Estimating nucleon substructure properties in a unified model of p-Pb and Pb-Pb collisions

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Quark Matter | Venice, Italy
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Collectivity in small systems, is it flow?

Narrative:

Multiparticle correlations in A-A collisions are explained by flow.

Justification:

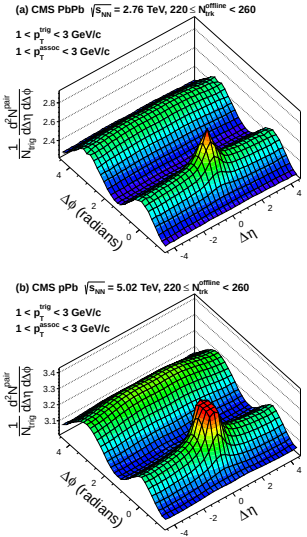
Quantitative agreement of hydrodynamic simulations remains strongest evidence for flow in A-A collisions. Agreement is global, self-consistent and highly non-trivial.

Natural questions:

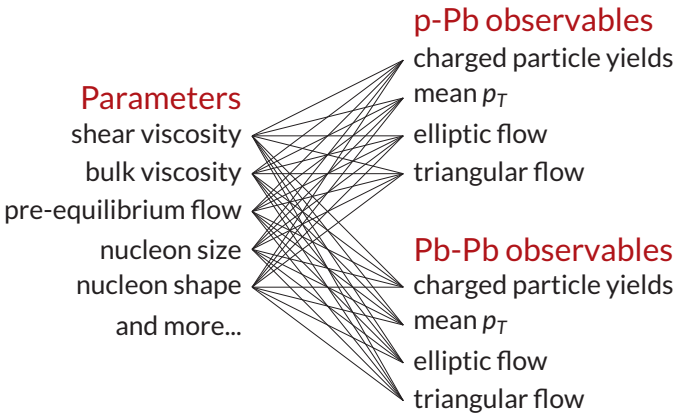
Can the same framework simultaneously describe p-A collisions without spoiling its description of A-A collisions? Does the framework continue to make sense?

This work:

Calibrate a successful hybrid model—based on viscous hydrodynamics and Boltzmann transport—to simultaneously describe p-Pb and Pb-Pb data. Does such a description exist?



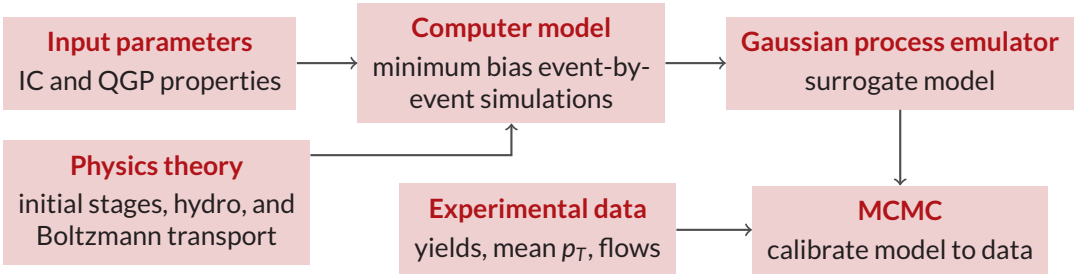
Challenge of rigorous model-to-data comparison



Many correlated input parameters—cannot calibrate by hand!

I. Bayesian parameter estimation

Bayesian parameter estimation

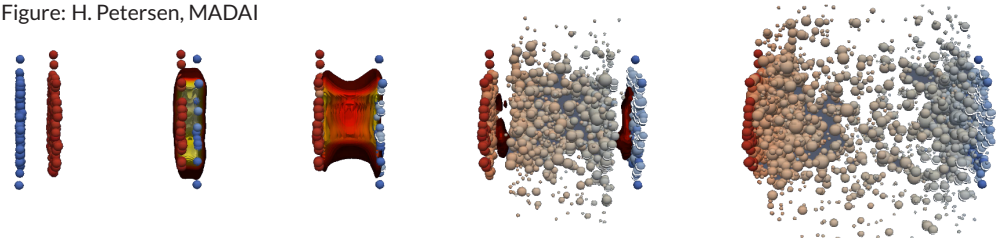


Objective:

- Explore parameter space of a given model
- Quantify posterior probability of every parameter region *given the model, data and* ***known uncertainties***

Heavy-ion collision model

Figure: H. Petersen, MADAI



TRENTo initial conditions

parametric entropy deposition

PRC.92.011901

FREESTREAM pre-flow

infinitely weak coupling limit

PRC.91.064906, PRC.80.034902

VISH2+1 viscous hydro

14-mom. approx w/ shear & bulk

PRC.77.064901, J.CPC.2015.08.039

FRZOUT sampler

shear & bulk corrections

(Bernhard thesis) arXiv:1804.06469

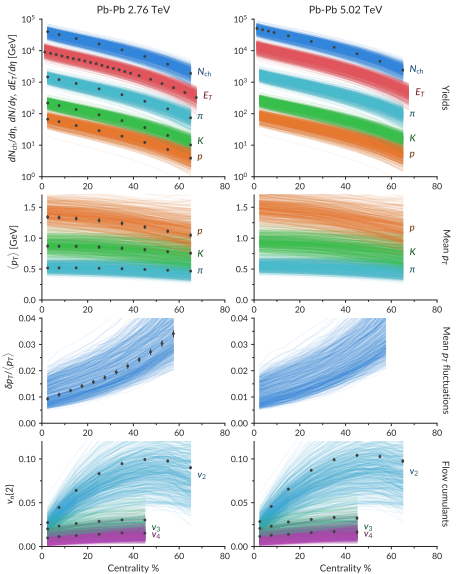
URQMD Boltzmann cascade

hadronic afterburner, simulate scatterings and decays

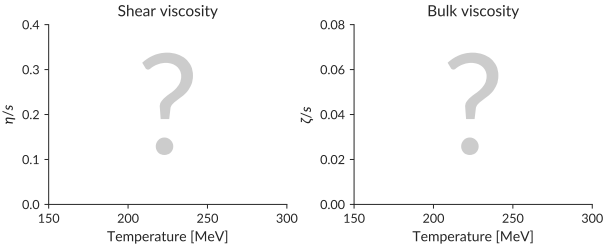
PPNP.98.00058, JPG.25.9.308

Final observables calculated as similar as possible to experiment

PARAMETER ESTIMATES FROM LEAD-LEAD COLLISIONS



- Combined analysis of 2.76 and 5.02 TeV beam energies (NEW!)
- Parametric initial conditions and medium parameters
- Quantitative estimates on η/s and ζ/s with meaningful uncertainties
- See thesis by Jonah Bernhard [1804.06469]



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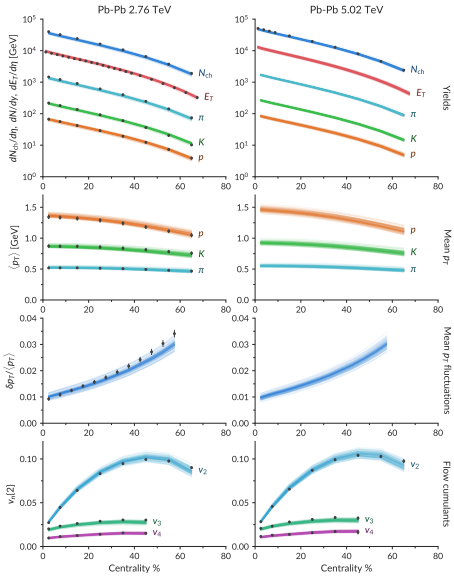

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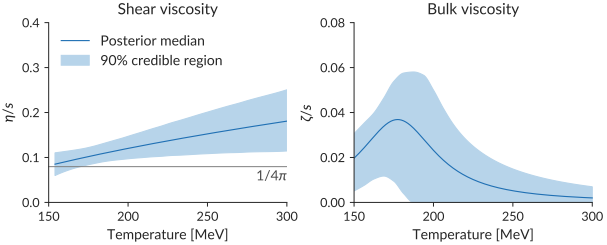
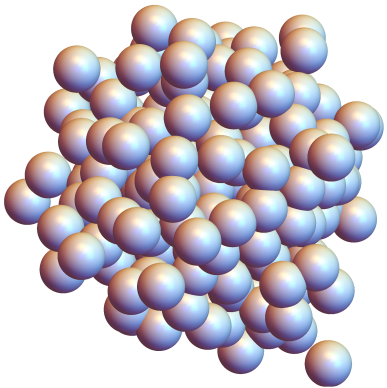


II. Adding nucleon substructure to the model

Nuclear structure



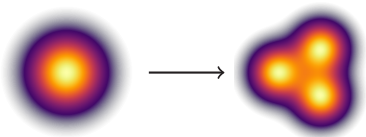
^{208}Pb nucleus

Original TRENTo model:

- Sample nucleon positions from spherical or deformed Woods-Saxon distributions
- Solid angles resampled to preserve minimum nucleon distance d_{min} [fm]
- Gaussian nucleons of width w [fm]

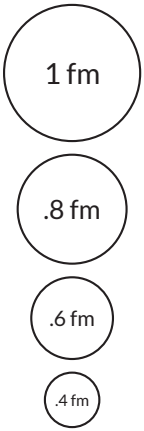
This work:

- Trade Gaussian nucleons for lumpy nucleons



NUCLEON SUBSTRUCTURE

Sampling radius



Constituent width

Constituent number

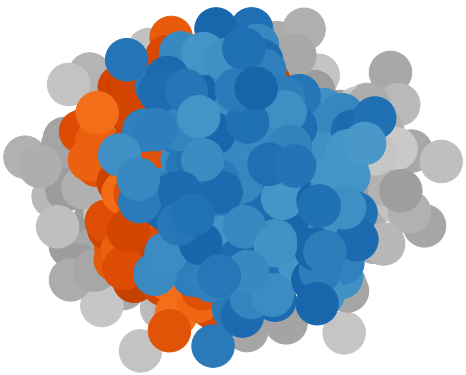
Free parameters:

- Sampling radius of constituent positions
- Constituent Gaussian width
- Number of constituents in each nucleon

Absent from this work:

- Spatial correlations, see talk by Alba Soto Ontoso

Cross sections

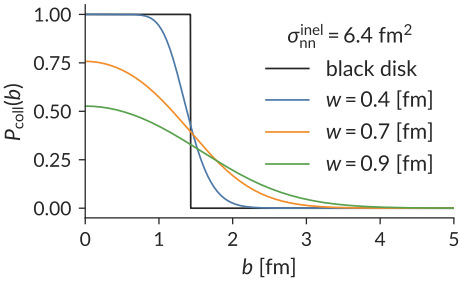


MC-Glauber cross sections
(one constituent)

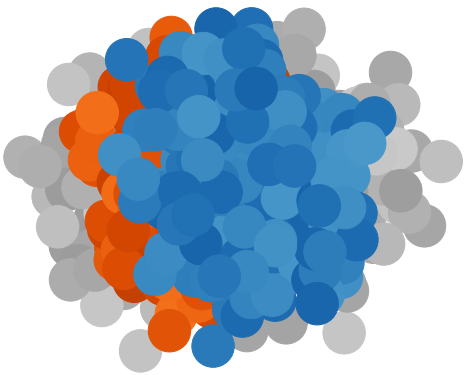
Original TRENTo model:

$$P_{\text{coll}} = 1 - \exp[-\sigma_{\text{eff}} T_{\text{pp}}(b)]$$

$$\sigma_{\text{nn}}^{\text{inel}} = \int 2\pi b\, db\, P_{\text{coll}}(b)$$



Cross sections



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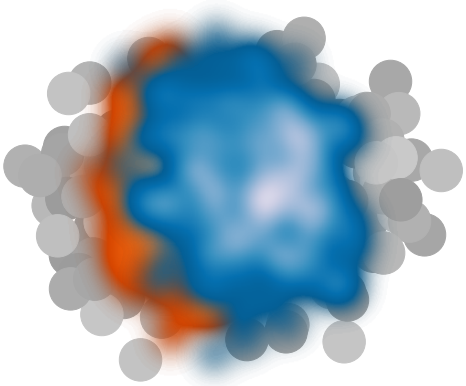
This work:

$$P_{\text{coll}} \to 1 - \prod_{i,j=1}^{n_{\text{part}}} [1 - P_{\text{coll}}(b_{ij})]$$

Solve for effective cross section parameter σ_{eff} numerically

PARTICIPANT THICKNESS FUNCTIONS



Participant matter

Original TRENTo model:

$$T_{\text{nucleus}}(x,y) = \sum_{i=1}^{N_{\text{part}}} \gamma_i \, T_{\text{proton}}(x - x_i, y - y_i)$$

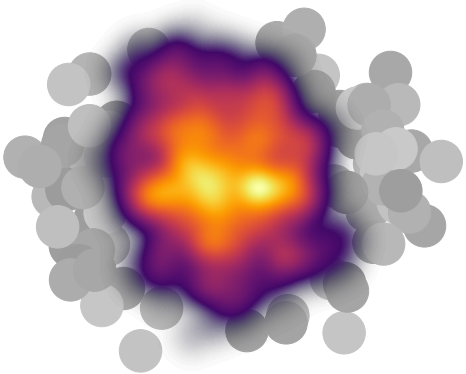
Random weight γ_i sampled from Gamma distribution with unit mean and variance $1/k$.

This work:

$$\gamma_i \, T_{\text{proton}} \to \sum_{j=1}^{M} \gamma_j \, T_{\text{constituent}}$$

where M is the number of constituents.

Entropy deposition



Entropy density

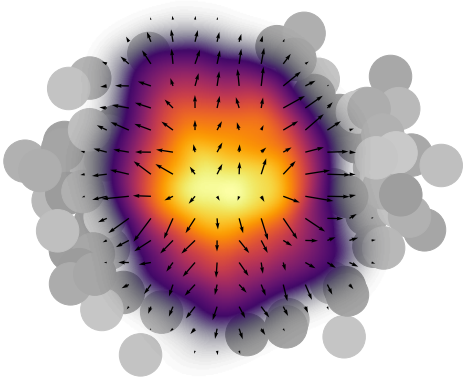
Generalized Mean Ansatz:

$$\frac{dS}{d\eta} \propto T_R(T_A, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

$$T_R = \begin{cases} \max(T_A, T_B) & p \to +\infty, \\ (T_A + T_B)/2 & p = +1, \\ \sqrt{T_A T_B} & p = 0, \\ 2\,T_A T_B/(T_A + T_B) & p = -1, \\ \min(T_A, T_B) & p \to -\infty. \end{cases}$$

T denotes participant thickness function

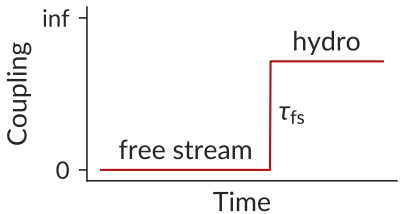
FREE STREAMING



Arrows: fluid velocity
weighted by energy density

Pre-equilibrium phase:

- Massless non-interacting parton gas matched to viscous hydrodynamics PRC.91.064906, PRC.80.034902.
- Must reinterpret initial entropy density as initial gluon density, $dN_g/y \sim dS/dy$
- Initialize hydro with non-zero u^μ and $\pi^{\mu\nu}$



III. Simultaneous calibration to p-Pb and Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

COMPUTER EXPERIMENT DESIGN

Parameter	Description	Range
Norm	Overall normalization	12–28
p	Entropy deposition parameter	-1 to $+1$
σ_{fluct}	Relative constituent fluct. std.	0–2
r	Constituent sampling radius	0.4–1.2 fm
χ_{struct}	Constituent structure parameter	0–1
m	Number of constituents	1–9
d^3_{min}	Nucleon exclusion volume	0–4.9 fm^3
τ_{fs}	Free streaming time	0.1–1.5 fm/c
η/s min	Shear viscosity at T_c	0–0.2
η/s slope	Slope above T_c	0–8 GeV^{-1}
η/s crv	Curvature above T_c	-1 to 1
ζ/s norm	Bulk viscosity peak height	0–0.1
ζ/s width	Bulk viscosity peak width	0–0.1 GeV
ζ/s temp	Bulk viscosity peak location	150–200 MeV
T_{switch}	Particlization temperature	135–165 MeV

IMPORTANT DETAILS:

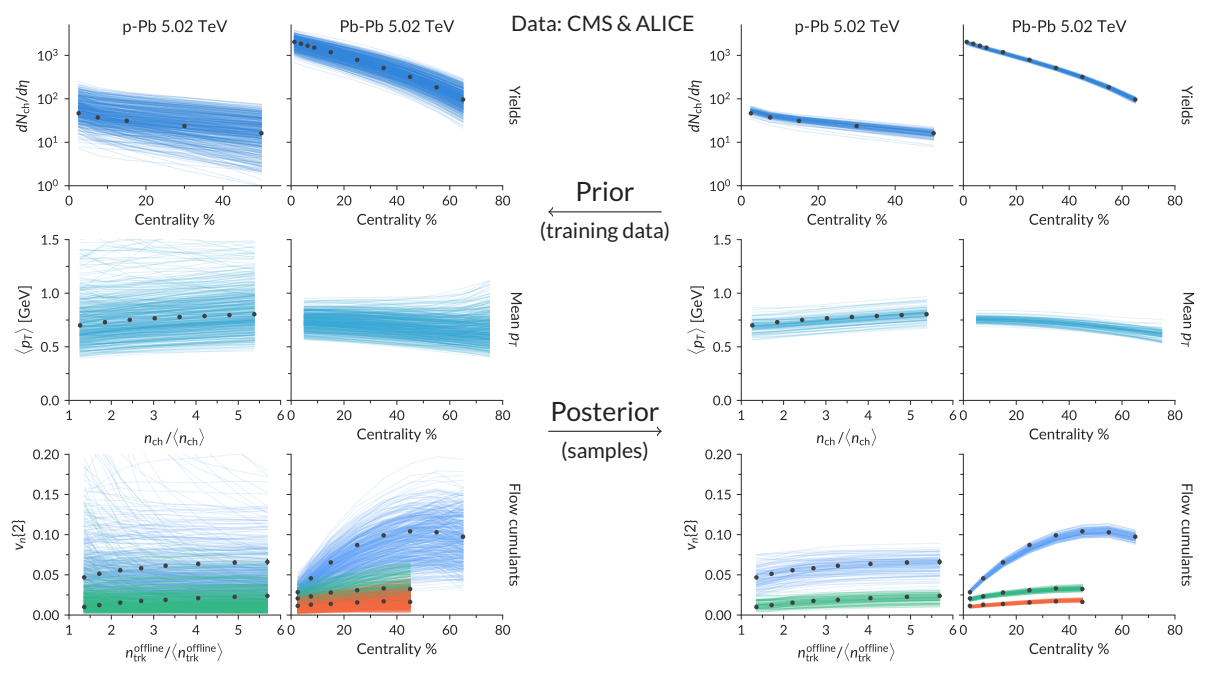
- Recast constituent width w as:

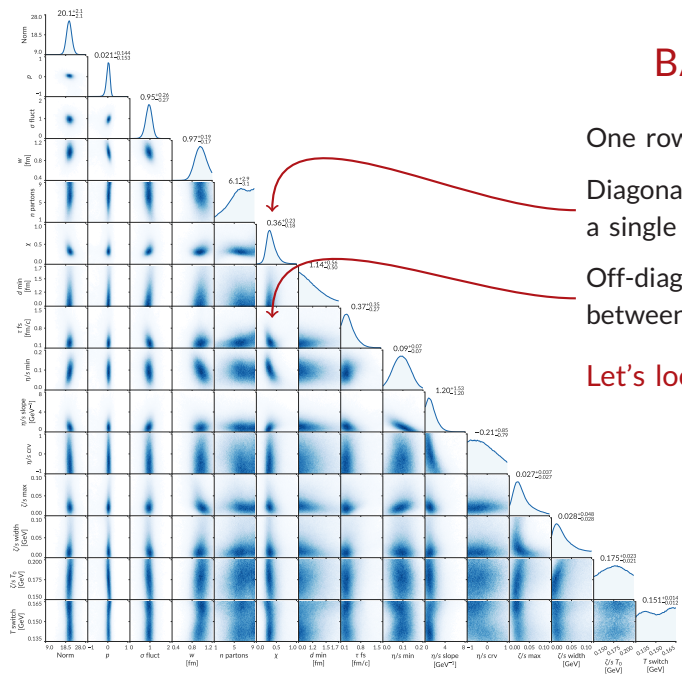
  $$w = w_{\text{min}} + \chi_{\text{struct}}(w_{\text{max}} - w_{\text{min}})$$

  $$w_{\text{min}} = 0.2\,\text{fm},\ w_{\text{max}} = r$$

- Constituent sampling radius is not equivalent to the proton radius in the proton c.o.m. frame, see PRC.94.024919
- Other parameters same as combined Pb+Pb analysis at 2.76 and 5.02 TeV

500 design points per collision system, $\mathcal{O}(10^4)$ events per design point!


Data: CMS & ALICE
p-Pb 5.02 TeV
Pb-Pb 5.02 TeV
Yields
$dN_{ch}/d\eta$
Centrality %
Prior
(training data)
Posterior
(samples)
Mean p_T
$\langle p_T \rangle$ [GeV]
$n_{ch}/\langle n_{ch} \rangle$
Flow cumulants
$v_n\{2\}$
$n_{trk}^{offline}/\langle n_{trk}^{offline} \rangle$



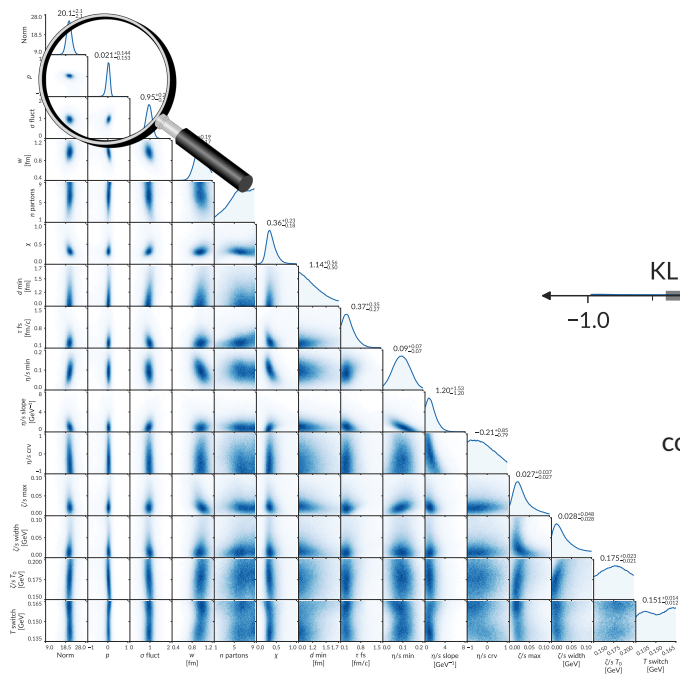
BAYESIAN POSTERIOR

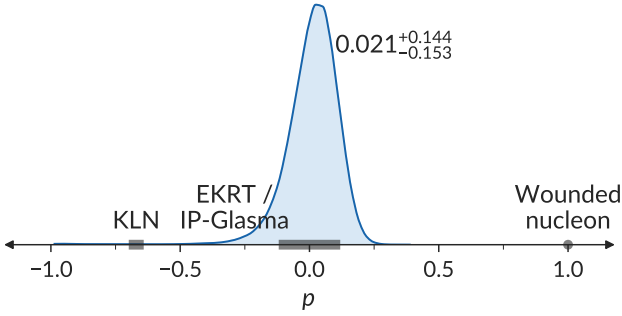
One row (column) for each parameter

Diagonal panels: marginal distribution of a single model parameter

Off-diagonal panels: joint distribution between a pair of model parameters

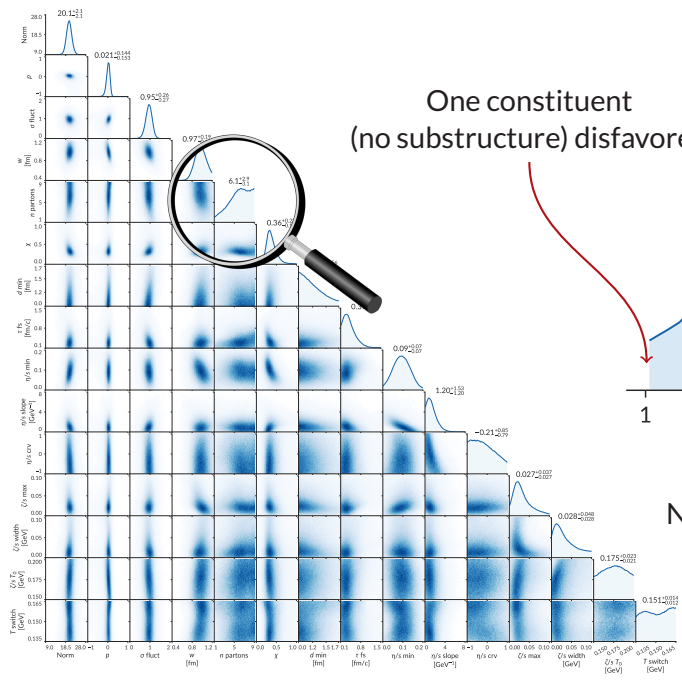
Let's look at some specific regions...



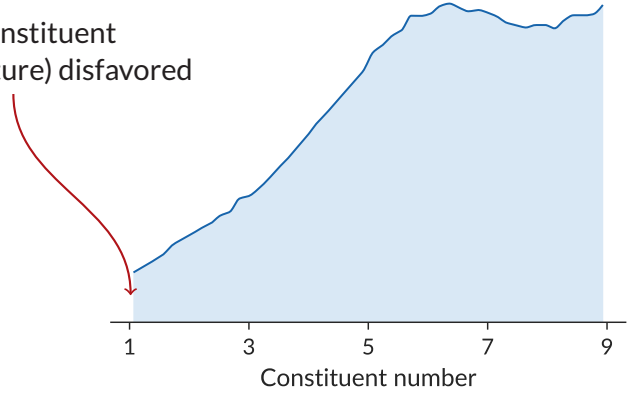


Initial entropy deposition consistent with geometric mean

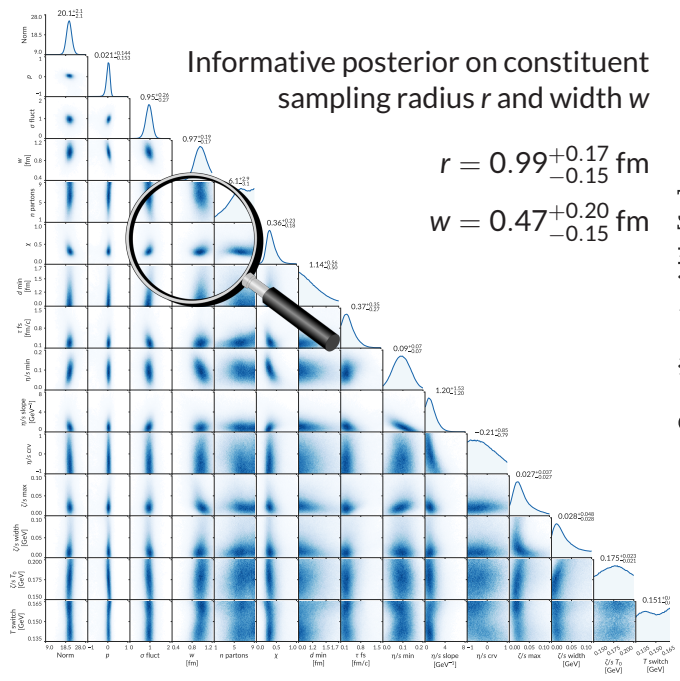
$$dS/dy \sim \sqrt{T_A T_B}$$

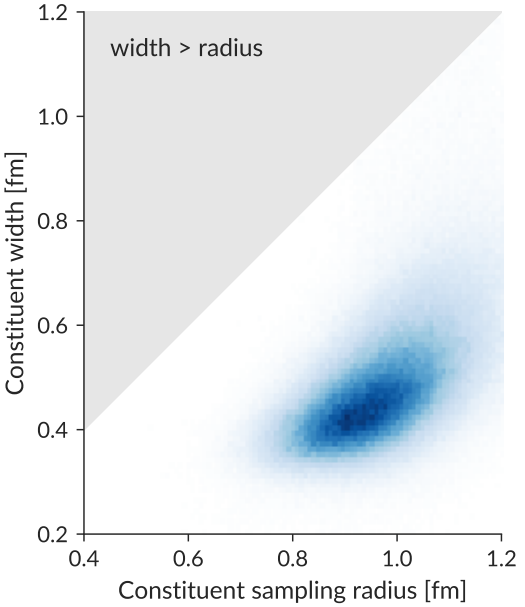


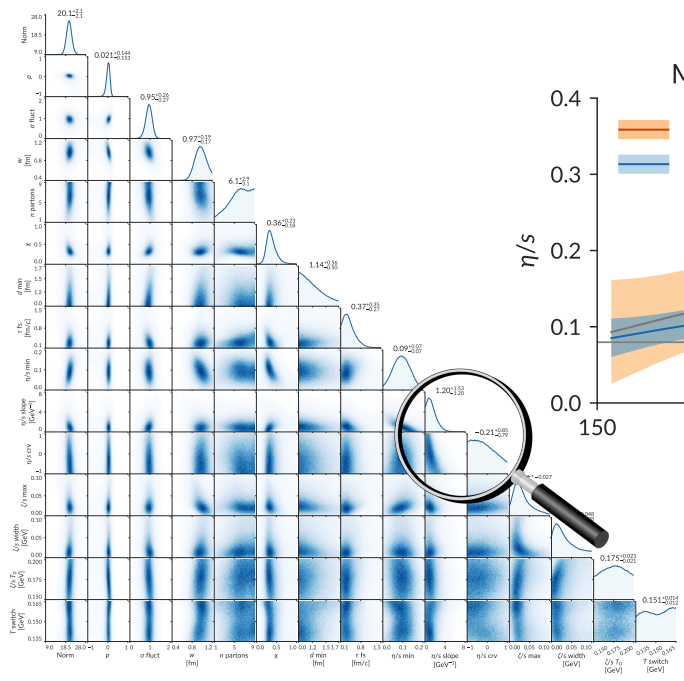
One constituent
(no substructure) disfavored

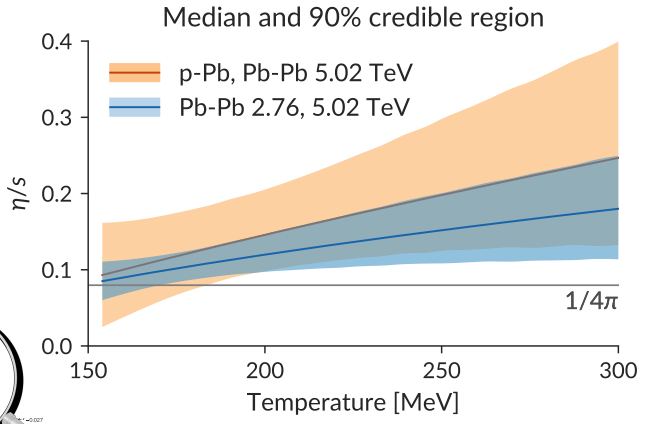


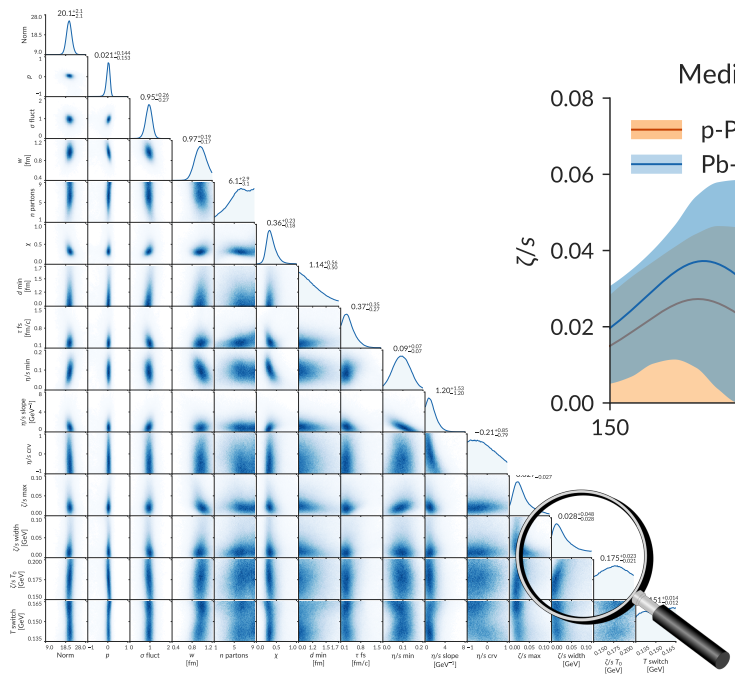
No strong preference for a specific constituent number

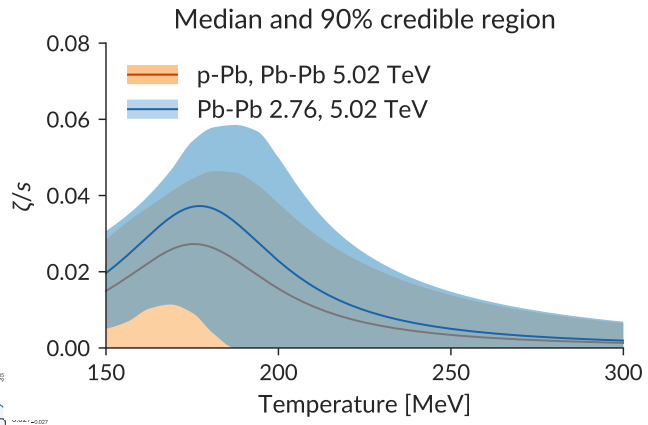

Informative posterior on constituent sampling radius r and width w
$r = 0.99^{+0.17}_{-0.15}$ fm
$w = 0.47^{+0.20}_{-0.15}$ fm


width > radius
Constituent width [fm]
Constituent sampling radius [fm]
1.2
1.0
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.0
1.2


Norm
p
σ fluct
w [fm]
n partons
χ
d min [fm]
τ fs [fm/c]
η/s min
η/s slope [GeV⁻¹]
η/s crv
ζ/s max
ζ/s width [GeV]
ζ/s T₀ [GeV]
T switch [GeV]
20.1
0.021
0.95
0.97
6.1
0.36
1.14
0.37
0.09
1.20
−0.21
−0.027
0.175
0.151


Median and 90% credible region
p-Pb, Pb-Pb 5.02 TeV
Pb-Pb 2.76, 5.02 TeV
η/s
1/4π
Temperature [MeV]
0.0
0.1
0.2
0.3
0.4
150
200
250
300


Norm
p
σ fluct
w [fm]
n partons
χ
d min [fm]
τ fs [fm/c]
η/s min
η/s slope [GeV⁻¹]
η/s crv
ζ/s max
ζ/s width [GeV]
ζ/s T₀ [GeV]
T switch [GeV]


Median and 90% credible region
p-Pb, Pb-Pb 5.02 TeV
Pb-Pb 2.76, 5.02 TeV
ζ/s
Temperature [MeV]
0.00
0.02
0.04
0.06
0.08
150
200
250
300

Concluding remarks

1. Calibrated model simultaneously describes p-Pb and Pb-Pb bulk observables
2. Nucleon substructure necessary to describe p-Pb and Pb-Pb flows (using hydro)
3. p-Pb observables constrain the constituent width and sampling radius, but not the constituent number
4. Varying the beam energy of A-A collisions provides better constraint on temperature dependence of the viscosity than varying system size

To do: run and analyze events using *maximum a posteriori* parameters

TRENTo with nucleon substructure:
`github.com/morelandjs/trento-partons`

Reproduce my results:
`github.com/morelandjs/hic-param-est-qm18`

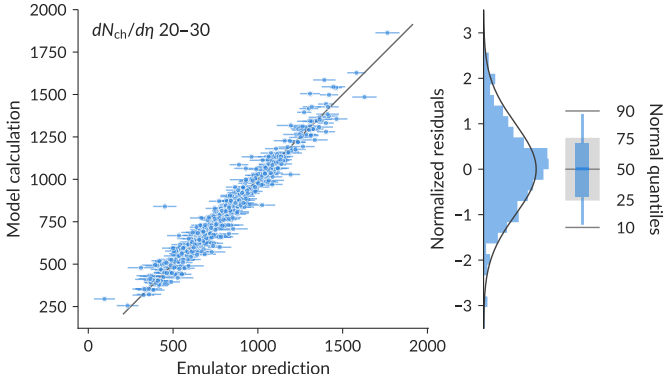
BACKUP SLIDES

EMULATOR VALIDATION

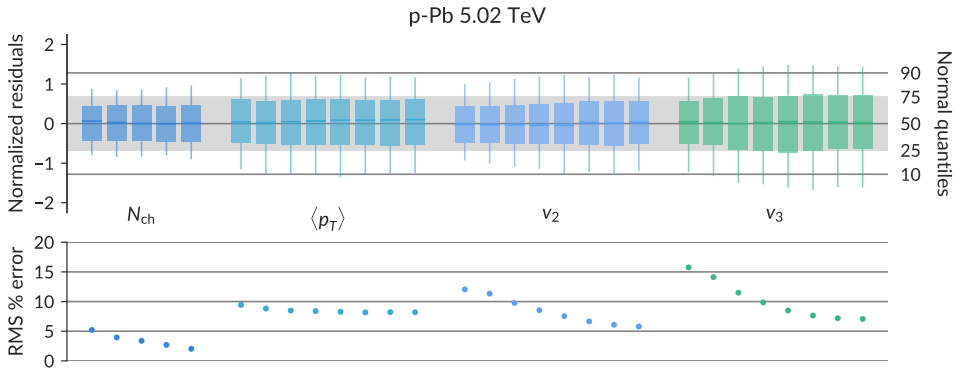
K-FOLD CROSS VALIDATION

Subdivide design into *k* partitions. Choose one for validation and use the rest for training. Repeat for all partitions.

Residuals should sample a unit normal distribution (far right).

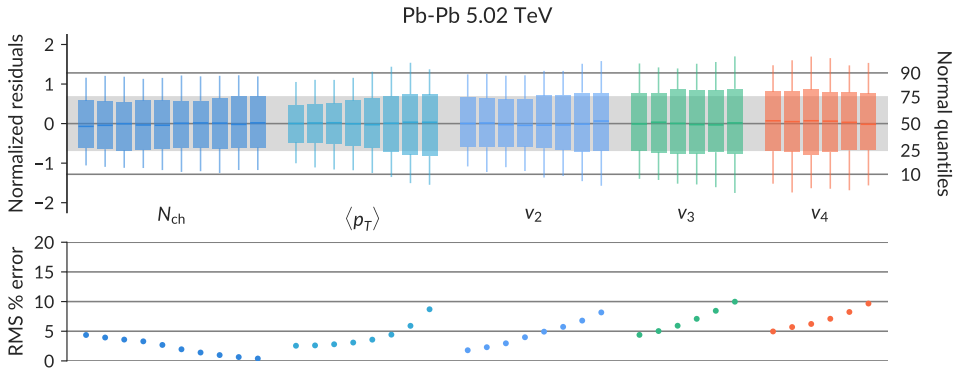


EMULATOR VALIDATION



- Model-data residuals should have zero mean and unit variance.
- Root-mean square error expressed as percentage of overall design variance.

EMULATOR VALIDATION



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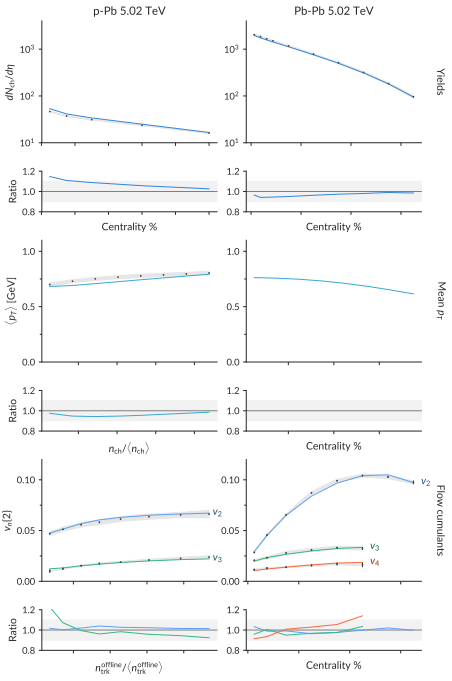

Maximum a posteriori (best fit) predictions from the calibrated emulator (not the model).

Parameter	Description	Value
Norm	Overall normalization	20.3
p	Entropy deposition parameter	0.04
σ_{fluct}	Relative constituent fluct. std.	0.92
r	Constituent sampling radius	0.86 fm
χ_{struct}	Constituent structure parameter	0.32
m	Number of constituents	6
d^3_{min}	Nucleon exclusion volume	1.1 fm^3
τ_{fs}	Free streaming time	0.42 fm/c
η/s min	Shear viscosity at T_c	0.11
η/s slope	Slope above T_c	0.98 GeV^{-1}
η/s crv	Curvature above T_c	-0.14
ζ/s norm	Bulk viscosity peak height	0.06
ζ/s width	Bulk viscosity peak width	0.017 GeV
ζ/s temp	Bulk viscosity peak location	0.18 MeV
T_{switch}	Particlization temperature	142 MeV