

Neutron stars meet constrains from high and low energy nuclear physics

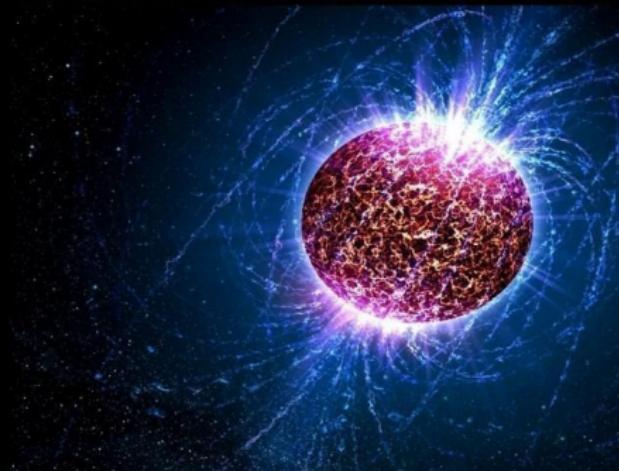
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Outline

Constraints on
realistic EoS

IST EoS

Modelling of
the neutron
stars

Conclusions

1 Constraints on realistic EoS

2 IST EoS



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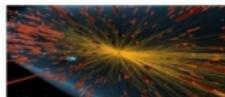
Constraints on the EoS

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HEP <ul style="list-style-type: none">- proton flow- hadron multiplicities 	Astro <ul style="list-style-type: none">- $2.01(4) M_{\text{sun}}$- observations of pulsars <p>M-R relation</p> 
Nucl. Phys. <ul style="list-style-type: none">- nuclear matter ground state 	Grav. Phys. <ul style="list-style-type: none">- NS+NS collisions- NS+BH <p>First NS+NS merger GW170817</p> 
General Requirements <ul style="list-style-type: none">- causality- thermodynamic consistency- multicomponent character (n, p, e, \dots)- electric neutrality- β-equilibrium- realistic interaction between the constituents	

Constrains on EoS: nuclear matter ground state

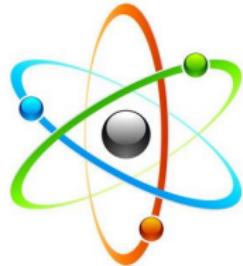
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- Lowest energy state \Rightarrow no excitations
zero temperature $T = 0$
- Dynamical equilibrium of nuclei and vacuum
zero pressure $p = 0$
- Size of large nuclei $R_A \simeq r_0 A^{1/3}$, $r_0 \simeq 1.15 \text{ fm}$
normal nuclear density $n_0 \simeq 0.16 \text{ fm}^{-3}$



- Binding energy per nucleon $\simeq 16 \text{ MeV}$:

energy density at ground state $\epsilon_0 \simeq 148 \text{ MeV fm}^{-3}$

$$\frac{E_B}{A} = \frac{Am - E_A}{A} = m - \frac{\epsilon_0}{n_0}$$

- Incompressibility factor $= 200 - 260 \text{ MeV}$:

E. Khan, Phys. Rev. C, 80, 011307 (2009); M. Dutra et al., Phys. Rev. C, 85, 035201 (2012)

$$K_0 = 9 \frac{dp}{dn}$$

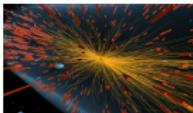
- Symmetry energy at saturation density $E_{sym}(n = n_B) \equiv J = 30 \pm 4 \text{ MeV}$

$$\text{Slope } L = 3n \frac{\partial E_{sym}}{\partial n} = 20 - 115 \text{ MeV}$$

Zhang, Z., Chen, L.-W., Phys. Lett. B, 726, 234 (2013)

Realistic EoS has to reproduce properties of ground state

Constrains on EoS: hard core radius of nucleons



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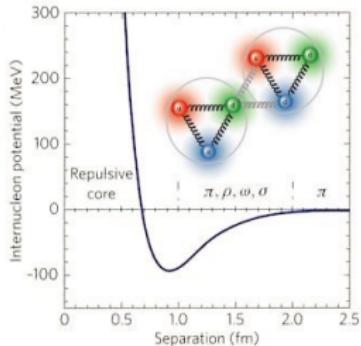
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Nucleon-nucleon scattering

hard core radius of nucleons is extracted
as a parameter of microscopic interaction
potential: $R \simeq 0.5 \text{ fm}$

A. Bohr and B. Mottelson, Nucl. Structure Benjamin, NY, 266 (1969)
M. Naghdi, Phys. Part. Nucl. 5, 924 (2014)

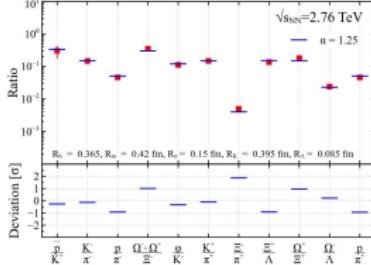


Hadron production in heavy ion collisions

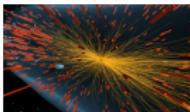
hard core radii of hadrons control the rate
of their production in thermal medium:

$R \simeq 0.3 - 0.5 \text{ fm}$

A. Andronic et al., Nucl. Phys. A 772, 167 (2006)
K. A. Bugaev et al., Nucl. Phys. A 970, 133 (2018)
V. V. Sagun et al., EPJ Web Conf., 137, 09007 (2017)



Constrains on EoS: Flow constraint



Constraints on
realistic EoS

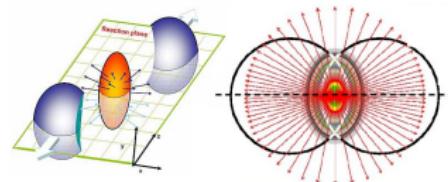
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Non-central nucleus-nucleus collisions

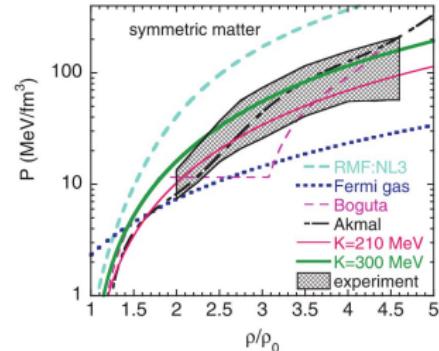
asymmetry of spatial distribution of nucleons at initial moment leads to strong anisotropy of particle flows



Hydrodynamic reconstruction of flows

anisotropic expansion is caused by gradient of pressure, which gives an access to EoS

P. Danielewicz, R. Lacey, W. G. Lynch, Science 298, 1593 (2002)



Constrains on EoS: Astro/Gravity observations

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Maximal observed mass of NS

J1614-2230 pulsar: $M_{\max} = 1.97(4)M_{\odot}$

P. B. Demorest et al., Nature, 467, 1081 (2010)

PSR J0348-0432 pulsar: $M_{\max} = 2.01(4)M_{\odot}$

J. Antoniadis et al., Science, 340, 448 (2013)

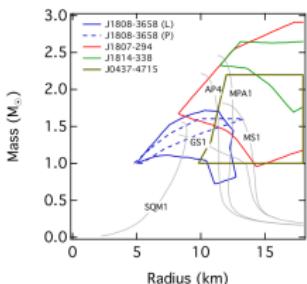


Observation of NS - ... binary systems

Shapiro Delay and radio timing constrain

R - M dependence of NSs

F. Özel, P. Freire, A. & A., 54, 401 (2006)



Coalescence of NS - ... binary systems

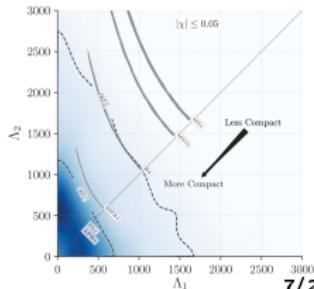
LIGO and Virgo collaborations, ApJL, 848, L12 (2017)

LIGO and Virgo collaborations, PRL 119, 161101 (2017)

First NS+NS merger GW170817

Love numbers and tidal polarizability are
highly sensitive to EoS

Ch. C. Moustakidis et al., Phys. Rev. C 95, 059904 (2017)



**Realistic EoS has to fulfil
these constraints**

EoS with hard core repulsion

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Conclusions

- Hard core reduces volume available for motion of particles by $V_{\text{excl}} = Nb$

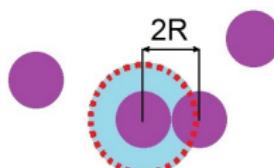
$$V \rightarrow V - V_{\text{excl}} \Rightarrow \underbrace{p = nT}_{\text{ideal gas EoS}} = \frac{NT}{V} \rightarrow \frac{NT}{V - V_{\text{excl}}} = \frac{NT}{V - Nb} = \underbrace{\frac{nT}{1 - nb}}_{\text{Van der Waals EoS}}$$

- Van der Waals EoS ($b = \text{const}$) in the Grand Canonical Ensemble

$$\begin{cases} p = p(T, \mu) \\ n = \frac{\partial p}{\partial \mu} \end{cases} \Rightarrow p = T \int_{\vec{k}} \exp\left(\frac{\mu - pb - \sqrt{m^2 + k^2}}{T}\right) = p_{\text{id}}(T, \mu - pb)$$

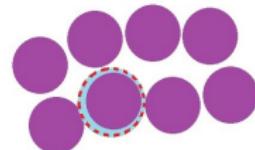
- Excluded volume (per particle) depends on density ($b \neq \text{const}$)

Low densities

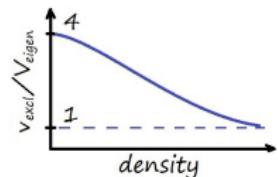


$$b \simeq \frac{1}{2} \cdot \frac{4\pi}{3} (2R)^3 = 4v$$

High densities



$$b \simeq v$$



The Induced Surface Tension (IST) EoS

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- Quantities of the Boltzmann ideal gas

$$p = nT, \quad n = \sum_i n_i^{id}, \quad n_i^{id} = \frac{p_i^{id}}{T}$$

- Virial expansion for one particle species

$$\frac{p}{T} = n + a_2 n^2 + a_3 n^3 + a_4 n^4 + \dots$$

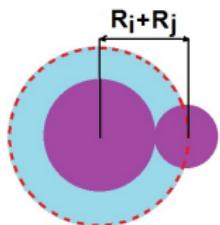
Ideal gas: $a_2 = 0, a_3 = 0, a_4 = 0, \dots$

Hard spheres: $a_2 = 4v, a_3 = 10v^2, a_4 = 18.365v^3, \dots$

R.K.Pathria, Statistical Mechanics, Pergamon Press, Oxford, 1972

- Excluded volume at low density ($v_i = \frac{4\pi}{3}R_i^3$ and $s_i = 4\pi R_i^2$)

$$a_2^{ij} = \frac{1}{2} \cdot \frac{4\pi}{3} (R_i + R_j)^3 = \frac{1}{2} \cdot (v_i + s_i R_j + R_i s_j + v_j)$$



- Virial expansion for many particle species

$$\frac{p}{T} = \sum_i \overset{\simeq n}{n_i^{id}} - \sum_{i,j} \overset{\simeq a_2 n^2}{a_2^{ij} n_i^{id} n_j^{id}} + \dots = \sum_i \underbrace{\frac{p_i^{id}}{T}}_{\text{bulk term}} \left(1 - v_i \sum_j \frac{p_j^{id}}{T} \right) - \underbrace{s_i \sum_j \frac{p_j^{id}}{T} R_j}_{\text{surface term}} + \dots$$



EoS with induced surface tension (IST)

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$$p = \sum_i p_i^{id} \left(1 - \underbrace{\frac{v_i}{T} \sum_j p_j^{id}}_{p + \mathcal{O}(n^2)} - \underbrace{\frac{s_i}{T} \sum_j p_j^{id} R_j}_{\Sigma + \mathcal{O}(n^2)} \right) + \dots$$

- Σ is conjugated to s_i – induced surface tension (IST)

$$\begin{cases} p = \sum_i p_i^{id} \left(1 - \frac{pv_i}{T} - \frac{\Sigma s_i}{T} \right) \\ \Sigma = \sum_i p_i^{id} \left(1 - \frac{pv_i}{T} - \frac{\alpha \Sigma s_i}{T} \right) R_i \end{cases}$$

α accounts for not uniqueness of extrapolation to high densities

- High density extrapolation (gives exponentials)

$$\begin{cases} p = \sum_i p_i^{id} \exp\left(-\frac{pv_i + \Sigma s_i}{T}\right) \\ \Sigma = \sum_i p_i^{id} \exp\left(-\frac{pv_i + \alpha \Sigma s_i}{T}\right) R_i \end{cases} \rightarrow \begin{cases} p = \sum_i p_i^{id} (\mu_i - pv_i - \Sigma s_i) \\ \Sigma = \sum_i p_i^{id} (\mu_i - pv_i - \alpha \Sigma s_i) R_i \end{cases}$$

V. Sagun, A. Ivanytskyi, K. Bugaev, I. Mishustin, Nucl. Phys. A 924, 24 (2014)

Physical Origin of the Induced Surface Tension

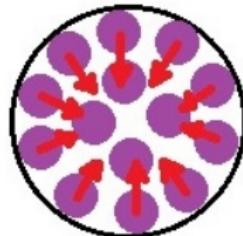
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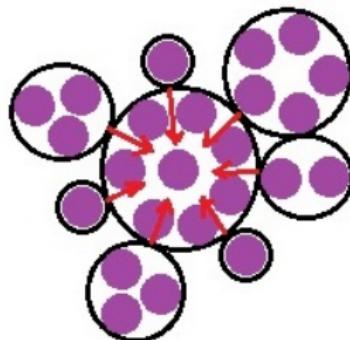
Conclusions

Vacuum

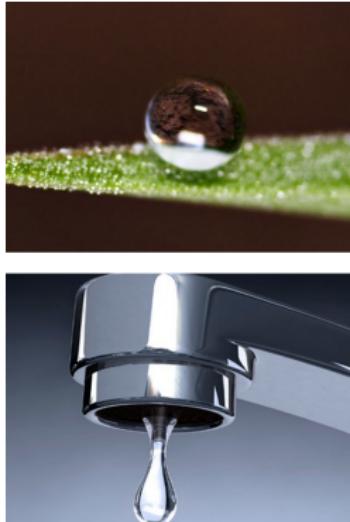


attraction of constituents
 \Rightarrow eigen surface tension

Medium



repulsion of clusters
 \Rightarrow induced surface tension



- Hard core repulsion only in part is accounted by eigen volume
- The rest corresponds to surface tension and curvature tension
Curvature tension can be accounted explicitly or implicitly
- Physical clusters tend to have spherical (in average) shape

Determination of α

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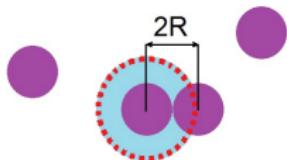
Conclusions

- One component EoS with IST and $\alpha > 1$

$$\begin{cases} p = p^{id} \exp\left(-\frac{pv + \Sigma s}{T}\right) \\ \Sigma = p^{id} \exp\left(-\frac{pv + \alpha \Sigma s}{T}\right) R \end{cases} \Rightarrow \begin{cases} p = p^{id} \exp\left(-\frac{pb}{T}\right) R \\ \Sigma = pR \exp\left(\frac{(1-\alpha)\Sigma s}{T}\right) \end{cases}$$

V. V. Sagun, A. I. Ivanytskyi, K. A. Bugaev, I. N. Mishustin, Nucl. Phys. A, 924, 24 (2014)

Excluded volume: $\frac{b}{v} = 1 + 3e^{\frac{(1-\alpha)\Sigma s}{T}} \rightarrow \begin{cases} 4, & \Sigma \rightarrow 0 \\ 1, & \Sigma \rightarrow \infty \end{cases}$



$\alpha > 1$ switches different regimes of excluded volume

- Virial expansion of one component EoS with IST

Second virial coefficient: $a_2 = 4V$ is reproduced always

Third virial coefficient: $a_3 = 10V^2 \Rightarrow \alpha = \frac{4}{3}$
 a_4 is not reproduced

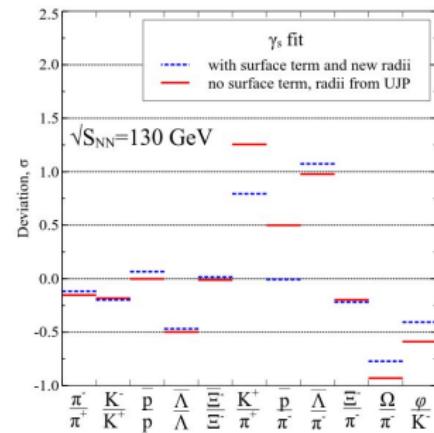
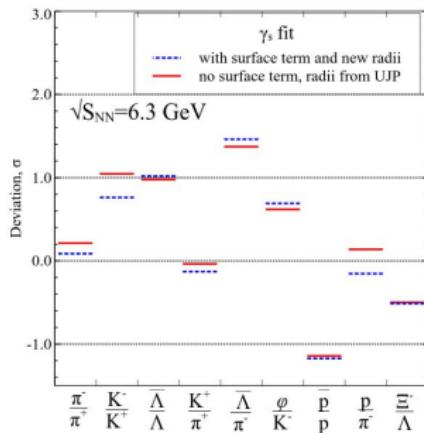
Fourth virial coefficient: $a_4 \simeq 18.365V^3 \Rightarrow \alpha \simeq 1.245$

a_3 – reproduced with 16% accuracy

$\alpha > 1.245$ reproduces two (3rd and 4th) virial coefficients

Hadron Resonance Gas Model

- Hadrons with masses ≤ 2.5 GeV (widths, strong decays, zero strangeness)
- 111 independent particle ratios measured at 14 energies (from 2.7 GeV to 200 GeV)
- 14×4 local parameters ($T, \mu_B, \mu_{I3}, \gamma_s$) + 5 global parameters (hard core radii)



$$R_b = 0.365 \text{ fm}, R_m = 0.42 \text{ fm}, R_\pi = 0.15 \text{ fm}, R_K = 0.395 \text{ fm}, R_\Lambda = 0.085 \text{ fm}$$

Overall $\chi^2/\text{dof} \simeq 1.038$

K.A. Bugaev, et al., NPA 970, p. 133-155, (2018)
 V.V. Sagun, Ukr. J. Phys. 59, 755 (2014)

Hadron Resonance Gas at ALICE Energies

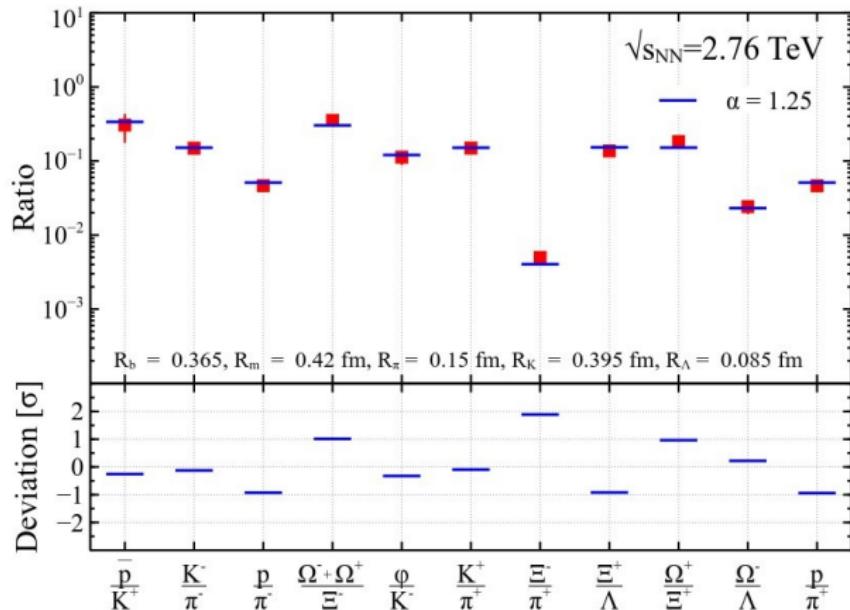
- 11 independent particle yields, 6 parameters (temperature + 5 hard core radii)
- Overall $\chi^2/dof \simeq 0.89$
- Freeze out temperature $T_{FO} = 148 \pm 7$ MeV

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MF interaction for nuclear matter

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Conclusions

- Thermodynamic consistency provides identity $\frac{\partial p}{\partial \mu} = n$

$$p(\mu) = p^{id}(\mu - U(x)) + p_{int}(x), \quad p_{int}(x) = \int_0^x dx' x' \frac{\partial U(x')}{\partial x'}$$

x – any quantity (density, asymmetry parameter, ...)

K. A. Bugaev and M. I. Gorenstein, Z. Phys. C 43, 261 (1989)

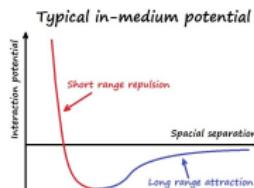
D.H. Rischke, et al. Z. Phys. C 51, 485 (1991)

Long range attraction (negative contribution to pressure)

$$p_{at}(x) = -\frac{\kappa}{\kappa+1} C_d^2 x^{1+\kappa}, \quad x = n_n^{id} + n_p^{id}$$

$\kappa < 1$, C_d^2 – fitted to flow constraint and properties of ground state

A. Ivanytskyi et al. 2017, arXiv: 1710.08218, accepted to PRC



- Repulsion due to symmetry energy (positive contribution to pressure)

binding energy of stable nuclei: $E_{sym} = a_{sym} \frac{(N-Z)^2}{A}$, $a_{sym} = 30 \pm 4$ MeV

$$p_{sym}(x) = \frac{A_{sym} x^2}{1 + (B_{sym} x)^2}, \quad x = n_n^{id} - n_p^{id}$$

A_{sym} , B_{sym} – fitted to a_{sym} and slope of symmetry energy at ground state

EoS for NS interiors

Constraints on realistic EoS

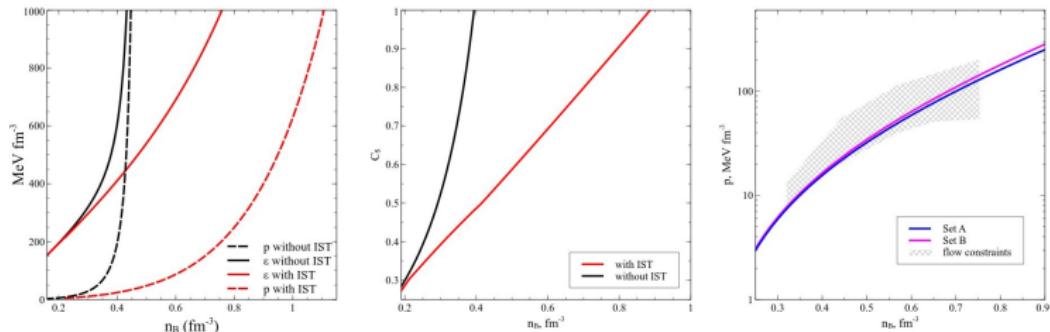
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$$\left\{ \begin{array}{l} p = \sum_{i=n,p} p_i^{id} (\mu_i - p v_i - \Sigma s_i + U_{at} \pm U_{sym}) + p^{id}(\mu_e) - p_{at} + p_{sym} \\ \Sigma = \sum_{i=n,p} p_i^{id} (\mu_i - p v_i - \alpha \Sigma s_i) R_i \end{array} \right.$$

- **Degrees of freedom:** neutrons, protons, electrons
- **Quantum statistics** is accounted by construction of p^{id} and n^{id}
- **Realistic interaction:** HC repulsion, MF attraction, symmetry energy repulsion
- **Virial coefficients** of classic hard spheres are reproduced
- **Causal behaviour** up to densities where QGP is expected and **Flow constraint**



V.V. Sagun, I. Lopes, A. Ivanytskyi, arxiv:1805.04976 (2018)

V. V. Sagun, I. Lopes, ApJ, 850, 75 (2017)

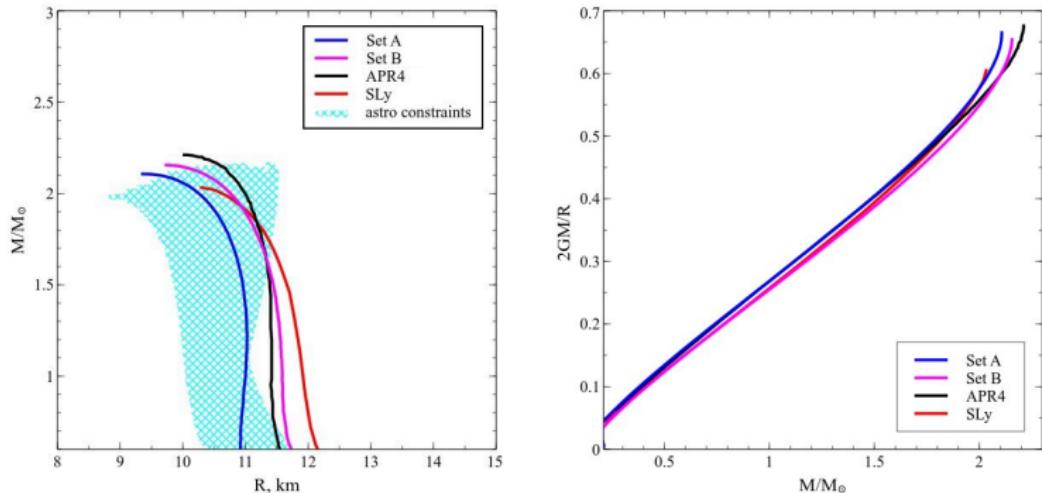
R-M relation and compactness of NS

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IST EoS PARAMETERS												
IST EoS Sets	r fm	α —	κ —	B^{sym} fm^3	A^{sym} $\text{MeV} \cdot \text{fm}^3$	C_4^2 $\text{MeV} \cdot \text{fm}^{3\kappa}$	U_0 MeV	K_0 MeV	J MeV	L MeV	M_{\max} M_\odot	
A (blue curve on Fig.)	0.477	1.245	0.25	3.5	18.267	147.652	161.236	205.616	30.0	113.448	2.107	
B (magenta curve on Fig.)	0.475	1.2	0.25	2.5	22.546	148.672	161.324	212.545	30.0	115.074	2.156	

Summary

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- Using a novel a thermodynamically self-consistent IST EoS the properties of the NS at zero-temperature limit were calculated.
- It was shown that the present EoS can be successfully applied to the description of the hadron multiplicities measured in A+A collisions, to studies of the nuclear matter phase diagram and to modelling of the NS interiors.
- IST EoS satisfies all astrophysical constraints, correctly reproduce properties of normal nuclear density, proton flow data, hadron multiplicities measured in A+A collisions and nuclear matter properties near the (3)CEP.
- The description of the compact stars with the IST provides with a strong constraint on the attraction contribution in EoS at zero temperature.
- The IST EoS gives a possibility to describe the strongly interacting matter phase diagram in a wide range of its thermodynamic parameters which helps to create a solid bridge between the astrophysical data, high-energy nuclear physics and gravitation physics.

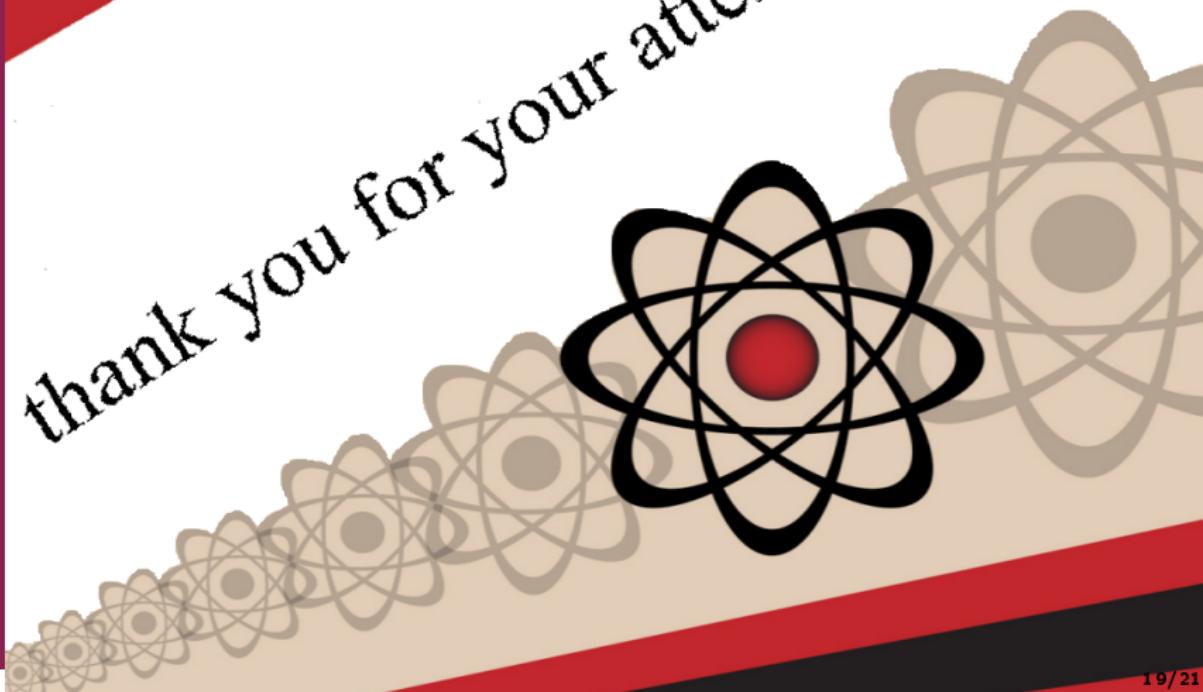
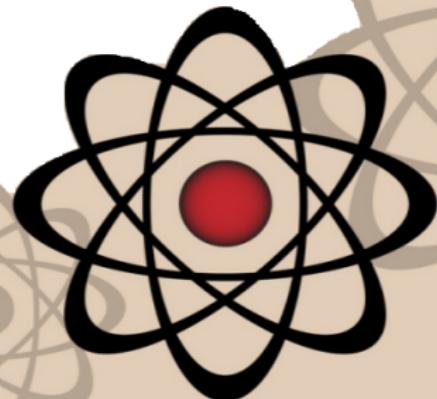
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thank you for your attention!



Why hard core (HC) repulsion of hadrons?

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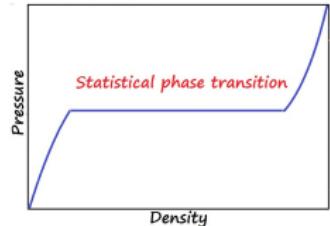
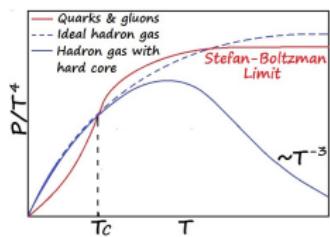
- Prevents phenomenological EoS of QCD from quark confinement at high temperatures

$$\text{ideal gas : } p \sim T^4$$

$$\text{hard core : } p \sim T$$

- Accounts for short range repulsion between the constituents (hadrons, nuclear fragments, etc.)
- Is necessary for statistical (not Van der Waals) liquid-gas phase transition in cluster models
- Important element in description of particle yields (ideal hadron gas is proven to be inadequate at high A+A collision energies)

J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993)
J. Cleymans, et al., Phys. Scripta 48, 277 (1993)



The Induced Surface Tension (IST) EoS

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- System of equations for pressure p and induced surface tension Σ :

$$\left\{ \begin{array}{l} p = \sum_i p_i^{id} \left(1 - \frac{pv_i}{T} - \frac{\Sigma s_i}{T} \right) \\ \Sigma = \sum_i p_i^{id} \left(1 - \frac{pv_i}{T} - \frac{\Sigma s_i}{T} \right) R_i \cdot \underbrace{\exp\left(\frac{(1-\alpha)S_i\Sigma}{T}\right)}_{\text{extrapolation to high densities}}, \quad \alpha = \text{const} \end{array} \right.$$

- Meaning of $\alpha > 1$: one component case

$$\Sigma = pR \exp\left(\frac{(1-\alpha)S\Sigma}{T}\right)$$

$$p = p^{id} \exp\left(-\frac{pb}{T}\right) R$$

$$b = v \left[1 + 3 \exp\left(\frac{(1-\alpha)S\Sigma}{T}\right) \right]$$

α switches excluded and eigen volume regimes

high order virial coefficients?

\Rightarrow low densities ($\Sigma \rightarrow 0$) : $b = 4v$
high densities ($\Sigma \rightarrow \infty$) : $b = v$



- Higher virial coefficients of hard spheres

- **Second virial coefficient** – reproduced for any α
- **Third virial coefficient** – reproduced for $\alpha = 1.245$ within 16%
- **Fourth virial coefficient** – reproduced for $\alpha = 1.245$

- IST EoS is causal up to $\simeq 7$ normal nuclear densities where quark matter is expected