

Hydrodynamic Fluctuations in Relativistic Heavy-Ion Collisions

Quark Matter 2018

Mayank Singh¹

Chun Shen², Scott McDonald¹, Sangyong Jeon¹, Charles Gale¹

¹McGill University

²Brookhaven National Laboratory

compute | calcul
canada | canada



McGill



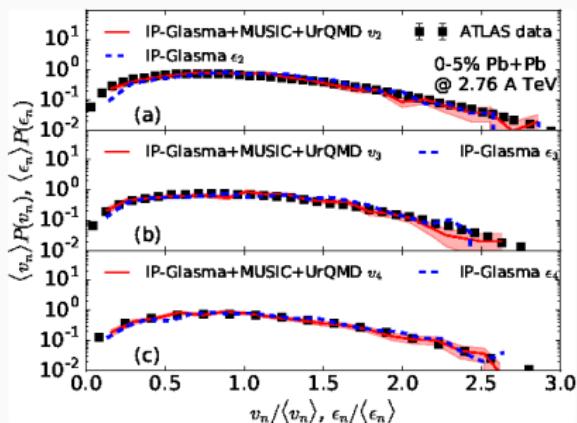
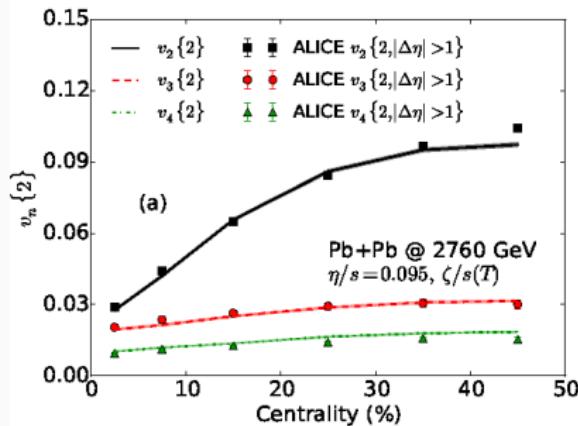
Outline

1. Motivation for hydrodynamic fluctuations
2. Solving Stochastic Hydrodynamics
3. Hydrodynamic Fluctuations in conjunction with Initial-State Fluctuations
4. Phenomenological Consequences

Motivation for hydrodynamic fluctuations

Why include thermal fluctuations

- Relativistic viscous hydrodynamics has been very successful



What is missing - The Fluctuation-Dissipation Theorem

Figures from S. McDonald et al. PRC, 2017

Stochastic Hydrodynamics

$$\partial_\mu (T_{\text{ideal}}^{\mu\nu} + \pi^{\mu\nu} + \Pi \Delta^{\mu\nu} + S^{\mu\nu}) = 0$$

$$(u \cdot \partial) \pi^{\mu\nu} = -\frac{1}{\tau_\pi} (\pi^{\mu\nu} - \eta \nabla^{\langle \mu} u^{\nu \rangle} + \dots)$$

$$(u \cdot \partial) \Pi = -\frac{1}{\tau_\Pi} (\Pi - \zeta (\partial \cdot u) + \dots)$$

$$(u \cdot \partial) S^{\mu\nu} = -\frac{1}{\tau_\pi} (S^{\mu\nu} - \xi^{\mu\nu} + \dots)$$

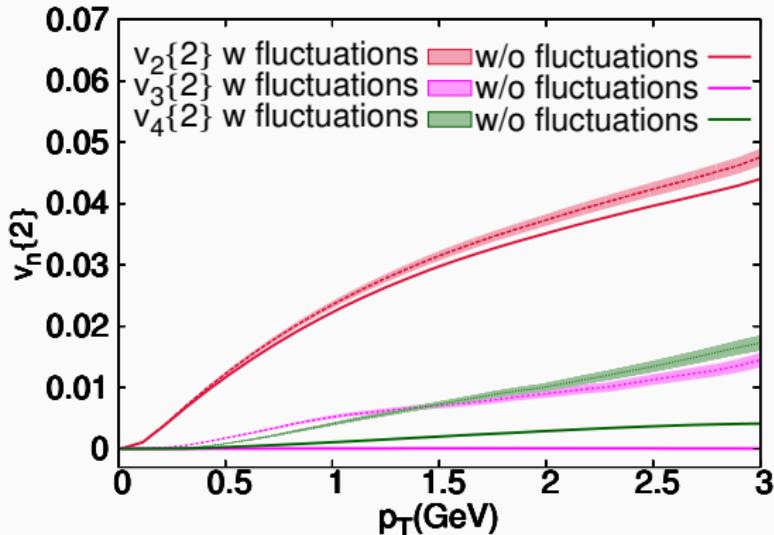
$$\langle \xi^{\mu\nu}(x) \xi^{\alpha\beta}(x') \rangle = 2\eta T \left[\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha} - \frac{2}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta} \right] \delta^4(x - x')$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu; \quad \nabla^\mu = \Delta^{\mu\nu} \partial_\nu; \quad \nabla^{\langle \mu} u^{\nu \rangle} = (\nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^{\mu\nu} \nabla_\alpha u^\alpha)$$

J.I. Kapusta, B. Mueller and M. Stephanov, PRC, 2012; C. Young, PRC, 2014

Why include thermal fluctuations

- Perturbative calculations
- Thermal fluctuations potentially affect multi-particle correlations

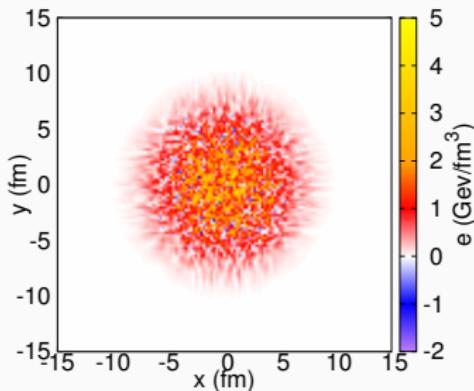
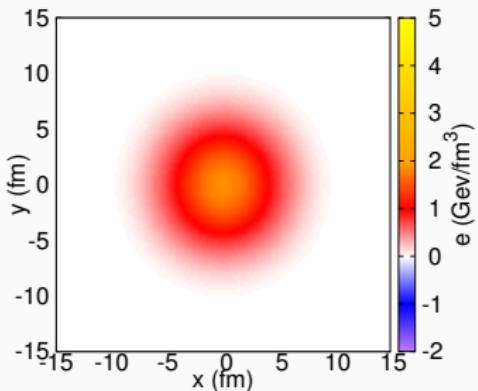


Our previous calculations using perturbative linearized fluctuations with smooth initial conditions

Solving Stochastic Hydrodynamics

Hydrodynamic Noise (we need to be careful)

- Arbitrarily large gradients
- Potentially negative energy densities

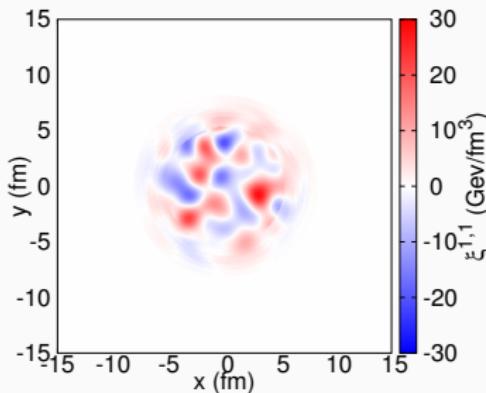
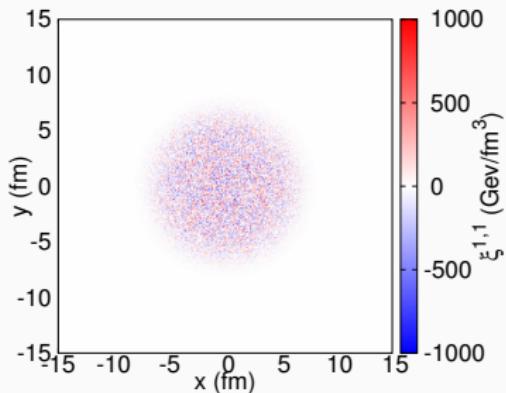


Energy density profile for our perturbative calculation

Removing high k modes

- We do not need all modes
- Modes above p_{cut} decay on small time scales¹

$$\sqrt{\frac{\omega}{T_\pi}} \ll p_{cut}$$

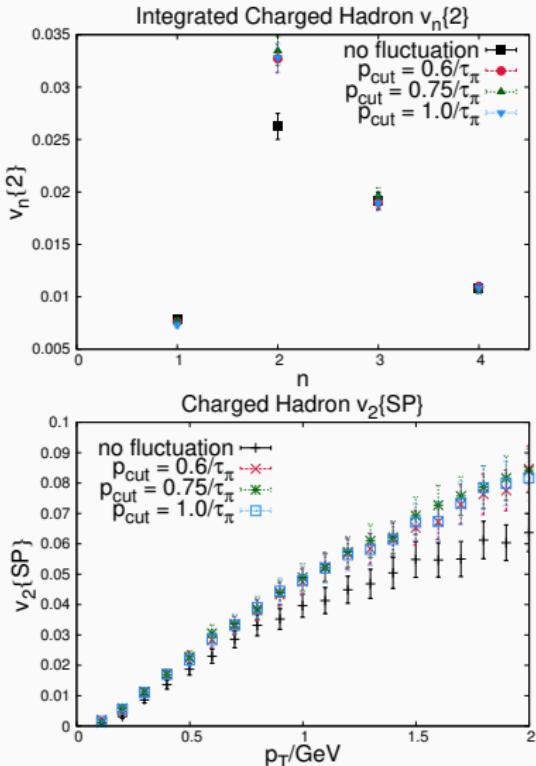


Noise Sampling before and after removal of high k modes

¹Y. Akamatsu, A. Mazeliauskas and D. Teaney, PRC, 2017

Choosing a p_{cut}

- A natural scale is $p_{cut} \sim 1/\tau_\pi$
- Charged hadron v_n are independent for three choices of p_{cut}
- Energy conservation is verified
- We use $p_{cut} = 0.75/\tau_\pi$ in calculations shown here



0-5% centrality. IP-Glasma + MUSIC + UrQMD
McGill University

Temporal evolution of thermal fluctuations

Evolution of fluctuations in an event with smooth glauber initial condition at 0fm impact parameter

Hydrodynamic Fluctuations in conjunction with Initial-State Fluctuations

Our Model

IP-Glasma initial conditions include sub-nucleonic color charge fluctuations and classical Yang-Mills evolution

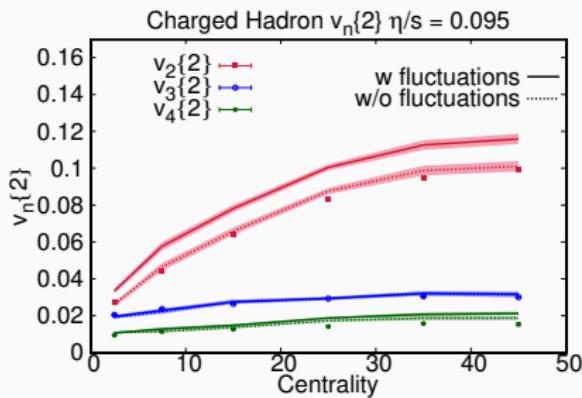
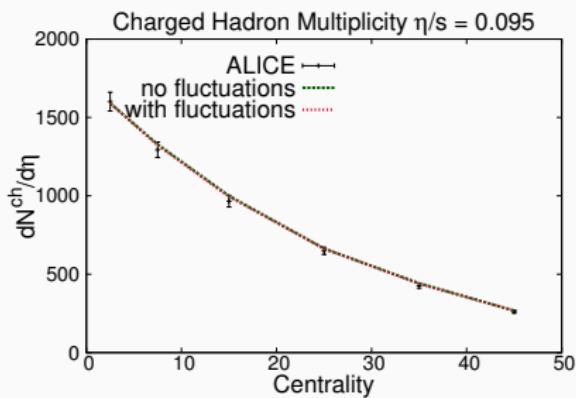
MUSIC is a second order 3+1D relativistic hydrodynamics package where stochastic terms were added

- $\tau_{sw} = 0.4 \text{ fm}$
- EOS: s95p-v1.2
- Constant η/s
- Temperature dependent bulk viscosity
- Shear Fluctuations
- $T_{sw} = 145 \text{ MeV}$

UrQMD does hadronic re-scatterings and resonance decays after freeze-out

Effects of fluctuations on observables

- Multiplicity is unaffected
- v_2 is the largest affected flow harmonic



ALICE, PRL,106, 032301, 2011

ALICE, PRL,107, 032301, 2011

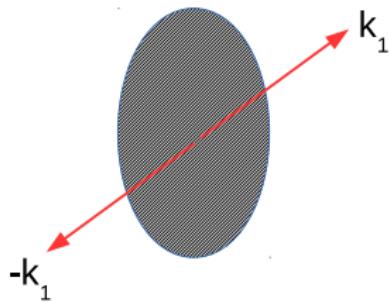
Fluctuations in Momentum Space

Let us look at the fluctuations in the momentum space

$$\langle \delta A(\mathbf{x}_1) \delta A(\mathbf{x}_2) \rangle \approx C(\mathbf{x}_1) \delta^3(\mathbf{x}_1 - \mathbf{x}_2)$$

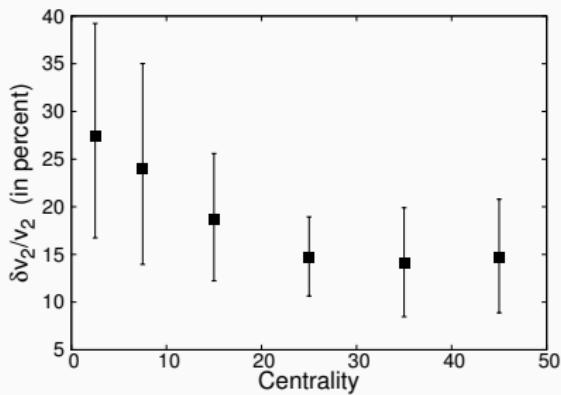
$$\begin{aligned}\langle \delta \tilde{A}(\mathbf{k}_1) \delta \tilde{A}(\mathbf{k}_2) \rangle &= \int d^3x_1 d^3x_2 C(\mathbf{x}_1) e^{-i(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)} \delta^3(\mathbf{x}_1 - \mathbf{x}_2) \\ &= \int d^3x C(\mathbf{x}) e^{-i(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{x}}\end{aligned}$$

The most dominant contribution would be $\mathbf{k}_2 = -\mathbf{k}_1$

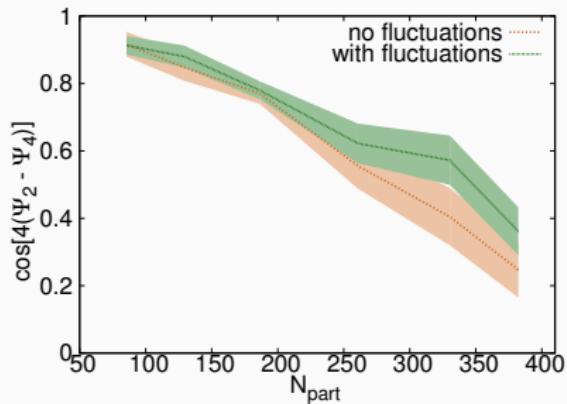


Effects of fluctuations on observables

- Event planes of v_2 and v_4 become more correlated at lower centralities
- Fluctuations mimic a geometric effect



Enhancement in v_2

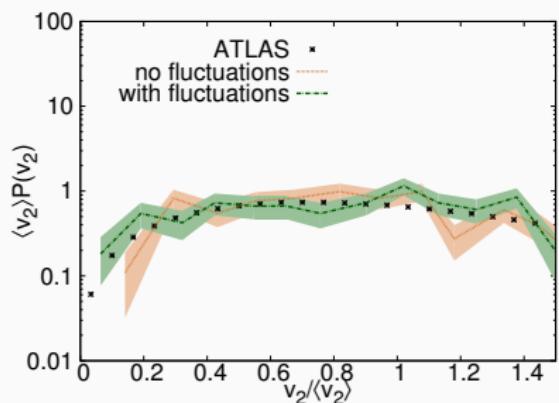


Ψ_2 and Ψ_4 correlator

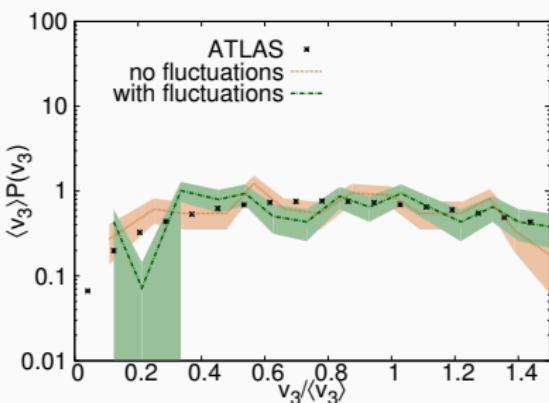
Effects of fluctuations on observables

- v_n distributions seems to be primarily determined by initial state fluctuations

0-5% centrality



v_2 distribution



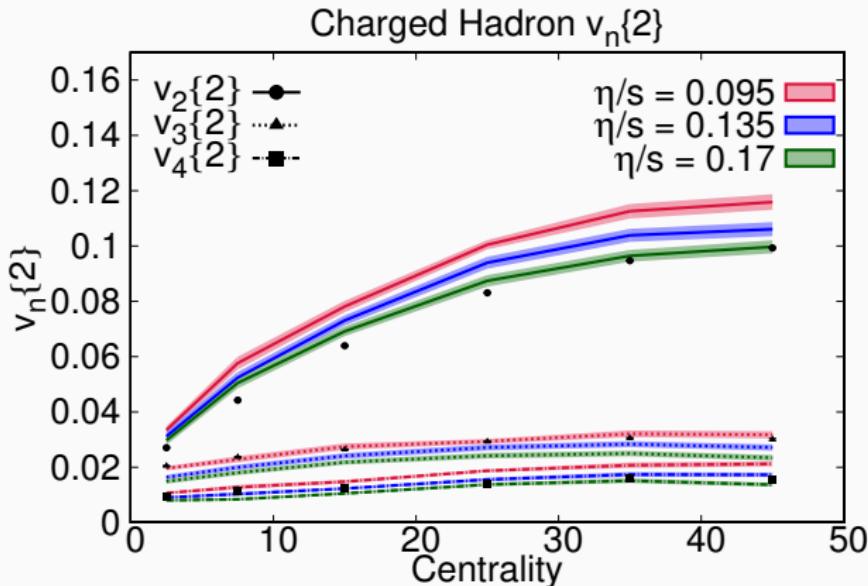
v_3 distribution

ATLAS, JHEP, 11, 183, 2013

Phenomenological Consequences

Readjusting Shear Viscosity

- Hydrodynamic fluctuations should be accounted for when extracting flow coefficients



Summary

- Hydrodynamic fluctuations are crucial when extracting the transport coefficients of QGP phenomenologically
- Thermal fluctuations tend to introduce back-to-back modes in momentum space
- v_n distribution seems to be primarily determined by initial state fluctuations
- Future work
 - Include Bulk viscous fluctuations
 - Explore effects of temperature dependent shear viscosity
 - Improve statistics for hadronic observables
 - Evaluate electromagnetic observables
 - Explore rapidity correlators (see talk by S. McDonald, yesterday, 16:50)
 - Study small systems

Backup

Removal of high k modes

- Noise term $\delta A(\mathbf{x})$ is sampled from the equal time correlator of the fluctuation-dissipation theorem
- Fourier transform of the noise is evaluated numerically

$$\delta \tilde{A}(\mathbf{k}) = \int_V d^3x A(\mathbf{x}) e^{-\mathbf{k} \cdot \mathbf{x}}$$

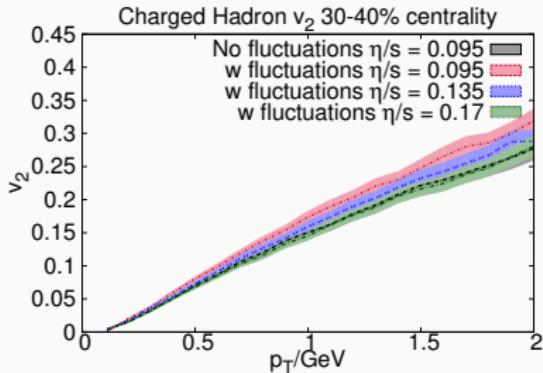
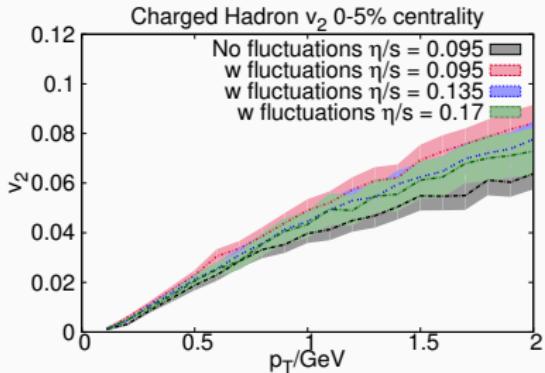
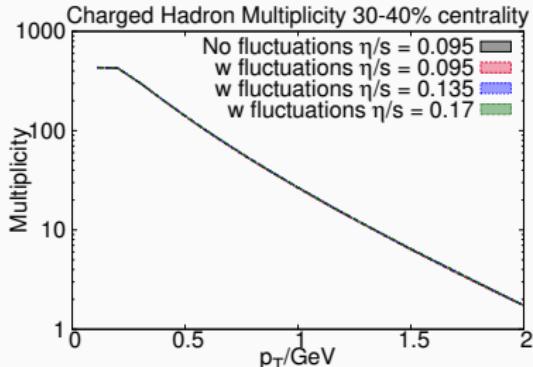
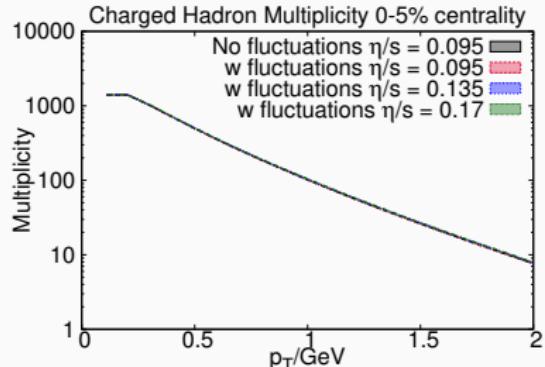
- p_{cut} is determined locally in each fluid cell

$$p_{cut}(\mathbf{x}) = 0.75 / \tau_\pi(\mathbf{x})$$

- Sampled noise is inverse fourier transformed for each \mathbf{x} using local $p_{cut}(\mathbf{x})$

$$\delta A'(\mathbf{x}) = \int_{-p_{cut}(\mathbf{x})}^{p_{cut}(\mathbf{x})} d^3k \delta \tilde{A}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{x}}$$

Differential Observables



Differential Observables

