Anisotropic fluid dynamics for heavy-ion collisions

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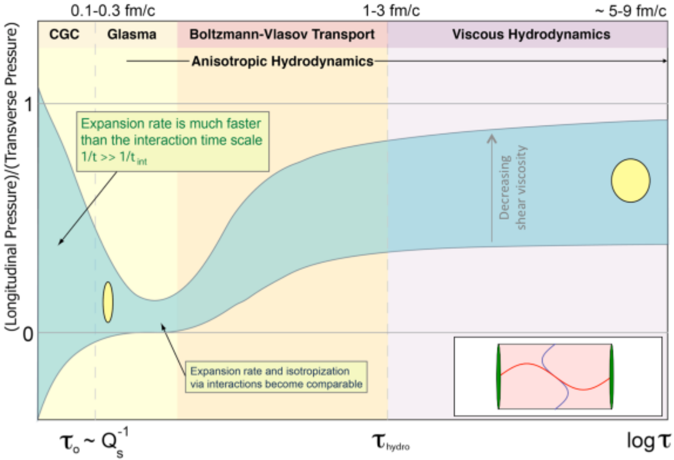
M. McNelis, D. Bazow and U. Heinz, arXiv:1803.01810 [nucl-th]

1. Anisotropic hydrodynamics
2. Relaxation equations
3. Comparison to standard viscous hydrodynamics
4. Conclusions and outlook

Motivation for anisotropic hydrodynamics



M. Strickland, Acta Phys. Polon. B45 (2014) 2355-2394

- Large longitudinal but small transverse expansion rate at early times $\rightarrow$ large pressure anisotropy $\pi^{zz}_{\mathrm{LRF}} \sim \mathcal{P}_L - \mathcal{P}_\perp$ throughout fireball evolution.

$\Longrightarrow$ Anisotropic hydrodynamics:
Florkowski & Ryblewski, Martinez & Strickland, Bazow et al., Tinti et al., Molnar & Niemi & Rischke …

Motivation for anisotropic hydrodynamics

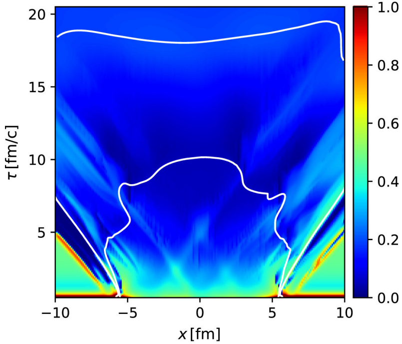
- $T^{\mu\nu}$ decomposition:

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}_L z^\mu z^\nu - \mathcal{P}_\perp \Xi^{\mu\nu} + 2 W_{\perp z}^{(\mu} z^{\nu)} + \pi_\perp^{\mu\nu}$$

- $\mathcal{P}_L$: longitudinal pressure.
- $\mathcal{P}_\perp$: transverse pressure.
- $W_{\perp z}^\mu, \pi_\perp^{\mu\nu}$: residual shear stress components.

$$\mathcal{P}_L = \mathcal{P}_{\text{eq}} + \Pi + \pi_{\text{LRF}}^{zz}$$
$$\mathcal{P}_\perp = \mathcal{P}_{\text{eq}} + \Pi - \tfrac{1}{2} \pi_{\text{LRF}}^{zz}$$

$$R_\pi^{-1} = \sqrt{\pi^{\mu\nu}\pi_{\mu\nu}} / \mathcal{P}_{\text{eq}}$$



Bazow et al., Comput. Phys. Commun. 225 (2018) 92

- Highly anisotropic expansion rate at early times $\to$ large pressure anisotropy $\pi_{\text{LRF}}^{zz} \sim \mathcal{P}_L - \mathcal{P}_\perp$.

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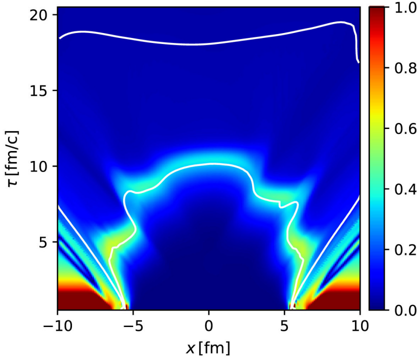

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- Large bulk viscosity near phase transition → large Π.

Hydrodynamic equations

- $\partial_\mu T^{\mu\nu} = 0$ evolve $\mathcal{E}$ and u^μ. $\mathcal{P}_{\rm eq}(\mathcal{E})$ given by lattice QCD.
- Six relaxation equations needed for $\mathcal{P}_L - \mathcal{P}_{\rm eq}$, $\mathcal{P}_\perp - \mathcal{P}_{\rm eq}$, $W^\mu_{\perp z}$ and $\pi^{\mu\nu}_\perp$.
- Starting point: model relaxation equations with Boltzmann equation in RTA:

$$p^\mu \partial_\mu f + m\, \partial_\mu m\, \partial^\mu_p f = C[f] \approx -\frac{(u \cdot p)(f - f_{\rm eq})}{\tau_r}$$

- Assume weakly-interacting gas of quasiparticles with medium-dependent mass m.
- A mean field B also necessary for thermodynamic consistency and energy conservation.
  (Biro et al. 1990; Gorenstein & Yang 1995; Alqahtani et al. 2015; Tinti et al. 2017)
- Goal: anisotropic hydrodynamic equations that are macroscopic!

Anisotropic distribution function

- Expand f around ellipsoidal Boltzmann distribution f_a.

$$f = f_a + \delta\tilde{f} \qquad f_a = \exp\left(-\frac{1}{\Lambda}\sqrt{m^2 + \frac{p^2_{\perp,\mathrm{LRF}}}{\alpha^2_\perp} + \frac{p^2_{z,\mathrm{LRF}}}{\alpha^2_L}}\right)$$

- $\Lambda(x)$: effective temperature.
- $\alpha_L(x)$, $\alpha_\perp(x)$: momentum anisotropy parameters.
- f_a captures $\mathcal{P}_L$ and $\mathcal{P}_\perp$ non-perturbatively (or equivalently, $\pi^{zz}_{\mathrm{LRF}} \sim \mathcal{P}_L - \mathcal{P}_\perp$, Π).
- $\delta\tilde{f}$ is a perturbative correction accounting for $W^\mu_{\perp z}$ and $\pi^{\mu\nu}_\perp$.

Generalized Landau matching conditions

- Unique separation of $f = f_a + \delta\tilde{f}$ requires fixing $(\Lambda, \alpha_L, \alpha_\perp)$.

- $\mathcal{E} \equiv \mathcal{E}_{\rm eq}(T)$ fixes the temperature T.

$$\delta\tilde{\mathcal{E}} = \int_p (u\cdot p)^2 \delta\tilde{f} \equiv 0 \qquad \text{(fixes } \Lambda\text{'s relation to } T\text{)}$$

$$\delta\tilde{\mathcal{P}}_L = \int_p (-z\cdot p)^2 \delta\tilde{f} \equiv 0 \qquad (\mathcal{P}_L \text{ matching fixes } \alpha_L)$$

$$\delta\tilde{\mathcal{P}}_\perp = \tfrac{1}{2}\int_p (-p\cdot\Xi\cdot p) \delta\tilde{f} \equiv 0 \qquad (\mathcal{P}_\perp \text{ matching fixes } \alpha_\perp)$$

- 14-moment approximation: $\delta\tilde{f} = \left(-\frac{(-z\cdot p)p_{\{\mu\}}W^{\mu}_{\perp z}}{\mathcal{J}_{4210}} + \frac{p_{\{\mu}p_{\nu\}}\pi^{\mu\nu}_{\perp}}{2\mathcal{J}_{4020}}\right) f_a$

- $\mathcal{J}_{nrqs}$ = momentum moments of distribution f_a. (arXiv:1803.01810)

Quasiparticle mass and mean field

- $m(T)$ and $B_{\rm eq}(T)$ fitted to QCD EoS.
- Mean field also has a viscous correction δB.
  - L. Tinti et al., Phys. Rev. D95 (2017) 054007

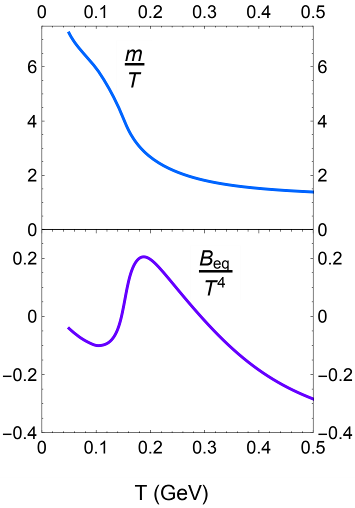
$$B = B_{\rm eq}(T) + \delta B$$

- $1^{\rm st}$ moment of the Boltzmann equation:

$$\partial_\mu \int_p p^\mu p^\nu f - m\,\partial^\nu m \int_p f = \int_p p^\nu C[f]$$

- Constraint of energy conservation:

$$\dot{B} = -\frac{\delta B}{\tau_\Pi} + \frac{\dot{m}}{m}(\mathcal{E} - 2\mathcal{P}_\perp - \mathcal{P}_L - 4B)$$



Macroscopic relaxation equations

- Macroscopic = transport coefficient × hydrodynamic gradient

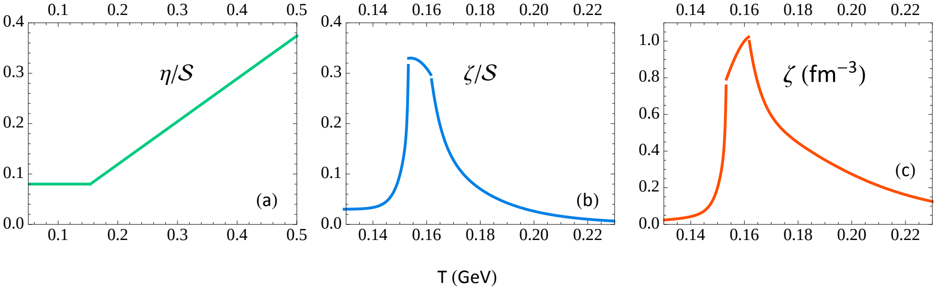
$$
\begin{aligned}
\dot{\mathcal{P}}_L &= -\frac{\bar{\mathcal{P}}-\mathcal{P}_{\rm eq}}{\tau_\Pi} - \frac{\mathcal{P}_L-\mathcal{P}_\perp}{3\tau_\pi/2} + \bar{\zeta}_z^L z_\mu D_z u^\mu + \bar{\zeta}_\perp^L \theta_\perp - 2 W^\mu_{\perp z} \dot{z}_\mu \\
&\quad + \bar{\lambda}_{Wu}^L W^\mu_{\perp z} D_z u_\mu + \bar{\lambda}_{W\perp}^L W^\mu_{\perp z} z_\nu \nabla_{\perp\mu} u^\nu - \bar{\lambda}_\pi^L \pi_\perp^{\mu\nu} \sigma_{\perp,\mu\nu} \\
\dot{\mathcal{P}}_\perp &= -\frac{\bar{\mathcal{P}}-\mathcal{P}_{\rm eq}}{\tau_\Pi} + \frac{\mathcal{P}_L-\mathcal{P}_\perp}{3\tau_\pi} + \bar{\zeta}_z^\perp z_\mu D_z u^\mu + \bar{\zeta}_\perp^\perp \theta_\perp + 1\cdot W^\mu_{\perp z} \dot{z}_\mu \\
&\quad + \bar{\lambda}_{Wu}^\perp W^\mu_{\perp z} D_z u_\mu - \bar{\lambda}_{W\perp}^\perp W^\mu_{\perp z} z_\nu \nabla_{\perp\mu} u^\nu + \bar{\lambda}_\pi^\perp \pi_\perp^{\mu\nu} \sigma_{\perp,\mu\nu} \\
\dot{W}^{\{\mu\}}_{\perp z} &= -\frac{W^\mu_{\perp z}}{\tau_\pi} + 2\bar{\eta}_u^W \Xi^{\mu\nu} D_z u_\nu - 2\bar{\eta}_\perp^W z_\nu \nabla^\mu_\perp u^\nu - \left(\bar{\tau}_z^W \Xi^{\mu\nu} + 1\cdot \pi_\perp^{\mu\nu}\right)\dot{z}_\nu \\
&\quad + \bar{\delta}_W^W W^\mu_{\perp z} \theta_\perp - \bar{\lambda}_{Wu}^W W^\mu_{\perp z} z_\nu D_z u^\nu + \bar{\lambda}_{W\perp}^W \sigma_\perp^{\mu\nu} W_{\perp z,\nu} + 1\cdot \omega_\perp^{\mu\nu} W_{\perp z,\nu} \\
&\quad + \bar{\lambda}_{\pi u}^W \pi_\perp^{\mu\nu} D_z u_\nu - \bar{\lambda}_{\pi\perp}^W \pi_\perp^{\mu\nu} z_\alpha \nabla_{\perp\nu} u^\alpha \\
\dot{\pi}_\perp^{\{\mu\nu\}} &= -\frac{\pi_\perp^{\mu\nu}}{\tau_\pi} + 2\bar{\eta}_\perp \sigma_\perp^{\mu\nu} - 2 W^{\{\mu}_{\perp z} \dot{z}^{\nu\}} - \bar{\delta}_\pi^\pi \pi_\perp^{\mu\nu} \theta_\perp - \bar{\tau}_\pi^\pi \pi_\perp^{\alpha\{\mu} \sigma^{\nu\}}_{\perp,\alpha} + 2\pi_\perp^{\alpha\{\mu} \omega^{\nu\}}_{\perp,\alpha} \\
&\quad + \bar{\lambda}_\pi^\pi \pi_\perp^{\mu\nu} z_\alpha D_z u^\alpha - \bar{\lambda}_{Wu}^\pi W^{\{\mu}_{\perp z} D_z u^{\nu\}} + \bar{\lambda}_{W\perp}^\pi W^{\{\mu}_{\perp z} z_\alpha \nabla^{\nu\}}_\perp u^\alpha
\end{aligned}
$$

Transport coefficients

- Relaxation equations contain two relaxation times:

$$\tau_\pi = \frac{\eta}{\mathcal{S}} \cdot \frac{\mathcal{S}}{\beta_\pi} \qquad \tau_\Pi = \frac{\zeta}{\mathcal{S}} \cdot \frac{\mathcal{S}}{\beta_\Pi} \qquad (\beta_\pi,\, \beta_\Pi = \text{isotropic thermodynamic integrals})$$

- $\eta/\mathcal{S}$ and $\zeta/\mathcal{S}$ modeled phenomenologically: (J. Bernhard et al., Phys. Rev. C94 (2016) 024907)



- Remaining transport coefficients approximated using quasiparticle kinetic theory.

Bjorken flow with Glasma-like initial conditions

- Anisotropic hydro + quasiparticle transport coefficients

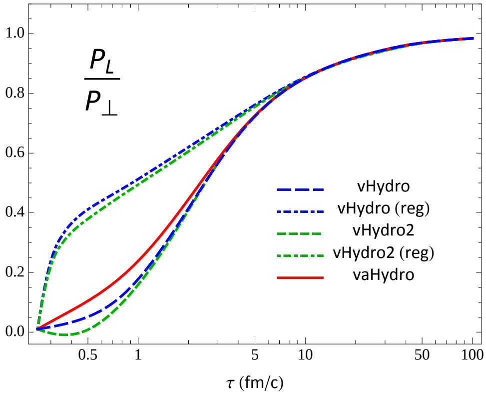
- Standard viscous hydro + quasiparticle transport coefficients

- Standard viscous hydro + $m/T \ll 1$ transport coefficients (current standard)

- $\mathcal{P}_L/\mathcal{P}_\perp$ very similar w/o regulation.

- Typically in (3+1)-d viscous hydro codes, $\pi^{\eta\eta}_{\mathrm{LRF}} \sim \mathcal{P}_L - \mathcal{P}_\perp$ is strongly regulated (alters the physics).

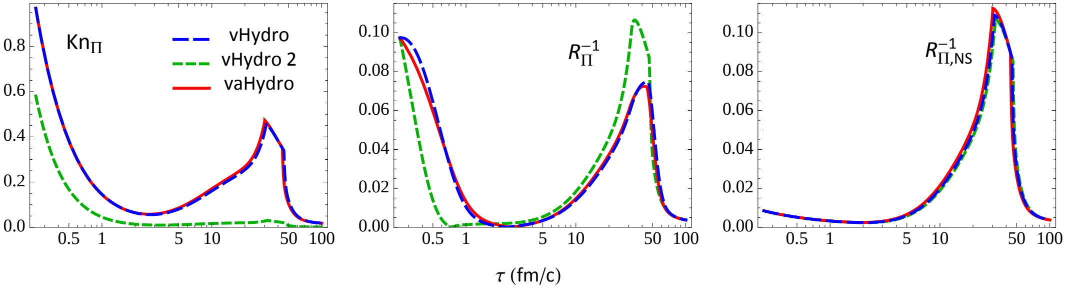
- In anisotropic hydro, $\approx$ no regulation needed for $\mathcal{P}_L$, $\mathcal{P}_\perp$.



$$\pi^{\mu\nu}_{\mathrm{reg}} \to \frac{\tanh\rho}{\rho}\pi^{\mu\nu} \qquad \rho = \sqrt{\frac{\pi^{\mu\nu}\pi_{\mu\nu}}{\mathcal{E}^2 + 3\mathcal{P}^2_{\mathrm{eq}}}}$$

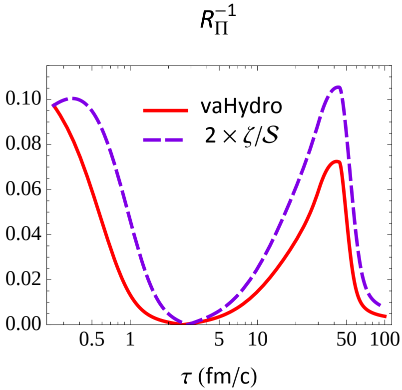
Evolution of bulk viscous pressure

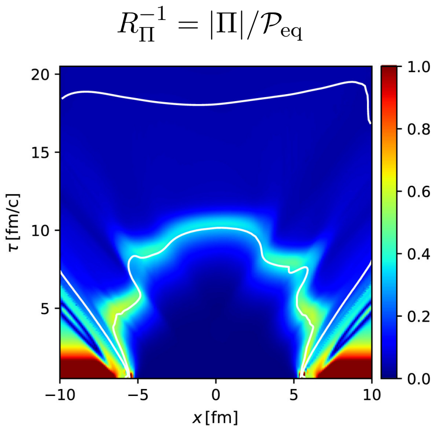
$\mathrm{Kn}_\Pi = \tau_\Pi/\tau$ $\quad R_\Pi^{-1} = |\Pi|/\mathcal{P}_{\mathrm{eq}}$ $\quad R_{\Pi,\mathrm{NS}}^{-1} = |\Pi_{\mathrm{NS}}|/\mathcal{P}_{\mathrm{eq}}$



- τ_Π much larger in quasiparticle model than in $m/T \ll 1$ limit.
- Relaxation dynamics plays important role for Π. ("critical slowing down")

Evolution of bulk viscous pressure





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- Advantageous to freeze out with f_a rather than $f = f_{\rm eq} + \delta f_\Pi$.
  - M. Alqahtani et al., Phys. Rev. C95 (2017) 034906; C96 (2017) 044910

Conclusion and outlook

So far achieved:

- Derived a complete set of (3+1)-d macroscopic anisotropic hydrodynamic equations that are useful for simulating heavy-ion collisions.

- Advantages over viscous hydrodynamics:
  - captures large $\mathcal{P}_L - \mathcal{P}_\perp$ at early times w/o need for regulation
  - accommodates a $\zeta/\mathcal{S}$ that peaks closer to the particlization surface ($T_{\rm sw} \approx 155\,\mathrm{MeV}$)

Ongoing efforts:

- Numerical implementation into a (3+1)-d vahydro code is ongoing.
  - improves over v1.0 developed by D. Bazow

- Interface anisotropic hydrodynamics with a modified anisotropic Cooper-Frye formula.

Open question:

- How to calculate the anisotropic transport coefficients from $1^{\rm st}$ principles?