Rapidity decorrelation from hydrodynamic fluctuations

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Outline

- Introduction
- Model & Settings
- Results
- Summary

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Introduction

Fluctuations in heavy ion collisions

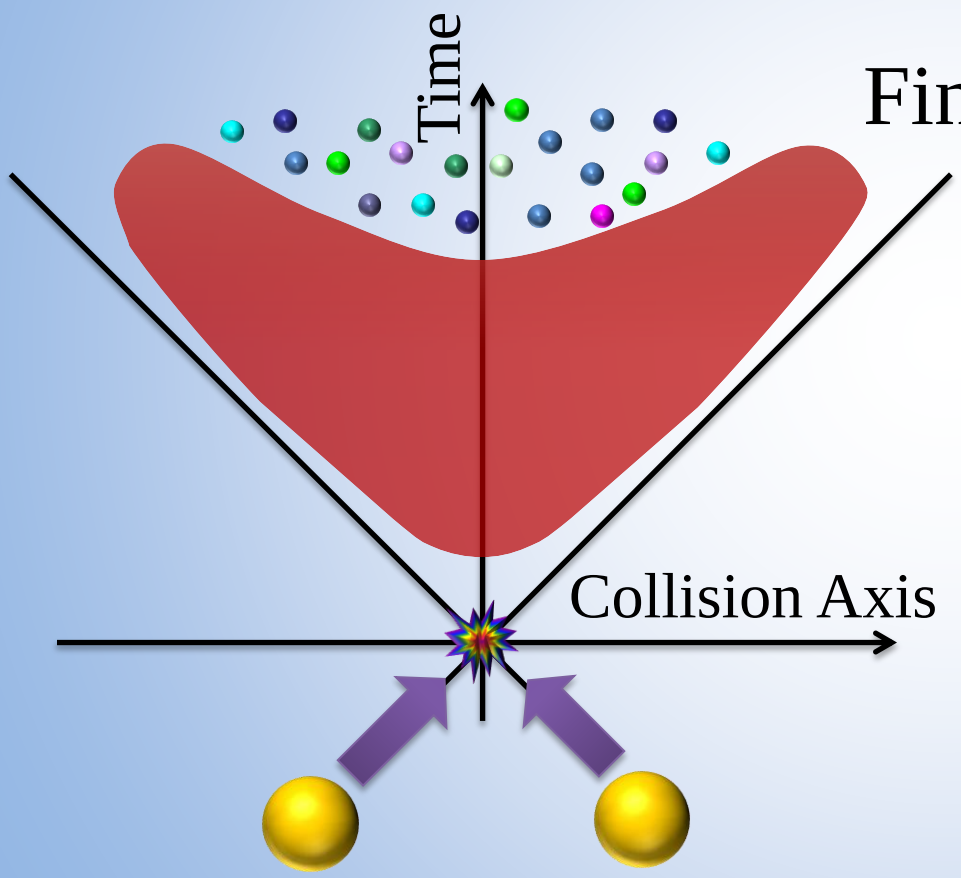


Figure: space-time evolution of high-energy nuclear collisions

Final observables

Hadron gas

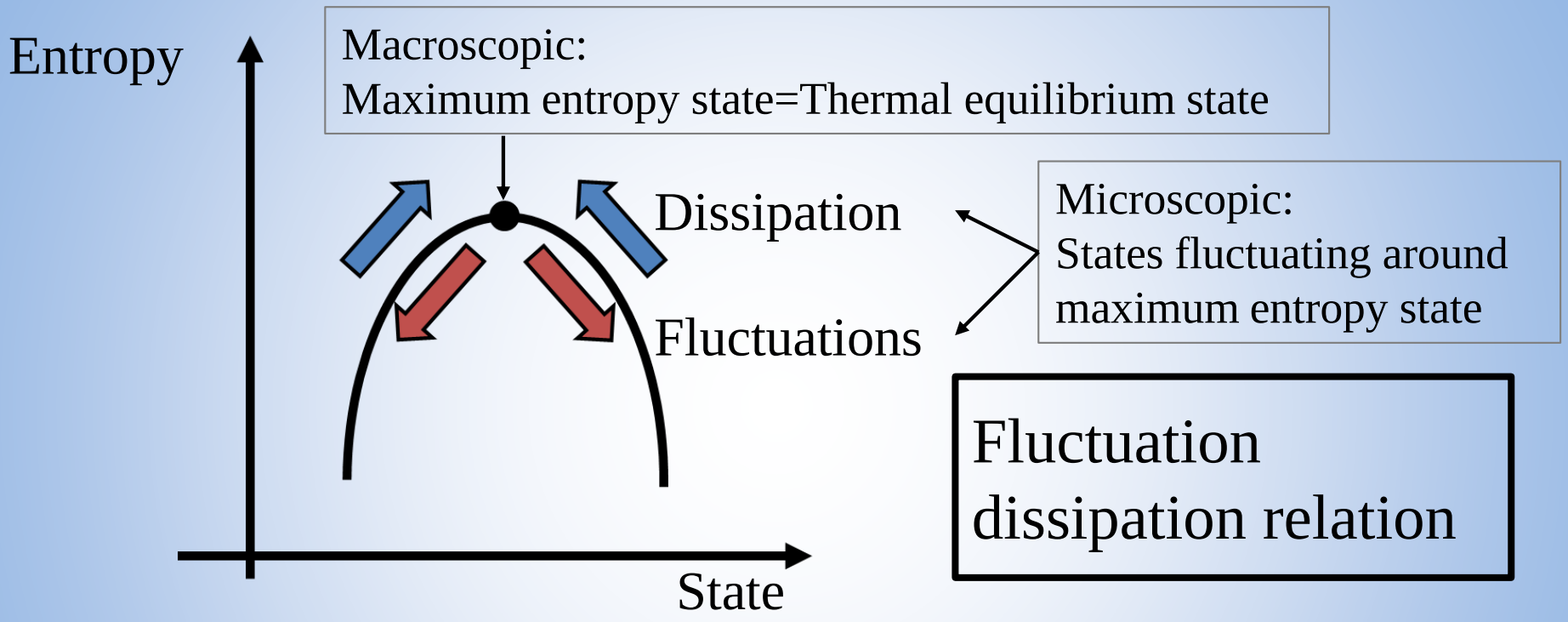
QGP fluid

Initial state

Thermal Fluctuations

Initial state fluctuations

Thermal Fluctuations



Fluctuation dissipation relation

Viscous hydro: Dissipation

Fluctuating hydro: Dissipation + Fluctuations

Purpose of study: Effects of hydrodynamic fluctuations on rapidity decorrelation

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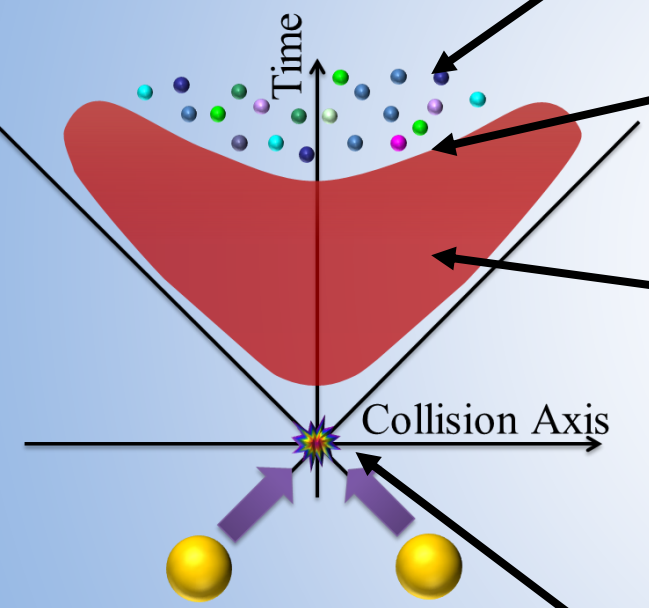
Integrated dynamical model



4.Hadron gas

Hadron cascade model (JAM)

3.Particlization

Cooper-Frye formula (T_{sw}= 155 MeV)

2.QGP fluid

Full 3D relativistic fluctuating hydrodynamic simulations

K. Murase: Ph. D thesis (2015).

EoS: s95p-v1.1 (Lattice QCD+HRG)

1.Initial state

Pb+Pb, $\sqrt{s_{\mathrm{NN}}}$=2.76 TeV

MC-Glauber model + Modified BGK model

T.Hirano et al., Prog. Part. Nucl. Phys. 70,108 (2013)

Hydrodynamic fluctuations

Shear stress tensor

Fluctuating hydro

Viscous hydro

$$\pi^{\mu\nu}(x) = 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \delta\pi^{\mu\nu}(x)$$

η: shear viscosity

u^{μ}: four fluid velocity

Thermodynamic force

Hydrodynamic fluctuations

Note: Relaxation term needed in actual simulations

Fluctuation dissipation relation for shear stress tensor

$$\pi^{\mu\nu} = 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \delta\pi^{\mu\nu}$$

Increase of entropy

Balance

Decrease of entropy

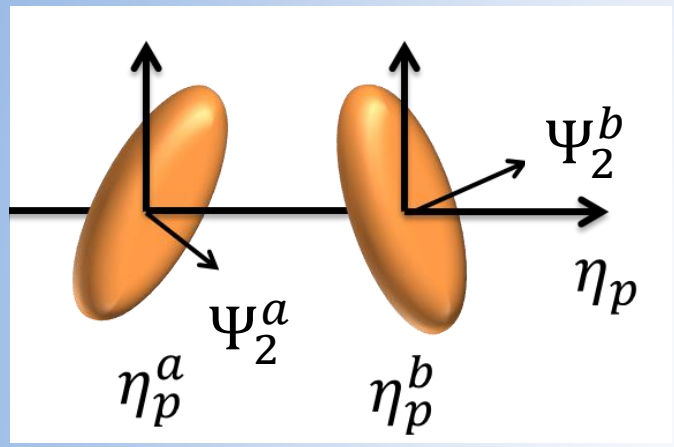
Fluctuation dissipation relation
=Stability condition of thermal system

$$\left\langle \delta\pi^{ij}\delta\pi^{ij} \right\rangle \sim 4T\eta\delta^4(x-x')$$

$$\delta^4(x-x') \Rightarrow \frac{1}{\Delta t}\frac{1}{(4\pi\lambda^2)^{\frac{3}{2}}}e^{-\frac{(\boldsymbol{x}-\boldsymbol{x}')^2}{4\lambda^2}}$$

λ: Gaussian width

Event plane fluctuations and decorrelations



Event plane angle $\Psi_n(\eta_p)$

$$\frac{dN_{\text{pair}}}{d\Delta\phi} \propto 1 + 2\sum V_{n\Delta}\cos(n\Delta\phi)$$

$\Psi_2(\eta_p) = \text{const}$

$\Rightarrow V_{2\Delta} = v_2^a v_2^b$

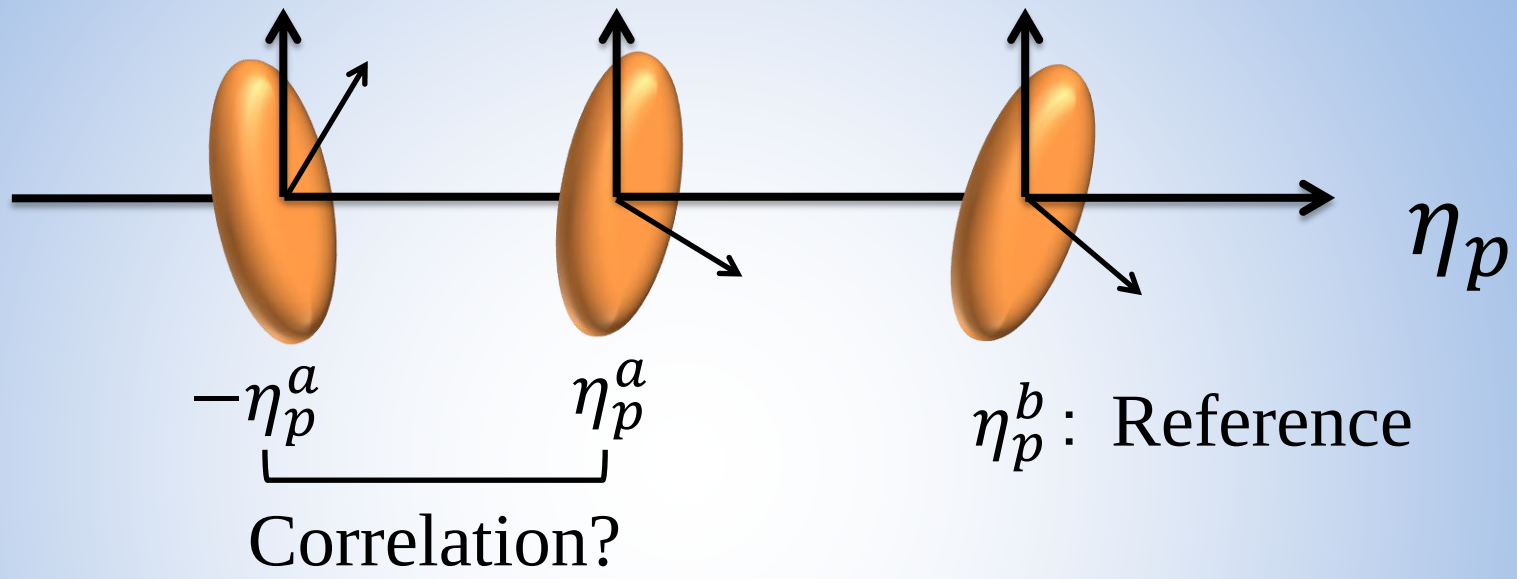
$\Psi_2(\eta_p) \neq \text{const}$

$\Rightarrow V_{2\Delta} \neq v_2^a v_2^b$

Factorization breaking

Evaluate rapidity decorrelations?

Factorization ratio



$$r_n(\eta_p^a, \eta_p^b) = \frac{V_{n\Delta}(-\eta_p^a, \eta_p^b)}{V_{n\Delta}(\eta_p^a, \eta_p^b)}, \quad V_{n\Delta} = \langle \cos(n\Delta\phi) \rangle$$

$r_n(\eta_p^a, \eta_p^b) \sim 1$
Unique event plane

$r_n(\eta_p^a, \eta_p^b) < 1$
Decorrelation

CMS Collaboration, Phys. Rev. C 92, 034911 (2015)

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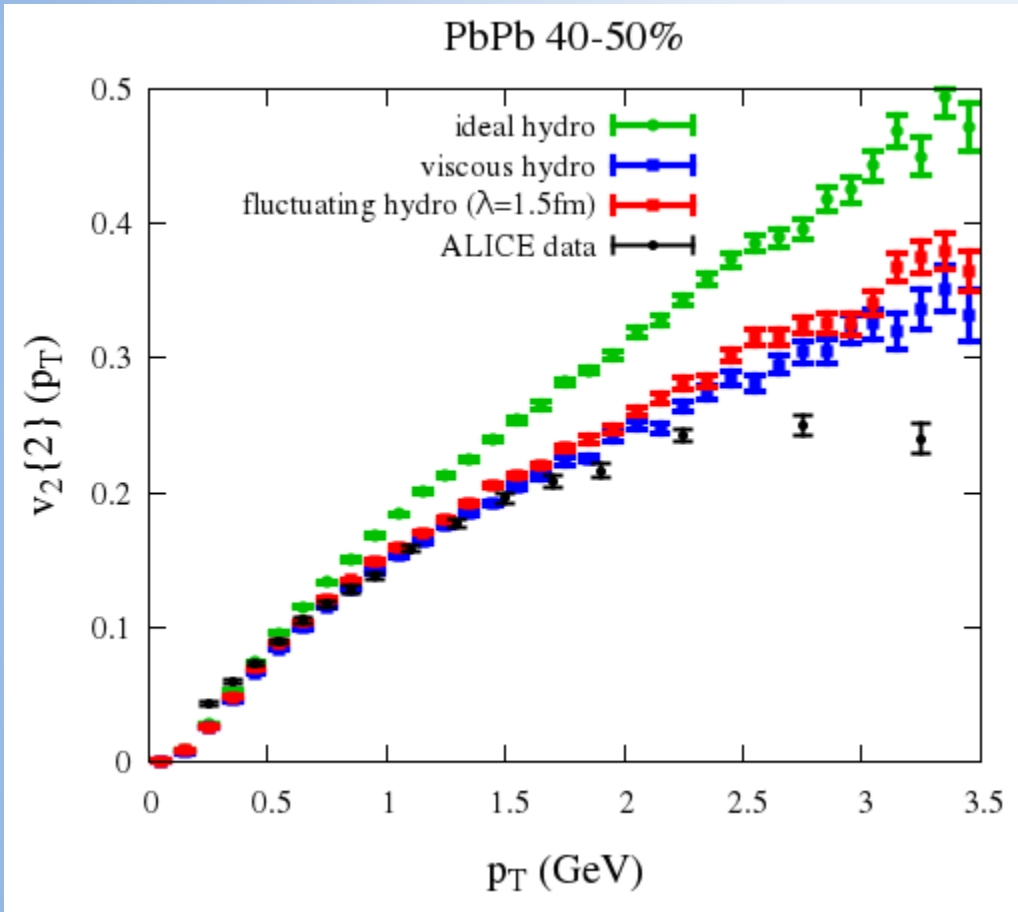
p_T-differential v_2



ALICE Collaboration,
Phys. Rev. Lett. 116 (2016) 132302

<u>Ideal hydro</u>

→ Larger than ALICE data

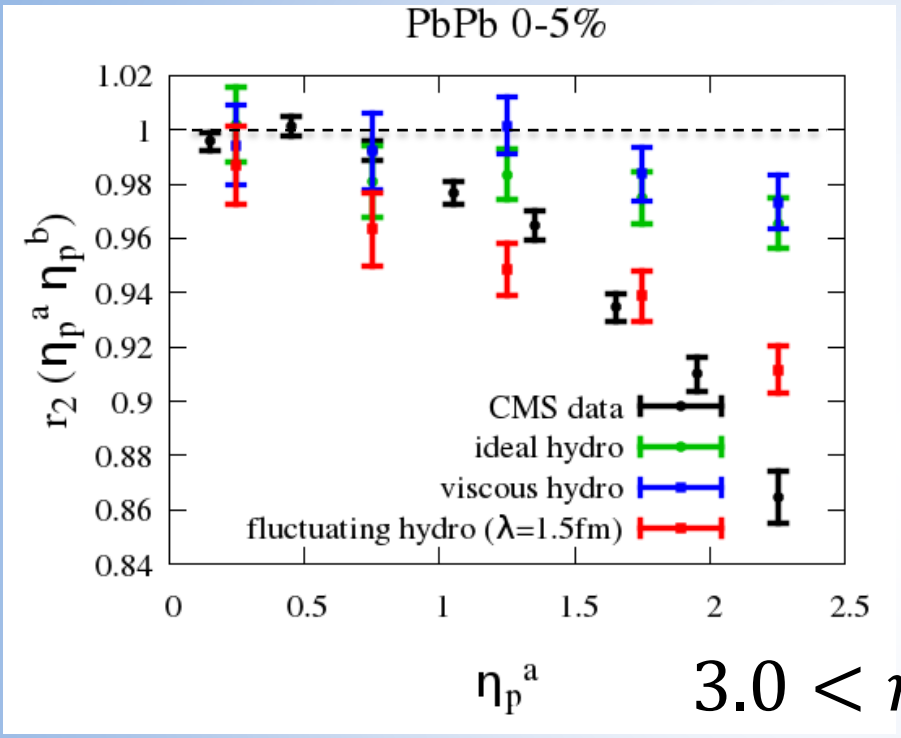
<u>Viscous & Fluctuating hydro ($\eta/s = 1/4\pi$)</u>

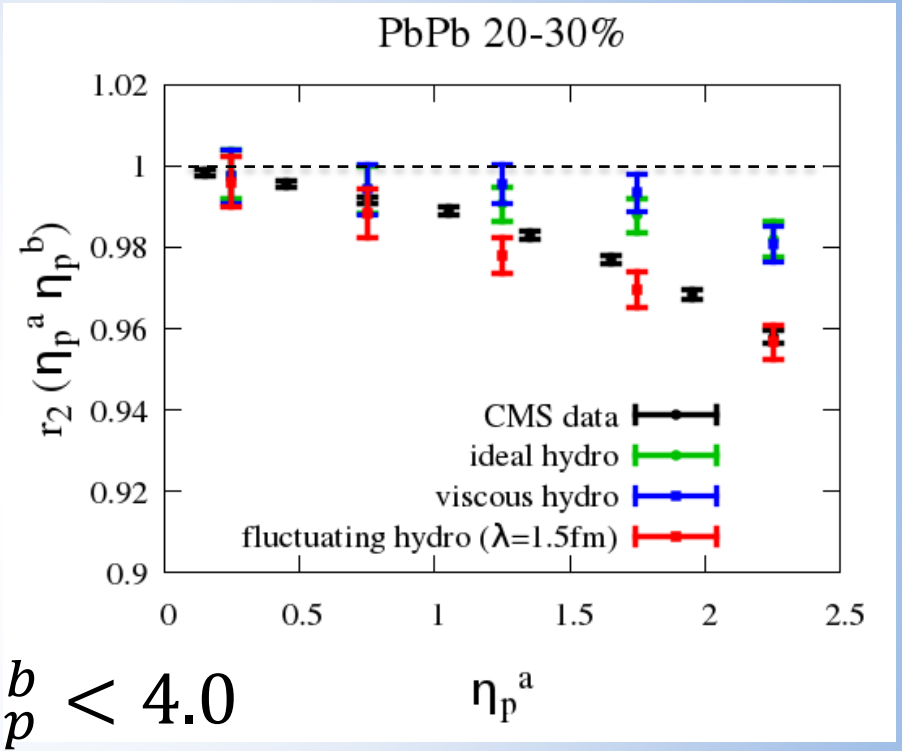
→ Good agreement with ALICE data below $p_T \sim 1.5$ GeV

Effect of fluctuations

→ What observable?

Factorization ratio $r_2(\eta_p^a, \eta_p^b)$





$3.0 < \eta_p^b < 4.0$

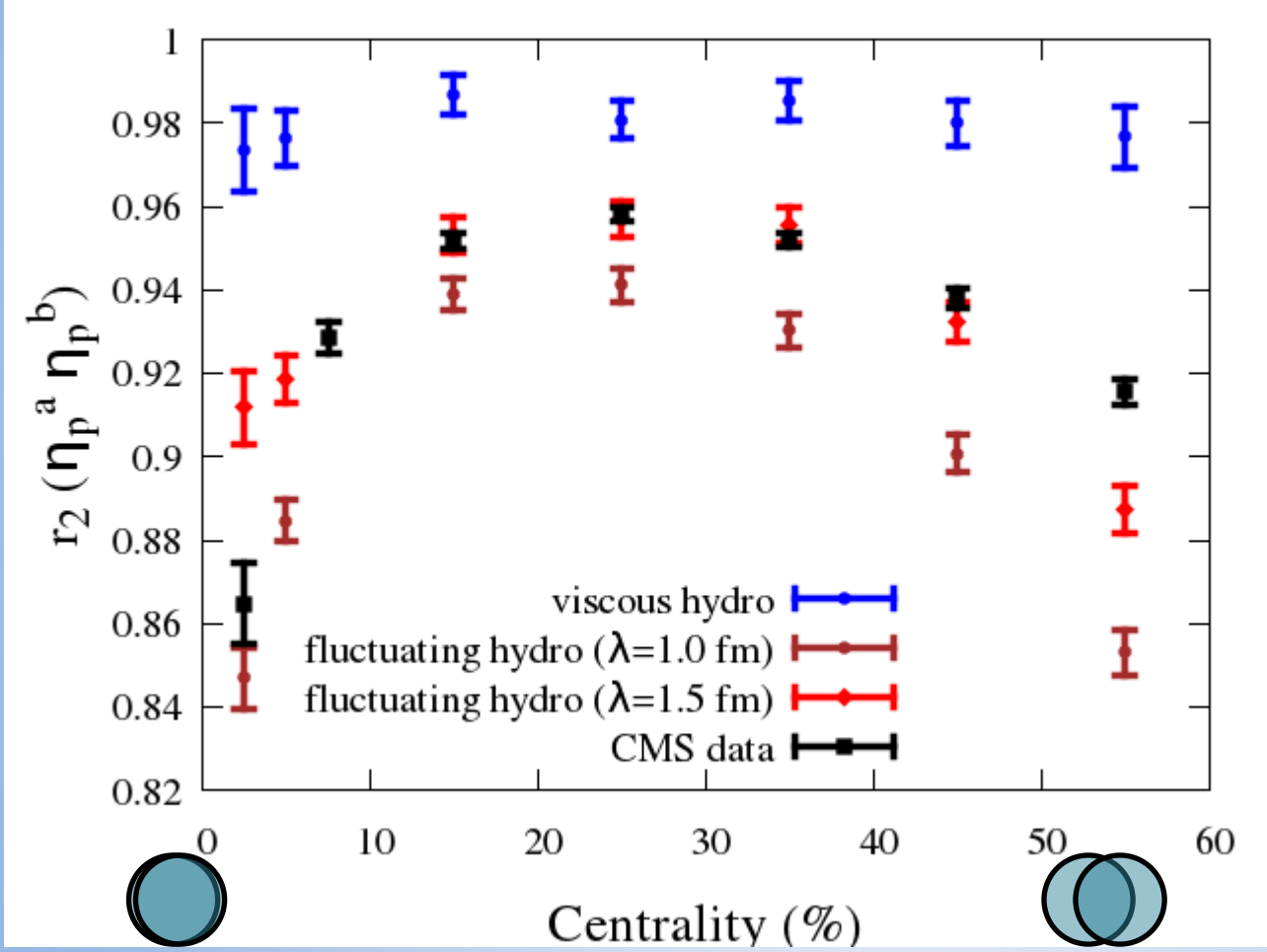
Ideal ≈ Viscous > CMS data ≈ Fluctuating hydro

Hydrodynamic fluctuations

→ Factorization more broken

CMS, Phys. Rev. C 92, 034911 (2015).

Factorization ratio $r_2\left(\eta_p^a, \eta_p^b\right)$



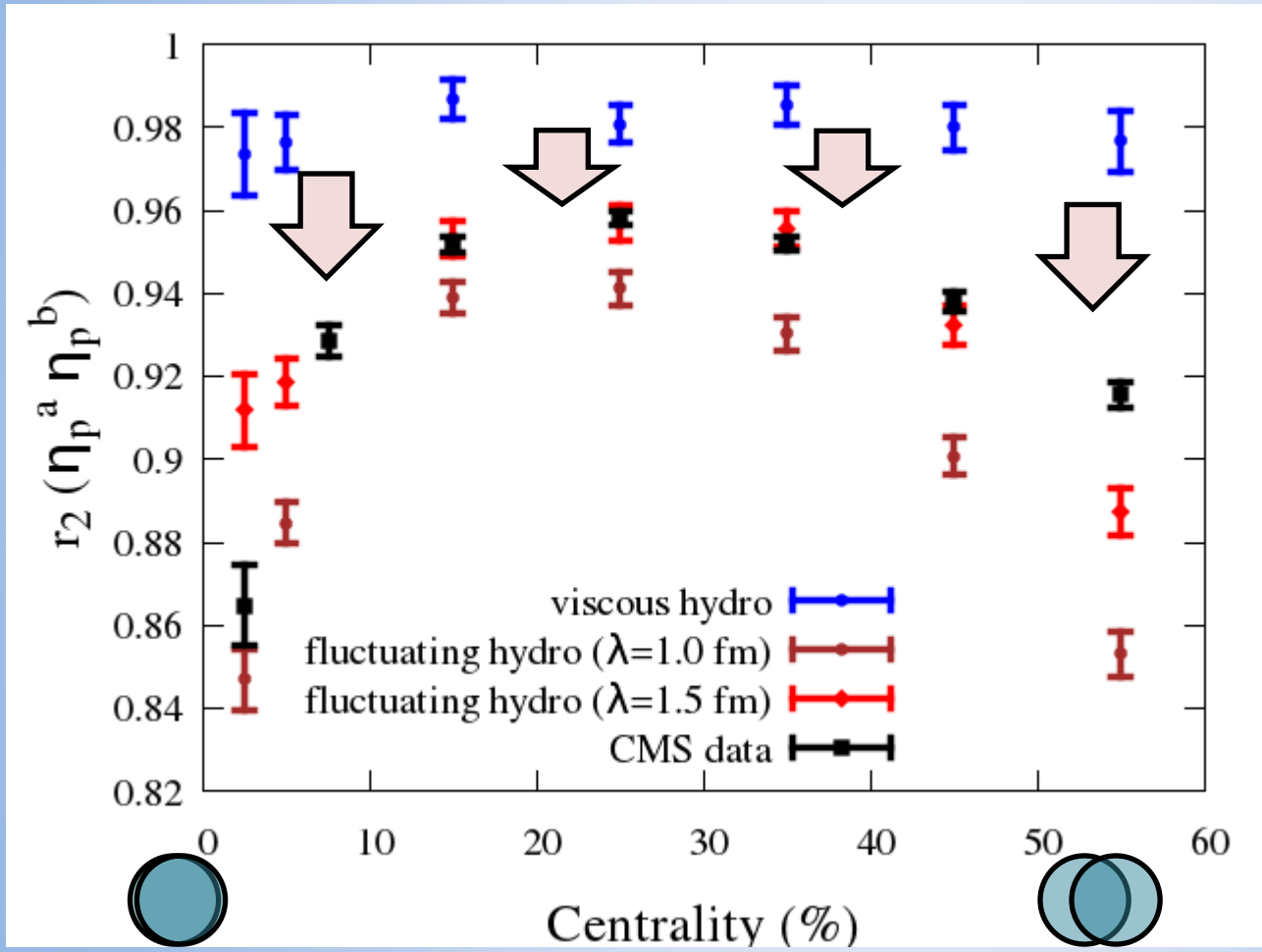
Viscous > CMS data

Central: Fluctuating hydro ($\lambda = 1.0$ fm) $\approx$ CMS data

Mid-central: Fluctuating hydro ($\lambda = 1.5$ fm) $\approx$ CMS data

$2.0 < \eta_p^a < 2.5,\ 3.0 < \eta_p^b < 4.0$

Factorization ratio $r_2\left(\eta_p^a, \eta_p^b\right)$



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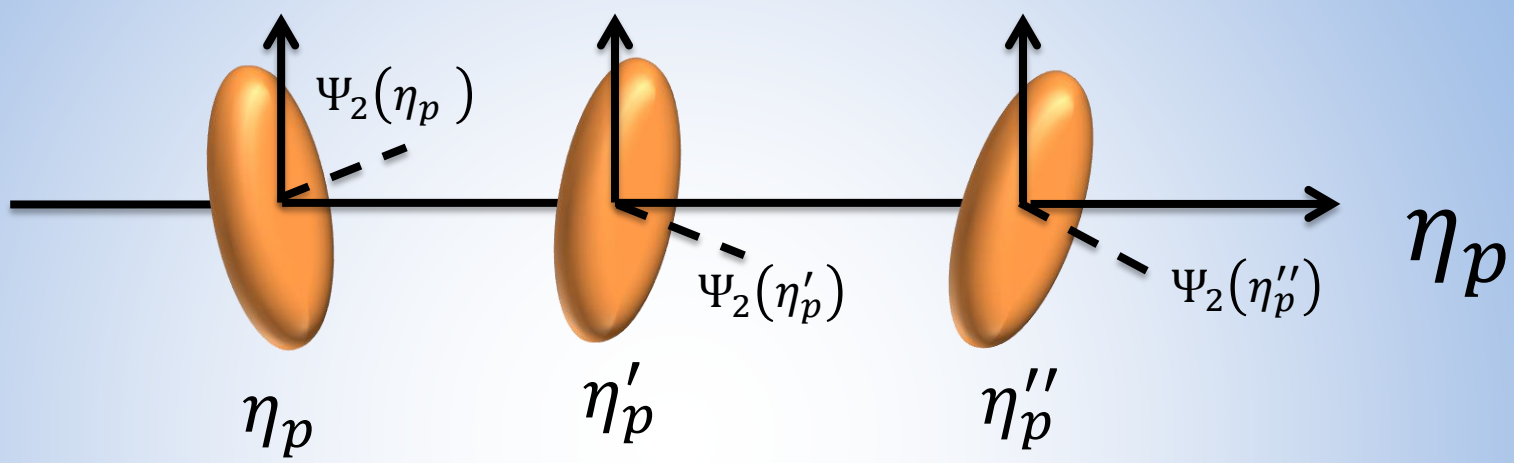
Viscous > CMS data

Central: Fluctuating hydro ($\lambda = 1.0$ fm) $\approx$ CMS data

Mid-central: Fluctuating hydro ($\lambda = 1.5$ fm) $\approx$ CMS data

<u>Decorrelation from fluctuations</u>

Legendre series



$$v_2(\eta_p) = \sum_{k=0}^{\infty} a_2^k \, P_k(\eta_p)$$

$$\Psi_2(\eta_p) = \sum_{k=0}^{\infty} b_2^k \, P_k(\eta_p)$$

P_k : Legendre polynomial

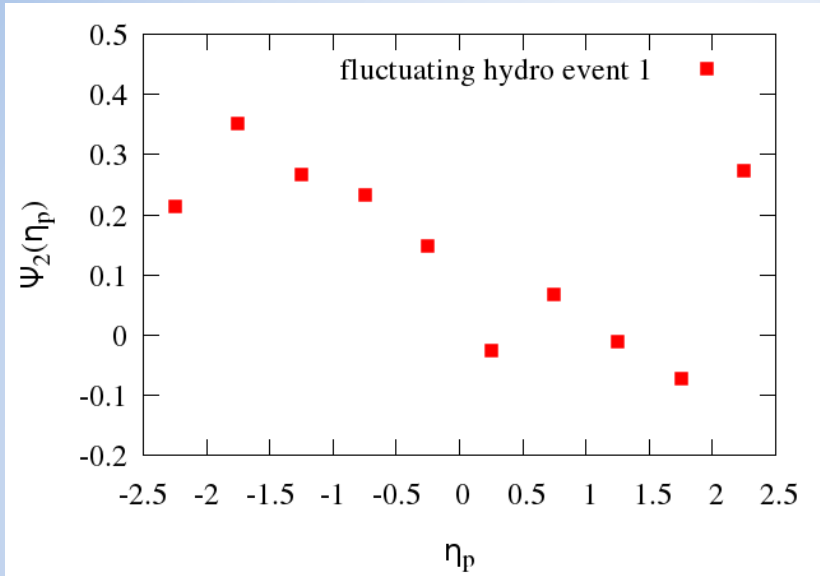
$P_1(\eta_p) = \eta_p$

$$P_2(\eta_p) = \frac{1}{2}\left(3{\eta_p}^2 - 1\right)$$

a_2^k, b_2^k: Legendre coefficients

$\Rightarrow$Quantity to understand η_p dependent

Legendre series



Hydro × Cascade
=4000 × 100

Averaged for each hydro event

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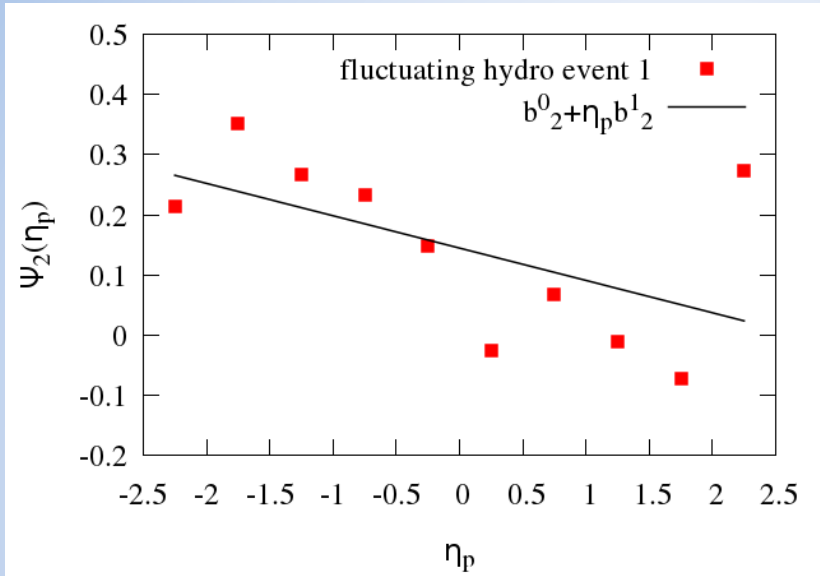

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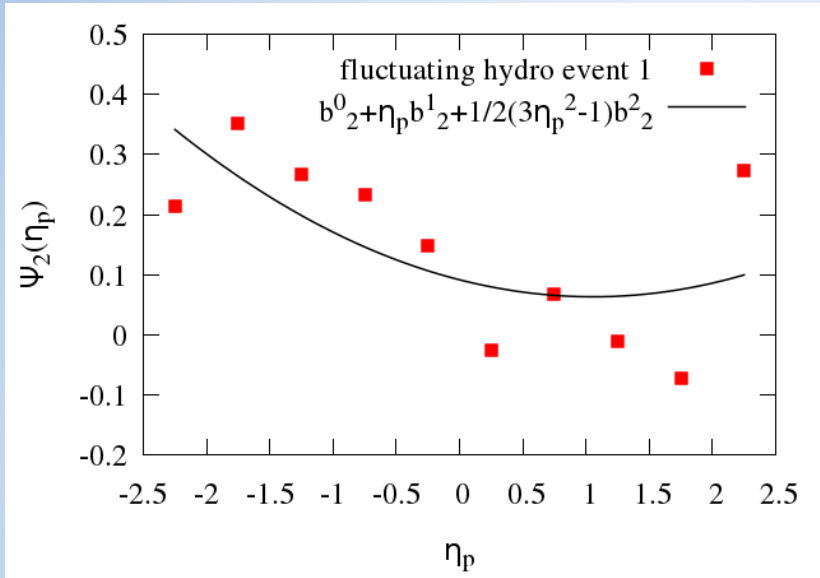

Hydro × Cascade
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P_k : Legendre polynomial

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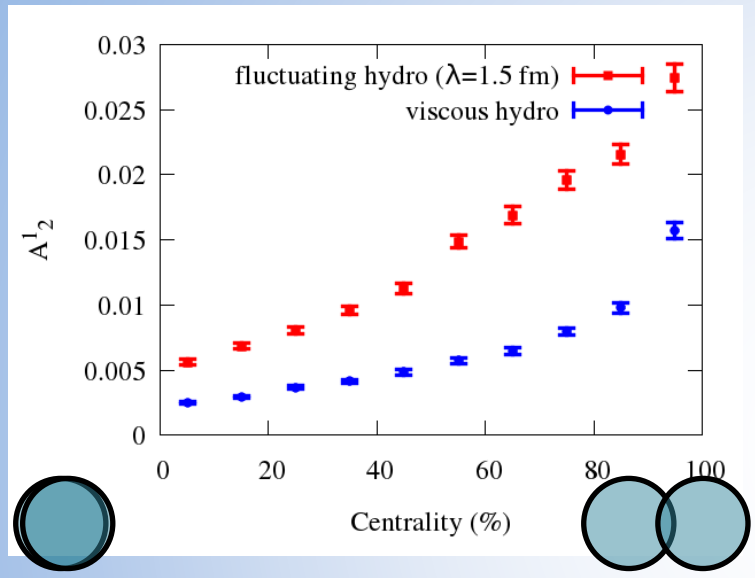
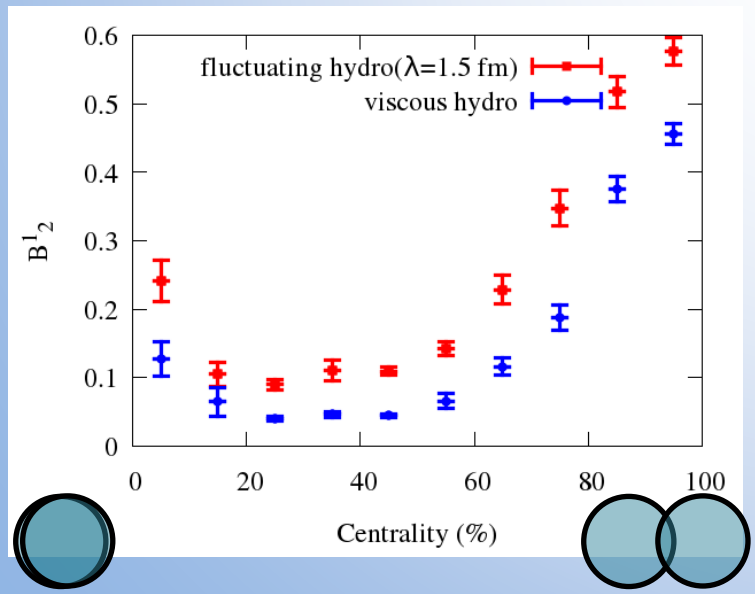
a_2^k, b_2^k: Legendre coefficients

⇒Quantity to understand η_p dependence

Legendre coefficients





Flow $|\boldsymbol{v}_2|$

$$A_2^1 = \sqrt{\langle (a_2^1)^2 \rangle}$$

Event plane angle Ψ_2

$$B_2^1 = \sqrt{\langle (b_2^1)^2 \rangle}$$

a_2^k, b_2^k: Legendre coefficients

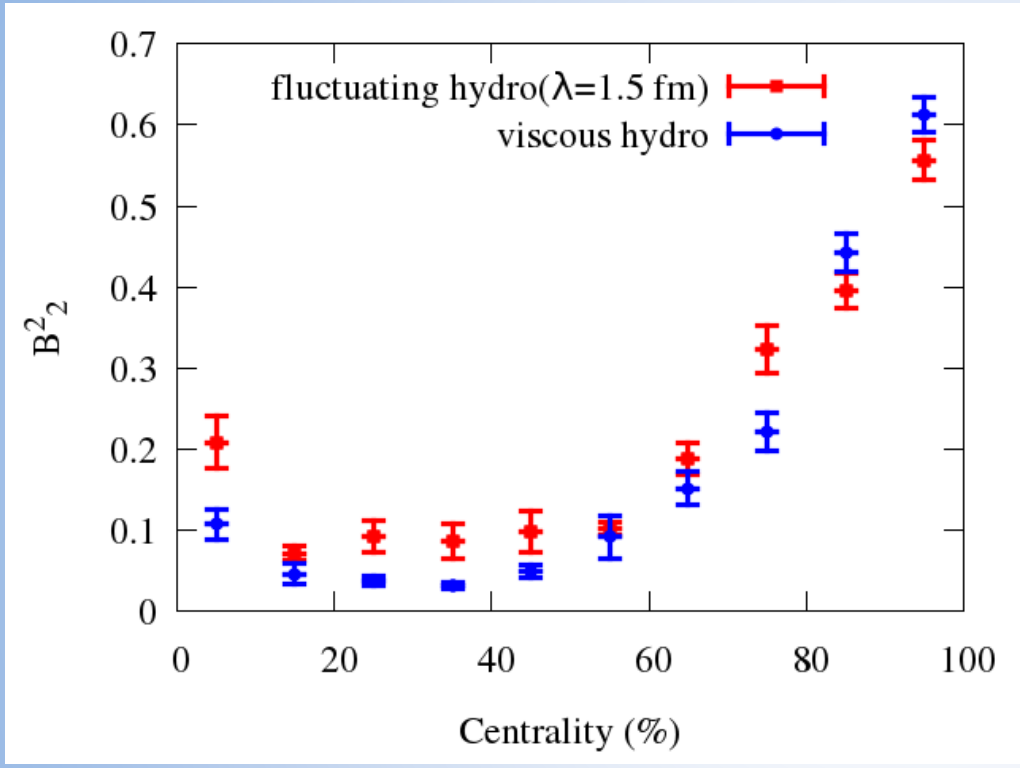
Fluctuating hydro $>$ Viscous hydro

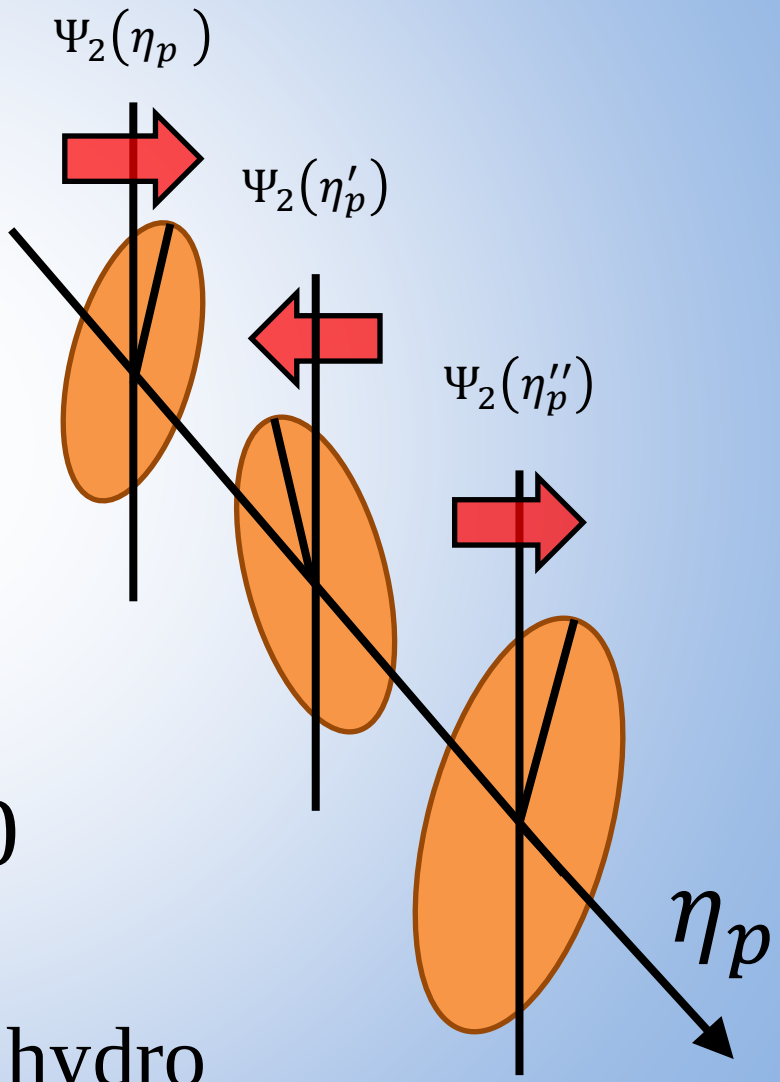
Hydrodynamic fluctuations

$\Rightarrow$increase η_p dependence

$\Rightarrow$increase Legendre coefficient

Legendre coefficients





$B_2^2 =$ "2nd order twist" $\neq 0$

Fluctuating hydro $>$ Viscous hydro

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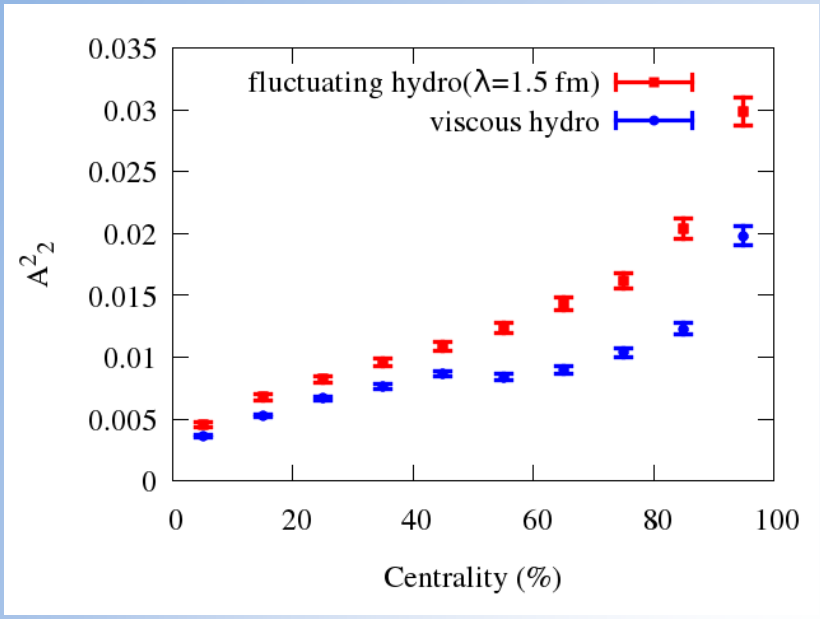
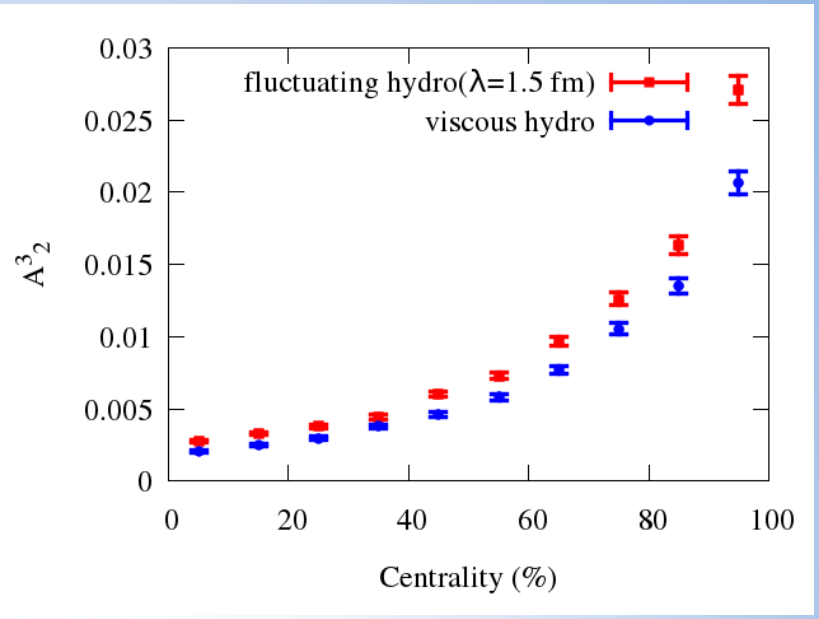
Summary

Integrated dynamical model based on full 3D fluctuating hydrodynamics

- ✓ **Factorization ratio** $r_n\left(\eta_p^a, \eta_p^b\right)$
  - <u>Fluctuating hydrodynamic model</u> factorization more broken
  - Importance of hydrodynamic fluctuations
- ✓ **Legendre coefficients** $A_2^k,\ B_2^k$
  - <u>Fluctuating hydrodynamic model</u> has larger $\eta_p$dependence than viscous hydrodynamic model
  - $B_2^2 = 2^{\text{nd}}$ order twist $\neq 0$

Back up


fluctuating hydro(λ=1.5 fm)
viscous hydro
A^2_2
Centrality (%)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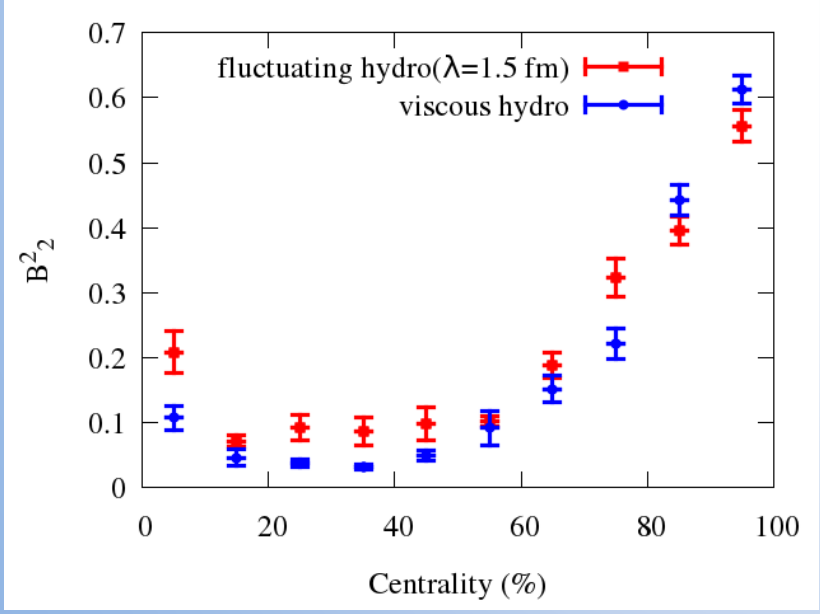
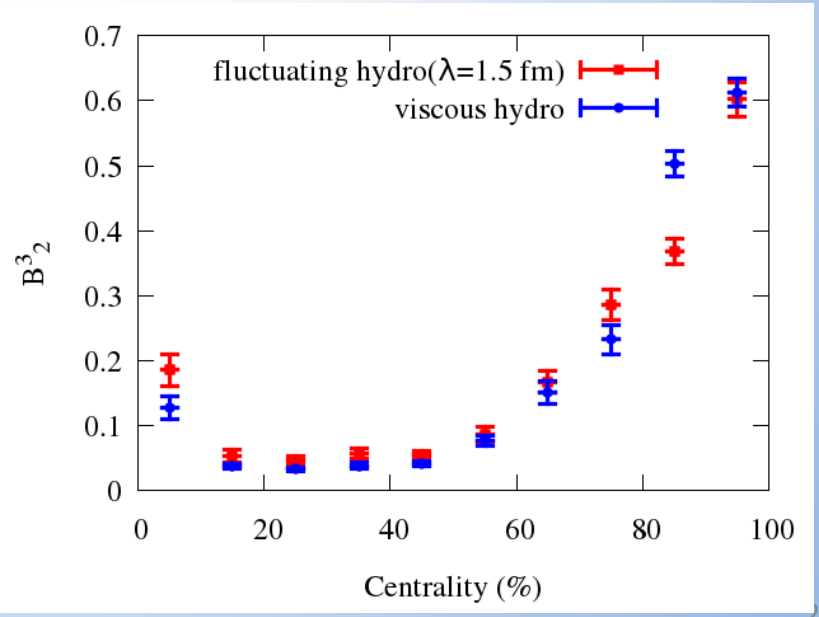
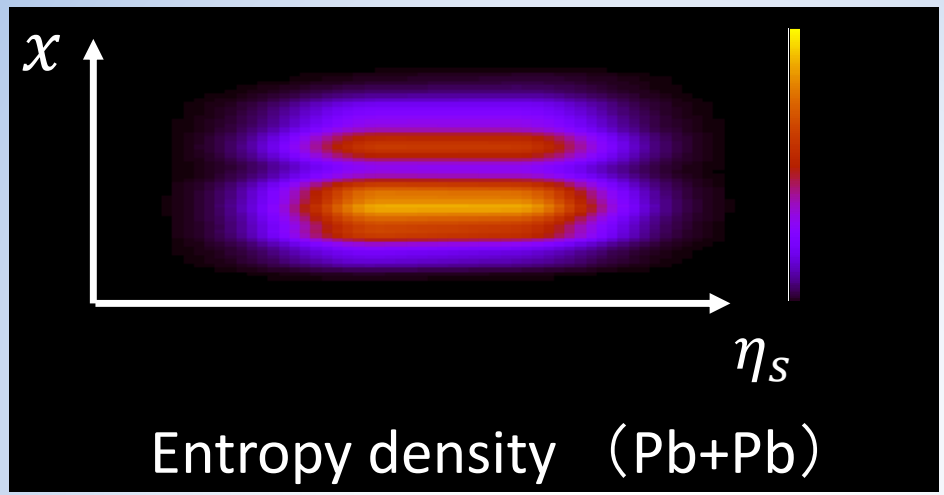

fluctuating hydro(λ=1.5 fm)
viscous hydro
B^2_2
Centrality (%)


fluctuating hydro(λ=1.5 fm)
viscous hydro
B^3_2
Centrality (%)

Effect of hydrodynamic fluctuation



Initial condition：

Hadronic string

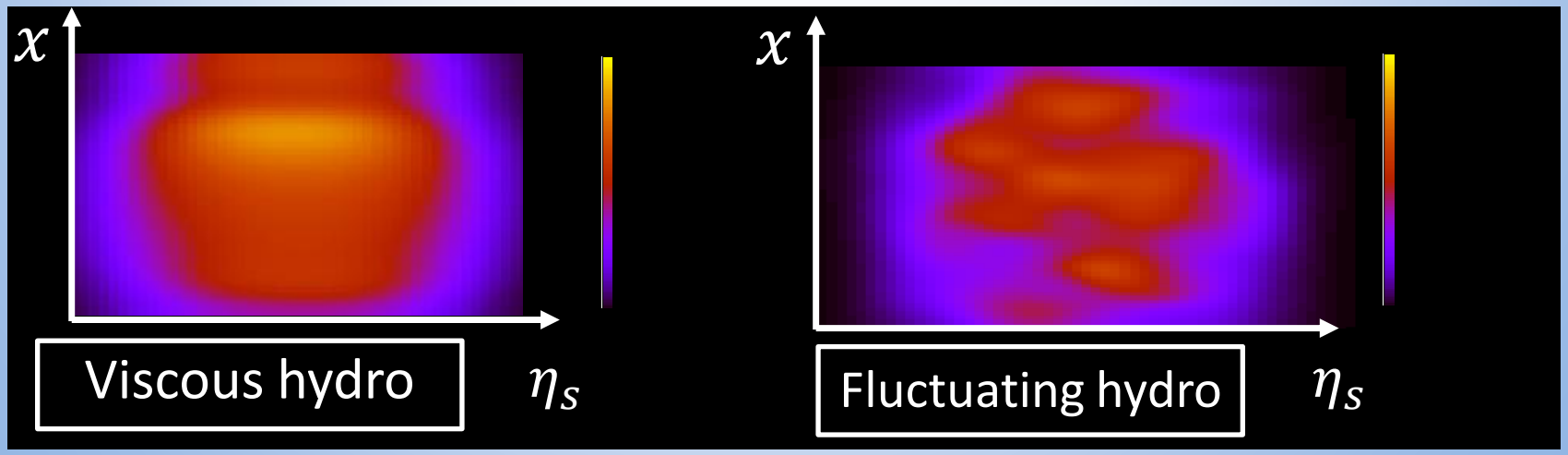
Hydrodynamic evolution：

Hydrodynamic fluctuation

⇒increase randomness

⇒Decorrelation?

Hadronic evolution



Initial Condition Setups

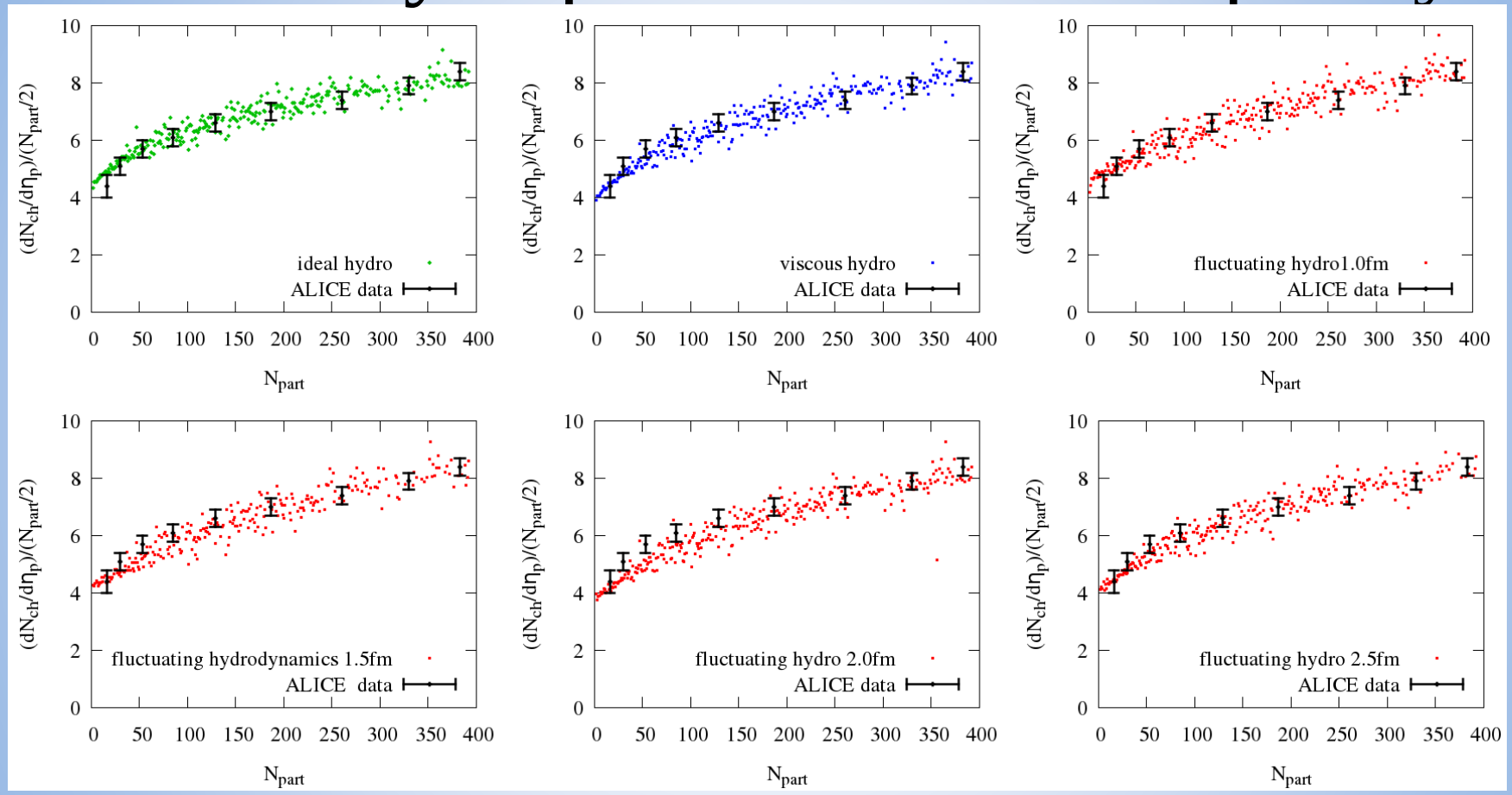
$$s_0(r_\perp) = \frac{C}{\tau_0}\left(\frac{1-\alpha}{2}\rho_{\mathrm{part}}(r_\perp) + \alpha\rho_{\mathrm{coll}}(r_\perp)\right)$$

λ: HF cutoff length scale (Gaussian width)

	η/s	HF	C/τ0	α
Ideal	0	None	62	0.08
Viscous	1/4π	None	49	0.13
Fluctuating	1/4π	λ=1.0 fm	31	0.20
	1/4π	λ=1.5 fm	41	0.16
	1/4π	λ=2.0 fm	42	0.16

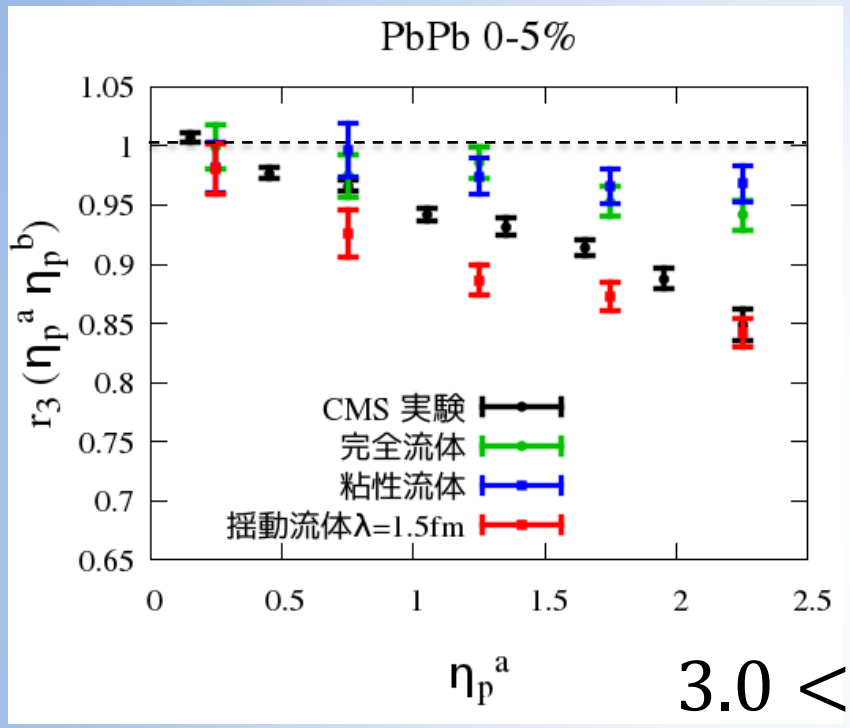
Hydro	Cascades
4k events	400 k (4k*100)

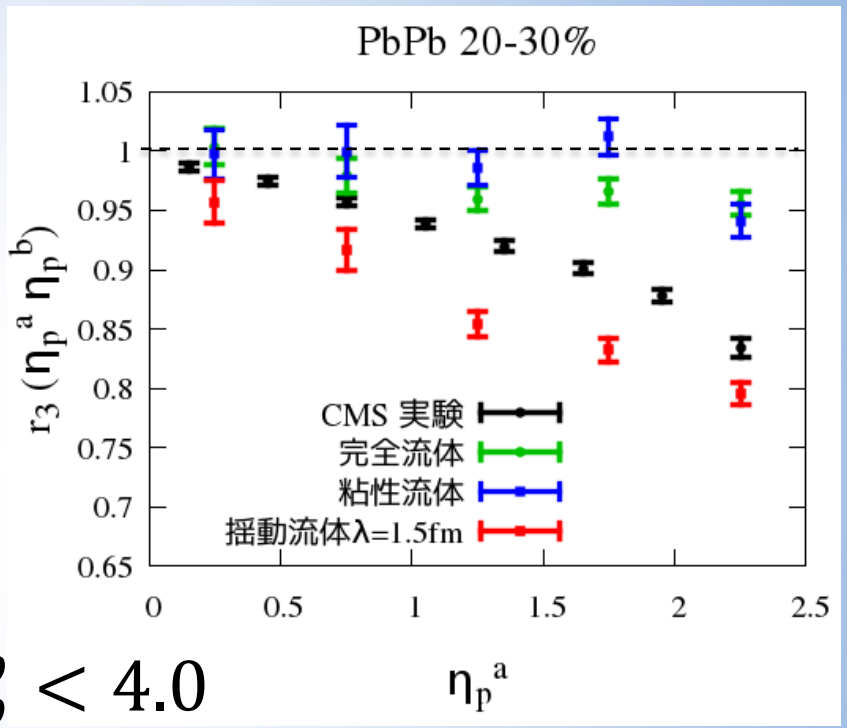
Centrality dependence of multiplicity



- Initial parameters tuning
- Centrality cut

Factorization ratio $r_3(\eta_p^a, \eta_p^b)$





$3.0 < \eta_p^b < 4.0$

Ideal ≈Viscous > CMS data ≈ Fluctuating hydro

Hydrodynamic fluctuations

→ Factorization more broken

CMS, Phys. Rev. C 92, 034911 (2015).

Hydrodynamic fluctuations

Shear stress tensor

Fluctuating hydro

Viscous hydro

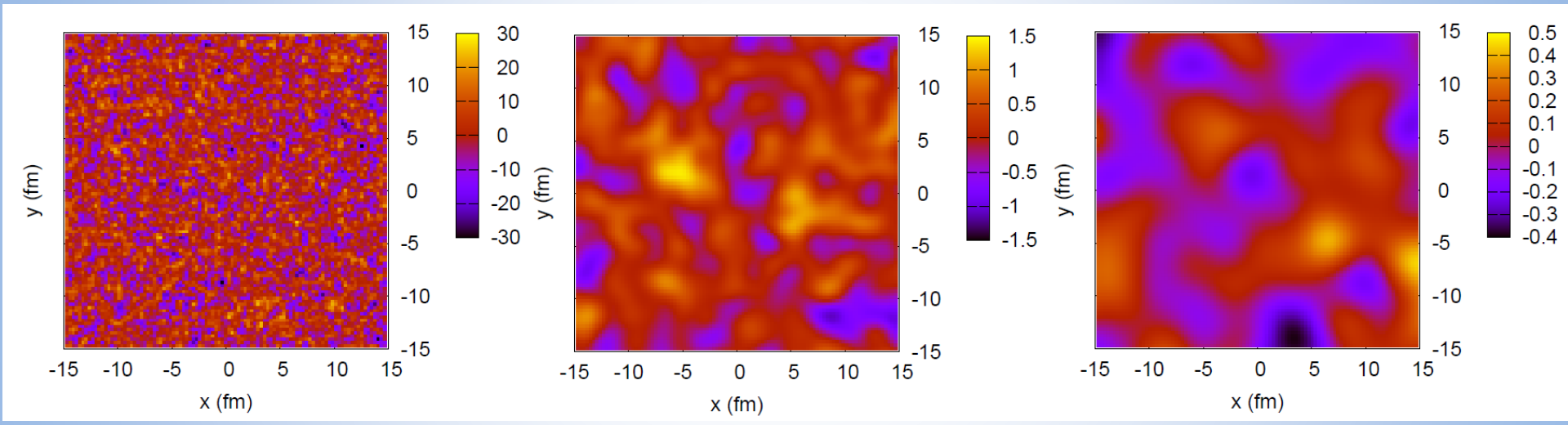
$$\pi^{\mu\nu}(x) = 2\eta\partial^{\langle\mu}u^{\nu\rangle} + \delta\pi^{\mu\nu}(x)$$

Actual Equation

$$\tau_\pi \Delta^{\mu\upsilon}{}_{\alpha\beta} u^\lambda \partial_\lambda \pi^{\alpha\beta} + \pi^{\mu\nu}\left(1 + \frac{4}{3}\tau_\pi \partial_\lambda u^\lambda\right)$$

$$= 2\eta\Delta^{\mu\upsilon}{}_{\alpha\beta}\partial^\alpha \pi^\beta + \delta\pi^{\mu\nu}$$

Cut of parameter



With out smearing

1.0 fm smearing

2.0 fm smearing

Settings of model

1. Initial conditions (see next slides):

- MC-Glauber model (Pb+Pb, 2.76 TeV)

  $\rho_{\mathrm{part}}(x,y), \rho_{\mathrm{coll}}(x,y) \rightarrow s(\tau = \tau_0, x, y, \eta_s)$

2. Full 3D fluctuating hydrodynamics:

- EoS: *s95p*-v1.1 (lattice QCD+resonance gas)
- Shear viscosity: $\eta/s = 1/4\pi$
- Relaxation time: $\tau_\pi = 3/4\pi T$
- Cutoff length scale (Gaussian width): $\lambda = 1$ fm

3. Particlization:

- Cooper-Frye formula with $T_{\mathrm{sw}} = 155$ MeV

4. Hadron cascade:

- JAM

Initial condition

$$s_0(\tau = \tau_0, r_\perp, \eta_s = 0) \propto \frac{1-\alpha}{2}\rho_{\mathrm{part}}(r_\perp) + \alpha\rho_{\mathrm{coll}}(r_\perp)$$

$\tau_0 = 0.6$ fm
$\alpha = 0.20$ (fluctuating hydro)

*longitudinal: modified BGK model

T.Hirano *et al.*, Phys.Lett.B **636**, 299 (2006).

Initial condition

$$s_0(\tau = \tau_0, r_\perp, \eta_s) \propto \left(\frac{1-\alpha}{2} \boxed{\rho_{\mathrm{part}}(\eta_s, r_\perp)} + \alpha \rho_{\mathrm{coll}}(r_\perp) \right)$$

$$\boxed{\rho_{\mathrm{part}}(\eta_s, r_\perp) = \frac{Y_b - \eta_s}{Y_b} \rho_{\mathrm{part,A}}(r_\perp) + \frac{Y_b + \eta_s}{Y_b} \rho_{\mathrm{part},B}(r_\perp)}$$

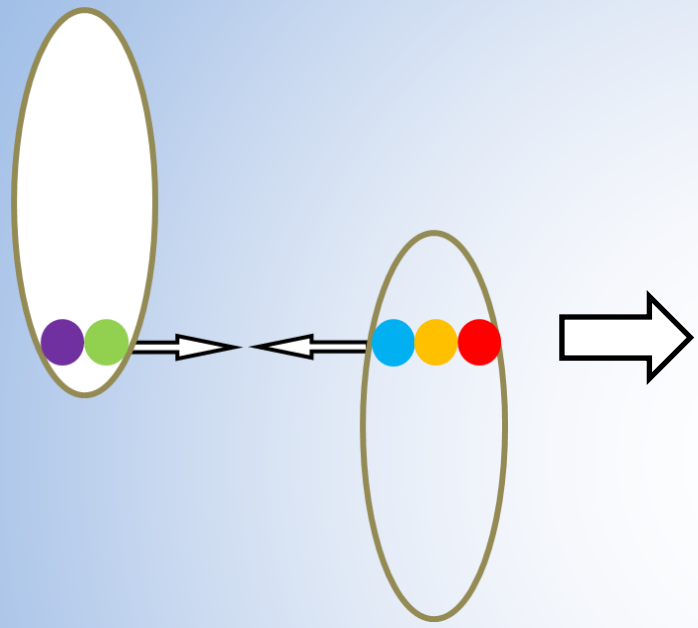
$\tau_0 = 0.6$ fm

$\alpha = 0.20$ (fluctuating hydro)

Y_b : beam rapidity

T.Hirano *et al.*, Phys.Lett.B **636**, 299 (2006).

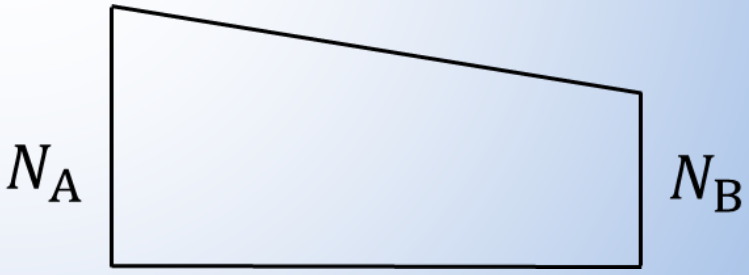
Modified BGK model



Hadronic string

$-Y_{\mathrm{beam}}$ ← rapidity → Y_{beam}

MC-Glauber model
Number of participant (N_A, N_B)



Entropy density

S.J.Brodsky, J.F.Gunion and J.H.Kuhn, Phys.Rev.Lett.**39**, 1120 (1977).
T.Hirano *et al.*, Phys.Lett.B **636**, 299 (2006).