Collectivity and Fluctuations in Au+Au with PHENIX

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We present results of $v_2$ and $v_3$ and their fluctuations using two complementary techniques:

- Multiparticle cumulants
- Event-by-event folding procedure

Multiparticle cumulants

- Insensitive to detector response
- Different $N_{\text{particle}}$ correlations have different response to fluctuations and non-flow
- Often assume small variance to extract $\sigma_v$
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Event-by-event distribution
• Truth $v_n$ distributions convolved with detector response
• All observables may be extracted from unfolded distribution directly
Analysis Methods

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Tracks for this analysis measured in PHENIX FVTX: 1 < |\eta| < 3
\ p_T \ \gtrsim \ 0.3 \ \text{GeV/c}
v$_2$ via Multiparticle Cumulants

v$_2$\{2\} from one sub event agrees with v$_2$\{2,|$\Delta$|$\eta$|>2\} in the most central events

More peripheral
- Lower multiplicity
- Non-flow relatively more dominant

https://arxiv.org/abs/1804.10024
v$_2$ via Multiparticle Cumulants

v$_2$\{2\} > v$_2$\{4\} \approx v$_2$\{6\} \approx v$_2$\{8\}

Consistent with expectations given Gaussian fluctuations and small variance

https://arxiv.org/abs/1804.10024
$v_2^2 = \langle v \rangle^2 + \sigma_v^2$

In small variance:

$\approx \langle v \rangle^2 - \sigma_v^2$
$v_2$ Relative fluctuations

\[ v_2\{2\}^2 = \langle v \rangle^2 + \sigma_v^2 \]

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\[ v_2\{4\}^2 \approx \langle v \rangle^2 - \sigma_v^2 \]

Compare with $\sigma_\epsilon / \langle \epsilon_2 \rangle$ from Glauber

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**Small variance not valid in very central**

**Non-linearity in hydro?**
$v_2$ Relative fluctuations

**PHOBOS 2010**

Phys. Rev. Lett. **104**, 142301

$|\eta| < 1$

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**PHENIX**

Au+Au $\sqrt{s_{_{NN}}} = 200$ GeV

$h^+ 1<|h|<3$

- MC Glauber, cumulant based estimate
- MC Glauber, direct calculation
- Data

Sys. Uncert. 2%

https://arxiv.org/abs/1804.10024
\[ v_3\{4\} = (-c_3\{4\})^{1/4} \]

Complex valued \( v_3\{4\} \)

https://arxiv.org/abs/1804.10024
$v_3$ Cumulants

Niseem Magdy QM2018 #588, May 15, 11:30

https://arxiv.org/abs/1804.10024
Flow vector measured event-by-event will have multiple sources of fluctuations

**Intrinsic**
- Impact parameter
- Nucleon positions

**Measurement/response**
- Detector resolution
- Acceptance
  - Finite particle sampling
Event-by-Event Flow

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Observed distribution has convolution of all fluctuations
→ Would like to unfold to intrinsic distribution
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ATLAS: J. HEP(2013) 2013: 183
CMS: https://arxiv.org/abs/1711.05594
Flow Vector Response

Observed to be Gaussian over 4 orders of magnitude
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If observed flow vector is Gaussian about truth, PDF of observed $|\vec{v}_n|$ is Bessel-Gaussian:

$$p(v_n^{\text{obs}} | v_n) \propto v_n^{\text{obs}} e^{-\frac{(v_n^{\text{obs}})^2 + v_n^2}{2\delta^2}} I_0 \left( \frac{v_n^{\text{obs}} v_n}{\delta^2} \right)$$

1 parameter
Unfolding

Given the Bessel-Gaussian response, we can try to unfold the observed distribution to the truth.

We employed SVD to decompose the response matrix:

$$\hat{A} = \hat{U} \hat{\Sigma} \hat{W}^T$$

In which case, the truth distribution is expressed as:

$$\hat{Q}_2 = \sum_{i=1}^{\text{Dim}(A)} \varphi_i \left( \frac{\hat{u}_i^T \cdot \hat{Q}^{\text{obs}}}{\sigma_i} \right) \hat{w}_i$$

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Fails the “Picard condition”

Procedure highly sensitive to statistical fluctuations in observed distribution

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If we make the **ansatz** that the intrinsic fluctuations are Gaussian, the truth $v_n$ distribution is also Bessel-Gaussian

$$p(v_n) = \frac{v_n}{\delta v_n^2} e^{-\frac{(v_n^2 + (v_n^{\text{RP}})^2)}{2\delta v_n^2}} I_0 \left( \frac{v_n v_n^{\text{RP}}}{\delta v_n^2} \right)$$

**Forward-fold** truth distributions through the response matrix

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*Forward-fold* truth distributions through the response matrix.

$v_2$ parameters tightly constrained.

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**Forward-fold** truth distributions through the response matrix

![Graph showing PHENIX Au+Au $v_n$](https://arxiv.org/abs/1804.10024)
$v_2$ Results

- Extracted $v_2$ in very good agreement
- Fluctuations agree in centrality where small variance expected to hold (>10%)
Fit to Observed $v_3$

Applying the same logic to $v_3$

$v_3$ has family of parameters

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Fit to Observed $v_3$

Applying the same logic to $v_3$

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\( v_3 \) Results

- Family of parameters give good constraints on \( <v_3> \) and \( \sigma_3 \)
- Relative fluctuation at constant value of 0.52 – a feature of the Bessel-Gaussian in the large fluctuation limit
$v_3$ Fluctuations

$n = 3$: Cumulants disagree with E-by-E
→ Driven by geometry fluctuations

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Glauber reproduces same behavior

PHENIX has measured cumulants and Bessel-Gaussian parameters for $v_2$ and $v_3$ at forward rapidity.

- $v_2$ results consistent with small variance except in most central and most peripheral

- $v_3$ results consistent with large variance limit

- $v_3(4)$ complex suggesting larger fluctuations at RHIC than LHC
Thank You!
FVTX efficiency

https://arxiv.org/abs/1804.10024
Bessel-Gaussian fluctuations

Observed relative fluctuation saturates in the large intrinsic fluctuation limit

https://arxiv.org/abs/1804.10024
$v_3$ method comparison

https://arxiv.org/abs/1804.10024
PHOBOS fluctuations

Phys. Rev. Lett. 104, 142301