Forward particle production in proton-nucleus collisions at NLO: solving the running-coupling puzzle

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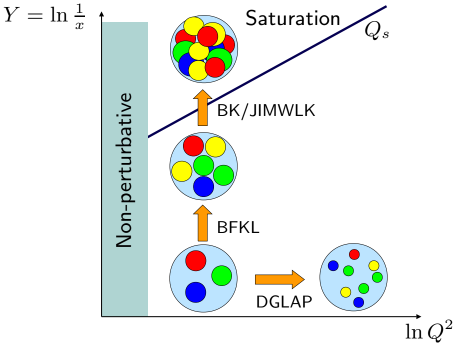
B. D., T. Lappi, Y. Zhu, PRD 93 (2016) 114016 [arXiv:1604.00225]
B. D., T. Lappi, Y. Zhu, PRD 95 (2017) 114007 [arXiv:1703.04962]
B. D., E. Iancu, T. Lappi, A.H. Mueller, G. Soyez, D.N. Triantafyllopoulos, Y. Zhu, PRD 97 (2018) 054020 [arXiv:1712.07480]

Our goal is to study QCD in the saturation regime

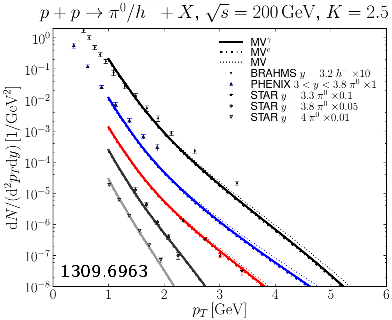


The production of forward particles is a crucial tool to probe small x values

Saturation effects stronger in pA collisions ($Q_s^2 \sim A^{1/3}$)

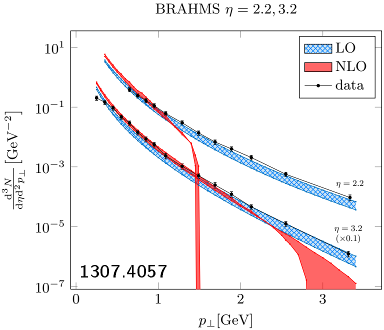
Here we study the inclusive production of forward hadrons in proton-nucleus collisions: $pA \to hX$

Typical calculation at LO:
(Lappi, Mäntysaari)



K factor needed to describe the data

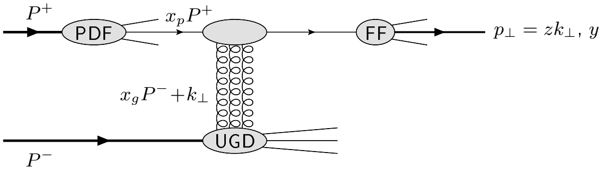
First numerical calculation at NLO:
(Staśto, Xiao, Zaslavsky)



Negative cross section above some $p_\perp$

Several proposals to solve the negativity problem at NLO, for example the kinematical constraint / Ioffe time cutoff (Altinoluk, Armesto, Beuf, Kovner, Lublinsky). Numerical implementation: Watanabe, Xiao, Yuan, Zaslavsky. Can extend the positivity range but doesn't solve the problem completely.

Single inclusive forward hadron production at LO in the $q \to q$ channel:



Dilute projectile: $x_p = \frac{k_\perp}{\sqrt{s}} e^y$, described by collinear PDFs

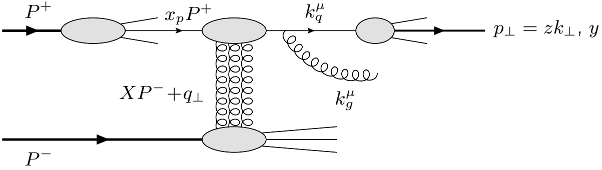
Dense target: $x_g = \frac{k_\perp}{\sqrt{s}} e^{-y} \ll 1$, described by unintegrated gluon distribution $\mathcal{F}$

$$\mathcal{F}(k_\perp) = \int \mathrm{d}^2\mathbf{b}\, \mathcal{S}(k_\perp)\ ,\ \mathcal{S}(k_\perp) = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} S(\mathbf{r})\ ,\ S(\mathbf{r} = \mathbf{x} - \mathbf{y}) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, V(\mathbf{x}) V^\dagger(\mathbf{y}) \right\rangle$$

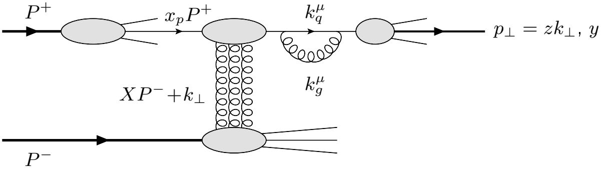
Rapidity (or x) dependence of S : governed by the Balitsky-Kovchegov equation

NLO corrections to the impact factor: Chirilli, Xiao, Yuan

Example of real $q \to q$ contribution:



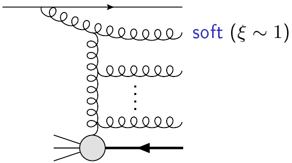
Example of virtual $q \to q$ contribution:



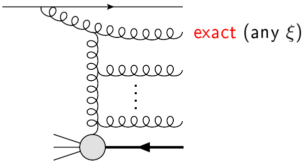
$1 - \xi = \frac{k_g^+}{x_p P^+}$ is the momentum fraction of the incoming quark carried by the gluon

Taking into account Balitsky-Kovchegov (BK) evolution: resummation of any number of soft gluons, already at LO

LO: all the gluons are soft:



NLO impact factor: the first gluon can be hard:

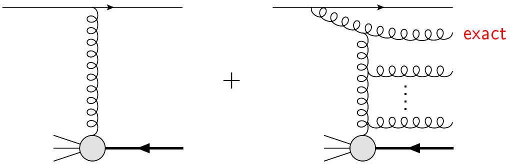


$\Rightarrow$ Need to avoid double counting between LO and NLO

Two possible solutions to avoid double counting:

1) Subtract the case where the gluon in the NLO impact factor is soft
Chirilli, Xiao, Yuan ('CXY')

2) Rearrange the terms to avoid doing a subtraction
Iancu, Mueller, Triantafyllopoulos



These two choices should be equivalent

The expression for the (quark production) multiplicity at NLO reads

$$\frac{\mathrm{d}N^{pA\to qX}}{\mathrm{d}^2\mathbf{k}\mathrm{d}y} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} \qquad \leftarrow \text{Lowest order}$$

$$+\frac{\alpha_s}{2\pi^2}\int_{x_p}^{\xi_{\mathrm{max}}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi}\frac{x_p}{\xi} q\left(\frac{x_p}{\xi}\right)\left\{C_F \mathcal{I}(k_\perp, \xi, X(\xi)) + \frac{N_c}{2}\mathcal{J}(k_\perp, \xi, X(\xi))\right\} \qquad \leftarrow \text{real}$$

$$-\frac{\alpha_s}{2\pi^2}\int_{0}^{\xi_{\mathrm{max}}} \mathrm{d}\xi \frac{1+\xi^2}{1-\xi} x_p q(x_p)\left\{C_F \mathcal{I}_v(k_\perp, \xi, X(\xi)) + \frac{N_c}{2}\mathcal{J}_v(k_\perp, \xi, X(\xi))\right\} \qquad \leftarrow \text{virt.}$$

with e.g.

$$\mathcal{J}(k_\perp, \xi, X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(q_\perp, X(\xi))$$

$$-\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\mathbf{k}-\xi\mathbf{q})\cdot(\mathbf{k}-\mathbf{l})}{(\mathbf{k}-\xi\mathbf{q})^2(\mathbf{k}-\mathbf{l})^2}\mathcal{S}(q_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

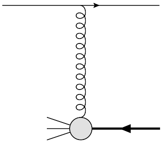
$$\mathcal{J}_v(k_\perp, \xi, X(\xi)) = \int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{k}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{k}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))$$

$$-\int \frac{\mathrm{d}^2\mathbf{q}}{(2\pi)^2}\frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2}\frac{2(\xi\mathbf{k}-\mathbf{q})\cdot(\mathbf{l}-\mathbf{q})}{(\xi\mathbf{k}-\mathbf{q})^2(\mathbf{l}-\mathbf{q})^2}\mathcal{S}(k_\perp, X(\xi))\mathcal{S}(l_\perp, X(\xi))$$

and $\dfrac{\mathrm{d}\sigma^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h} = \displaystyle\int \mathrm{d}^2\mathbf{b}\frac{\mathrm{d}N^{pA\to hX}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_h}$

The NLO cross section

In the previous expressions:



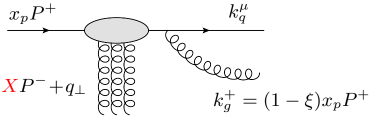
- $x_p q(x_p) \dfrac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2}$ represents the lowest order contribution

  (no BK evolution. x_0: initial condition)

- $X(\xi)$ is the rapidity scale at which the dipole correlators are evaluated

  At LO: the P^- fraction needed from the target is $\dfrac{k_\perp}{\sqrt{s}} e^{-y} \equiv x_g$

  At NLO:



$$X = \frac{k_\perp}{\sqrt{s}} e^{-y} \left(1 + \frac{\xi}{1-\xi} \frac{(q_\perp - k_\perp)^2}{k_\perp^2}\right)$$

$$\approx \frac{x_g}{1-\xi} \equiv X(\xi) \text{ when } k_\perp \gtrsim Q_s$$

The limit $\xi < 1 - \frac{x_g}{x_0} \equiv \xi_{\max}$ enforces $X(\xi) < x_0$

We can write the sum of the LO and N_c terms as

$$\frac{dN^{\text{LO}+N_c}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_0)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\mathcal{K}(k_\perp, \xi, X(\xi)) \equiv \frac{dN^{\text{LO}+N_c,unsub}}{d^2\mathbf{k}dy},$$

$$\mathcal{K}(k_\perp, \xi, X) = \frac{N_c}{(2\pi)^2}(1+\xi^2)\left[\theta(\xi - x_p)\frac{x_p}{\xi}q\left(\frac{x_p}{\xi}\right)\mathcal{J}(k_\perp, \xi, X) - x_p q(x_p)\mathcal{J}_v(k_\perp, \xi, X)\right].$$

At large $k_\perp$ the function $\mathcal{K}(k_\perp, \xi, X)$ is positive and so is the cross section.

Using the integral BK equation,

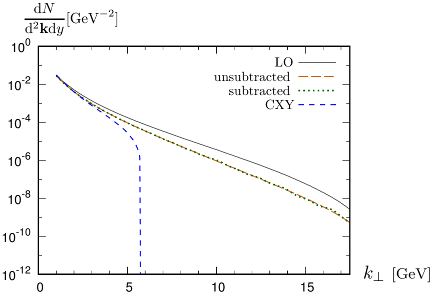
$$\mathcal{S}(k_\perp, x_g) = \mathcal{S}(k_\perp, x_0) + 2\alpha_s N_c \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{J}(k_\perp, 1, X(\xi)) - \mathcal{J}_v(k_\perp, 1, X(\xi))\right],$$

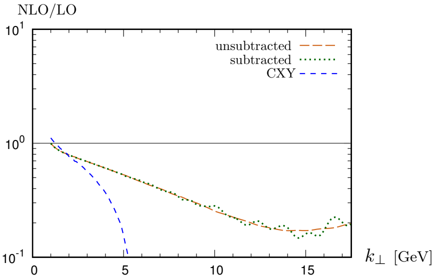
the LO+N_c terms can be rewritten as

$$\frac{dN^{\text{LO}+N_c,sub}}{d^2\mathbf{k}dy} = x_p q(x_p)\frac{\mathcal{S}(k_\perp, x_g)}{(2\pi)^2} + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi}\left[\mathcal{K}(k_\perp, \xi, X(\xi)) - \mathcal{K}(k_\perp, 1, X(\xi))\right].$$

The 'CXY' approximation corresponds to making the replacements $X(\xi) \to x_g$ and $\xi_{\text{max}} \to 1$ in this subtracted version

Results for the LO+N_c NLO corrections at fixed coupling ($\alpha_s = 0.2$):

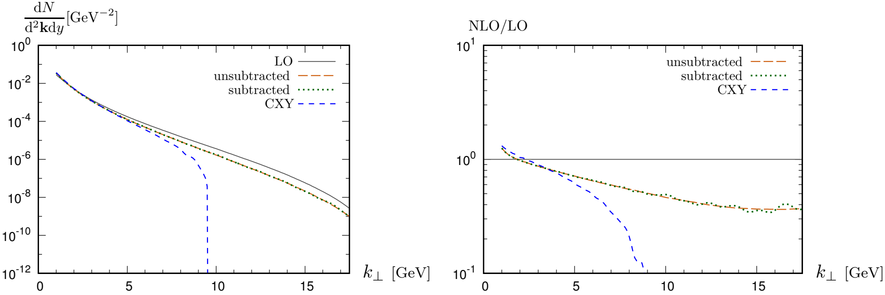




The 'subtracted' and 'unsubtracted' expressions give the same (positive) results

The 'CXY' approximation leads to negative results for $k_\perp \gtrsim 5$ GeV.

Total (LO+C_F+N_c) multiplicity ($\alpha_s = 0.2$):



Similar conclusions (the C_F terms are positive at large $k_\perp$)

Conclusions at fixed coupling

The negativity issue observed in the first implementation of the NLO impact factor can be attributed to approximations made in the LO subtraction

In the 'subtracted' formulation, we add and subtract a large contribution. If we use the CXY approximation what we add and subtract is no longer the same which can make the final result negative

Without this approximation the cross section has a physical behavior at all $k_\perp$
$\Rightarrow$ Problem solved? No!

So far we discussed only the fixed coupling case. But the running of the coupling is an important effect that has to be taken into account in realistic calculations

The equivalence between the 'subtracted' and 'unsubtracted' formulations holds only if one uses the same coupling α_s when computing the cross section and when solving the BK equation

In practice the BK equation is usually solved in coordinate space, while the cross section is written in momentum space

Fixed coupling BK equation:

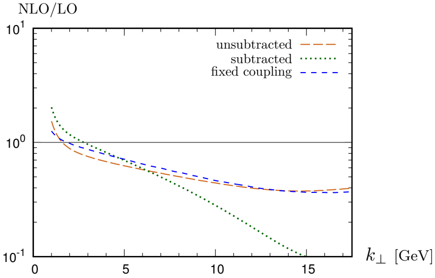
$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s N_c \int \frac{d^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X) \right]$$

rcBK with the simple parent dipole running coupling prescription:

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = 2\alpha_s(\mathbf{r}^2) N_c \int \frac{d^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{r}^2}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r},X) - S(\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X) \right]$$

Using $\alpha_s(\mathbf{r}^2)$ when solving BK seems to be reasonable since $r_\perp$ is Fourier-conjugate to $k_\perp$

Using $\alpha_s(\mathbf{r}^2)$ when solving BK and $\alpha_s(\mathbf{k}^2)$ for the explicit α_s factors in the cross section:



The 'subtracted' and 'unsubtracted' expressions are no longer equivalent since we don't use exactly the same α_s in the cross section and when solving BK

- 'Subtracted' version: very different results compared to fixed coupling, can be negative at large $k_\perp$
- 'Unsubtracted' version: physical results but does not have the correct LO limit

Possible way to use consistently a coordinate-space running coupling: rewrite the cross section expression in coordinate space

We write $\mathcal{J} = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}$ and $\mathcal{J}_v = \int \mathrm{d}^2\mathbf{r} e^{-i\mathbf{k}\cdot\mathbf{r}} \widetilde{\mathcal{J}}_v$, with

$$\widetilde{\mathcal{J}}(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{\mathbf{x}\cdot(\mathbf{x}-\mathbf{r})}{\mathbf{x}^2(\mathbf{r}-\mathbf{x})^2} \left[S(\mathbf{r}-(1-\xi)\mathbf{x},X) - S(\xi\mathbf{x},X)S(\mathbf{r}-\mathbf{x},X)\right]$$

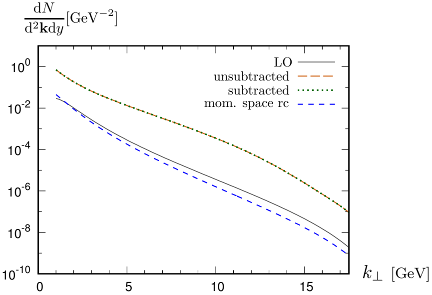
$$\widetilde{\mathcal{J}}_v(\mathbf{r},\xi,X) = 2\int \frac{\mathrm{d}^2\mathbf{x}}{(2\pi)^2} \frac{1}{\mathbf{x}^2} \left[S(\mathbf{r}+(1-\xi)\mathbf{x},X) - S(\mathbf{x},X)S(\mathbf{r}-\xi\mathbf{x},X)\right]$$

(and similarly for the C_F terms)

In these notations the BK equation reads

$$\frac{\partial S(\mathbf{r},X)}{\partial \ln X} = -2\alpha_s N_\mathsf{c} \left[\widetilde{\mathcal{J}}(\mathbf{r},1,X) - \widetilde{\mathcal{J}}_v(\mathbf{r},1,X)\right]$$

Results using $\alpha_s(\mathbf{r}^2)$ both in BK and in the cross section:



The 'subtracted' expression gives the same results as the 'unsubtracted' one

Completely different results compared to fixed coupling or $\alpha_s(k_\perp)$, absurdly large NLO corrections

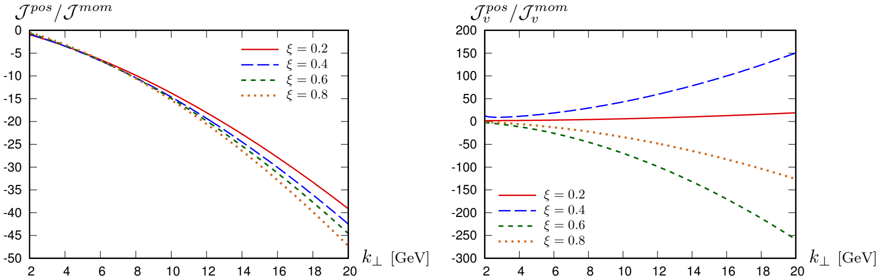
Similar situation with simple generalizations of the Balitsky or smallest dipole prescriptions $\rightarrow$ the problem is more general

The problem comes from large daughter dipoles contributions $x_\perp \gg r_\perp$. Indeed, in this limit we have for example

$$\mathcal{J}(\mathbf{k},\xi) \sim \frac{\bar{\alpha}_s}{2\pi^2}\int \mathrm{d}^2\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}} \underbrace{\int \frac{\mathrm{d}^2\mathbf{x}}{\mathbf{x}^2}\,[S((1-\xi)\mathbf{x}) - S(-\xi\mathbf{x})S(\mathbf{x})]}_{\mathbf{r}-\text{independent}} = 0 \text{ for } k_\perp \neq 0$$

On the contrary, if we move the coupling under the integral and replace $\alpha_s \to \alpha_s(\mathbf{r}^2)$, we get a large contribution from this region

Numerically we can study how $\mathcal{J}$ and $\mathcal{J}_v$ are modified when using $\alpha_s(\mathbf{r}^2)$ instead of $\alpha_s(\mathbf{k}^2)$:

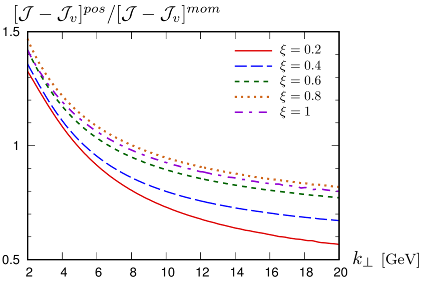


However we know that such severe issues don't appear when solving BK with the parent dipole prescription. Why?

The BK equation involves the difference between $\widetilde{\mathcal{J}}$ and $\widetilde{\mathcal{J}}_v$ at $\xi = 1$:

$$\frac{\partial S(\mathbf{r}, X)}{\partial \ln X} = -2\alpha_s N_c \left[\widetilde{\mathcal{J}}(\mathbf{r}, 1, X) - \widetilde{\mathcal{J}}_v(\mathbf{r}, 1, X)\right]$$

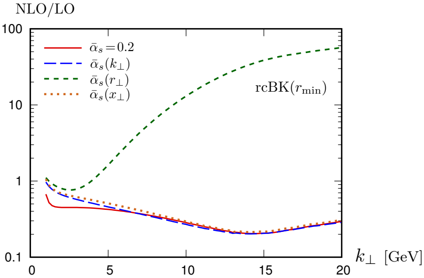
Thus the spurious contributions generated by large daughter dipoles cancel:



But in the cross section we have $\left[\widetilde{\mathcal{J}} \times q(x_p/\xi) - \widetilde{\mathcal{J}}_v \times q(x_p)\right] \Rightarrow$ No cancellation

Based on this we expect that the daughter dipole prescription, $\alpha_s(\mathbf{x}^2)$, should lead to physical results: the spurious contributions generated by large daughter dipoles will remain independent of $\mathbf{r}$, and will thus be eliminated by the Fourier transform

With this prescription the cross section indeed has a physical behavior, similar to the results with fixed or momentum space running coupling:



⇒ By using the daughter dipole prescription in both the BK evolution and the cross section, we can get meaningful results while keeping the equivalence between the 'subtracted' and 'unsubtracted' formulations

So far we focused on the terms proportional to N_c in the cross section

The C_F terms are affected by the same large daughter dipoles problem as the N_c terms when using a coordinate space running coupling

However, here the problem looks even more severe: because of the subtraction of the collinear divergence in the C_F terms, we have no control on the size of the daughter dipoles anymore, so we cannot use the coupling $\alpha_s(\mathbf{x}^2)$ as in the N_c terms

Thus the only reasonable choice seems to be a coupling running with a transverse scale, such as $\alpha_s(\mathbf{k}^2)$

If one insists on using the same coupling everywhere, the most practical way seems to perform the whole calculation in momentum space

Conclusions

The negativity problem at NLO originally observed for this process is now understood and solved as long as fixed coupling is considered

Running coupling: additional complications appear

- Mixed coordinate/momentum space calculation: mismatch between 'subtracted' and 'unsubtracted' formulations
- 'Naive' coordinate space formulation: non-physical results due to spurious large daughter dipoles contributions which should cancel in the end
- The problem can be avoided for the N_c terms by using the daughter dipole prescription
- The use of a momentum space running coupling seems mandatory for the C_F terms

Directions towards phenomenology:

- Add the $q \to g$, $g \to q$ and $g \to g$ channels + fragmentation functions
- Use NLO BK for the rapidity evolution of the dipole correlators
- The initial condition for the BK evolution of the target must be obtained by a fit (e.g. to HERA DIS data) also performed at NLO accuracy