FLUCTUATIONS OF CONSERVED CHARGES IN THE CANONICAL ENSEMBLE

CONFRONTING EXPERIMENTAL RESULTS WITH THEORY

Peter Braun-Munzinger[1], Anar Rustamov[1,2,3], Johanna Stachel[2]

GSI/EMMI[1], Universität Heidelberg[2], BSU/NNRC[3]

Quark Matter 2018

Venezia

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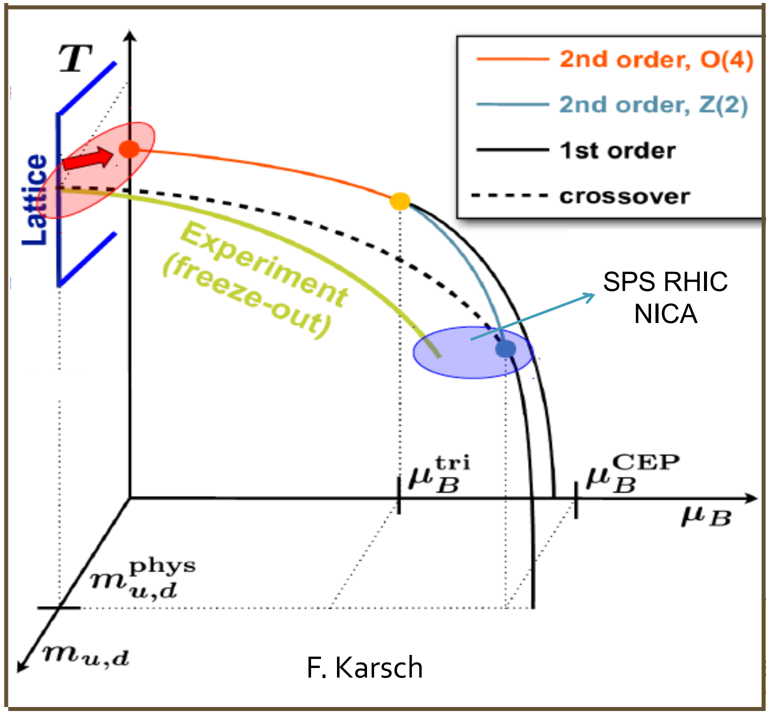
- Why Fluctuations
- Canonical suppression
- Comparison to experiments
- Critical fluctuations

Why Fluctuations



- To probe the structure of strongly interacting matter
  - Locate phase boundaries
  - Search for critical phenomena
  - …

E-by-E fluctuations are predicted within Grand Canonical Ensemble

$$\frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle^2} = \frac{T\chi}{V}, \quad \chi = -\frac{1}{V}\frac{\partial V}{\partial P}$$

direct link to the EoS

fingerprints of criticality for $m_{u,d} = 0$ survive at crossover with $m_{u,d} \neq 0$

A. Bazavov et al., Phys.Rev. D85 (2012) 054503

probing the response of the system to external perturbations

Baselines from theory (in GCE)

for a thermal system in a fixed volume V within the Grand Canonical Ensemble

$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

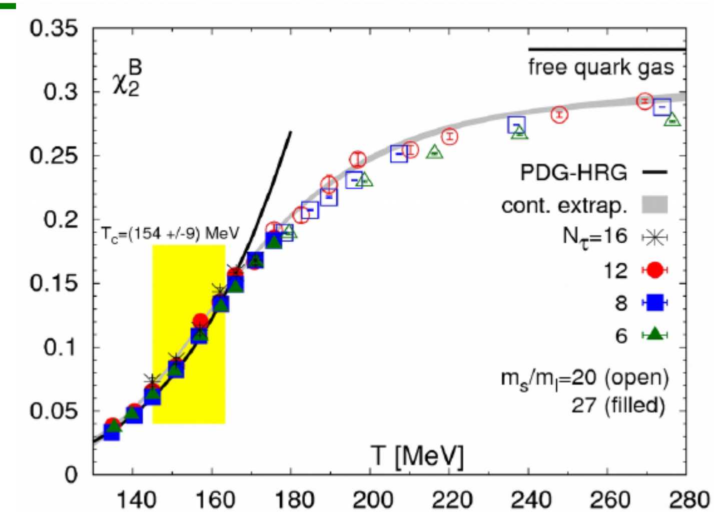
$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial(\mu_N/T)^n} \qquad \frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_{B,Q,S})$$

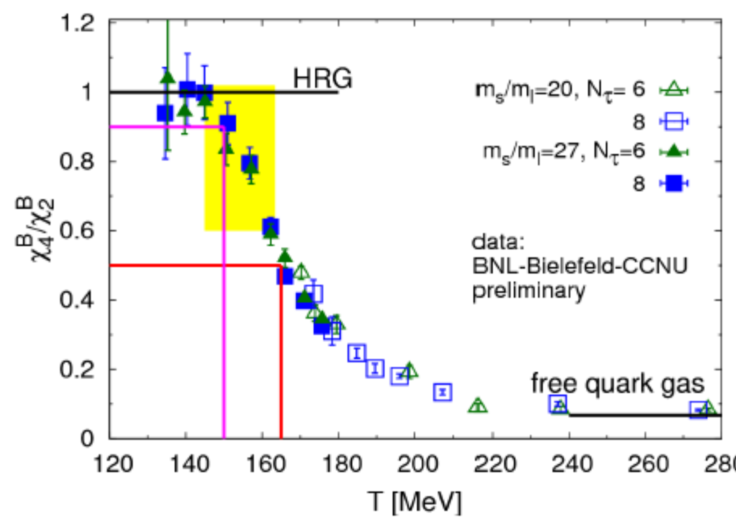
- *In experiments*
  - ***Volume (participants) fluctuates from E-to-E***
  - ***Global conservation laws are important***

$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \equiv \gamma_2 \sigma^2 \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \qquad \frac{\kappa_3(\Delta N_B)}{\kappa_2(\Delta N_B)} \equiv \gamma_1 \sigma \neq \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$

V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114





smaller than in HRG for T > 150 MeV

F. Karsch, QM17, arXiv:1706.01620

O. Kaczmarek, QM17, arXiv:1705.10682

The strategy

- Estimation of the effects of global conservation laws
  - Canonical suppression for cumulants in finite acceptance
- Subtraction of volume fluctuations

Fluctuations in GCE

Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh\left(\mu/T\right)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

z – single baryon partition function

Uncorrelated Poisson limit: $\langle N_B N_{\bar{B}}\rangle = \langle N_B\rangle\langle N_{\bar{B}}\rangle$

Net-Baryons → Skellam

$$\kappa_n = \langle N_B\rangle + (-1)^n\langle N_{\bar{B}}\rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \tanh\left(\frac{\mu}{T}\right) = \frac{\langle N_B\rangle - \langle N_{\bar{B}}\rangle}{\langle N_B\rangle + \langle N_{\bar{B}}\rangle}$$

Fluctuations in CE

Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh(\mu/T)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

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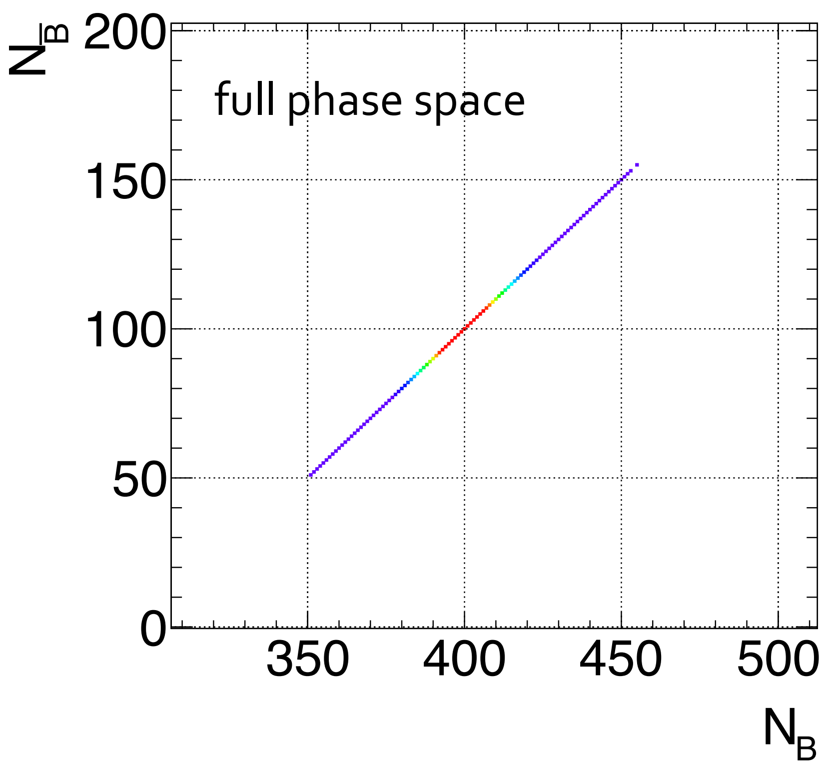
$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B-N_{\bar{B}}-B)=I_B(2z)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

- Non-Poisson single particles → **Canonical Suppression**
- Strong correlations $\langle N_B N_{\bar{B}}\rangle \neq \langle N_B\rangle\langle N_{\bar{B}}\rangle$
- **Net-Baryons do not fluctuate!**

K. Redlich and L. Turko, Z. Phys. C5 (1980) 201, V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902
P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich, V. Skokov, NPA 880 (2012), A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

Baryon number conservation (implementation)

$\langle N_B \rangle = 400, \langle N_{\overline{B}} \rangle = 100$



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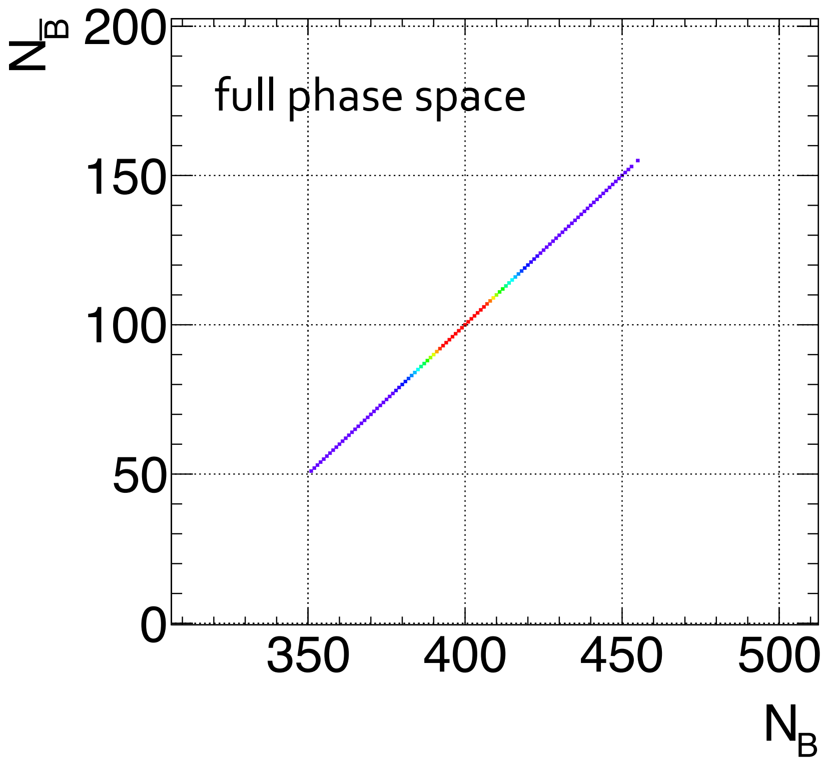
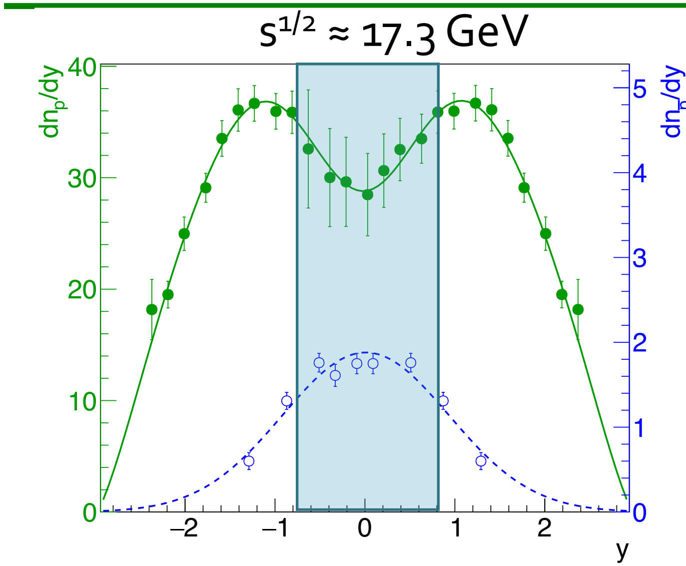
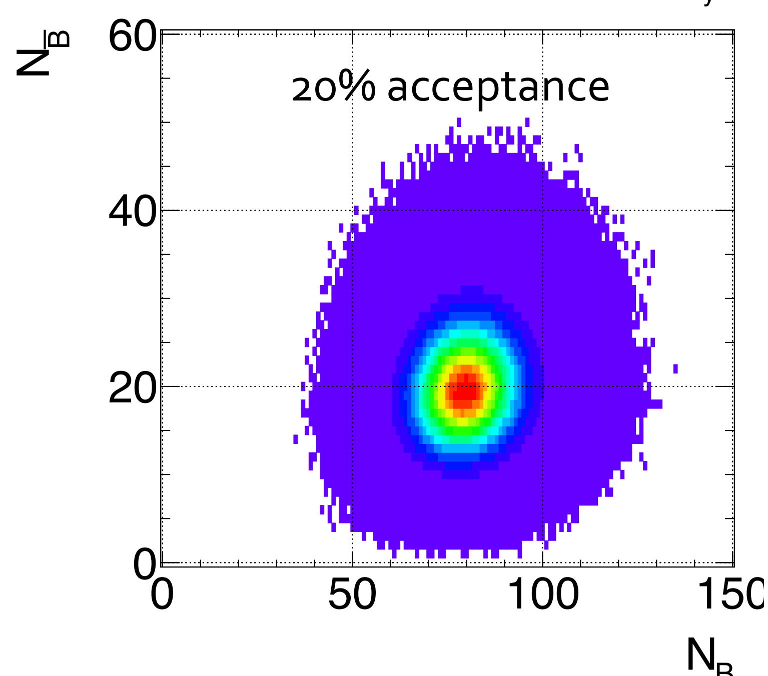
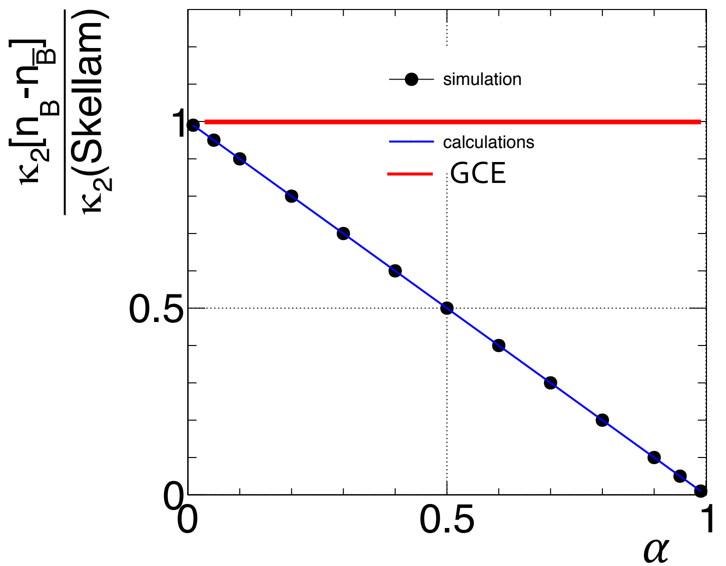






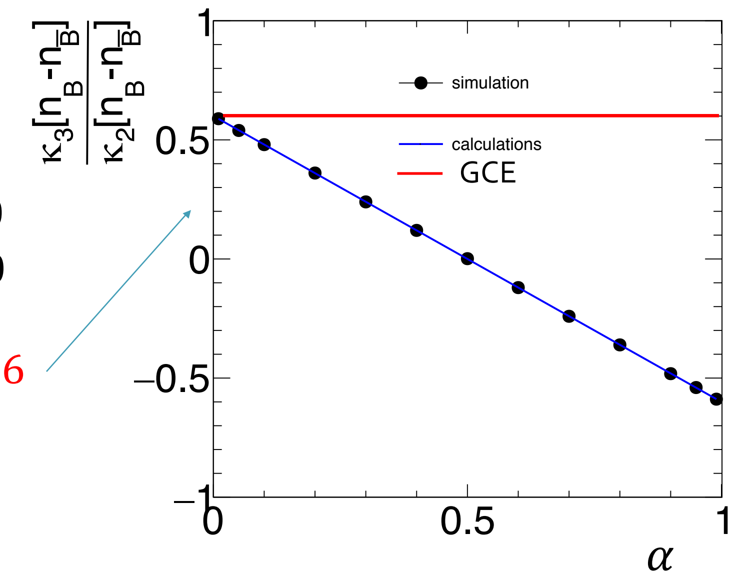
- **fluctuations of net-baryons appear only inside finite acceptance**

Fluctuations in CE (10^8 Events)



$\langle N_B \rangle = 400$
$\langle N_{\bar{B}} \rangle = 100$

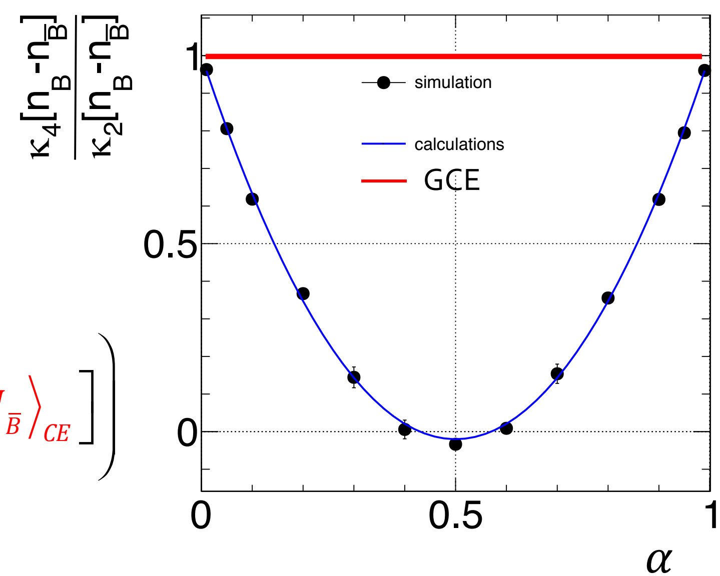
GCE: $\frac{k_3}{k_2} = 0.6$



$$\frac{\kappa_2}{\kappa_2(Skellam)} = 1 - \alpha \qquad \alpha = \frac{\langle N_B \rangle^{accepted}}{\langle N_B \rangle^{4\pi}}$$

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1 - 2\alpha) \xrightarrow{\langle n_{\bar{B}} \rangle \to 0} (1 - 2\alpha)$$

$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle N_B + N_{\bar{B}} \rangle_{CE}}\left[\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE} - \langle N_B \rangle_{CE} \langle N_{\bar{B}} \rangle_{CE}\right]\right)$$



Results From ALICE

Contribution from global baryon number conservation

$$\frac{\kappa_2(p-\bar{p})}{\kappa_2(Skellam)} = 1-\alpha \qquad \alpha = \frac{\langle p \rangle^{measured}}{\langle B \rangle^{4\pi}}$$

P. Braun-Munzinger, A. R., J. Stachel,
arXiv:1612.00702, NPA 960 (2017) 114

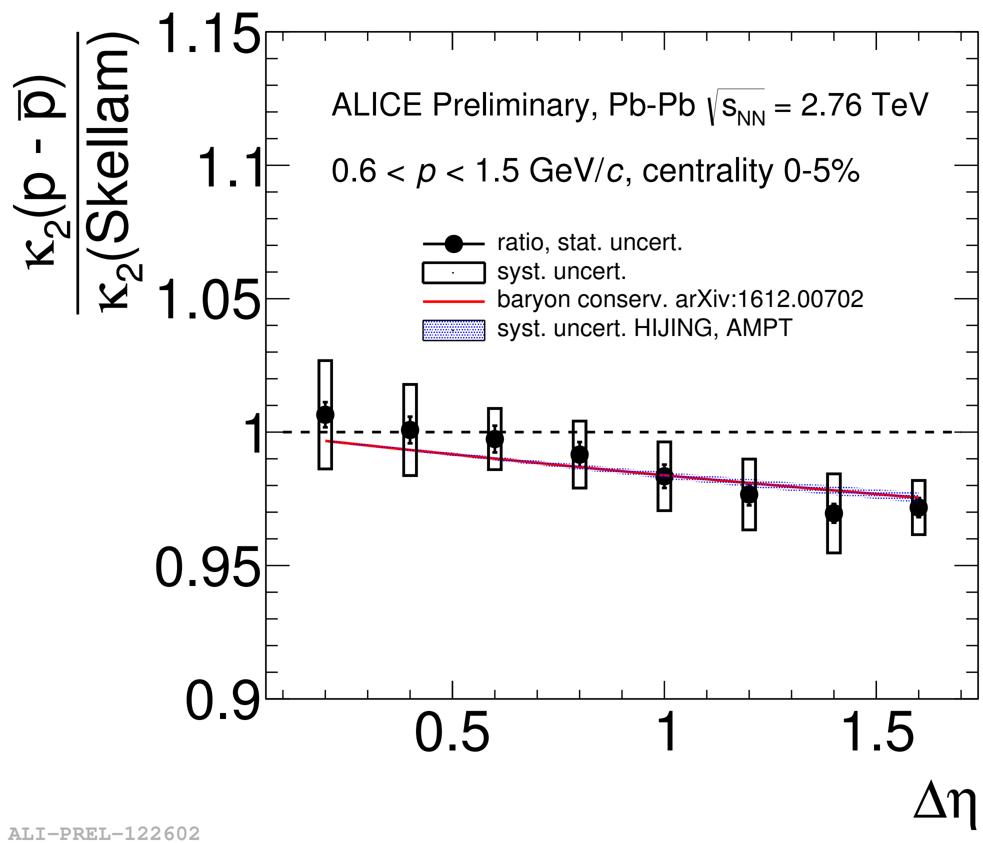
Inputs for $\langle B \rangle^{acc}$ from:

Phys. Lett. B 747, 292 (2015)
P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel

extrapolation from $\langle B \rangle^{acc}$ to $\langle B \rangle^{4\pi}$

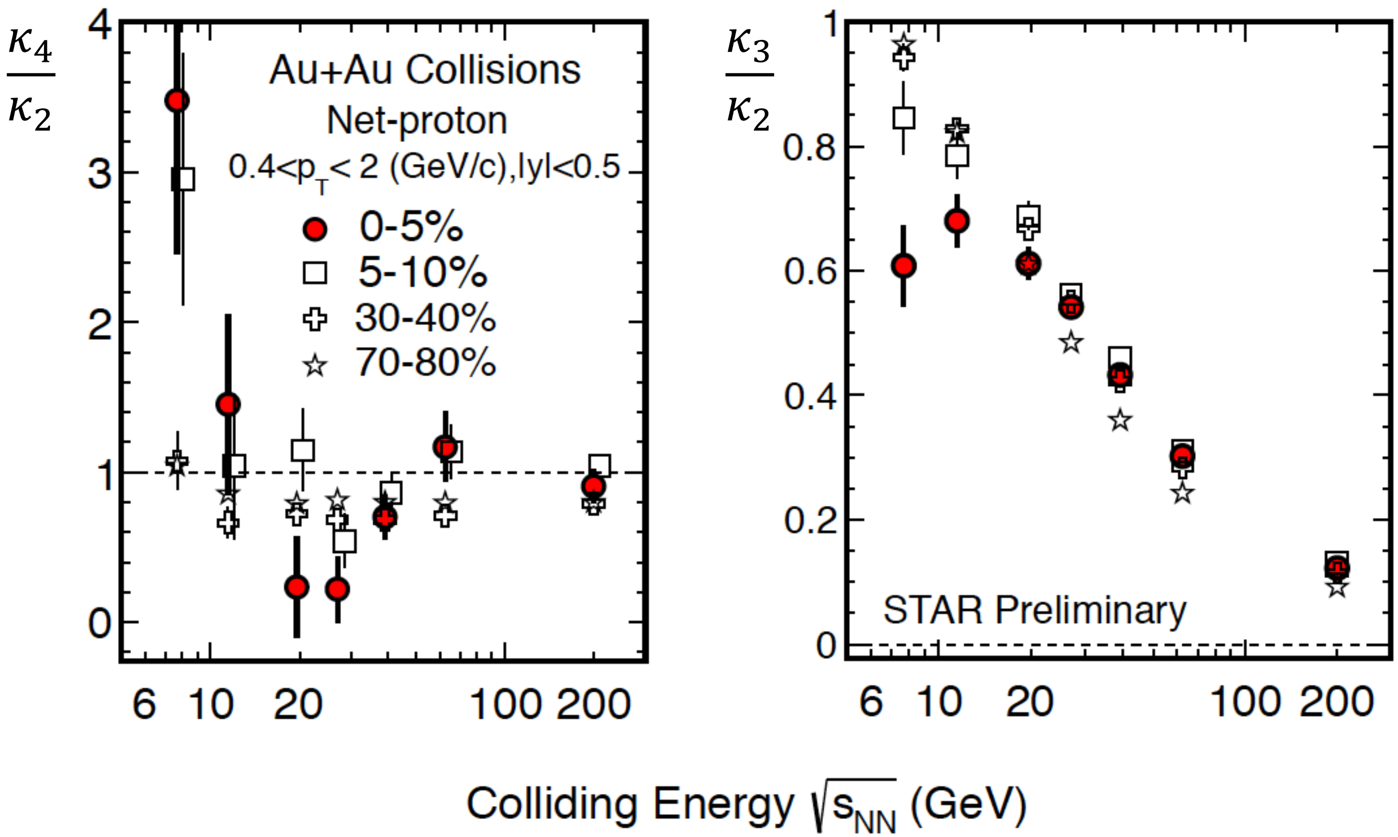
using HIJING and AMPT models

A. R., QM2017, arXiv:1704.05329



The deviation from Skellam is due to the global baryon number conservation.

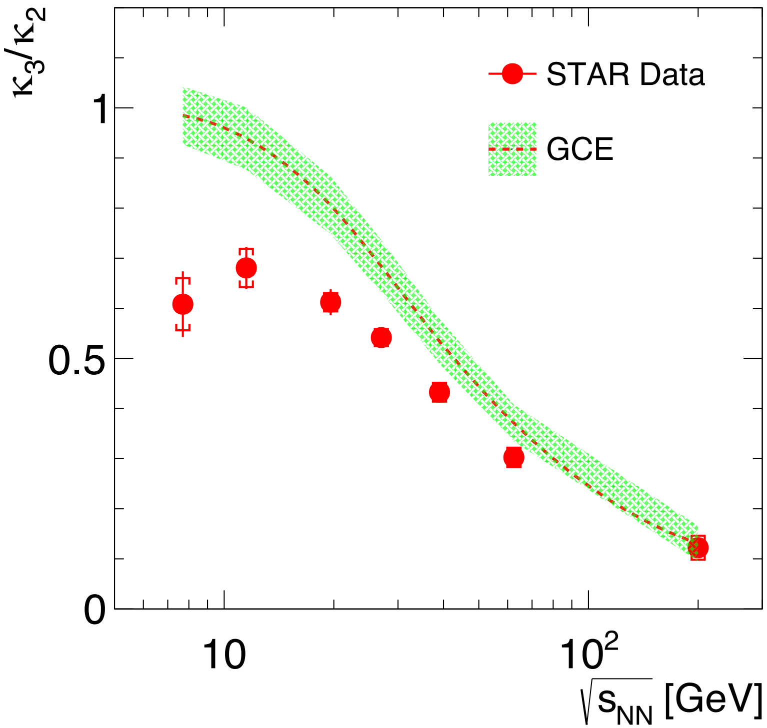
Results from STAR



- Approach to unity for the highest energies
- Non-monotonic behavior below 39 GeV
- Drop at 7.7 GeV for central events

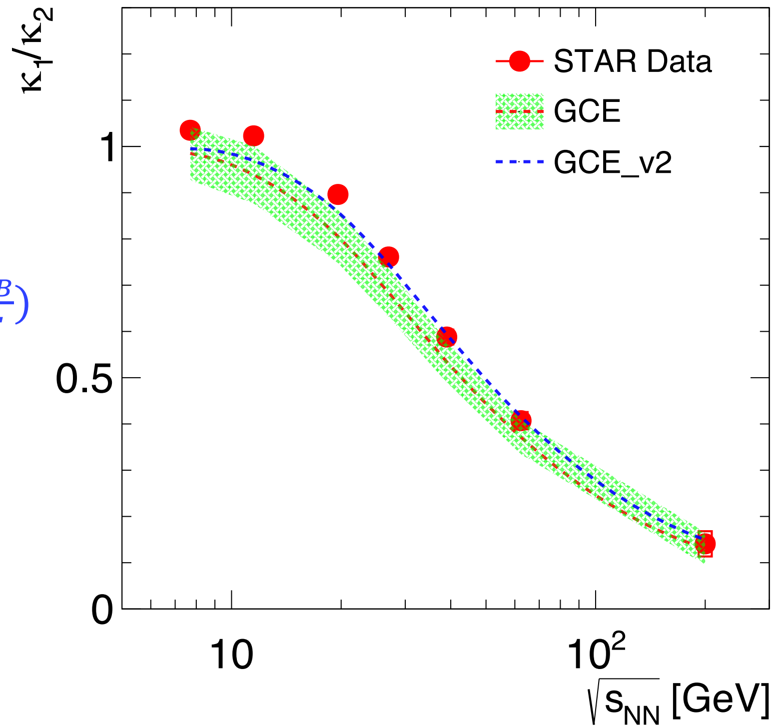
X. Luo, PoS CPOD2014, 019 (2015)
STAR: PRL 112, 032302 (2014)

Results from STAR



GCE: $\frac{\langle n_p \rangle - \langle n_{\bar{p}} \rangle}{\langle n_p \rangle + \langle n_{\bar{p}} \rangle}$

GCE_v2 : $tanh(\frac{\mu_B}{T})$

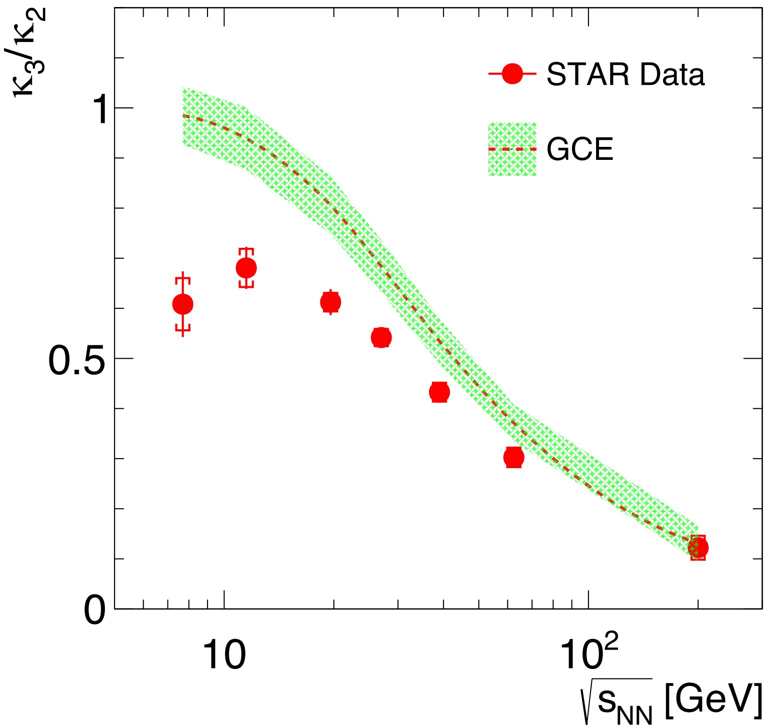


- why do κ_3/κ_2 and κ_1/κ_2 look so different?
- **critical phenomenon or non-dynamical effects?**

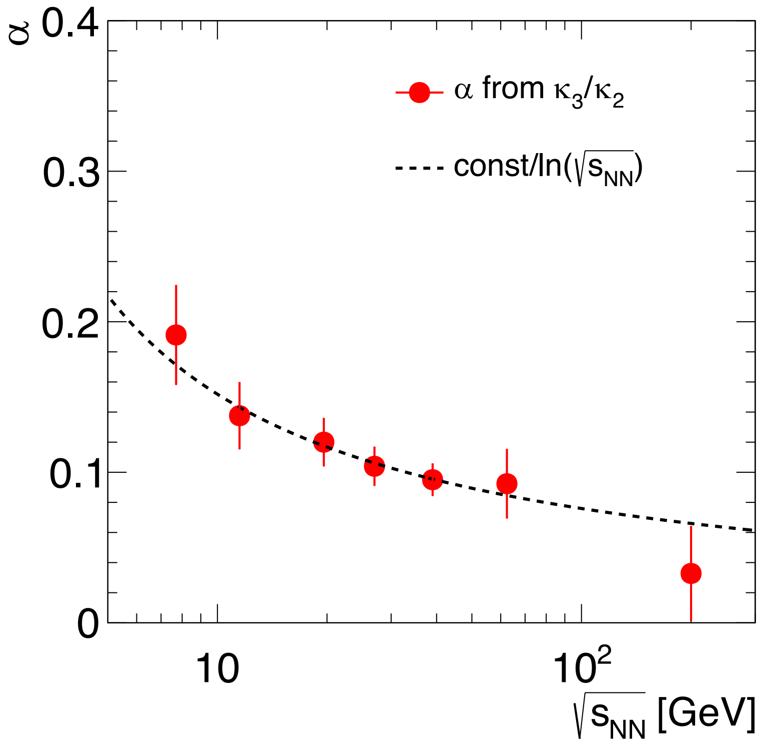
	κ_3/κ_2	κ_1/κ_2
vol. fluct	↗	↘
conserv. laws	↘	↗

conservation laws dominate!

The α parameter



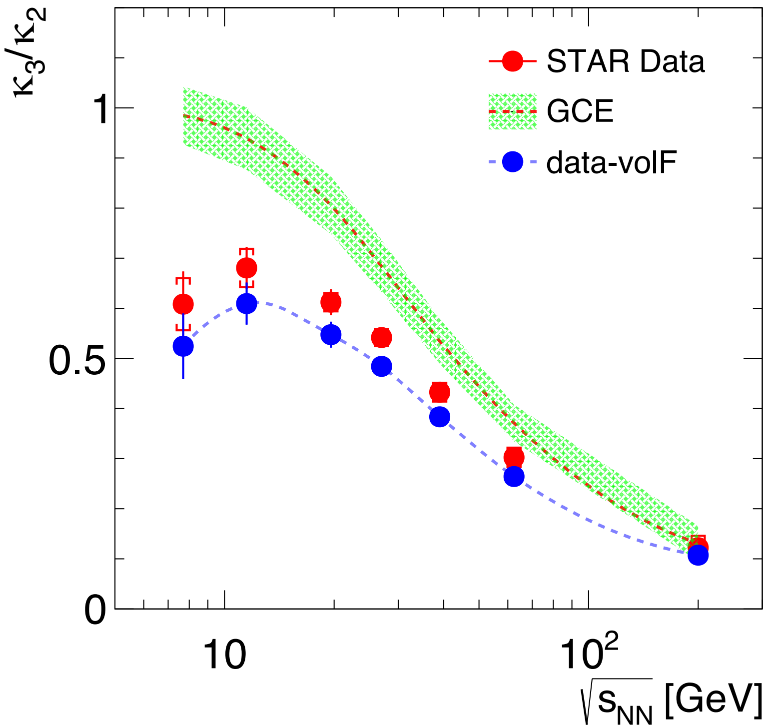
GCE: $\frac{\langle n_p \rangle - \langle n_{\bar{p}} \rangle}{\langle n_p \rangle + \langle n_{\bar{p}} \rangle}$



$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}} (1 - 2\alpha) \xrightarrow{\langle n_{\bar{p}} \rangle \to 0} (1 - 2\alpha)$$

$\langle n_p \rangle, \langle n_{\bar{p}} \rangle$ - also taken from STAR data

The α parameter



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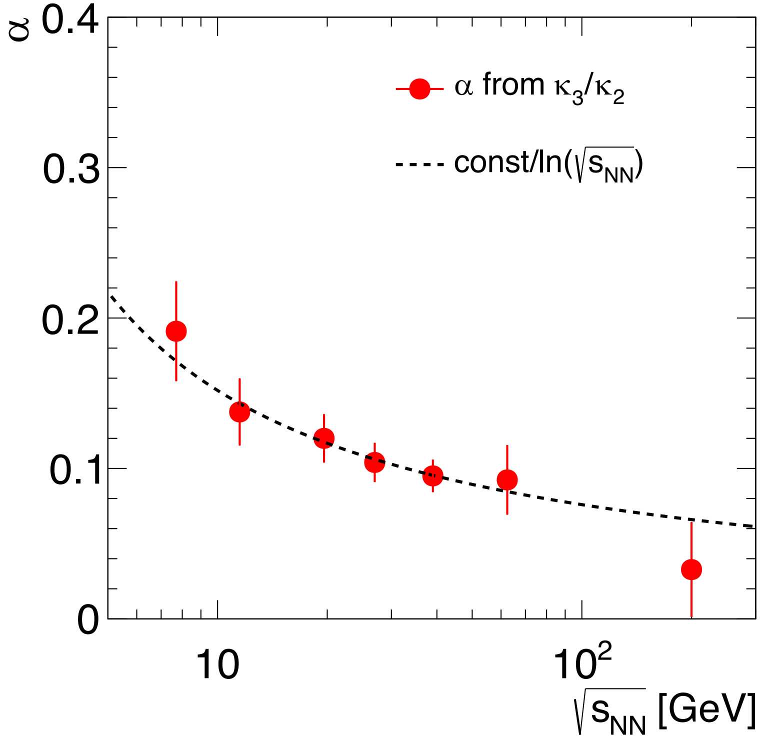
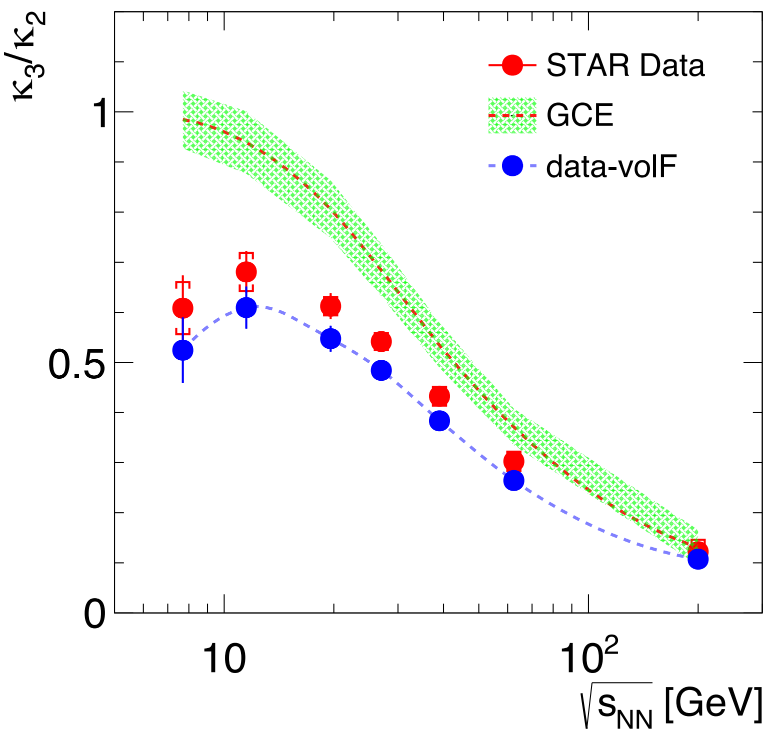


$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_p - n_{\bar{p}}\rangle_{CE}}{\langle n_p + n_{\bar{p}}\rangle_{CE}}(1-2\alpha) \xrightarrow{\langle n_{\bar{p}}\rangle \to 0} (1-2\alpha)$$

Volume fluctuations: P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

The α parameter



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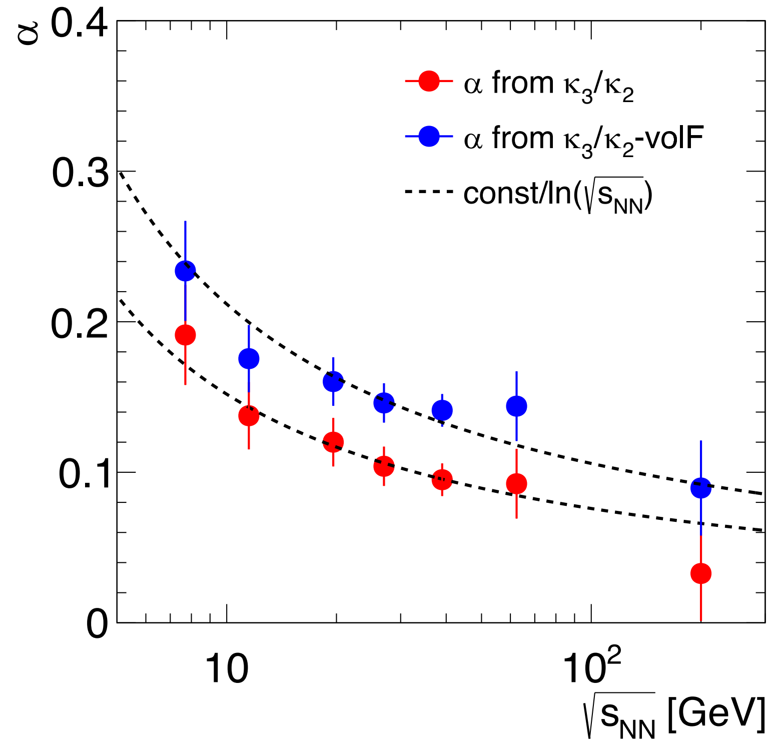
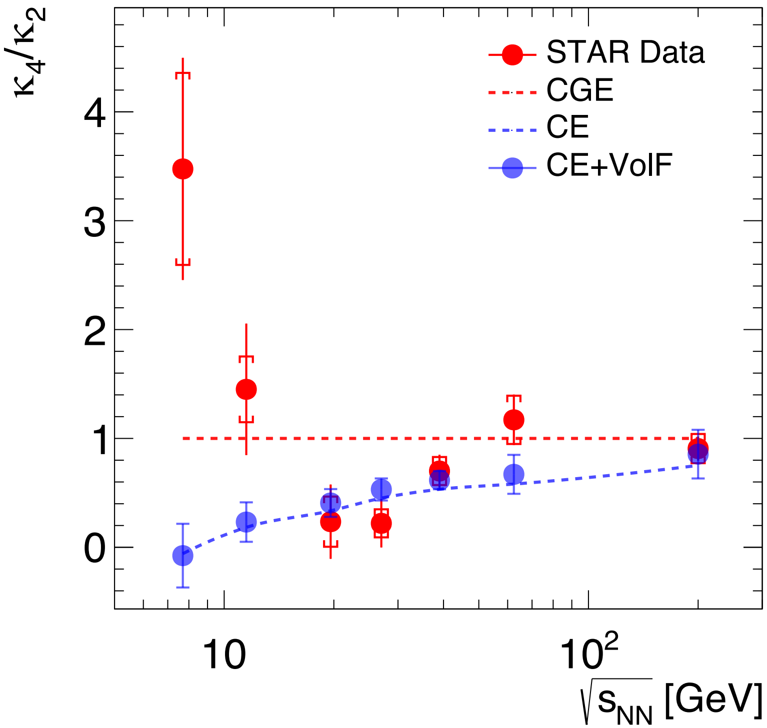
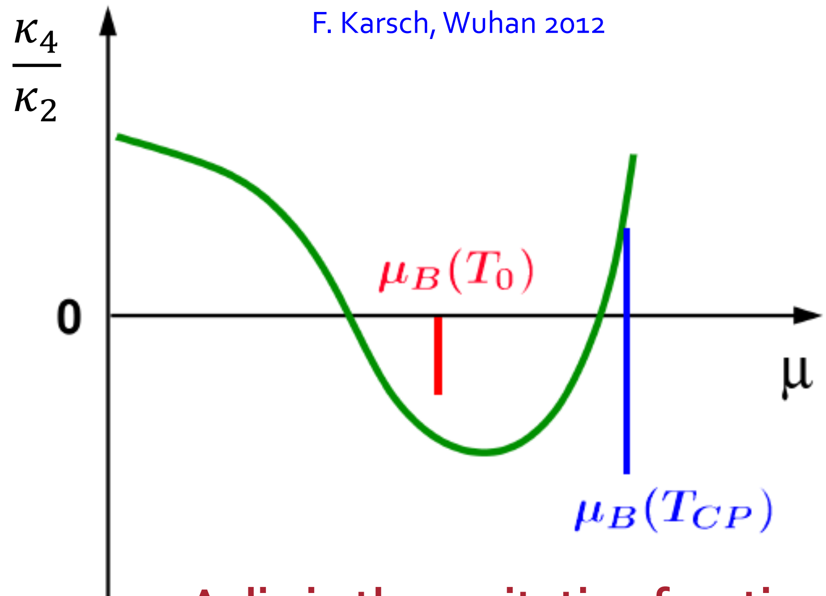


$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_p - n_{\bar{p}} \rangle_{CE}}{\langle n_p + n_{\bar{p}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{p}} \rangle \to 0} (1-2\alpha)$$

Volume fluctuations: P. Braun-Munzinger, A. R., J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

Predictions for κ_4/κ_2



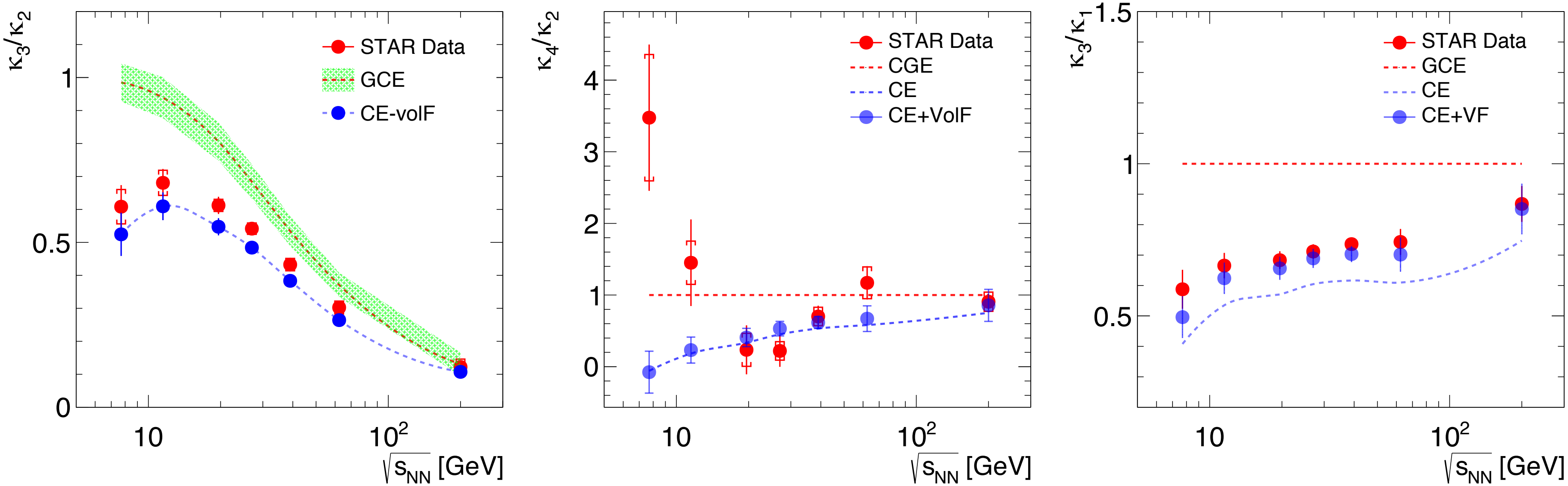


A dip in the excitation function seems to be generic

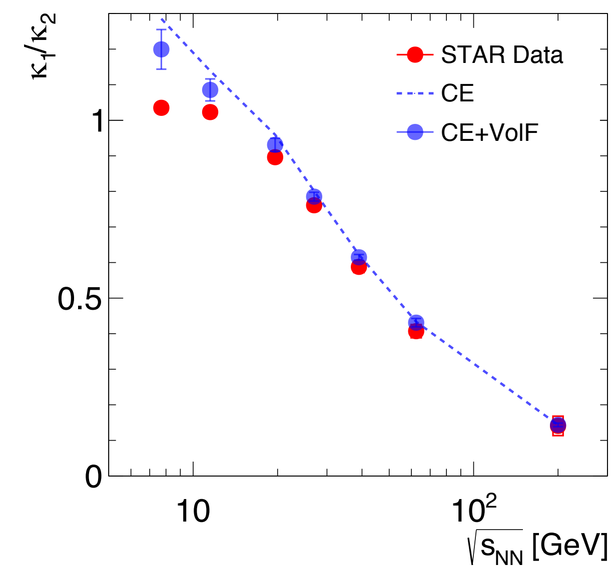
$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_p + n_{\bar{p}}\rangle_{CE}}\left[\langle n_p\rangle_{GCE}\langle n_{\bar{p}}\rangle_{GCE} - \langle n_p\rangle_{CE}\langle n_{\bar{p}}\rangle_{CE}\right]\right)$$

above 11.5 GeV CE suppression describes the data

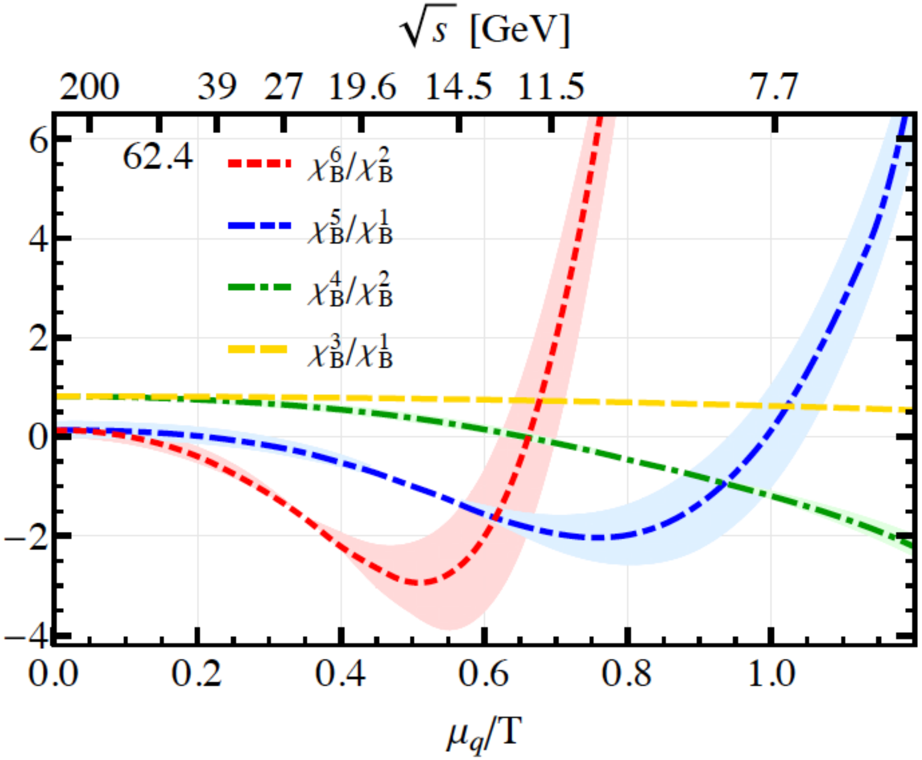
The global picture



above 11.5 GeV CE suppression for cumulants accounts for measured deviations from GCE

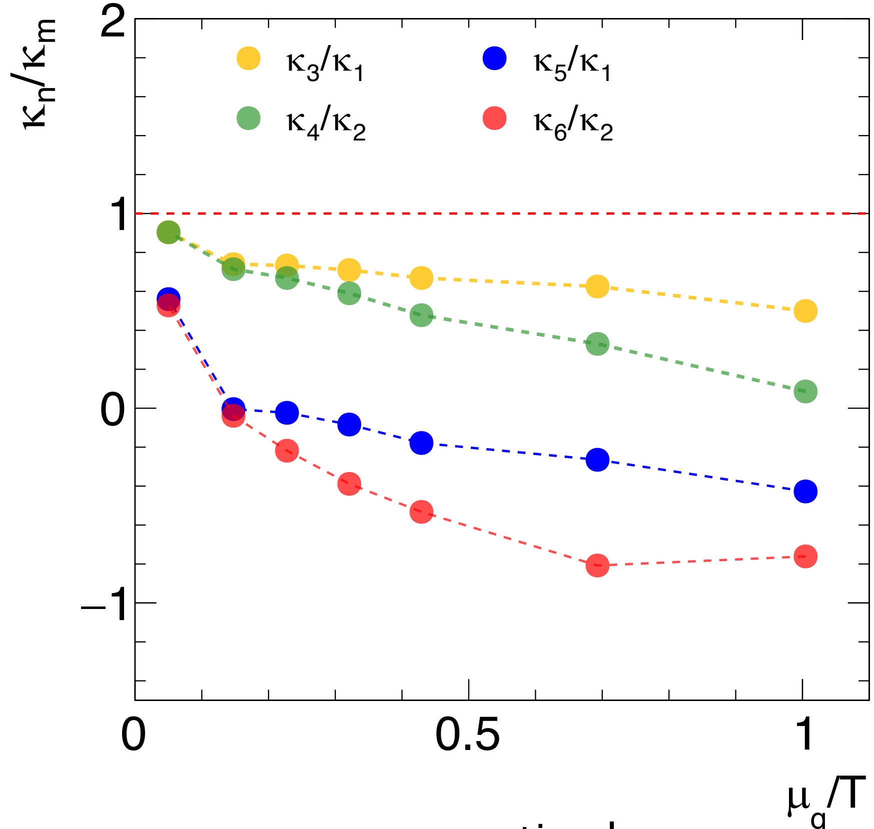


Comparison to critical fluctuations



critical fluctuations

G. A. Almasi, B. Friman, K. Redlich, P.R.D96 (2017) 1, 014027.



conservation laws

- **Data have to be corrected for conservation laws and volume fluctuations**
- **Qualitative differences emerge above 4th order cumulants!**

Conclusions

- First development of the statistical event generator in GCE and CE
- The measured second cumulants of net-protons at ALICE are, after accounting for baryon number conservation, in agreement with the corresponding second cumulants of the Skellam distribution.
- All net-proton cumulants from STAR show deviations from the Skellam baseline.
- Above 11.5 GeV these deviations can be consistently described with the global baryon number conservation + unavoidable fluctuations of participating nucleons

Before making any quantitative statements the data on cumulants have to be corrected for conservation laws and volume fluctuations