2} & \beta_{x_1 \bar{x}_2 x_2 \bar{x}_1} \\ \beta_{x_1 \bar{x}_1 x_2 \bar{x}_2} & \alpha_{x_1 \bar{x}_2 x_2 \bar{x}_1} \end{pmatrix} \begin{pmatrix} \langle D_{x_1 \bar{x}_1} D_{x_2 \bar{x}_2} \rangle \\ \langle Q_{x_1 \bar{x}_2 x_2 \bar{x}_1} \rangle \end{pmatrix}_V$$

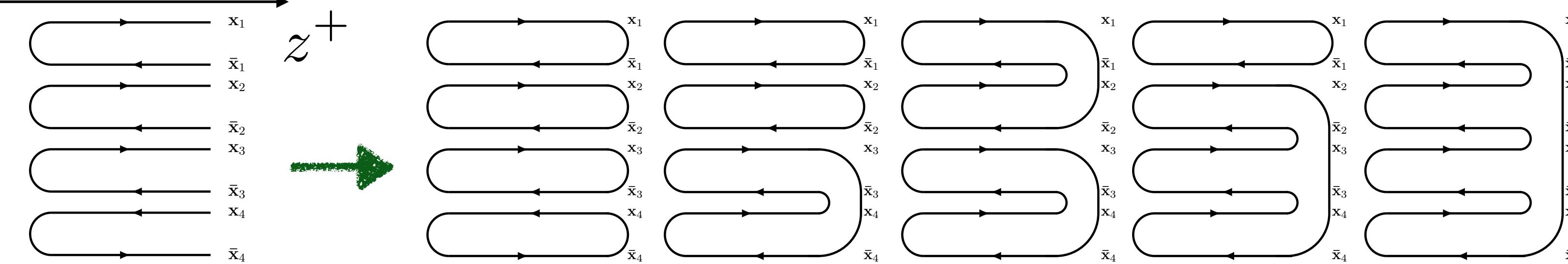
Which can be solved to all orders in gluon exchanges

Straightforward to generalize

$$\frac{d^4 N}{d^2 \mathbf{p}_1 \cdots d^2 \mathbf{p}_4} \simeq \int \langle DDDD \rangle$$

Four dipole correlators

Closed set of five topologically distinct configurations

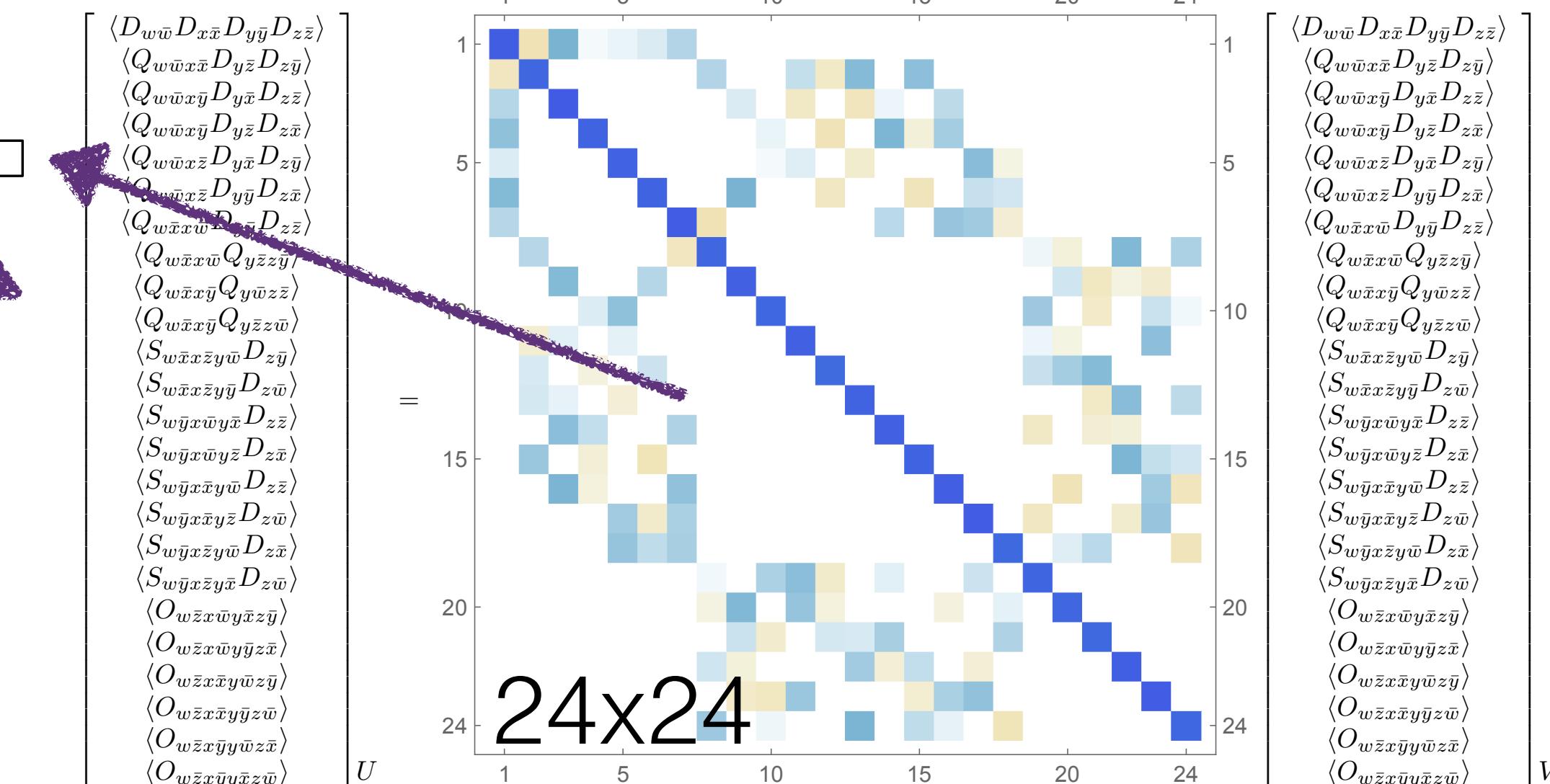
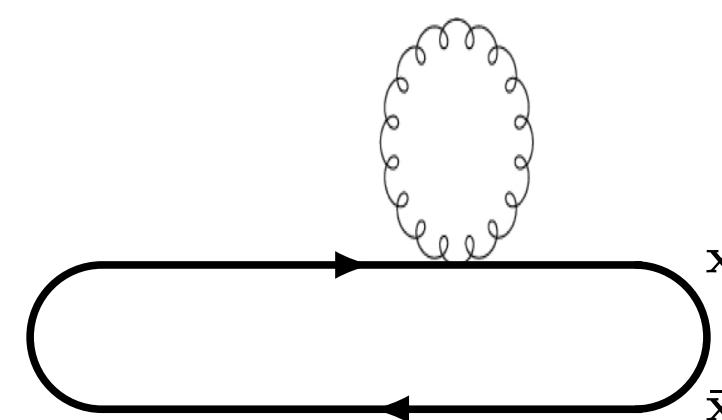


Permutations for each topology for closing on $z^+ = +\infty$

Define single gluon exchange matrix in terms of $L_{x,y}$

$$\left\langle \frac{d^4 N}{d^2 \mathbf{p}_1 \dots d^2 \mathbf{p}_4} \right\rangle \simeq \int \langle DDDD \rangle \sim e^{\square}$$

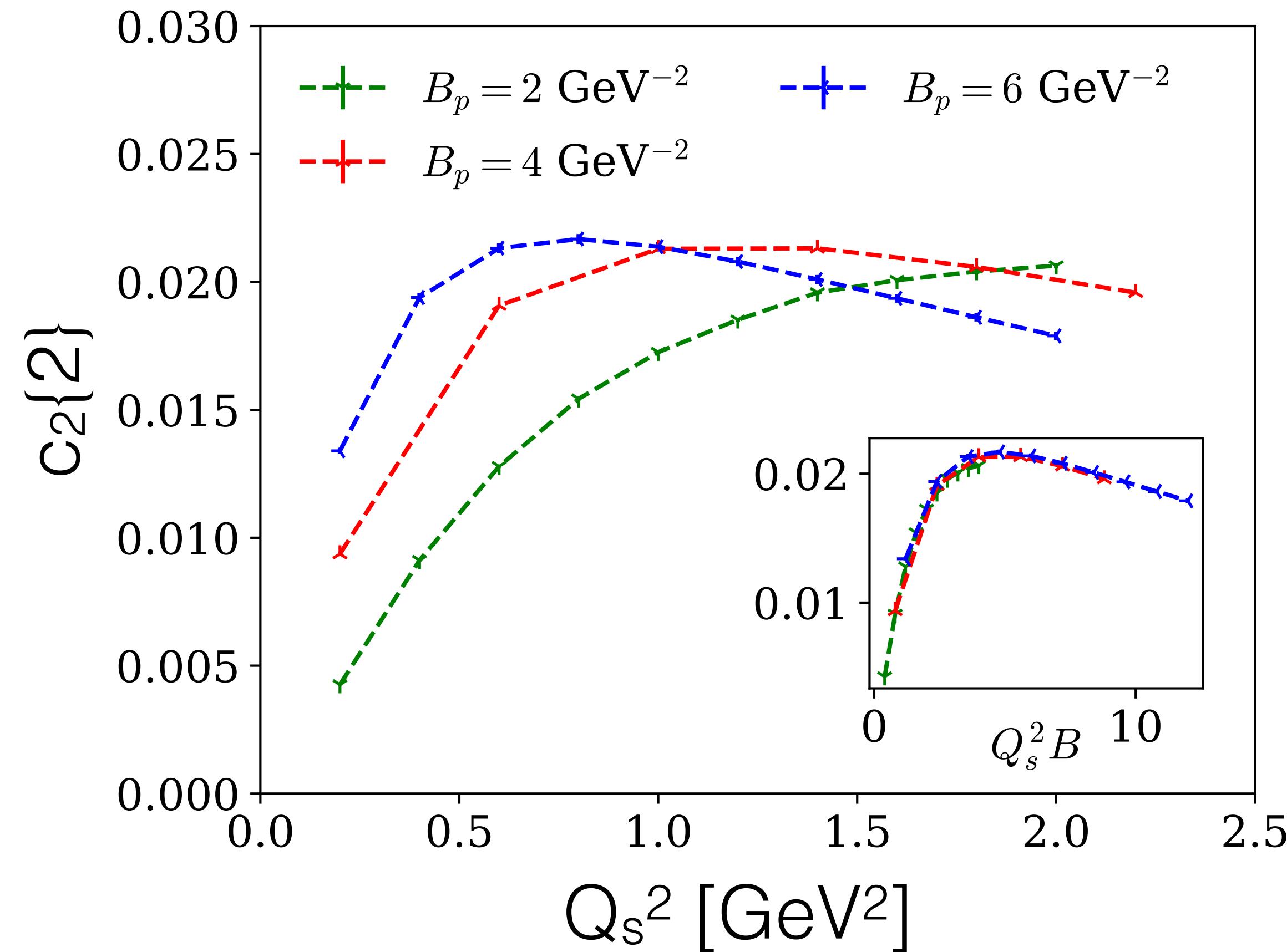
Also includes tadpole contribution



Algorithm can be used to compute other configurations, arbitrary number of Wilson lines

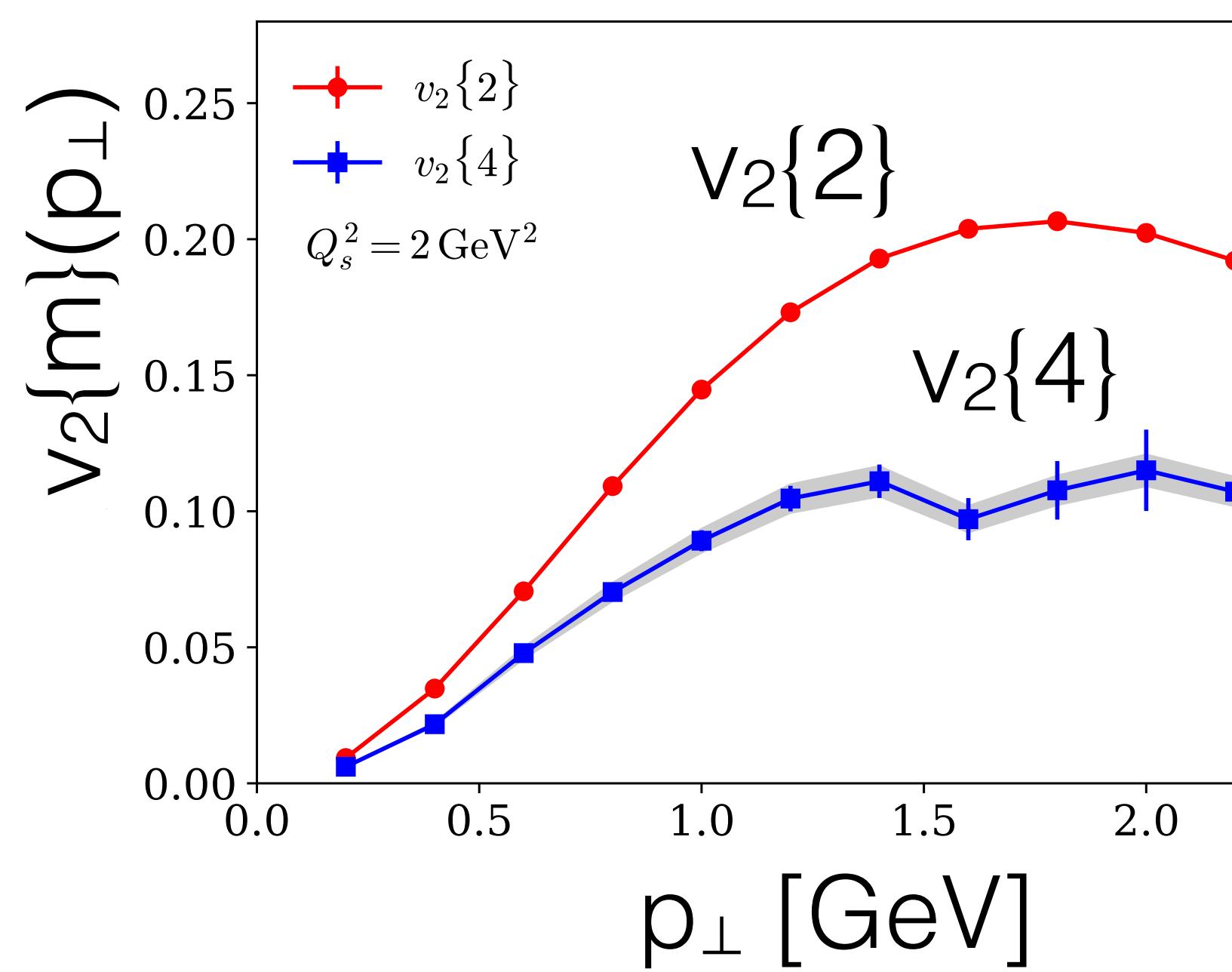
Scale dependence

For a fixed p_{\perp}^{\max} , single scale in problem, $Q_s^2 B_p$

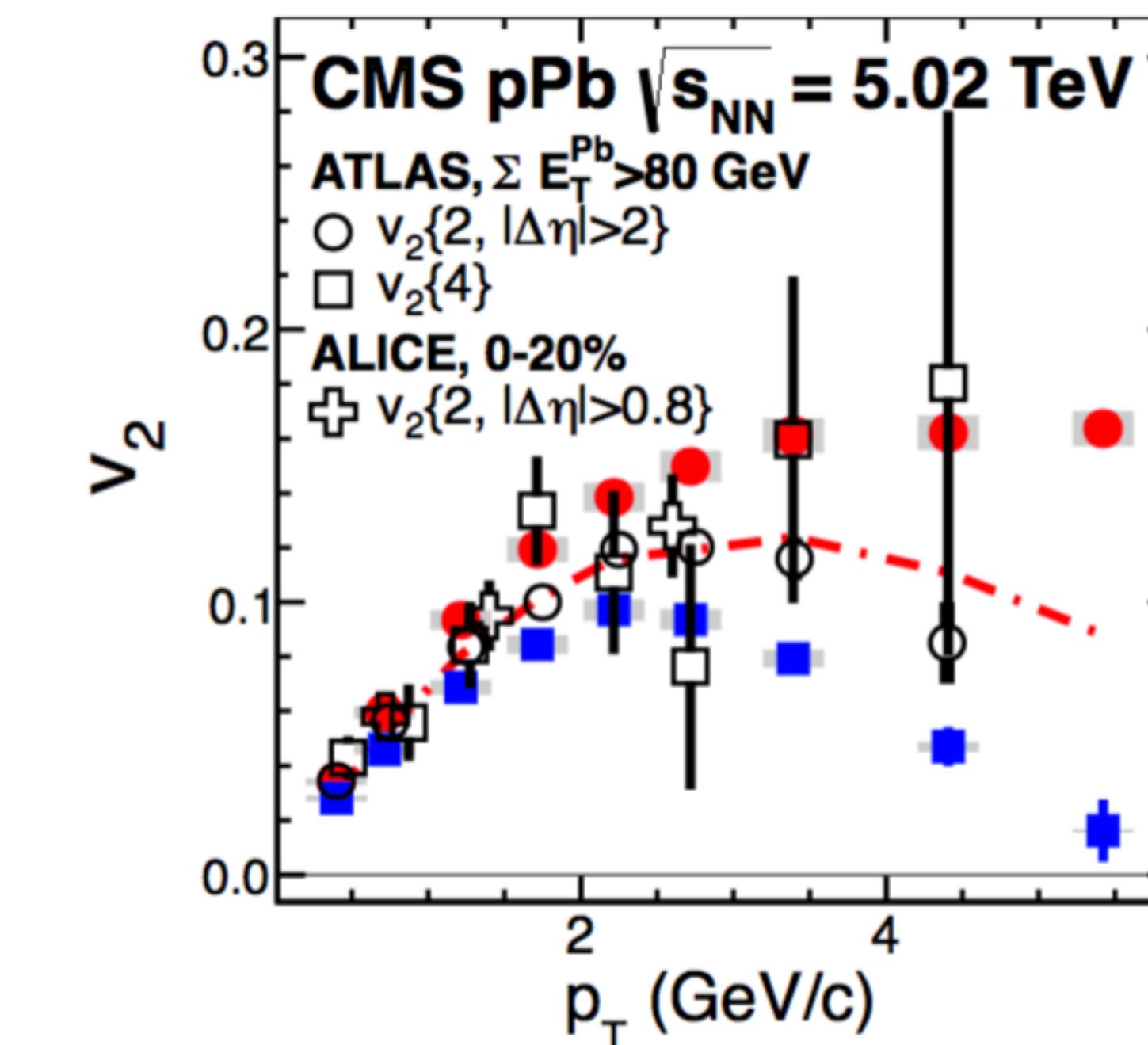


Multi-particle correlations

Integrating momentum of m-1 particles



Dusling, MM, Venugopalan PRD 97 (2018)



CMS PLB 724 (2013) 213

Similar characteristic shape

Dilute-dense for gluons

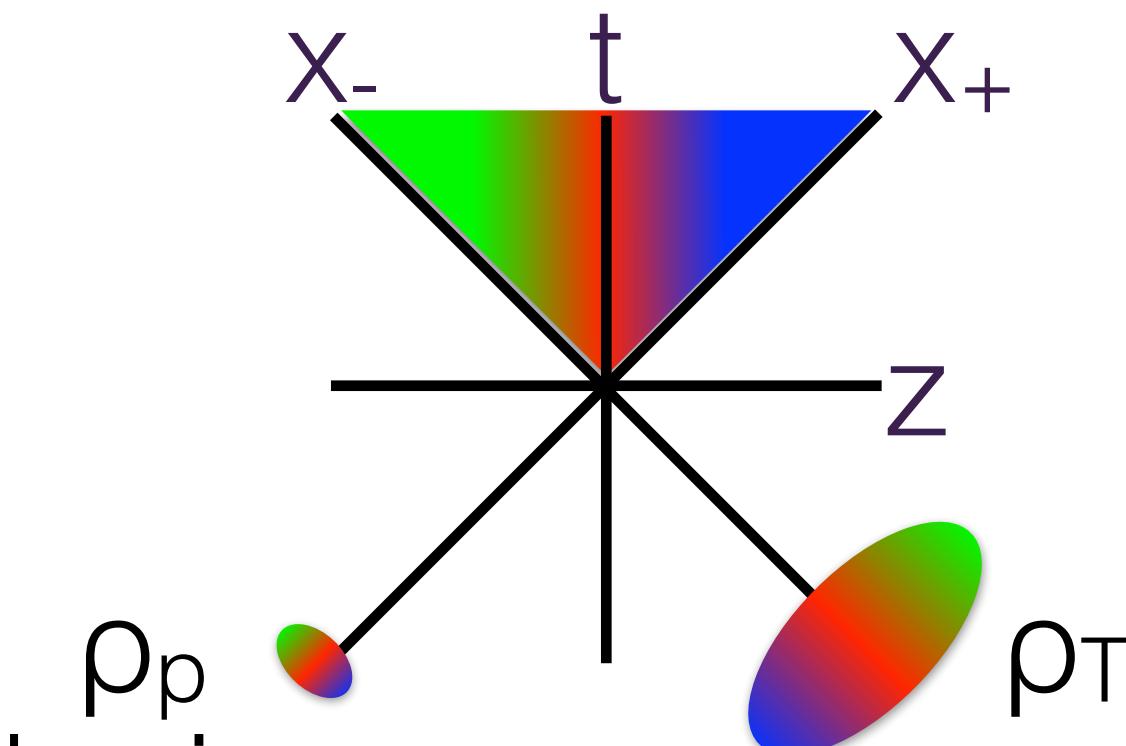
Solve CYM with static color sources

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

Dilute-dense limit: $\rho_T \gg \rho_p$

Dumitru, McLerran NPA 700 (2002), Blaziot, Gelis, Venugopalan NPA 743 (2004)



Calculate for A^μ to all order in ρ_T , first order in ρ_p

-> analytically accessible

Need NLO in ρ_p to generate v_3

McLerran, Skokov NPA 959 (2017)

Single inclusive spectrum readily calculable

$$\frac{dN}{d^2kdy} \Big|_{\rho_p, \rho_T} = \frac{1}{2(2\pi)^3} \frac{1}{|\mathbf{k}|^2} (\delta_{ij}\delta_{lm} + \epsilon_{ij}\epsilon_{lm}) \Omega_{ij}^a(\mathbf{k}) [\Omega_{lm}^a(\mathbf{k})]^*$$

$$\Omega_{ij}^a(\mathbf{x}) = g \left[\frac{\partial_i}{\partial^2} \rho_p^b(\mathbf{x}) \right] \partial_j U^{ab}(\mathbf{x})$$

Projectile Target

Dilute-dense for gluons

Solve CYM with static color sources

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

Dilute-dense limit: $\rho_T \gg \rho_p$

Dumitru, McLerran NPA 700 (2002), Blaziot, Gelis, Venugopalan NPA 743 (2004)

Calculate for A^μ to all order in ρ_T , first order in ρ_p

-> analytically accessible

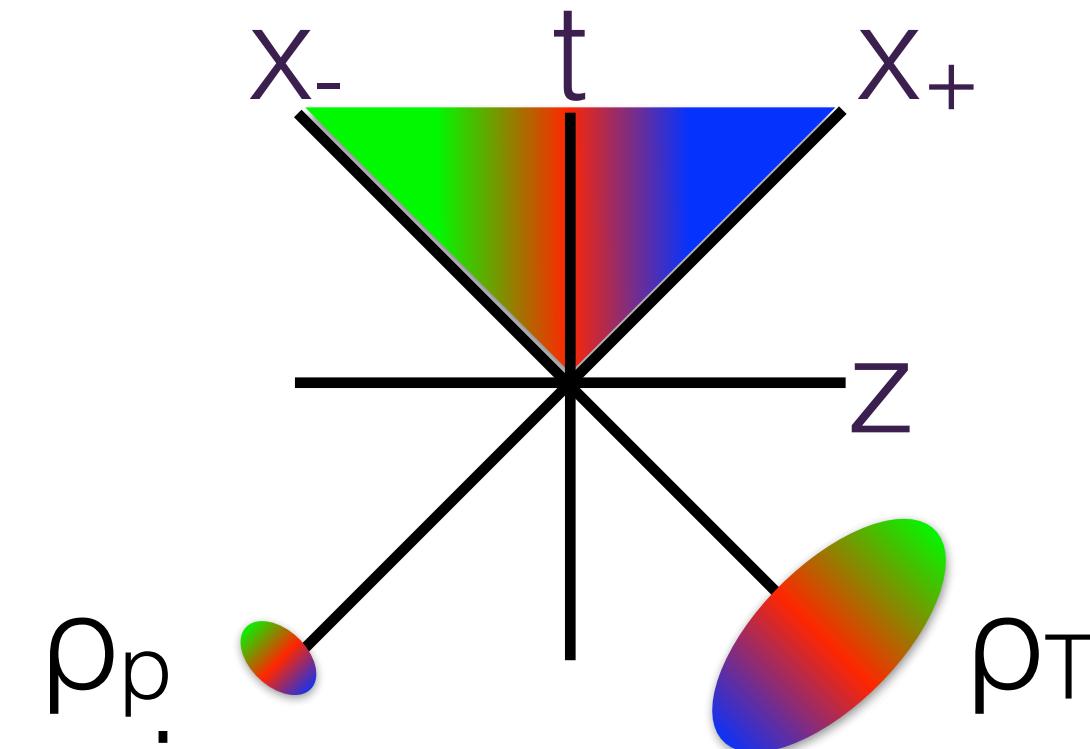
Need NLO in ρ_p to generate v_3

McLerran, Skokov NPA 959 (2017)

Multi-particle distribution then easily accessible

$$\frac{d^2N}{d^2k_1 dy_1 \dots d^2k_n dy_n} = \left\langle \left\langle \frac{dN}{d^2k_1 dy_1} \Big|_{\rho_p, \rho_T} \dots \frac{dN}{d^2k_n dy_n} \Big|_{\rho_p, \rho_T} \right\rangle_p \right\rangle_T$$

Only well defined for ensemble over $W[\rho_T, \rho_p]$

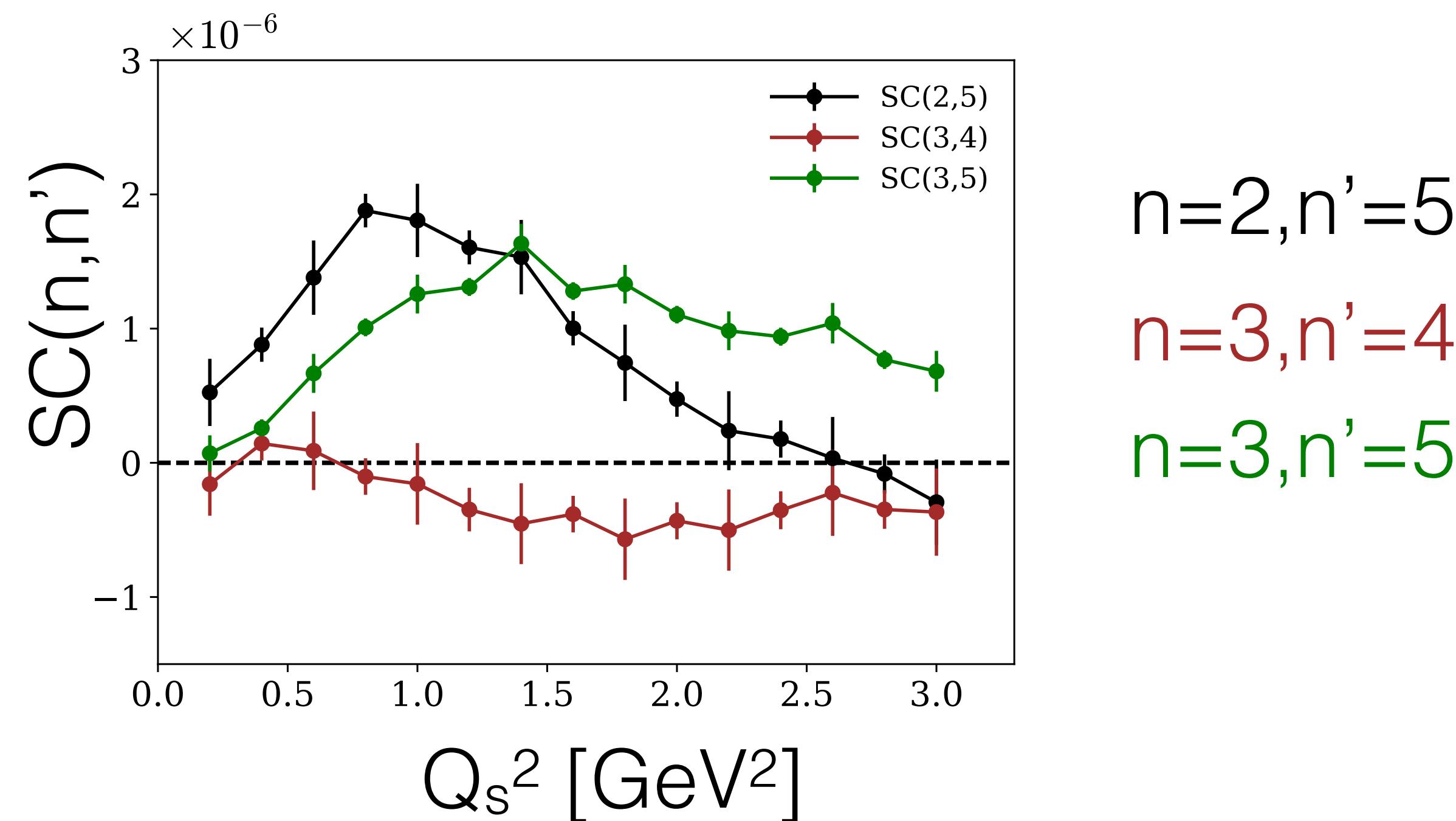


Symmetric Cumulants

$$SC(n, n') = \langle e^{i(n(\phi_1 - \phi_3) - n'(\phi_2 - \phi_4))} \rangle - \langle e^{in(\phi_1 - \phi_3)} \rangle \langle e^{in'(\phi_2 - \phi_4)} \rangle$$

Bilandzic et al, PRC 89, no. 6, 064904 (2014)

Prediction for higher moments in small systems



Collectivity in small systems

Without a proper definition...

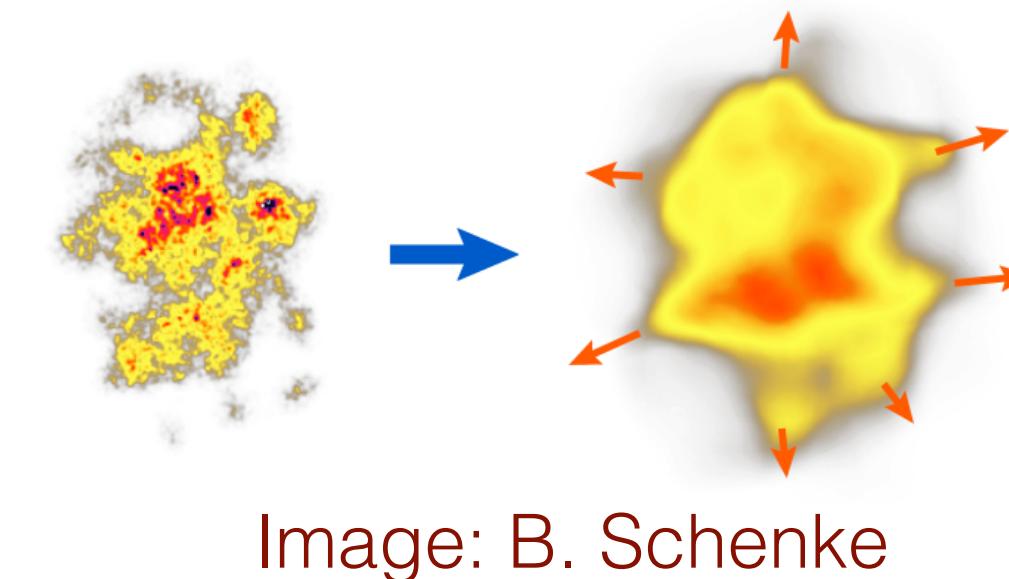


Image: B. Schenke

Define multi-particle azimuthal cumulants

Borghini, Dinh, Ollitault PRC 64, 054901 (2001)

$$c_n\{2\} = \langle e^{ni(\phi_1 - \phi_2)} \rangle$$

$$c_n\{4\} = \langle e^{ni(\phi_1 + \phi_2 - \phi_3 - \phi_4)} \rangle^2 - 2\langle e^{ni(\phi_1 - \phi_2)} \rangle^2$$

...



$$v_n\{2\} = (c_n\{2\})^{1/2}$$

$$v_n\{4\} = (-c_n\{4\})^{1/4}$$

Glauber-like modeling (std. hydro initial conditions)

$$\epsilon\{2\} \geq \epsilon\{4\} \approx \epsilon\{6\} \approx \dots \xrightarrow{\text{Linear response}} v_n\{m\} \approx c_n \epsilon_n\{m\}$$

Yan, Ollitrault PRL 112 (2014) 082301, Bzdak, Skokov NPA 943 (2015)

Define collectivity as: $v_2\{2\} \geq v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \dots$

Glauber IP-Sat model

For data-guided initial conditions, consider initial conditions based on very successful IP-Glasma model

Schenke, Tribedy, Venugopalan PRL 108 (2012), PRC 86 (2012)

Sample nucleons through Monte-Carlo Glauber
IP-Sat model provides $Q_s^2(x, \mathbf{b})$ for each nucleon

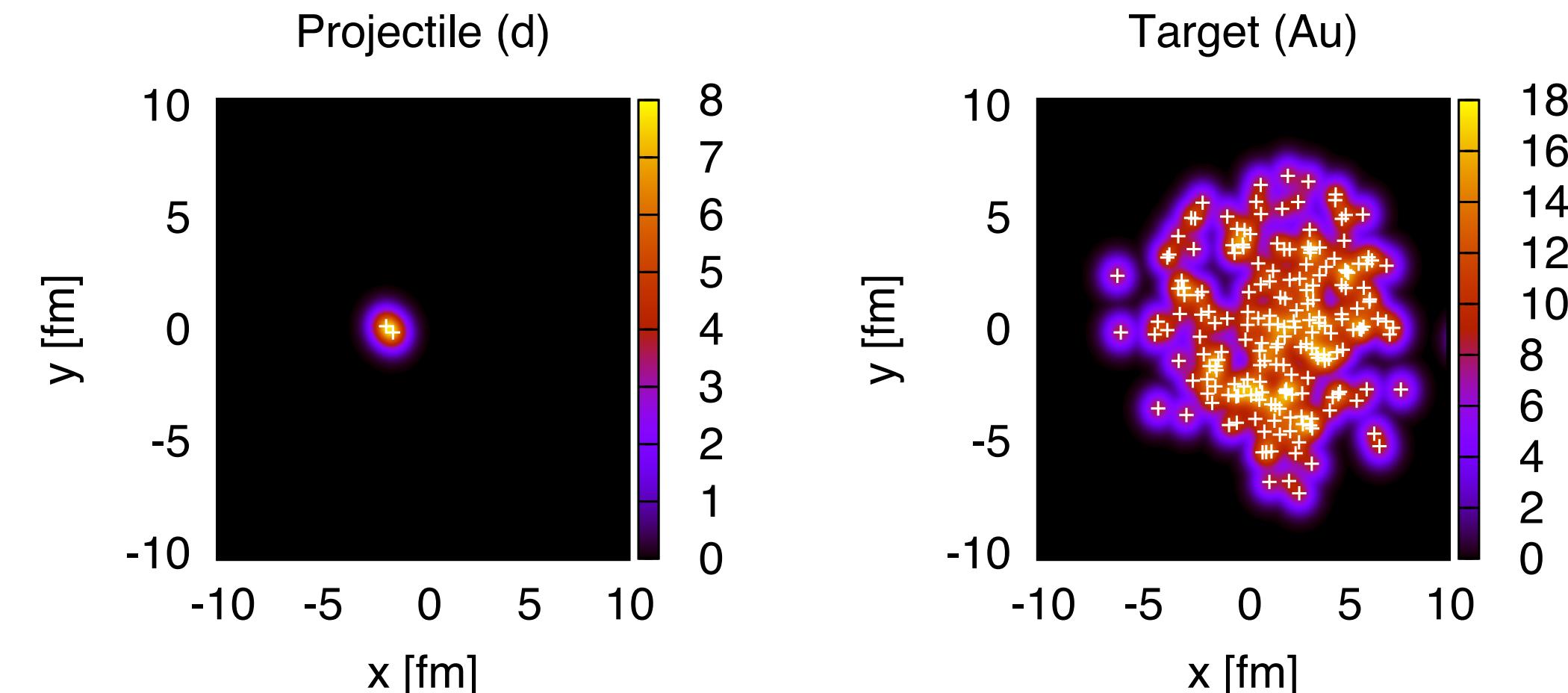
Kowalski, Teaney, Phys.Rev. D68 (2003) 114005

Based on dipole model fits to HERA DIS data

Fluctuations of proton can be constrained by exclusive J/ Ψ production HERA data

Mäntysaari, Schenke, PRL 117 (2016)
052301; PRD 94 (2016) 034042

Color charge density



Color Glass Condensate

McLerran, Venugopalan, PRD 49 (1994), Iancu, Venugopalan hep-ph/0303204

CGC is an effective field theory in the non-linear regime of QCD ($Q_s^2(x) \gg \Lambda_{\text{QCD}}^2$) describing dynamical gluon *fields* (small- x partons) effected by static color sources (large- x partons)

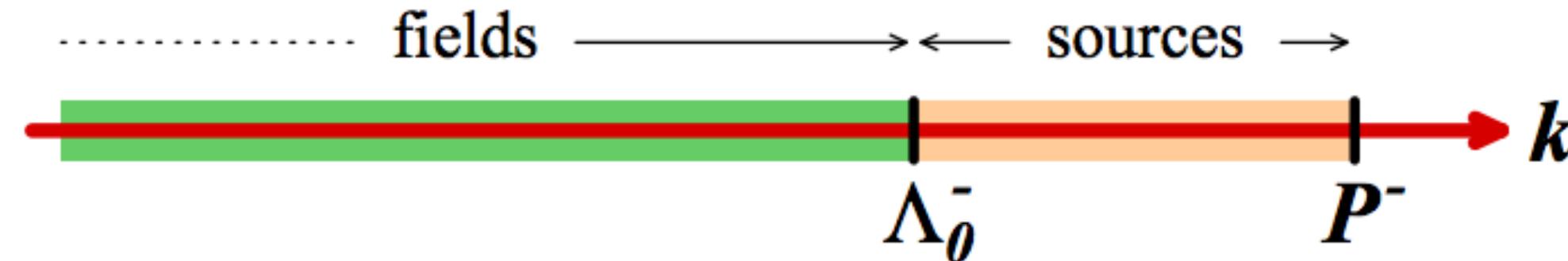
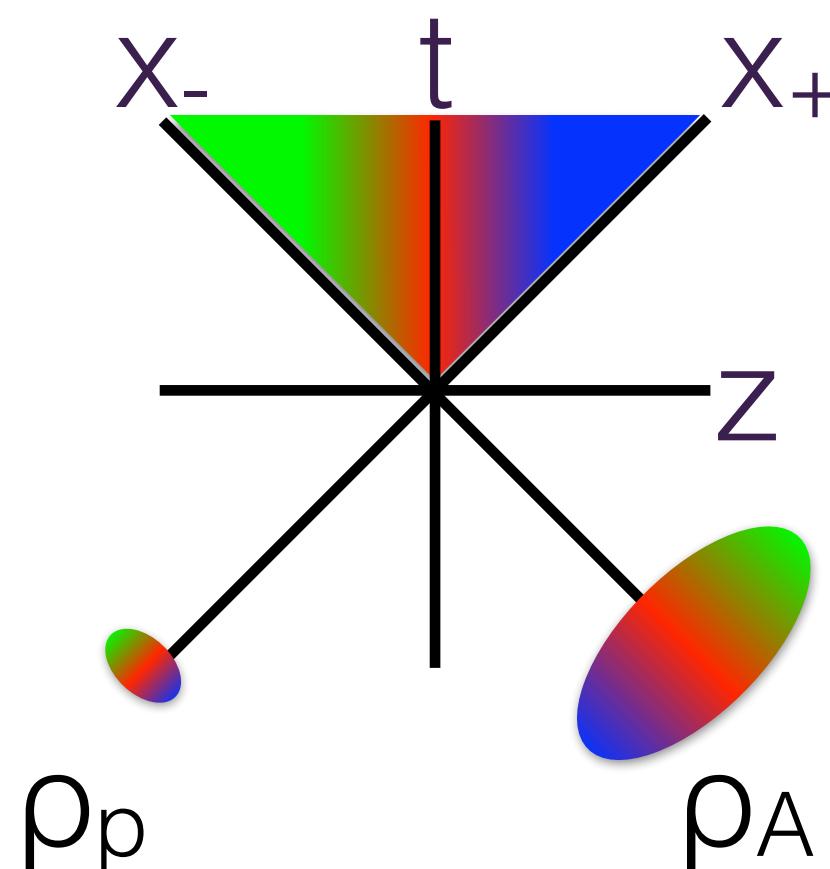


Fig: Gelis, Iancu, Jalilian-Marian, Venugopalan ARNPS. 60 (2010)



Classical background field: $[D_\mu, F^{\mu\nu}] = J^\nu$

Static color sources:

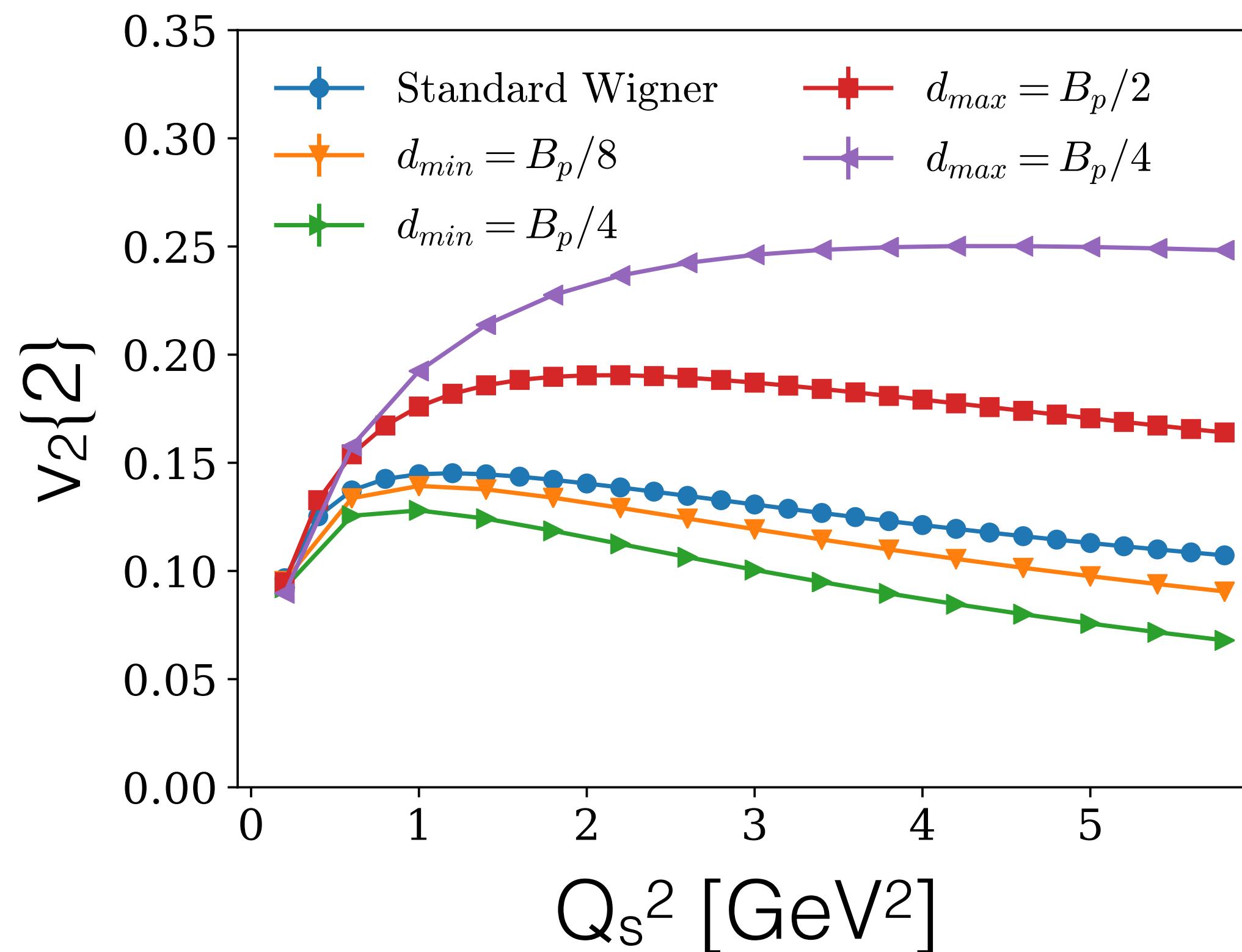
$$J^\nu = g\delta^{\nu+}\delta(x^-)\rho_{p,a}(\mathbf{x}_\perp) + g\delta^{\nu-}\delta(x^+)\rho_{A,a}(\mathbf{x}_\perp)$$

McLerran-Venugopalan (MV) Model: interactions between nucleons is a Gaussian random walk in color space

$$\langle \rho^a(\mathbf{x}_\perp) \rho^b(\mathbf{y}_\perp) \rangle = g^2 \delta^{ab} \mu^2 \delta^{(2)}(\mathbf{x}_\perp - \mathbf{y}_\perp)$$

Role of the projectile

In order to study role of choice of the projectile,
consider hard minimum cutoff (d_{\min}) and hard
maximum distance cutoff (d_{\max}) between quarks



By keeping quarks
separated by a
maximum distance,
correlations decrease

Confining quarks to a
smaller separation
increases correlations

Long range in rapidity?

To make meaningful comparison to experiment, correlations should be long range in rapidity

Model is based on hybrid framework, valid at forward rapidity

Dumitru, Jalilian-Marian PRL 89 (2002), Kovchegov, Wertepny NPA 906 (2013), Kovner, Lublinsky IJMPE 22 (2013)

Valence partons in projectile long lived and have a boost invariant wave function, coherence length $\Delta y \sim 1/\alpha_s \sim \infty$

Quantum corrections can change this picture, however beyond scope of hybrid model

Dusling, Gelis Lappi, Venugopalan NPA 836 (2010)

Valid for large-x quarks, taking $x_q \geq 0.01$

From $x_q = \frac{p_\perp}{\sqrt{s}} e^y$ taking $p_\perp = 3 \text{ GeV}$ $\sqrt{s} = 5.02 \text{ TeV}$

Framework valid for $y \geq 2.8$

Rapidity dependence

Assume eikonal
quarks

$$\text{Initial} \quad k^\mu = (k^+ = \frac{\sqrt{s}}{\sqrt{2}}x_q, 0, \mathbf{0}_\perp)$$

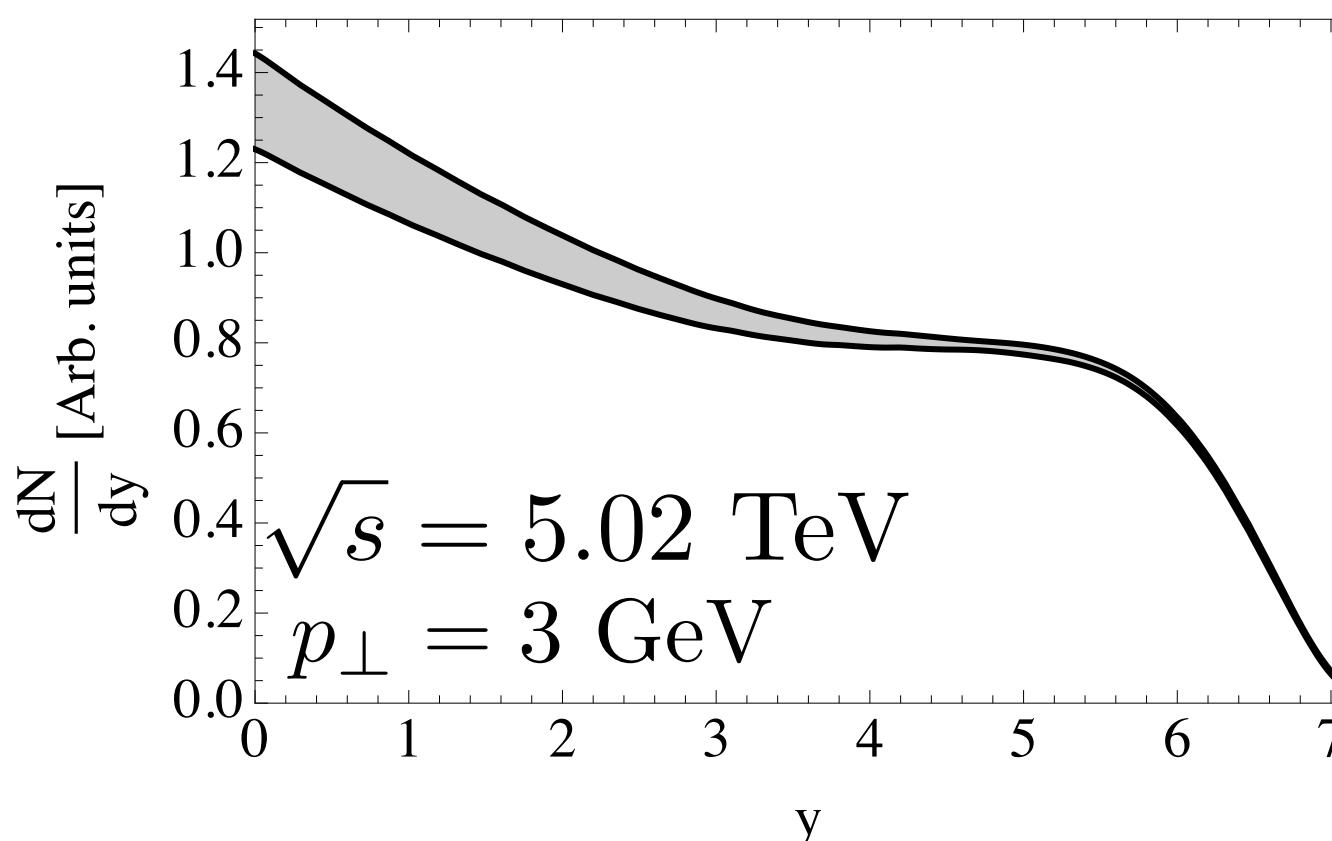
$$\text{Final} \quad p^\mu = (p^+ = \frac{p_\perp}{\sqrt{2}}e^y, 0, \mathbf{p}_\perp)$$

$$\frac{dN}{d^3\mathbf{p}} = \frac{dN}{dp^+ d^2\mathbf{p}_\perp} = \delta(p^+ - k^+) \frac{dN}{d^2\mathbf{p}_\perp}.$$

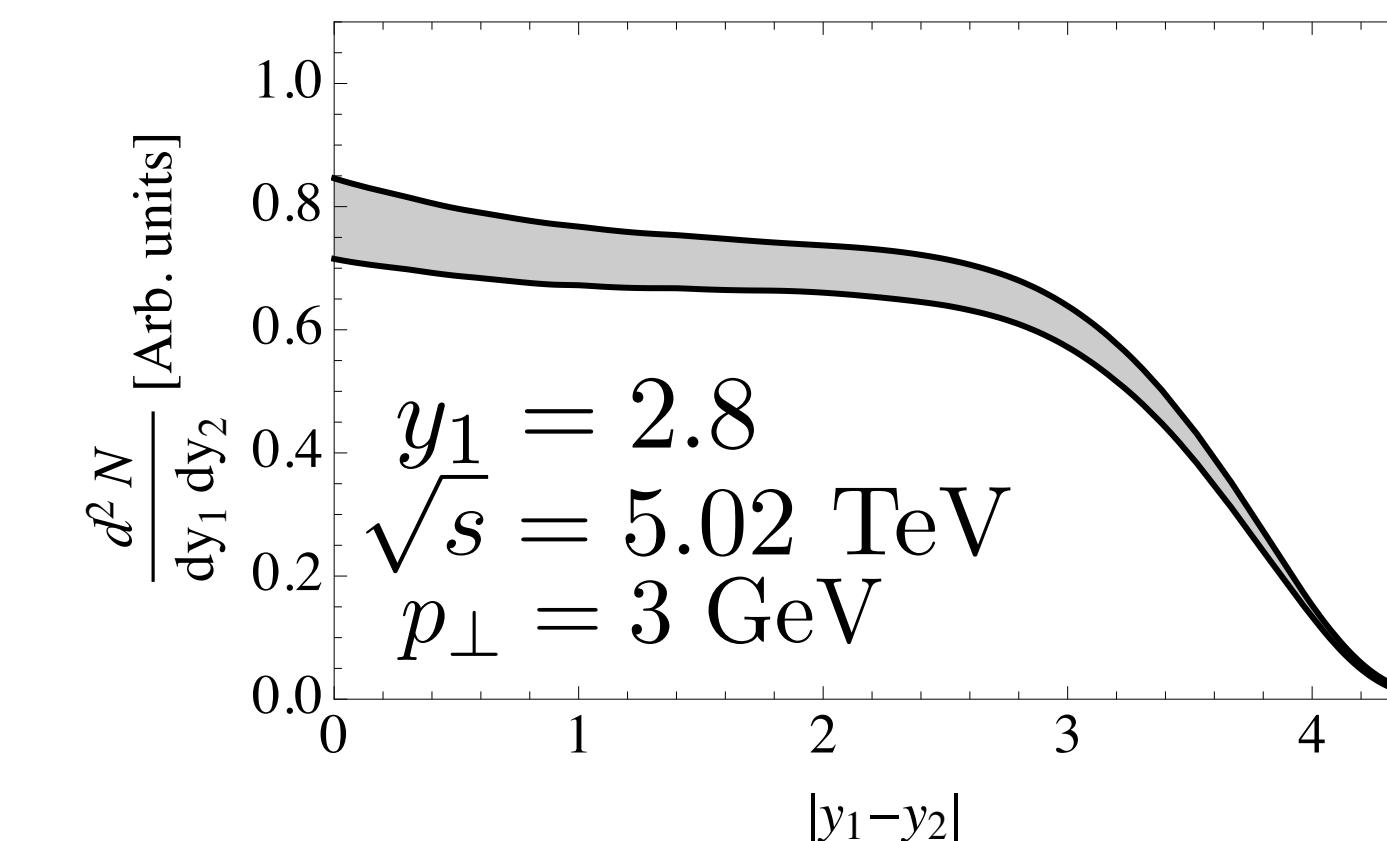
Straightforward to add rapidity dependence

$$\frac{dN^{pA \rightarrow q+X}}{dy d^2\mathbf{p}_\perp} = x'_q f(x'_q) \frac{dN^{qA \rightarrow q+X}}{dy d^2\mathbf{p}_\perp}$$

$$\frac{d^2N^{pA \rightarrow q+X}}{dy_1 d^2\mathbf{p}_{1\perp} dy_2 d^2\mathbf{p}_{2\perp}} = x'_{q,1} f(x'_{q,1}) x'_{q,2} f(x'_{q,2}) \frac{d^2N^{qA \rightarrow q+X}}{dy_1 d^2\mathbf{p}_{1\perp} dy_2 d^2\mathbf{p}_{2\perp}}$$



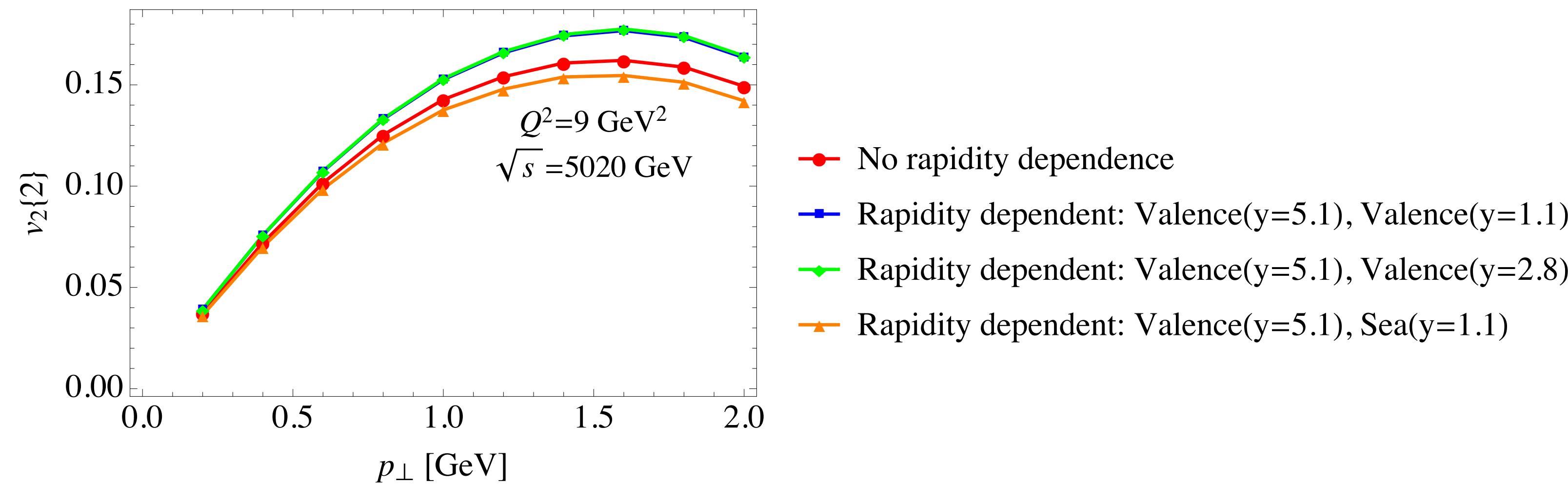
Single quark



Two quarks

Rapidity dependence

Compare $v_2\{2\}(p_T)$ for with rapidity dependent distributions



Only quantitative, not qualitative, differences when considering both small and large x quarks