



# Characterizing hydrodynamic fluctuations in heavy-ion collisions from effective field theory approach

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# Why effective action of hydrodynamics?

- Hydrodynamics describes long wavelength modes
- Conservation equations
  - Energy  $\nabla_\mu T^{\mu\nu} = 0$
  - Currents  $\nabla_\mu J^\mu = 0$
- Noise (Fluctuation) contributes to the conservation equations
- Incorporated phenomenologically
  - e.g. gaussian noise



## Studies of fluctuation in QGP - examples

- Relativistic Theory of Hydrodynamic Fluctuations with Applications to Heavy Ion Collisions - J. I. Kapusta, B. Müller, M. Stephanov
- A kinetic regime of hydrodynamic fluctuations and long time tails for a Bjorken expansion – Y. Akamatsu, A. Mazeliauskas, D. Teaney
- Thermal noise in a boost-invariant matter expansion in relativistic heavy-ion collisions – C. ChattoPadhyay, R. S. Bhalerao, S. Pal
- .....

# Effective action for Hydrodynamics - features

- Conservation equation as E.O.M.

$$\frac{\delta \mathcal{L}}{\delta \phi} = 0$$

- Includes higher order interaction vertices systematically
  - c.f. Chiral lagrangian
  - pertubative corrections through loop diagrams



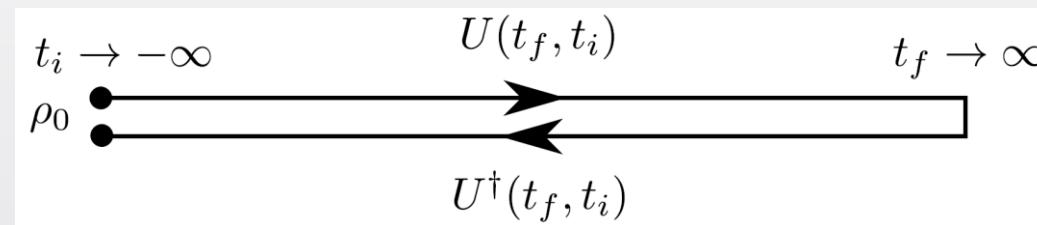
# Effective hydro action - aim & proposal

- Derive E.O.M. including resummed correction from fluctuation
- Compute 1PI effective action as in quantum field theory
  - e.g. Weinberg-Coleman potential



## Formulating an effective action - Idea

- Integrate out all degrees of freedom  $\psi$  but hydrodynamics modes  $\chi$  of microscopic action  $S[\Psi]$



- Schwinger-Keldysh formalism (Closed time path, CTP)
  - Real time correlation e.g.  $\langle T^{\mu\nu} \rangle$
  - Dissipation effect

$$Tr(\rho_0 \dots \dots) = \int D\Psi_1 D\Psi_2 (\dots \dots) e^{iS[\Psi_1] - iS[\Psi_2]} = \int D\chi_1 D\chi_2 (\dots \dots) e^{iS_{hydro}[\chi_1, \chi_2; \rho_0]}$$



## Properties of CTP

$$e^{W[\phi_1, \phi_2]} = \text{Tr}(\rho_0 \mathcal{P} e^{i \int dt (\mathcal{O}_1(t)\phi_1(t) - \mathcal{O}_2(t)\phi_2(t))})$$

- Another convenient basis
  - Average field
  - Noise
- Two point functions are

$$\begin{aligned}\phi_r &= \frac{1}{2}(\phi_1 + \phi_2) \\ \phi_a &= (\phi_1 - \phi_2)\end{aligned}$$

$$\langle \mathcal{O}_r(t_1) \mathcal{O}_a(t_2) \rangle = G_R(t_1, t_2) \quad \langle \mathcal{O}_a(t_1) \mathcal{O}_r(t_2) \rangle = G_A(t_1, t_2) \quad \langle \mathcal{O}_r(t_1) \mathcal{O}_r(t_2) \rangle = G_S(t_1, t_2)$$

- Kubo-Martin-Schwinger condition leads to the Fluctuation-dissipation theorem

$$G_S = \frac{i}{2} \coth \frac{\beta_0 \omega}{2} (G_A - G_R)$$



## Example : Brownian motion in a potential

- Langevin equation

$$(\partial_t^2 + \nu \partial_t) x_r(t) - F(x_r(t)) = \xi(t)$$

- $\langle \xi(t) \rangle = 0$
- $\langle \xi(t) \xi(t') \rangle = D(T, \nu) \delta(t - t')$



## Example : Brownian motion in CTP

$$I = \int dt \mathcal{L} = \int dt x_a (\partial_t^2 + \nu \partial_t) x_r - x_a F(x_r) + \frac{i\kappa}{2} x_a^2$$

- Introduce  $\xi$  as conjugate variable of  $x_a$

$$e^W = \int D\xi Dx_a Dx_r e^{iI} = \int D\xi Dx_r \delta\left((\partial_t^2 + \nu \partial_t)x_r - F(x_r) - \xi\right) e^{-\frac{1}{2}\int dt \xi \kappa^{-1} \xi}$$

# 1PI effective action - Brownian motion

- Integrate out fluctuation about a background with source terms

$$e^{i\Gamma[\bar{x}_r, \bar{x}_a, \bar{J}_r, \bar{J}_a]} = \int Dx_r Dx_a e^{i(\int dt \mathcal{L}[x_r, x_a] + x_a \bar{J}_r + x_r \bar{J}_a)}$$

$$\Gamma[\bar{x}_r, \bar{x}_a, \bar{J}_r, \bar{J}_a] = \int dt \mathcal{L}[\bar{x}_r, \bar{x}_a] + \bar{x}_a \bar{J}_r + \bar{x}_r \bar{J}_a + \frac{i}{2} \ln \det \hat{M}$$

- E.O.M. of  $\Gamma[\bar{x}_r, \bar{x}_a, \bar{J}_r, \bar{J}_a]$

$$\left. \frac{\delta \Gamma[\bar{x}_r, \bar{x}_a, 0, 0]}{\delta \bar{x}_a} \right|_{\bar{x}_a=0} = 0$$

- The background is the solution including fluctuation correction



## 1-loop resummed EOM

- Two coupled equations

$$(\partial_t^2 + \nu \partial_t) \bar{x}_r - F(\bar{x}_r) = \text{tr} \left( \hat{L}(\bar{x}_r, \partial_t)^{-1} F''(\bar{x}_r) \right)$$

$$\hat{L}(\bar{x}_r, \partial_t) \langle \bar{x}_r(t), \bar{x}_r(t') \rangle = \delta(t - t')$$

$$\hat{L} = \left( (\partial_t^2 + \nu \partial_t) - F'(\bar{x}_r) \right) (i\kappa)^{-1} \left( (\partial_t^2 + \nu \partial_t) \bar{x}_r - F'(\bar{x}_r) \right)$$

- c.f. perturbative correction: expand about solution of classical E.O.M.

# Hydrodynamics fluctuation – Bjorken flow

- The Lagrangian is

$$\mathcal{L} = \frac{1}{2} T^{\mu\nu}(\epsilon(X_r), u^\mu(X_r)) G_{a\mu\nu} + \frac{i}{2} \eta^{\mu\alpha\nu\beta}(\epsilon(X_r), u^\mu(X_r)) G_{a\mu\nu} G_{a\alpha\beta} + O(G_a^3)$$

$$G_{a\mu\nu} = g_{a\mu\nu} + \nabla_\mu X_{a\nu} + \nabla_\nu X_{a\mu}$$

- Equation of motion

$$\partial_\tau \epsilon + \frac{\epsilon + p}{\tau} = \frac{4\eta_{vis}}{3\tau}$$



# 1PI effective action - hydro

$$\delta X^T \hat{M} \delta X = -i(\delta X_r^\mu \quad \delta X_a^\mu) \begin{pmatrix} A_{\mu\nu} & B_{\mu\nu} \\ \tilde{B}_{\mu\nu} & C_{\mu\nu} \end{pmatrix} \begin{pmatrix} \delta X_r^\nu \\ \delta X_a^\nu \end{pmatrix}$$

- Fluctuation corrected E.O.M.

$$\partial_\tau \bar{\epsilon} + \frac{\bar{\epsilon} + \bar{p}}{\tau} - \frac{4\bar{\eta}_{vis}}{3\tau} = -\frac{i}{2} \left[ \text{tr} \left( (BC^{-1}\tilde{B})_0^{-1} \frac{\delta A}{\delta \bar{x}_a} \right) + \text{tr} \left( B_0^{-1} \frac{\delta B}{\delta \bar{x}_a} \right) + \text{tr} \left( \tilde{B}_0^{-1} \frac{\delta \tilde{B}}{\delta \bar{x}_a} \right) \right] \Big|_{\bar{x}_a=0}$$



# Summary

- CTP formalism provides a good framework for effective field theory of hydrodynamics
- This formalism reproduces the conventional perturbation treatment of hydrodynamics
- The fluctuation correction to E.O.M. is captured by the 1PI effective action