Thermalization and hydrodynamics in Bjorken and Gubser flows

Ulrich Heinz



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Quark Matter 2018, Venice, May 14-19, 2018

M. Martinez, M. McNelis, UH, PRC 95 (2017) 054907

C. Chattopadhyay, UH, S. Pal, G, Vujanovic, arXiv:1801.07755

M. McNelis, D. Bazow, UH, arXiv:1803:01810







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Hydrodynamics - a theory with predictive power

After tuning initial conditions and viscosity at RHIC to obtain a good description of all soft hadron data simultaneously (Song et al. 2010) the first LHC spectra and elliptic flow measurements were successfully **pre**dicted:



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Hydrodynamics describes Pb+Pb, p+Pb and p+p at the LHC simultaneously!

R.D. Weller, P. Romatschke, Phys. Lett. B 774 (2017) 351



[Requires fluctuating proton substructure (gluon clouds clustered around valence quarks (K. Welsh et al. PRC94 (2016) 024919))]

But when you look under the hood you find...

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Large shear stress throughout the QGP phase!



VISH2+1 (from H. Song's PhD thesis (2009))

\Longrightarrow Large deviations from local equilibrium and momentum isotropy!

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Heavy-ion collisions provide a particular challenge:

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions
- However, the kinematics of ultra-relativistic heavy-ion collisions introduces a complication that severely limits the applicability of standard viscous relativistic fluid dynamics:

Large viscous stresses caused by (1) large initial anisotropies between the longitudinal and transverse expansion rates and by (2) critical dynamics near the quark-hadron phase transition

These should be treated non-perturbatively!

Earlier work ("standing on the shoulders of giants"): Florkowski & Ryblewski, Martinez & Strickland, Bazow et al., Tinti et al., Molnar & Niemi & Rischke, and others

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Boltzmann Equation in Relaxation Time Approximation (RTA):

$$p^{\mu}\partial_{\mu}f(x,p) = C(x,p) = rac{p \cdot u(x)}{ au_{\mathrm{rel}}(x)} \Big(f_{\mathrm{eq}}(x,p) - f(x,p)\Big)$$

For conformal systems $\tau_{\rm rel}(x) = c/T(x) = 5\eta/(ST) \equiv 5\bar{\eta}/T(x)$.

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Macroscopic currents:

$$j^{\mu}(x) = \int_{p} p^{\mu} f(x,p) \equiv \langle p^{\mu} \rangle; \quad T^{\mu\nu}(x) = \int_{p} p^{\mu} p^{\nu} f(x,p) \equiv \langle p^{\mu} p^{\nu} \rangle$$

here
$$\int_{p} \cdots \equiv \frac{g}{(2\pi)^3} \int \frac{d^3p}{E_p} \cdots \equiv \langle \dots \rangle$$

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Hydrodynamics for strongly anisotropic expansion:

Account for large viscous flows by including their effect already at leading order in the Chapman-Enskog expansion:

Expand the solution f(x, p) of the Boltzmann equation as

$$f(x,p) = f_0(x,p) + \delta f(x,p) \qquad \Big(\big| \delta f/f_0 \big| \ll 1 \Big),$$

$$f_0(x, p) = f_0\left(rac{\sqrt{
ho_\mu\Omega^{\mu
u}(x)
ho_
u} - ilde{\mu}(x)}{ ilde{T}(x)}
ight),$$

where $p_{\mu}\Omega^{\mu\nu}(x)p_{\nu} = m^2 + (1+\xi_{\perp}(x))p_{\perp,\text{LRF}}^2 + (1+\xi_{L}(x))p_{z,\text{LRF}}^2$

- $\tilde{T}(x)$, $\tilde{\mu}(x)$ are the effective temperature and chemical potential in the LRF, Landau matched to energy and particle density, *e* and *n*.
- ξ_{⊥,L} parametrize the momentum anisotropy in the LRF, Landau matched to the transverse and longitudinal pressures, P_⊥ and P_L. (McNelis, Bazow, UH, arXiv:1803.01810)
- P_{\perp} and P_L encode the bulk viscous pressure, $\Pi = (2P_{\perp} + P_L)/3 P_{eq}$, and the largest shear stress component, $P_L - P_{\perp}$.

A variety of hydrodynamic approximations:

Different hydrodynamic approaches can be characterized by the different assumptions they make about the dissipative corrections and/or the different approximations they use to derive their dynamics from the underlying Boltzmann equation:

- Ideal hydro: local momentum isotropy $(\xi_{\perp,L} = 0)$, $\Pi^{\mu\nu} = V^{\mu} = 0$.
- Navier-Stokes (NS) theory: local momentum isotropy $(\xi_{\perp,L} = 0)$, ignores microscopic relaxation time by postulating instantaneous constituent relations for $\Pi^{\mu\nu}$, V^{μ} .
- Israel-Stewart (IS) theory: local momentum isotropy $(\xi_{\perp,L} = 0)$, evolves $\Pi^{\mu\nu}$, V^{μ} dynamically, keeping only terms linear in $\text{Kn} = \lambda_{\text{mfp}}/\lambda_{\text{macro}}$
- Denicol-Niemi-Molnar-Rischke (DNMR) theory: improved IS theory that keeps nonlinear terms up to order Kn^2 , $Kn \cdot Re^{-1}$ when evolving $\Pi^{\mu\nu}$, V^{μ} .
- Third-order Chapman-Enskog expansion (Jaiswal 2013): local momentum isotropy ($\xi_{\perp,L} = 0$), keeping terms up to third order when evolving $\Pi^{\mu\nu}$, V^{μ} .
- Anisotropic hydrodynamics (aHydro): allows for leading-order local momentum anisotropy (ξ_{⊥,L} ≠ 0), evolved according to equations obtained from low-order moments of BE, but ignores residual dissipative flows: Π^{μν} = V^μ = 0.
- Viscous anisotropic hydrodynamics (vaHydro): improved aHydro that additionally evolves residual dissipative flows \$\Pi^\mu\$, \$\V^\mu\$ with \$\mu\$s or \$\DNMR\$ theory.

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Testing the various hydrodynamic approximations against exact solutions of the underlying microscopic dynamics

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BE for systems with highly symmetric flows: I. Bjorken flow

• Longitudinal boost invariance, transverse homogeneity ("physics on the light cone", no transverse flow) $\implies u^{\mu} = (1, 0, 0, 0)$ in Milne coordinates (τ, r, ϕ, η) where $\tau = (t^2 - z^2)^{1/2}$ and $\eta = \frac{1}{2} \ln[(t-z)/(t+z)] \implies v_z = z/t$

• Metric:
$$ds^2 = d\tau^2 - dr^2 - r^2 d\phi^2 - \tau^2 d\eta^2$$
, $g_{\mu\nu} = \text{diag}(1, -1, -r^2, -\tau^2)$

 Symmetry restricts possible dependence of distribution function f(x, p) (Baym '84, Florkowski et al. '13, '14):

 $f(x, p) = f(\tau; p_{\perp}, w)$ where $w = tp_z - zE = \tau m_{\perp} \sinh(y-\eta)$.

RTA BE simplifies to ordinary differential equation

 $D(\tau_2,\tau_1) = \exp\left(-\int_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\rm rel}(\tau'')}\right).$

$$\partial_{\tau}f(\tau; \mathbf{p}_{\perp}, \mathbf{w}) = -\frac{f(\tau; \mathbf{p}_{\perp}, \mathbf{w}) - f_{\mathrm{eq}}(\tau; \mathbf{p}_{\perp}, \mathbf{w})}{\tau_{\mathrm{rel}}(\tau)}.$$

Solution:

$$f(\tau; \boldsymbol{p}_{\perp}, \boldsymbol{w}) = D(\tau, \tau_0) f_0(\boldsymbol{p}_{\perp}, \boldsymbol{w}) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\mathrm{rel}}(\tau')} D(\tau, \tau') f_{\mathrm{eq}}(\tau'; \boldsymbol{p}_{\perp}, \boldsymbol{w})$$

where

Venice, 5/16/18 10 / 24

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BE for systems with highly symmetric flows: II. Gubser flow

Longitudinal boost invariance, azimuthally symmetric radial dependence ("physics on the light cone" with azimuthally symmetric transverse flow) (Gubser '10, Gubser & Yarom '11) $\Rightarrow u^{\mu} = (1, 0, 0, 0)$ in de Sitter coordinates $(\rho, \theta, \phi, \eta)$ where $\rho(\tau, r) = -\sinh^{-1}\left(\frac{1-q^2\tau^2+q^2r^2}{2q\tau}\right) \text{ and } \theta(\tau, r) = \tan^{-1}\left(\frac{2qr}{1+q^2\tau^2-q^2r^2}\right).$ 10 2 8 0 97 $^{-2}$ 8 $\overline{10}$ 0 2 4 6 qrUlrich Heinz (OSU, CERN & EMMI) Thermalization and hydrodynamics Venice, 5/16/18 11 / 24

BE for systems with highly symmetric flows: II. Gubser flow

- Longitudinal boost invariance, azimuthally symmetric radial dependence ("physics on the light cone" with azimuthally symmetric transverse flow) (Gubser '10, Gubser & Yarom '11) ⇒ u^μ = (1,0,0,0) in de Sitter coordinates (ρ, θ, φ, η) where ρ(τ, r) = - sinh⁻¹ (1-q²r²+q²r²/2qr) and θ(τ, r) = tan⁻¹ (2qr/(1+q²τ²-q²r²}).
 ⇒ v_z = z/t and v_r = 2q²τr/(1+q²τ²+q²r²) where q is an arbitrary scale parameter.
 Metric: ds² = ds²/τ² = dρ² - cosh²ρ (dθ² + sin² θ dφ²) - dη², g_{µν} = diag(1, - cosh² ρ, - cosh² ρ sin² θ, -1)
- Symmetry restricts possible dependence of distribution function f(x, p)

$$f(x,p) = f(
ho; \hat{p}_{\Omega}^2, \hat{p}_{\eta})$$
 where $\hat{p}_{\Omega}^2 = \hat{p}_{\theta}^2 + \frac{\hat{p}_{\phi}^2}{\sin^2 \theta}$ and $\hat{p}_{\eta} = w$.

• With $T(\tau, r) = \hat{T}(\rho(\tau, r))/\tau$ RTA BE simplifies to the ODE

$$\frac{\partial}{\partial \rho} f(\rho; \hat{p}_{\Omega}^{2}, \hat{\rho}_{\varsigma}) = -\frac{\hat{T}(\rho)}{c} \left[f(\rho; \hat{p}_{\Omega}^{2}, \hat{\rho}_{\varsigma}) - f_{\rm eq}(\hat{p}^{\rho}/\hat{T}(\rho)) \right].$$

Exact solution (formally similar to an analogous solution for Bjorken flow): $f(\rho; \hat{\rho}_{\Omega}^{2}, w) = D(\rho, \rho_{0}) f_{0}(\hat{\rho}_{\Omega}^{2}, w) + \frac{1}{c} \int_{\rho_{0}}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho_{0}}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho_{0}}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho_{0}}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \hat{\rho}_{\Omega}^{2}, w) = \frac{1}{2} \int_{\rho}^{\rho} d\rho' \hat{T}(\rho') D(\rho, \rho') f_{eq}(\rho'; \rho') f_{eq}(\rho') f_{eq}(\rho'; \rho') f_{eq}(\rho'; \rho') f_{eq}(\rho'; \rho') f_{eq}(\rho'; \rho') f_{eq}(\rho') f_{$

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Venice, 5/16/18 12 / 24

Hydrodynamic equations for systems with Gubser flow:

The exact solution for f can be worked out for any "initial" condition $f_0(\hat{\rho}_{\Omega}^2, w) \equiv f(\rho_0; \hat{\rho}_{\Omega}^2, w)$. We here use equilibrium initial conditions, $f_0 = f_{eq}$.

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- By taking hydrodynamic moments, the exact f yields the exact evolution of all components of $T^{\mu\nu}$. Here, $\Pi^{\mu\nu}$ has only one independent component, $\pi^{\eta\eta}$.

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- By taking hydrodynamic moments, the exact f yields the exact evolution of all components of $T^{\mu\nu}$. Here, $\Pi^{\mu\nu}$ has only one independent component, $\pi^{\eta\eta}$.
- This exact solution of the BE can be compared to solutions of the various hydrodynamic equations in de Sitter coordinates, using identical initial conditions.
 - Ideal: $\hat{T}_{ideal}(\rho) = \frac{\hat{T}_0}{\cosh^2/3(\rho)}$ NS: $\frac{1}{\hat{T}} \frac{d\hat{T}}{d\rho} + \frac{2}{3} \tanh \rho = \frac{1}{3} \overline{\pi}(\rho) \tanh \rho$ (viscous *T*-evolution) with $\overline{\pi} \equiv \hat{\pi}_{\eta}^{\eta}/(\hat{T}\hat{S})$ and $\hat{\pi}_{NS} = \frac{4}{3}\hat{\eta} \tanh \rho = \frac{4}{15}\hat{\tau}_{rel} \tanh \rho$ IS: $\frac{d\overline{\pi}}{d\rho} + \frac{\overline{\pi}}{\hat{\tau}_{rel}} = \frac{4}{15} \tanh \rho = -\frac{4}{3}\overline{\pi}^2 \tanh \rho$ DNMR: $\frac{d\overline{\pi}}{d\rho} + \frac{\overline{\pi}}{\hat{\tau}_{rel}} = \frac{4}{15} \tanh \rho + \frac{10}{21}\overline{\pi} \tanh \rho \frac{4}{3}\overline{\pi}^2 \tanh \rho$ 3rd-order CE: $\frac{d\overline{\pi}}{d\rho} + \frac{\overline{\pi}}{\hat{\tau}_{rel}} = \frac{4}{15} \tanh \rho + \frac{10}{21}\overline{\pi} \tanh \rho \frac{412}{147}\overline{\pi}^2 \tanh \rho$ a Hydro: see M. Nopoush et al., PRD 91 (2015) 045007
 va Hydro: $\frac{d\overline{\pi}}{d\rho} + \frac{\overline{\pi}}{\hat{\tau}_{rel}} = \frac{5}{12} \tanh \rho + \frac{4}{3}\overline{\pi} \tanh \rho \frac{4}{3}\overline{\pi}^2 \tanh \rho \frac{4}{3}\mathcal{F}(\overline{\pi})$ (M. Martinez et al., PRC 95 (2017) 054907)

Optimal evolution of the momentum deformation parameter ξ ?

- "Standard" viscous hydrodynamics (IS or DNMR): expansion around local equilibrium $\implies \xi \equiv 0$
- Anisotropic hydrodynamics:

expansion around a locally momentum-anisotropic state $\Longrightarrow \xi \neq 0$

■ P_L-matching (Tinti 2015; Molnar, Niemi, Rischke, 2016):

Additional Landau matching condition that matches ξ evolution to that of the longitudinal pressure $P_L \implies$ no $\delta \tilde{f}$ corrections to P_L . In this case ξ can be eliminated, and the evolution equations can be written entirely in terms of macroscopic variables, as in standard viscous hydrodynamics

- **NSR approach** (Nopoush, Strickland, Ryblewski 2015): obtain ξ evolution equation from second moments of the BE $\implies P_L$ evolution not fully captured by ξ evolution.
- **NLO-NSR approach** (Martinez, McNelis, UH 2017): Same ξ evolution but includes residual $\delta \tilde{f}$ contribution to P_L This captures the missing part of the pressure anisotropy.



*For Bjorken flow see E. Molnar, H. Niemi, D. Rischke, PRC 94 (2016) 125003.

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Venice, 5/16/18 15 / 24







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Shear stress evolution: Bjorken vs. Gubser

Chattopadhyay, UH, Pal, Vujanovic, arXiv:1801.07755



Bjorken:



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Hydrodynamic attractors: shear stress

Chattopadhyay, UH, Pal, Vujanovic, arXiv:1801.07755



Viscous heating: Bjorken vs. Gubser

Chattopadhyay, UH, Pal, Vujanovic, arXiv:1801.07755



Bjorken:

Gubser:

Hydrodynamic attractors: entropy production

Chattopadhyay, UH, Pal, Vujanovic, arXiv:1801.07755



Summary

- Viscous relativistic hydrodynamics provides a robust, reliable, efficient and accurate description of QGP evolution in heavy-ion collisions.
- It is valid even when the expansion is fast and highly anisotropic, causing large local momentum anisotropies ⇒ local momentum isotropy and thermalization not strictly required.
- While some first-order viscous corrections are large in nuclear collisions, especially in small systems, they can be handled efficiently in an optimized anisotropic hydrodynamic approach that accounts for local momentum anisotropies at leading order; residual dissipative flows remain small.
- New exact solutions of the Boltzmann equation enable powerful tests of the efficiency and accuracy of various hydrodynamic expansion schemes, providing strong support for the validity and robustness of second-order viscous hydrodynamics (especially their anisotropic variants).
- While anisotropic hydrodynamics evolves the hydrodynamic moments of the underlying phase-space distributions quite faithfully, it tends to overpredict viscous heating by O(10%). Presumably worse in other hydrodynamic approximation schemes.

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Thank you!

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Venice, 5/16/18 24 / 24

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