Rapidity decorrelation from hydrodynamic fluctuations

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Outline

• Introduction
• Model & Settings
• Results
• Summary
Introduction

Fluctuations in heavy ion collisions

Figure: space-time evolution of high-energy nuclear collisions

Initial state fluctuations

Collision Axis

Time

Final observables

Hadron gas

QGP fluid

Initial state

Thermal Fluctuations
Thermal Fluctuations

Entropy

Macroscopic:
Maximum entropy state = Thermal equilibrium state

Dissipation
Fluctuations

Microscopic:
States fluctuating around maximum entropy state

Fluctuation dissipation relation

State

Viscous hydro: Dissipation

Fluctuating hydro: Dissipation + Fluctuations

Purpose of study: Effects of hydrodynamic fluctuations on rapidity decorrelation
Outline

• Introduction
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Integrated dynamical model

1. Initial state
- Pb+Pb, $\sqrt{s_{NN}}=2.76$ TeV
- MC-Glauber model + Modified BGK model

2. QGP fluid
- Full 3D relativistic fluctuating hydrodynamic simulations
  - EoS: s95p-v1.1 (Lattice QCD+HRG)

3. Particlization
- Cooper-Frye formula ($T_{sw}=155$ MeV)

4. Hadron gas
- Hadron cascade model (JAM)

Hydrodynamic fluctuations

Shear stress tensor

Fluctuating hydro

Viscous hydro
\[ \pi^{\mu\nu}(x) = 2\eta \partial^{\langle \mu} u^{\nu \rangle} + \delta\pi^{\mu\nu}(x) \]

\( \eta \): shear viscosity
\( u^\mu \): four fluid velocity

Thermodynamic force
Hydrodynamic fluctuations

Note: Relaxation term needed in actual simulations
Fluctuation dissipation relation for shear stress tensor

\[ \pi^{\mu\nu} = 2\eta \partial^{\mu}u^{\nu} + \delta \pi^{\mu\nu} \]

Increase of entropy \quad Balance \quad Decrease of entropy

Fluctuation dissipation relation
= Stability condition of thermal system

\[ \langle \delta \pi^{ij}\delta \pi^{ij} \rangle \sim 4T\eta \delta^4(x - x') \]

\[ \delta^4(x - x') \Rightarrow \frac{1}{\Delta t} \frac{1}{(4\pi \lambda^2)^{3/2}} e^{-\frac{(x-x')^2}{4\lambda^2}} \]

\( \lambda \): Gaussian width
Event plane fluctuations and decorrelations

Event plane angle $\Psi_n(\eta_p)$

$$\frac{dN_{\text{pair}}}{d\Delta\phi} \propto 1 + 2 \sum V_{n\Delta} \cos(n\Delta\phi)$$

$\Psi_2(\eta_p) = \text{const}$

$\Rightarrow V_{2\Delta} = \nu_2^a \nu_2^b$

$\Psi_2(\eta_p) \neq \text{const}$

$\Rightarrow V_{2\Delta} \neq \nu_2^a \nu_2^b$

Factorization breaking

Evaluate rapidity decorrelations?
Factorization ratio

\[ r_n(\eta_p^a, \eta_p^b) = \frac{V_{n\Delta}(-\eta_p^a, \eta_p^b)}{V_{n\Delta}(\eta_p^a, \eta_p^b)} \]

where

\[ V_{n\Delta} = \langle \cos(n\Delta\phi) \rangle \]

- \( r_n(\eta_p^a, \eta_p^b) \sim 1 \)
  - Unique event plane
- \( r_n(\eta_p^a, \eta_p^b) < 1 \)
  - Decorrelation

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\[ p_T \text{-differential } \nu_2 \]

Ideal hydro
→ Larger than ALICE data
Viscous & Fluctuating hydro \((\eta/s = 1/4\pi)\)
→ Good agreement with ALICE data below \( p_T \sim 1.5 \) GeV

Effect of fluctuations
→ What observable?

ALICE Collaboration,
Factorization ratio $r_2(\eta_p^a, \eta_p^b)$

$\eta_p^a \ 3.0 < \eta_p^b < 4.0 \quad \eta_p^a$

Ideal $\approx$ Viscous $>$ CMS data $\approx$ Fluctuating hydro

Hydrodynamic fluctuations $\rightarrow$ Factorization more broken

Factorization ratio $r_2(\eta_p^a, \eta_p^b)$

Viscous > CMS data

Central:
Fluctuating hydro ($\lambda = 1.0 \text{ fm}$) 
$\approx$ CMS data

Mid-central:
Fluctuating hydro ($\lambda = 1.5 \text{ fm}$) 
$\approx$ CMS data

$2.0 < \eta_p^a < 2.5$, $3.0 < \eta_p^b < 4.0$
Factorization ratio $r_2(\eta_p^a, \eta_p^b)$

Viscous $>$ CMS data

Central:
Fluctuating hydro ($\lambda = 1.0$ fm)
$\approx$ CMS data

Mid-central:
Fluctuating hydro ($\lambda = 1.5$ fm)
$\approx$ CMS data

Decorrelation from fluctuations

$2.0 < \eta_p^a < 2.5$, $3.0 < \eta_p^b < 4.0$
Legendre series

\[ v_2(\eta_p) = \sum_{k=0}^{\infty} a_2^k P_k(\eta_p) \]

\[ \Psi_2(\eta_p) = \sum_{k=0}^{\infty} b_2^k P_k(\eta_p) \]

\[ P_k : \text{Legendre polynomial} \]

\[ P_1(\eta_p) = \eta_p \]

\[ P_2(\eta_p) = \frac{1}{2} (3\eta_p^2 - 1) \]

\( a_2^k, b_2^k : \text{Legendre coefficients} \)

⇒ Quantity to understand \( \eta_p \) dependent
Legendre series

\[ v_2(\eta_p) = \sum_{k=0}^{\infty} a_2^k P_k(\eta_p) \]

\[ \Psi_2(\eta_p) = \sum_{k=0}^{\infty} b_2^k P_k(\eta_p) \]

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\[ P_2(\eta_p) = \frac{1}{2} (3\eta_p^2 - 1) \]

\[ a_2^k, b_2^k : \text{Legendre coefficients} \]

\[ \Rightarrow \text{Quantity to understand } \eta_p \text{ dependence} \]

Hydro $\times$ Cascade $= 4000 \times 100$

Averaged for each hydro event
Legendre series

\[
\begin{align*}
\nu_2(\eta_p) &= \sum_{k=0}^{\infty} a_2^k P_k(\eta_p) \\
\Psi_2(\eta_p) &= \sum_{k=0}^{\infty} b_2^k P_k(\eta_p)
\end{align*}
\]

\(P_k\): Legendre polynomial

\[
\begin{align*}
P_1(\eta_p) &= \eta_p \\
P_2(\eta_p) &= \frac{1}{2} (3\eta_p^2 - 1)
\end{align*}
\]

\(a_2^k, b_2^k\): Legendre coefficients

\(\Rightarrow\) Quantity to understand \(\eta_p\) dependence

Hydro \(\times\) Cascade

= \(4000 \times 100\)

Averaged for each hydro event

\[
\begin{align*}
\Psi_2(\eta_p) &= \sum_{k=0}^{\infty} b_2^k P_k(\eta_p)
\end{align*}
\]
Legendre series

Hydro $\times$ Cascade
$= 4000 \times 100$

Averaged for each hydro event

$v_2(\eta_p) = \sum_{k=0}^{\infty} a^k_2 P_k(\eta_p)$

$\Psi_2(\eta_p) = \sum_{k=0}^{\infty} b^k_2 P_k(\eta_p)$

$P_k : \text{Legendre polynomial}$

$P_1(\eta_p) = \eta_p$

$P_2(\eta_p) = \frac{1}{2} (3\eta_p^2 - 1)$

$a^k_2, b^k_2 : \text{Legendre coefficients}$

$\Rightarrow \text{Quantity to understand } \eta_p \text{ dependence}$
Legendre coefficients

Flow $|v_2|$

$$A_2^1 = \sqrt{\langle (a_2^1)^2 \rangle}$$

Event plane angle $\Psi_2$

$$B_2^1 = \sqrt{\langle (b_2^1)^2 \rangle}$$

$a_k^2, b_k^2$: Legendre coefficients

Fluctuating hydro $>\text{Viscous hydro}$

Hydrodynamic fluctuations

$\Rightarrow$ increase $\eta_p$ dependence

$\Rightarrow$ increase Legendre coefficient
Legendre coefficients

$B^2_2 = \text{“2nd order twist”} \neq 0$

Fluctuating hydro $> Viscous hydro$
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Summary

Integrated dynamical model based on full 3D fluctuating hydrodynamics

✓ **Factorization ratio** $r_n(\eta_p^a, \eta_p^b)$
  - Fluctuating hydrodynamic model factorization more broken
  - Importance of hydrodynamic fluctuations

✓ **Legendre coefficients** $A_2^k, B_2^k$
  - Fluctuating hydrodynamic model has larger $\eta_p$ dependence than viscous hydrodynamic model
  - $B_2^2 = 2^{\text{nd}} \text{ order twist} \neq 0$
Back up
Effect of hydrodynamic fluctuation

Initial condition: Hadronic string

Hydrodynamic evolution: Hydrodynamic fluctuation
⇒ increase randomness
⇒ Decorrelation?

Entropy density \((\text{Pb+Pb})\)
Initial Condition Setups

\[ s_0(r_\perp) = \frac{c}{\tau_0} \left( \frac{1 - \alpha}{2} \rho_{\text{part}}(r_\perp) + \alpha \rho_{\text{coll}}(r_\perp) \right) \]

\( \lambda: \) HF cutoff length scale (Gaussian width)

<table>
<thead>
<tr>
<th></th>
<th>( \eta/s )</th>
<th>HF</th>
<th>( C/\tau_0 )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>0</td>
<td>None</td>
<td>62</td>
<td>0.08</td>
</tr>
<tr>
<td>Viscous</td>
<td>( 1/4\pi )</td>
<td>None</td>
<td>49</td>
<td>0.13</td>
</tr>
<tr>
<td>Fluctuating</td>
<td>( 1/4\pi )</td>
<td>( \lambda = 1.0 \text{ fm} )</td>
<td>31</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>( 1/4\pi )</td>
<td>( \lambda = 1.5 \text{ fm} )</td>
<td>41</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>( 1/4\pi )</td>
<td>( \lambda = 2.0 \text{ fm} )</td>
<td>42</td>
<td>0.16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hydro</th>
<th>Cascades</th>
</tr>
</thead>
<tbody>
<tr>
<td>4k events</td>
<td>400 k (4k*100)</td>
</tr>
</tbody>
</table>
Centrality dependence of multiplicity

- Initial parameters tuning
- Centrality cut
Factorization ratio $r_3(\eta_p^a, \eta_p^b)$

Ideal $\approx$ Viscous $>\approx$ CMS data $\approx$ Fluctuating hydro

Hydrodynamic fluctuations $\rightarrow$ Factorization more broken

Hydrodynamic fluctuations

Shear stress tensor

Fluctuating hydro

Viscous hydro
\[ \pi^{\mu\nu}(x) = 2\eta \partial^{\langle \mu} u^{\nu \rangle} + \delta\pi^{\mu\nu}(x) \]

Actual Equation

\[ \tau_{\pi} \Delta^{\mu\nu}_{\alpha\beta} u^{\lambda} \partial_\lambda \pi^{\alpha\beta} + \pi^{\mu\nu} \left(1 + \frac{4}{3} \tau_{\pi} \partial_\lambda u^{\lambda}\right) = 2\eta \Delta^{\mu\nu}_{\alpha\beta} \partial^\alpha \pi^\beta + \delta\pi^{\mu\nu} \]
Cut of parameter

With out smearing  1.0 fm smearing  2.0 fm smearing
Settings of model

1. Initial conditions (see next slides):
   • MC-Glauber model (Pb+Pb, 2.76 TeV)
     \[ \rho_{\text{part}}(x, y), \rho_{\text{coll}}(x, y) \rightarrow s(\tau = \tau_0, x, y, \eta_s) \]

2. Full 3D fluctuating hydrodynamics:
   • EoS: \textit{s}95\textit{p}-v1.1 (lattice QCD+resonance gas)
   • Shear viscosity: \( \eta/s = 1/4\pi \)
   • Relaxation time: \( \tau_\pi = 3/4\pi T \)
   • Cutoff length scale (Gaussian width): \( \lambda = 1 \text{ fm} \)

3. Particlization:
   • Cooper-Frye formula with \( T_{\text{sw}} = 155 \text{ MeV} \)

4. Hadron cascade:
   • JAM
Initial condition

\[ s_0(\tau = \tau_0, r_\perp, \eta_s = 0) \propto \frac{1 - \alpha}{2} \rho_{\text{part}}(r_\perp) + \alpha \rho_{\text{coll}}(r_\perp) \]

\[ \tau_0 = 0.6 \text{ fm} \]
\[ \alpha = 0.20 \text{ (fluctuating hydro)} \]

*longitudinal: modified BGK model

Initial condition

\[ s_0(\tau = \tau_0, r_\perp, \eta_s) \propto \left( \frac{1 - \alpha}{2} \rho_{\text{part}}(\eta_s, r_\perp) + \alpha \rho_{\text{coll}}(r_\perp) \right) \]

\[ \rho_{\text{part}}(\eta_s, r_\perp) = \frac{Y_b - \eta_s}{Y_b} \rho_{\text{part},A}(r_\perp) + \frac{Y_b + \eta_s}{Y_b} \rho_{\text{part},B}(r_\perp) \]

\[ \tau_0 = 0.6 \, \text{fm} \]
\[ \alpha = 0.20 \, (\text{fluctuating hydro}) \]
\[ Y_b : \text{beam rapidity} \]

Modified BGK model

MC-Glauber model
Number of participant \((N_A, N_B)\)

Hadronic string

\(-y_{\text{beam}} \leftrightarrow \text{rapidity} \rightarrow y_{\text{beam}}\)

Entropy density