Nonequilibrium quark production in the expanding QCD plasma

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In collaboration with

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Phys. Rev. D 97, 034013 [arXiv: 1711.03445]

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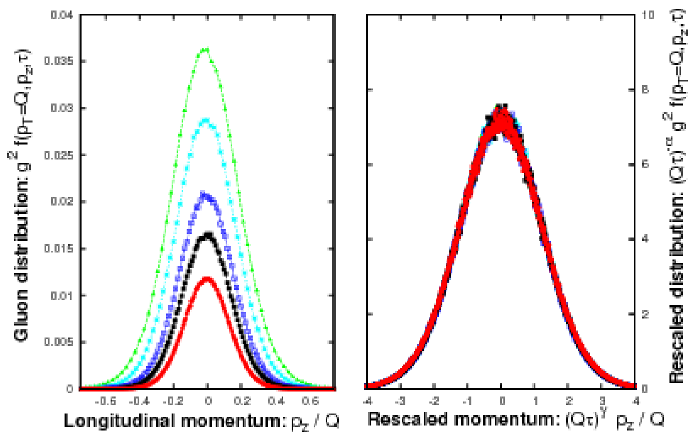
Early stage of heavy-ion collisions

First-principles-based description is possible at sufficiently high energies.

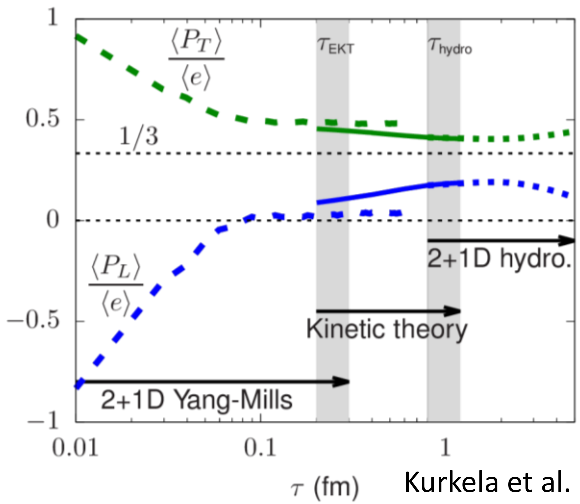
Weak coupling $g \ll 1$ but strongly correlated $A \sim Q_s/g$.

→ **Classical-statistical field theory**

- Classical-statistical simulations confirmed the "bottom-up" thermalization scenario.
- The results have been implemented into effective kinetic theory descriptions for hydrodynamization at later stage.



Berges et al. PRD89, 114007 (2014)



Kurkela et al. arXiv:1805.00961

[See talk by A. Mazeliauskas]

Much progress in the pure glue sector.

How about quarks?

Quark production

- Chemical equilibrium in QGP?
- Quarks play important roles in connection with experimental observables:
  - Photon and dilepton production
  - Chiral Magnetic Effect

Real-time lattice simulations of quark production in the longitudinally expanding QCD plasma

- ☐ How intense and quick is the quark production from overoccupied gluon plasma?
- ☐ What is the effect of the longitudinal expansion?
- ☐ How does the quark production depend on quark mass?

Real-time lattice simulations

By systematic weak-coupling expansion around strong gauge fields, real-time evolution equations for classical-statistical gauge fields and dynamical quantum quark fields can be derived from the Schwinger-Keldysh path-integral formalism. Kasper et al. PRD90, 025016 (2014)

Classical Yang-Mills equation for fluctuating initial conditions

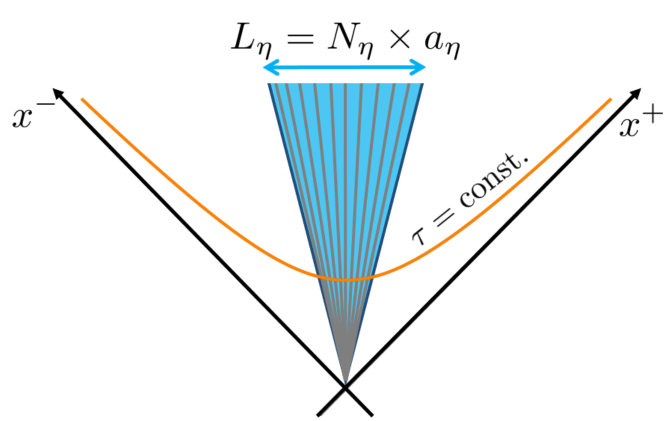
$$[D_\mu, F^{\mu\nu}] = J^\nu$$

backreaction from quarks to the gauge fields

Dirac equation for quark mode functions

$$\left[i\gamma^\mu(\partial_\mu + igA_\mu) - m\right]\psi_{\boldsymbol{p},s,c} = 0$$

- The coupled equations are solved for each gauge configuration on the lattice in the expanding geometry.
- The time evolution of quark and gluon number densities are extracted by using Coulomb-type gauge condition.



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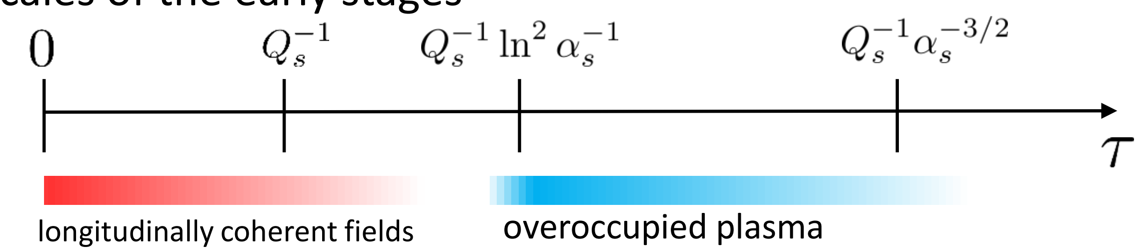
Time scales of the early stages



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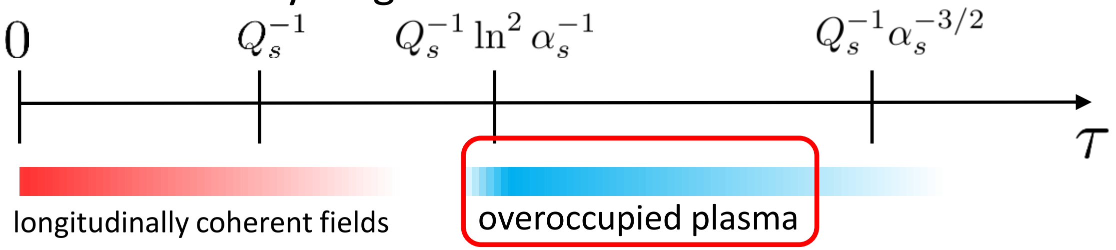
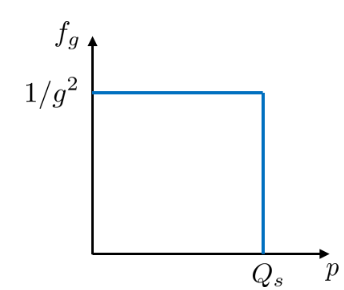
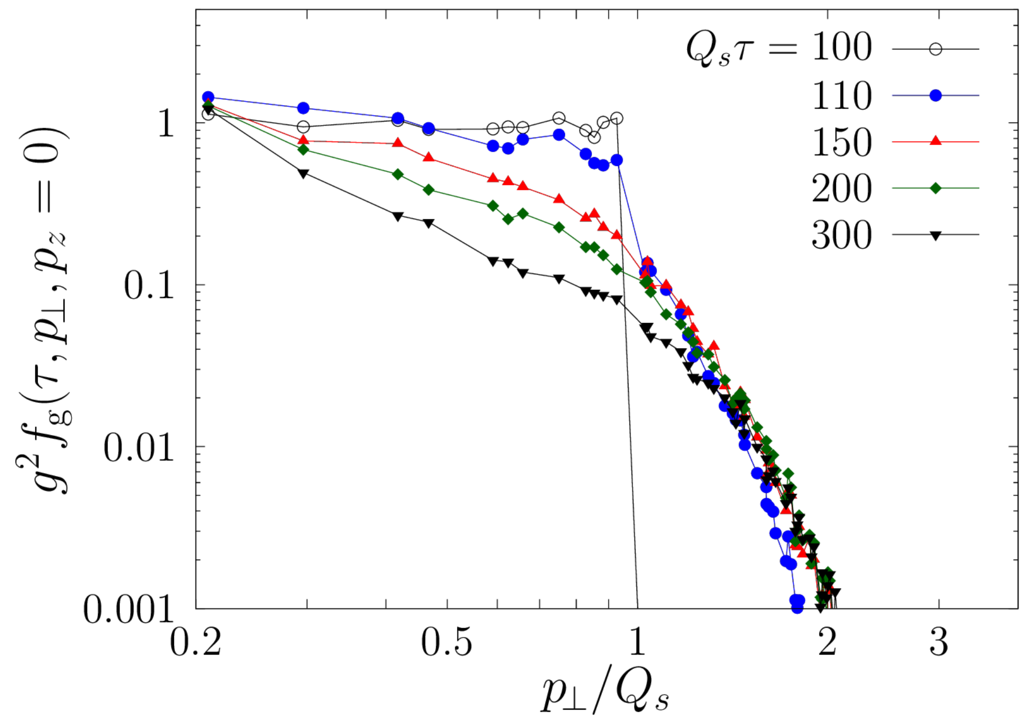




Time evolution of the gluon distribution

$N_c = 2\,,\; g = 10^{-2}$



- IR and UV cascades
- Decrease due to the longitudinal expansion and momentum broadening
- At later times, it reaches non-thermal fixed point

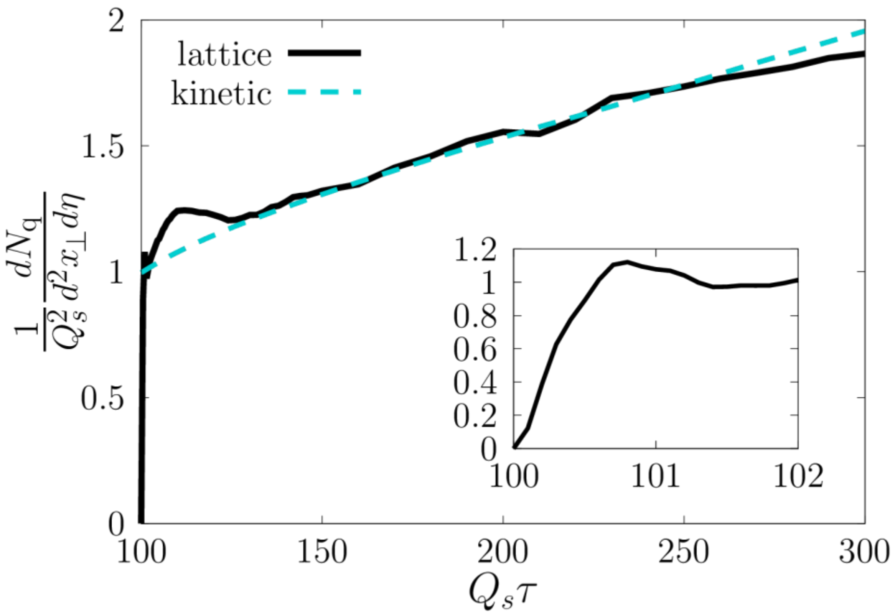
$$f_g(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} f_S\left(p_\perp, (Q_s\tau)^{1/3} p_z\right)$$

Berges et al. PRD89, 114007 (2014)

Time evolution of the quark number density

Integrated quark number density per unit transverse area and per unit rapidity

$$\frac{dN_{\rm q}}{d^2x_\perp d\eta} = \nu_{\rm q}\,\tau \int \frac{d^2p_\perp dp_z}{(2\pi)^3} f_{\rm q}(\tau, p_\perp, p_z) \qquad \nu_{\rm q} = 2\cdot 2N_f N_c$$



$m/Q_s = 0.1$

$N_f = 1$

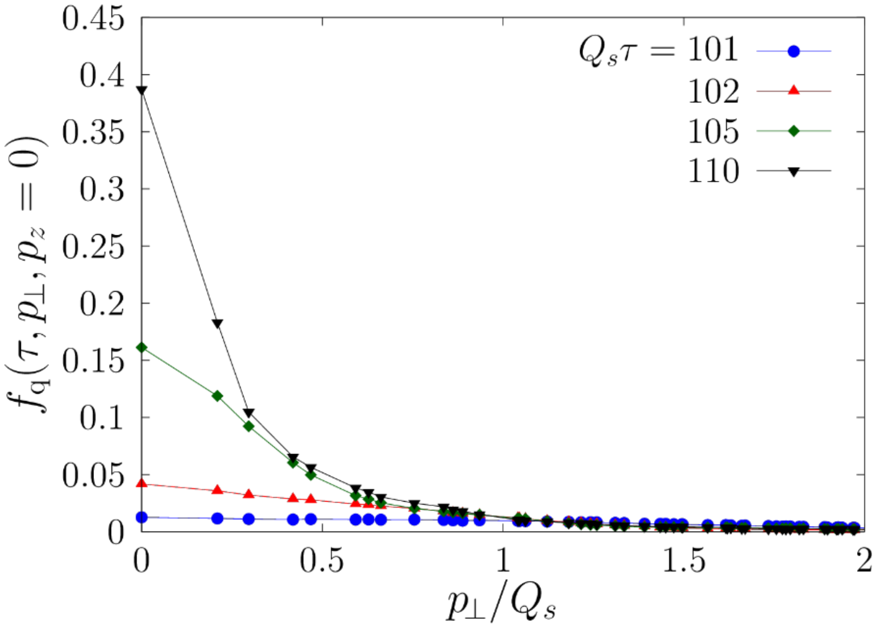
- rapid increase at early time by nonperturbative production or initial quench

For $Q_s \sim 1$ GeV, $\frac{1}{Q_s^2}\frac{dN_{\rm q}}{d^2x_\perp d\eta} = 1 \Rightarrow \frac{dN_{\rm q}}{d^2x_\perp d\eta} = 25\ /{\rm fm}^2$

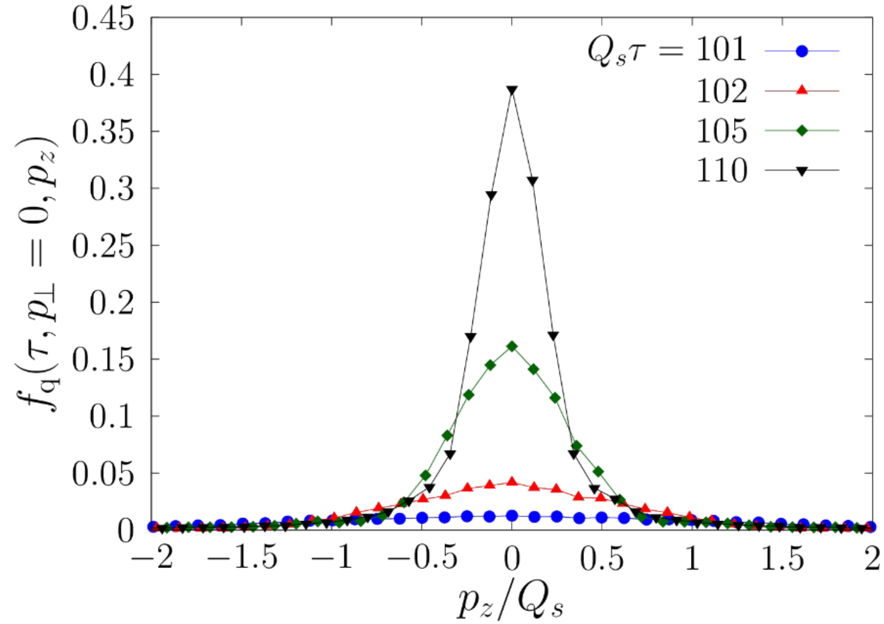
- nearly linear increase at later times ⇒ well explained by the kinetic theory

Time evolution of the quark distribution

Earlier times



transverse momentum



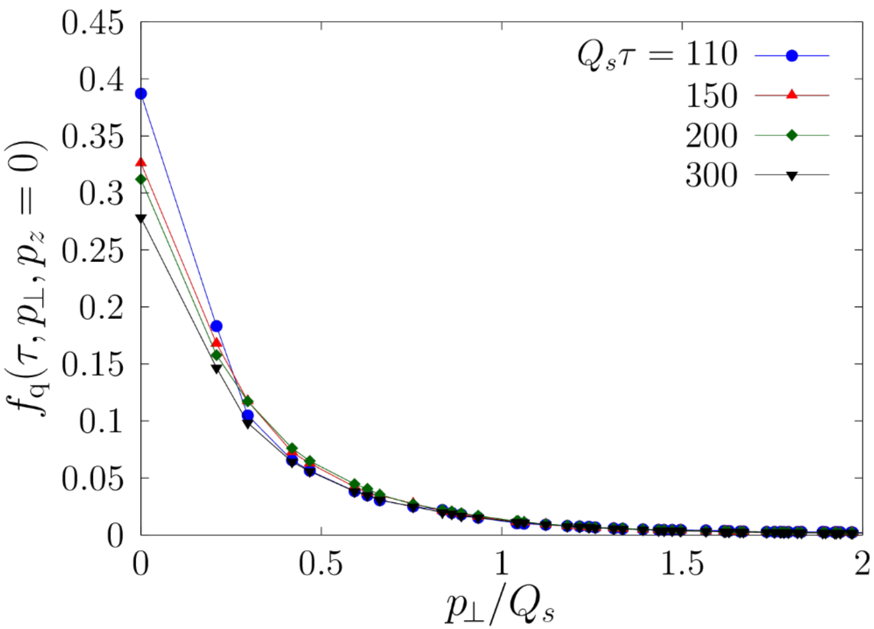
longitudinal momentum

- Occupation number of the order of one is developed at this stage.
- Quark production is an order-one effect even in weak coupling.

$$gA \sim \mathcal{O}(1)$$

Time evolution of the quark distribution

Later times



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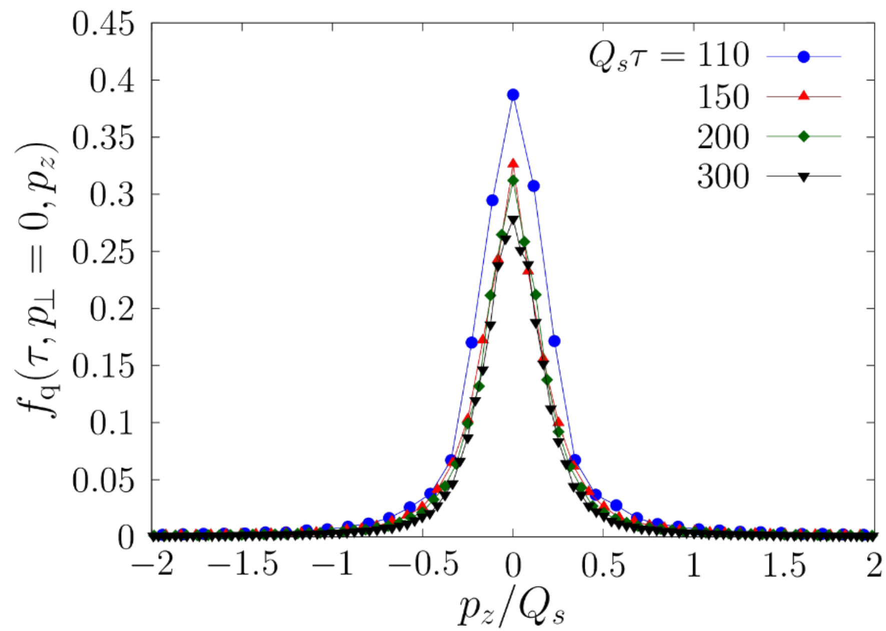


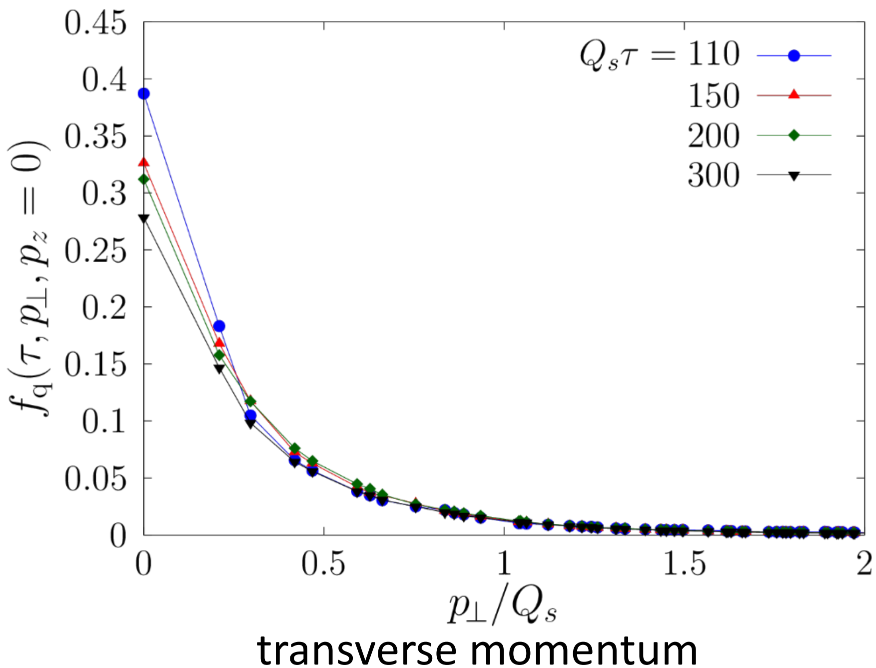
longitudinal momentum

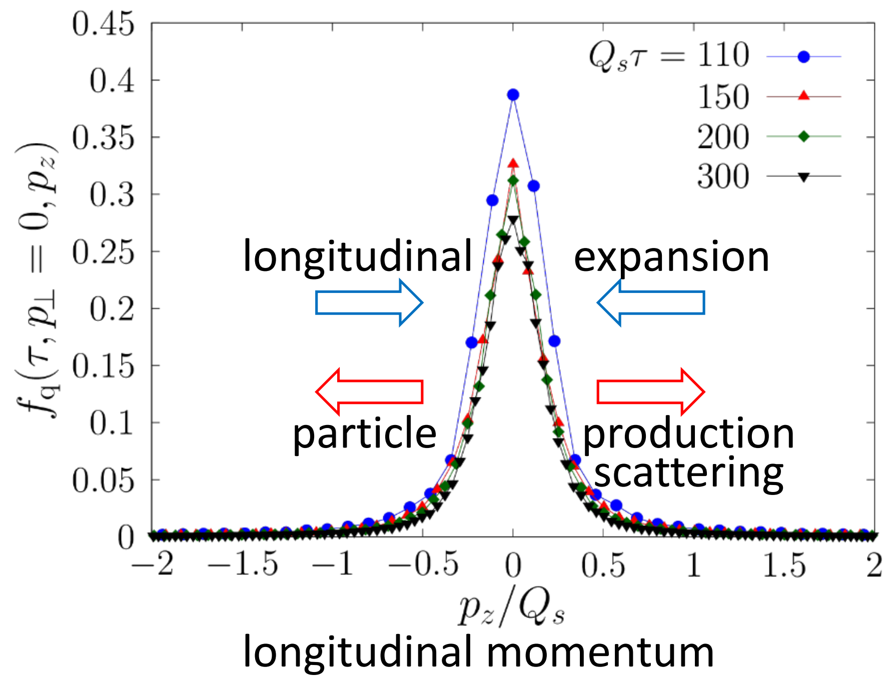
- Occupation numbers show slight decrease due to the expansion of the system and momentum broadening.
- The width of the longitudinal distribution is almost constant.

  In the case of the free streaming (free particles in the expanding system), the longitudinal momentum decreases as $p_z \sim 1/\tau$.

Time evolution of the quark distribution

Later times





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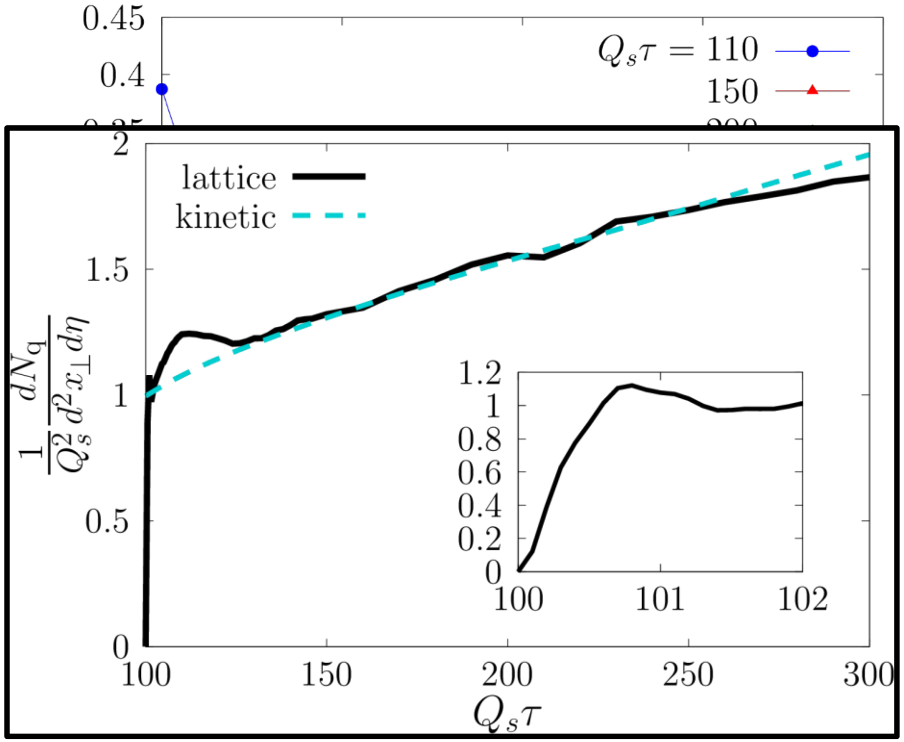

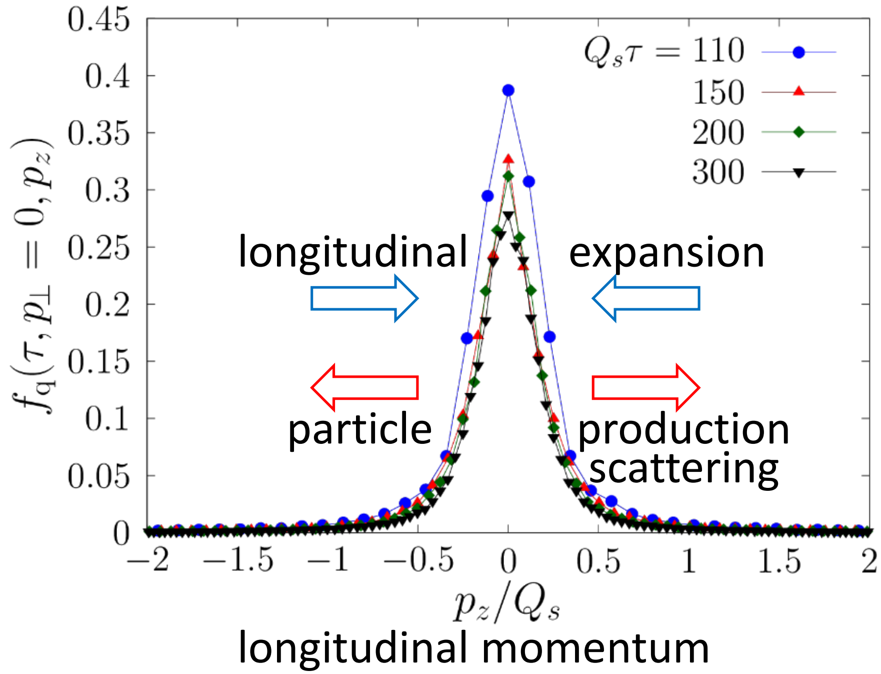

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Comparison to a kinetic estimate

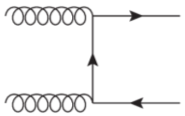
Boltzmann equation for quarks in the expanding geometry

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f_{\mathrm{q}}(\tau, \boldsymbol{p}) = C_{\mathrm{q}}[f_{\mathrm{g}}, f_{\mathrm{q}}]$$

Total production rate

$$\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \nu_{\mathrm{q}}\,\tau \int \frac{d^2p_\perp dp_z}{(2\pi)^3} C_{\mathrm{q}}$$

Consider only 2-2 scattering processes for simplicity



Small-angle approximation

$$\boxed{\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \frac{g^4}{4\pi}\frac{(N_c^2-1)^2}{N_c} N_f \mathcal{L} I_c\,\tau \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}}(1-2f_{\mathrm{q}}) - f_{\mathrm{q}}\right]}$$

$$I_c = \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}} + f_{\mathrm{q}}\right]$$

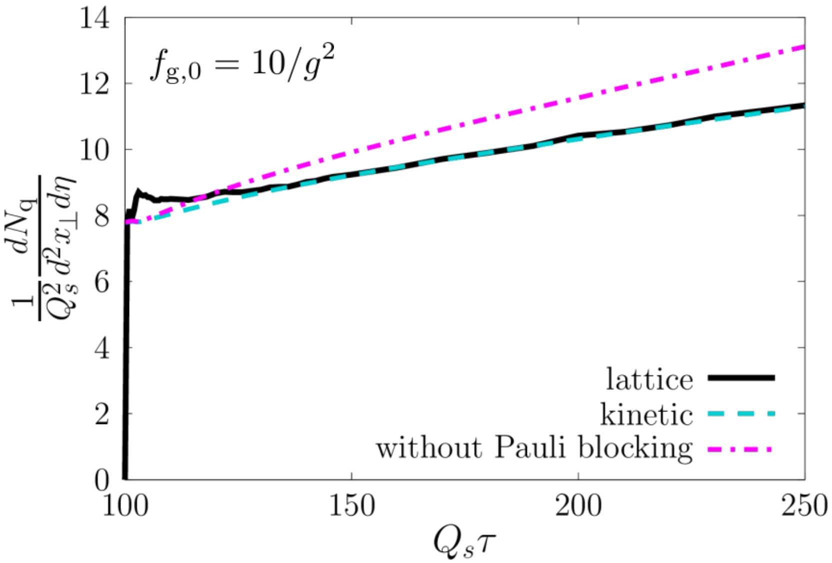
$$\mathcal{L} = \int_{m_D}^{Q_s} \frac{dq}{q}$$

$$m_D^2 = 4g^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[N_c f_{\mathrm{g}} + N_f f_{\mathrm{q}}\right]$$

Comparison to a kinetic estimate

Substitute $f_{\mathrm{g/q}}$ obtained from the lattice calculations into the kinetic formula

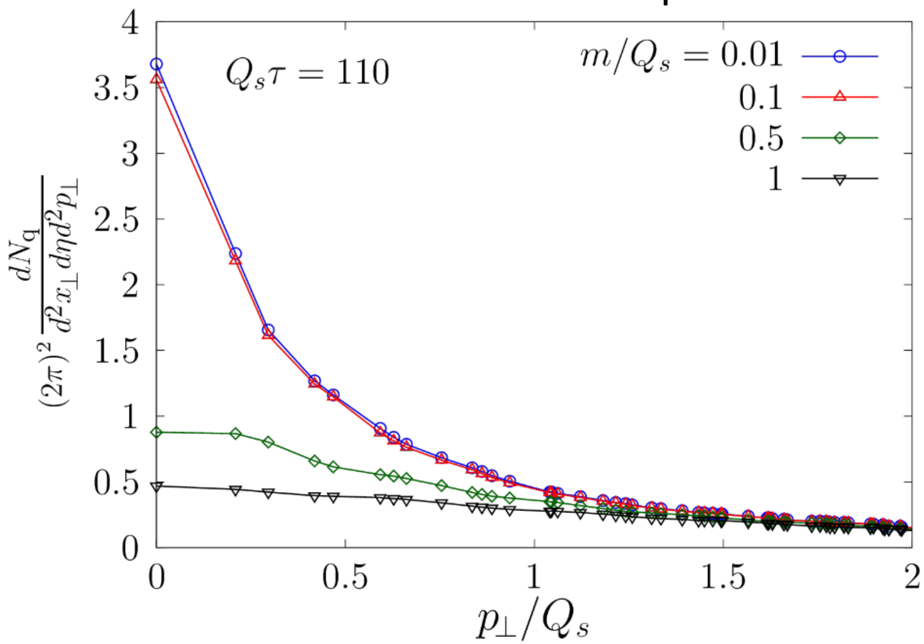
$$\frac{dN_{\mathrm{q}}}{d\tau d^2x_\perp d\eta} = \frac{g^4}{4\pi}\frac{(N_c^2-1)^2}{N_c} N_f \mathcal{L} I_c \tau \int \frac{d^3p}{(2\pi)^3}\frac{1}{p}\left[f_{\mathrm{g}}\underbrace{(1-2f_{\mathrm{q}})}_{\text{Pauli blocking}} - f_{\mathrm{q}}\right]$$



- Good agreement even for the initial gluon occupancy of $10/g^2$.
- The kinetic theory is normally not justified for such high density.
- The Pauli blocking is correctly described by the lattice calculations.

Quark mass dependence

Transverse momentum spectrum



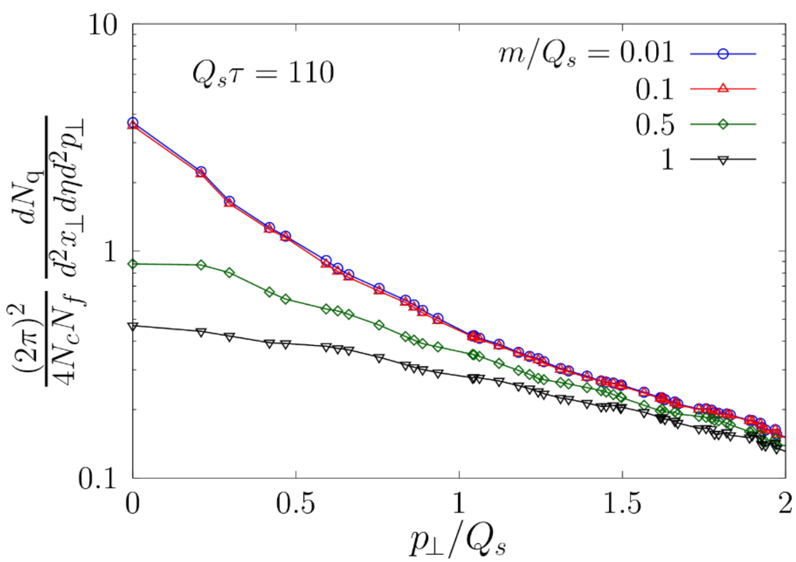
$Q_s \sim 1$ GeV $\rightarrow$

$$
\begin{aligned}
m/Q_s &= 0.01 \gtrsim \text{u,d} \\
&= 0.1 \quad \sim \text{s} \\
&= 1 \quad\;\; \sim \text{c}
\end{aligned}
$$

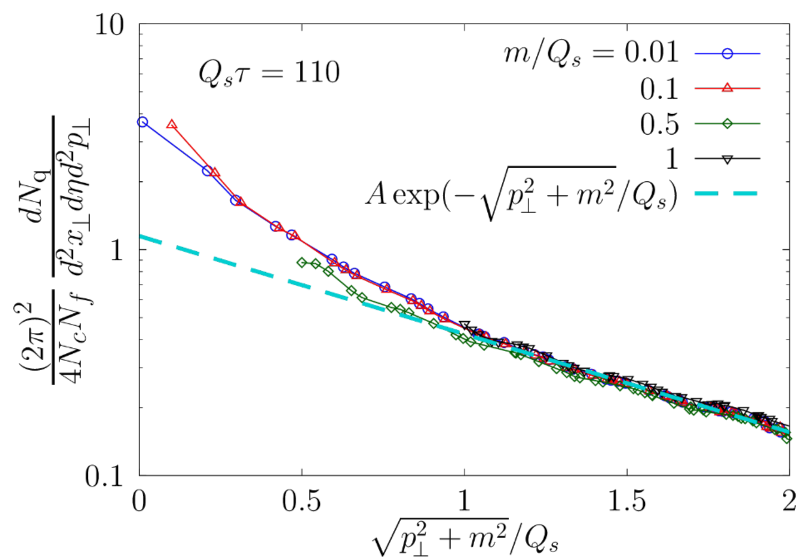
- Natural mass ordering.
- Lighter quarks $m/Q_s \leq 0.1$ are almost degenerated.

mT scaling

Transverse momentum spectrum



as a function of the **transverse momentum**



as a function of the **transverse mass**

$$m_T = \sqrt{p_\perp^2 + m^2}$$

- For $m_T \gtrsim Q_s$, all the spectra for different masses lie on top of each other.
- Its shape is not inconsistent with an exponential $\exp(-m_T/Q_s)$, which resembles the Boltzmann distribution.

Summary

- The nonequilibrium evolution of dynamical quark fields and overoccupied classical-statistical gauge fields has been computed by the real-time lattice simulation technique.

- Intense and rapid quark production is obtained.

  For $Q_s \sim 1\ \mathrm{GeV}$, $\dfrac{dN_{\mathrm{q}}}{d^2x_\perp d\eta} = 25\ /\mathrm{fm}^2\ \times 3$ flavors at $\Delta\tau = 0.2\ \mathrm{fm}/c$

- The transverse momentum spectra of quarks produced in the early stage satisfy the mT scaling at high mT tails.

- The lattice results for quark production rate at later times appear to be consistent with a simple kinetic estimate, although the kinetic theory is not a priori justified in such a dense system.