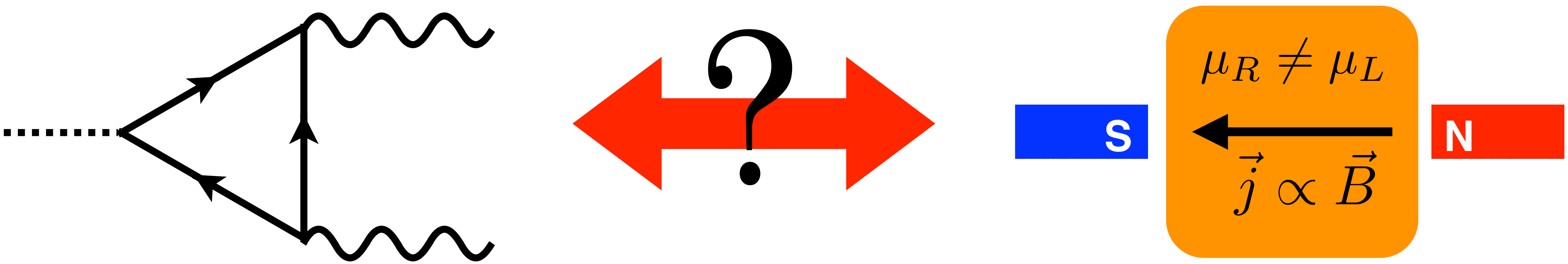
Anomlaous hydrodynamics from projection operator method



Masaru Hongo (RIKEN, iTHMES)

Quark Matter 2018 The 27th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, 2018 May 16th, Lido

Collaborators: Noriyuki Sogabe, Naoki Yamamoto

Outline

MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?

APPROACH:

Mori's method as a generalization of current algebra

Anomalous commutation:

RESULT:

Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

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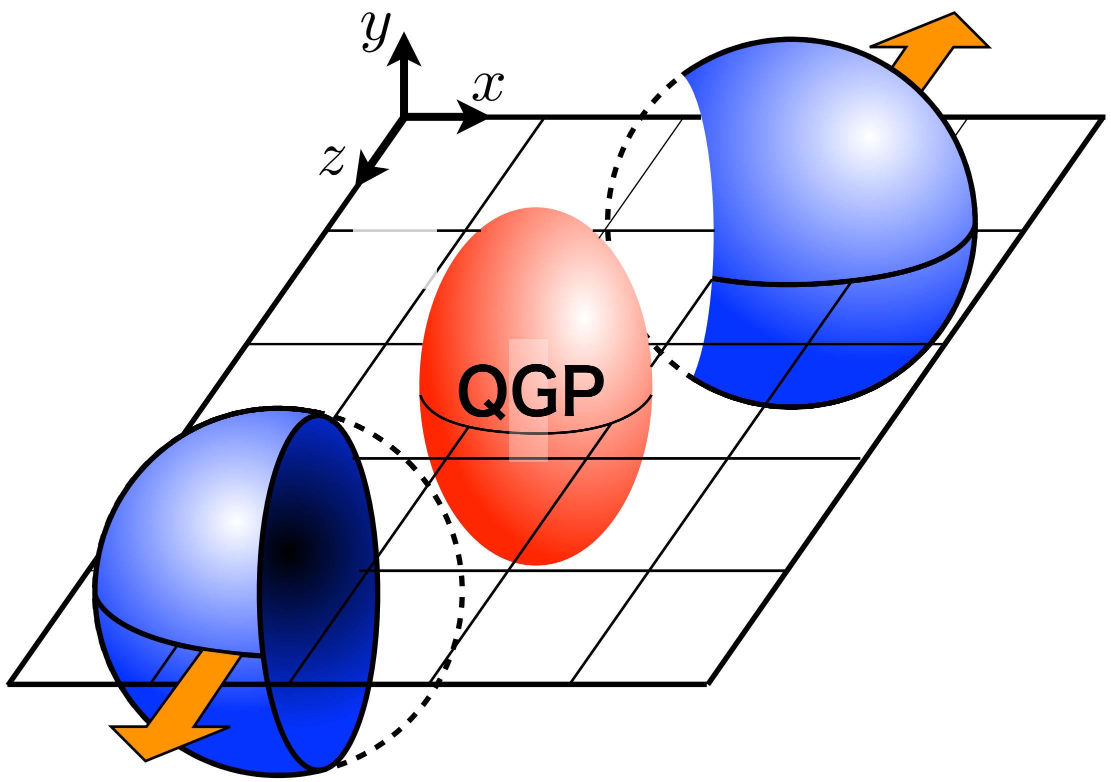
QGP as Chiral fluid



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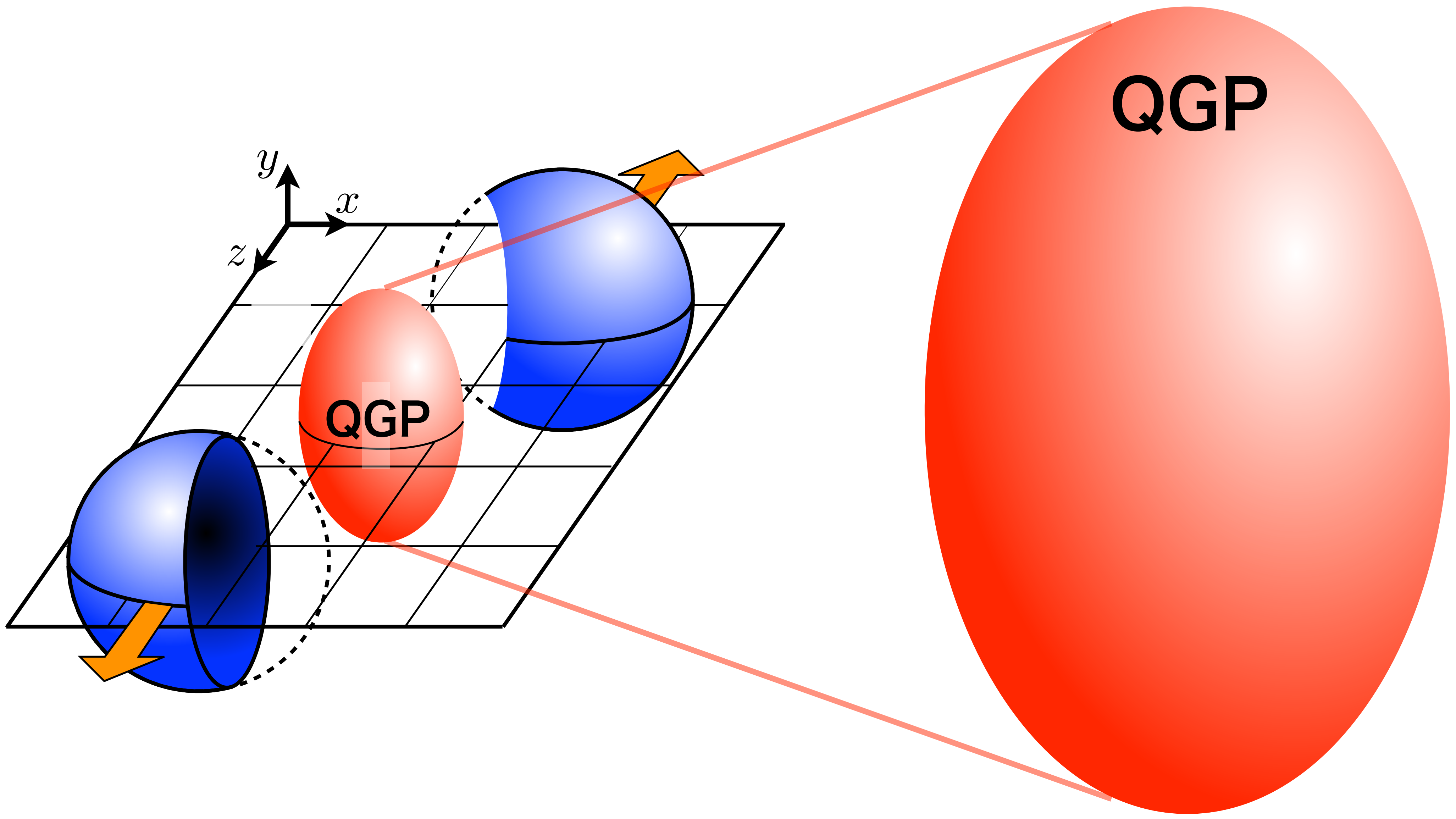

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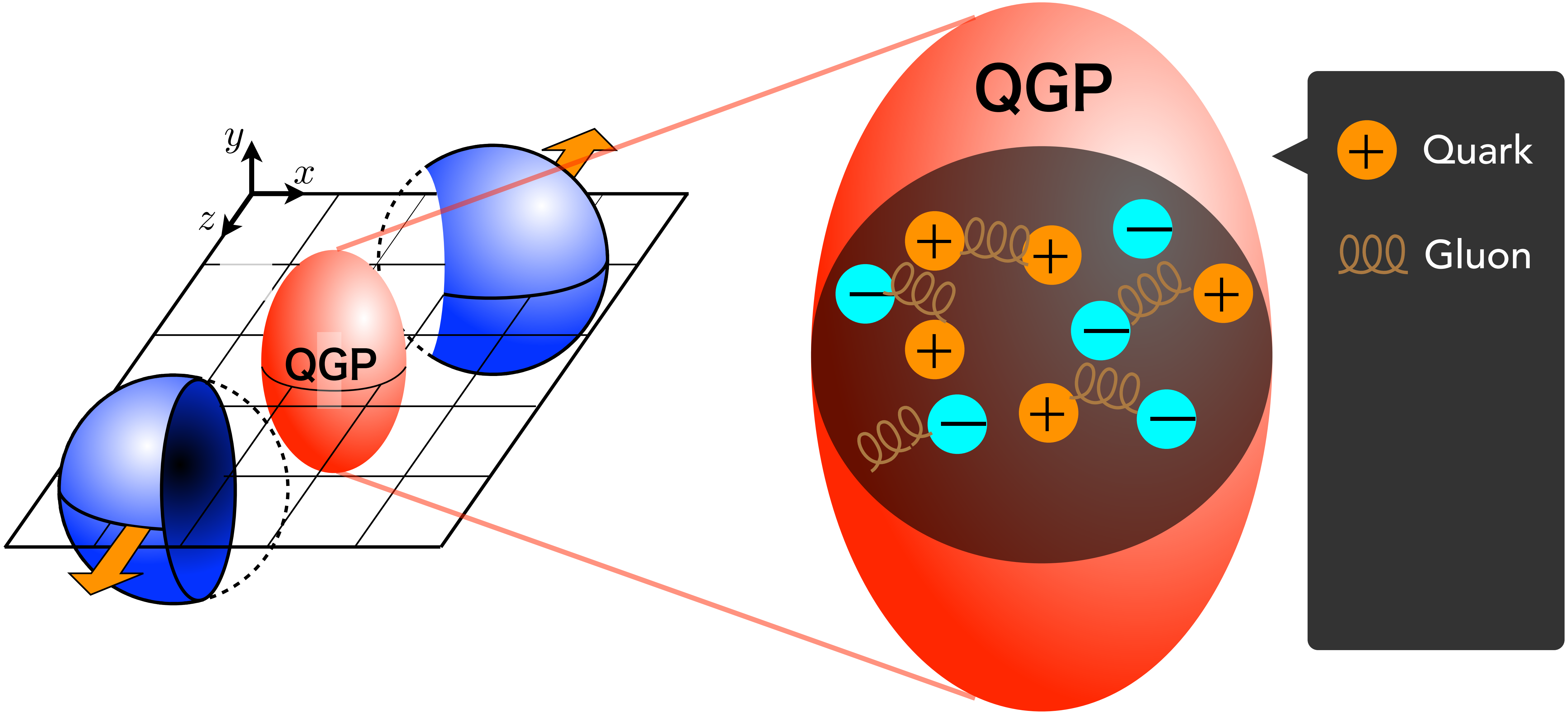

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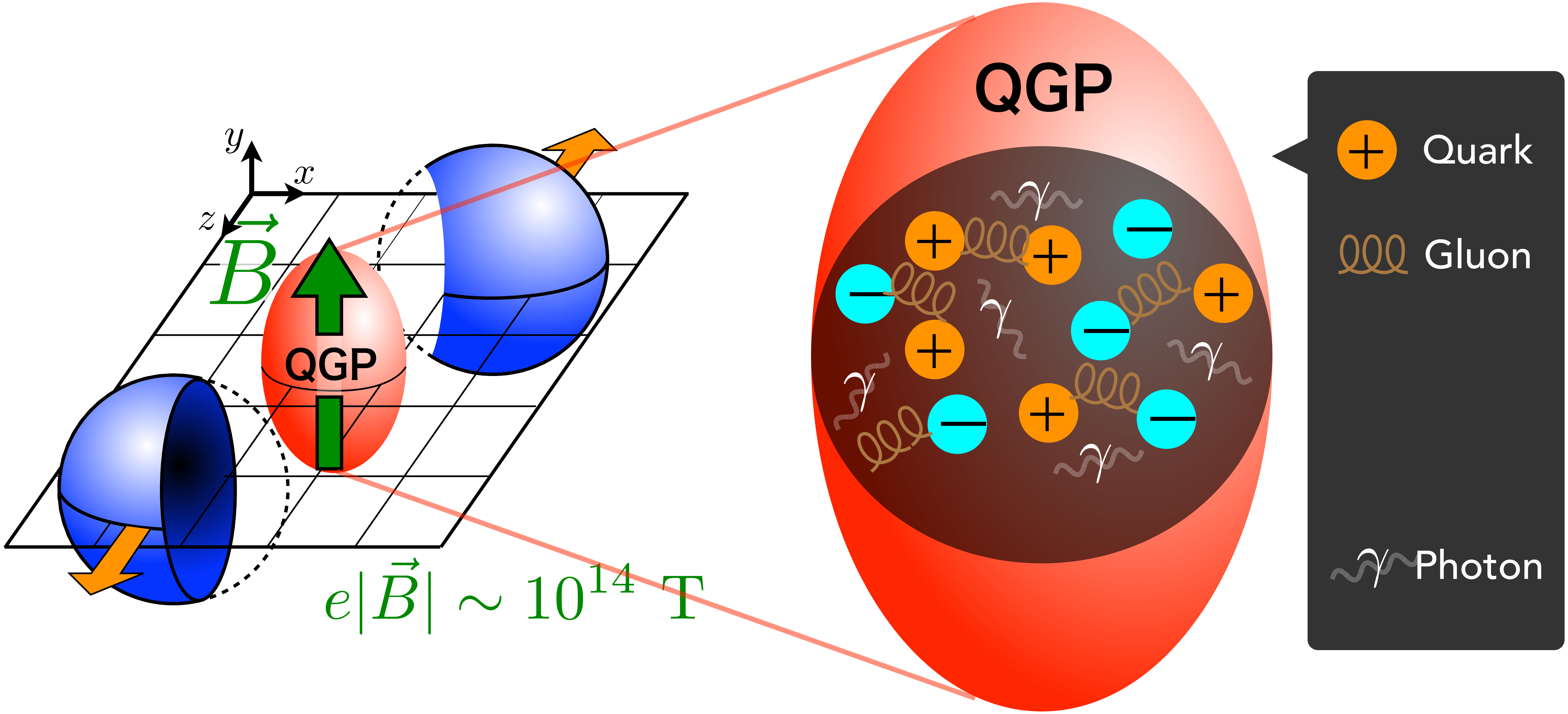
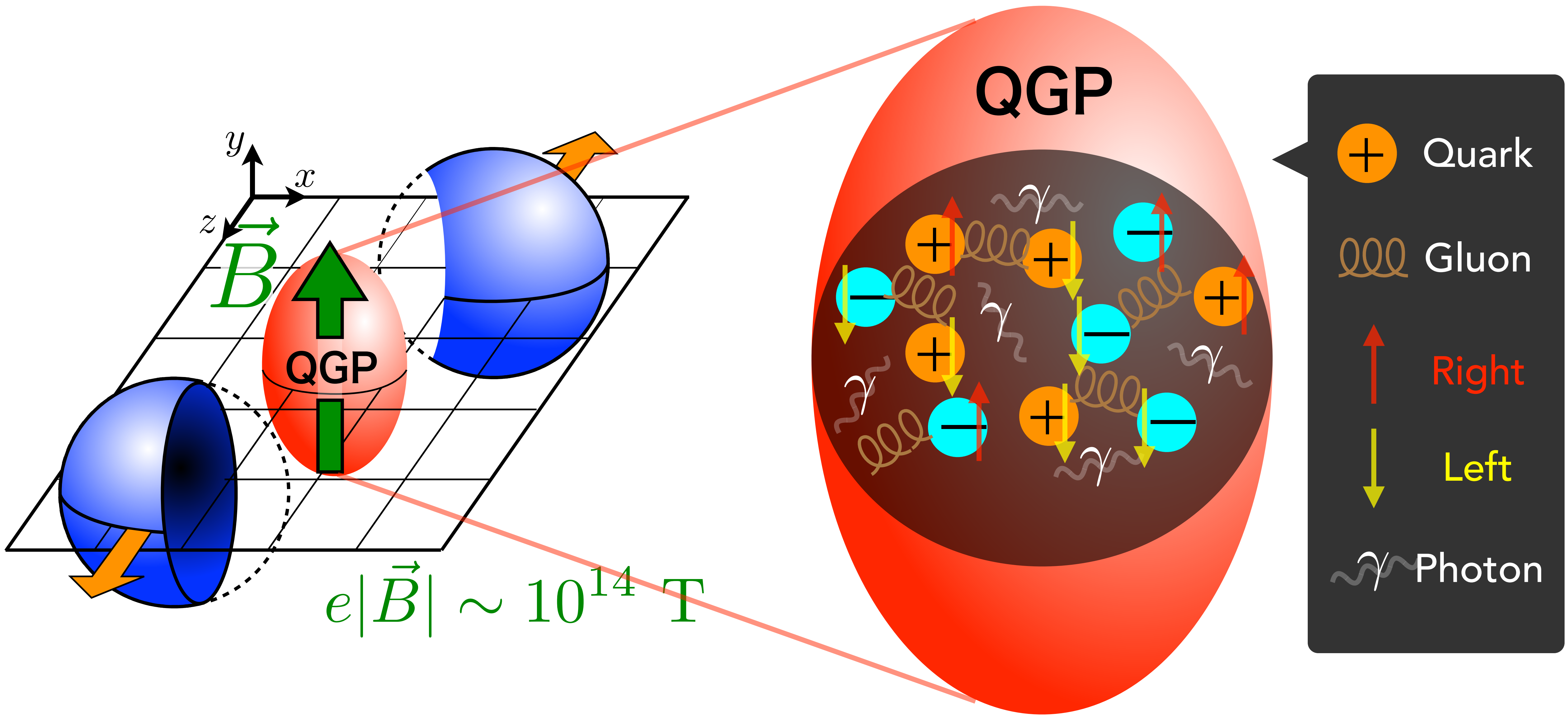


- Existence of extremely strong magnetic field

QGP as Chiral fluid

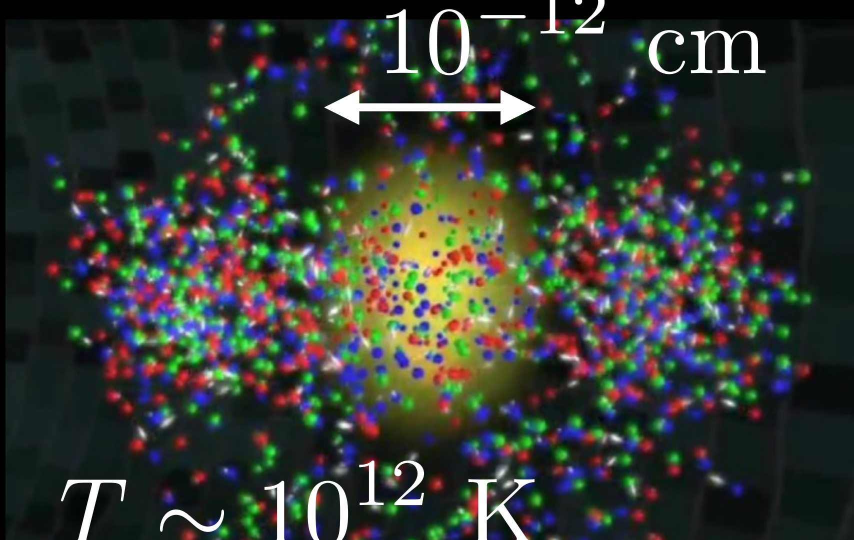


- Existence of extremely strong magnetic field
- Chirality drastically affect hydrodynamic transport

Hydrodynamics is

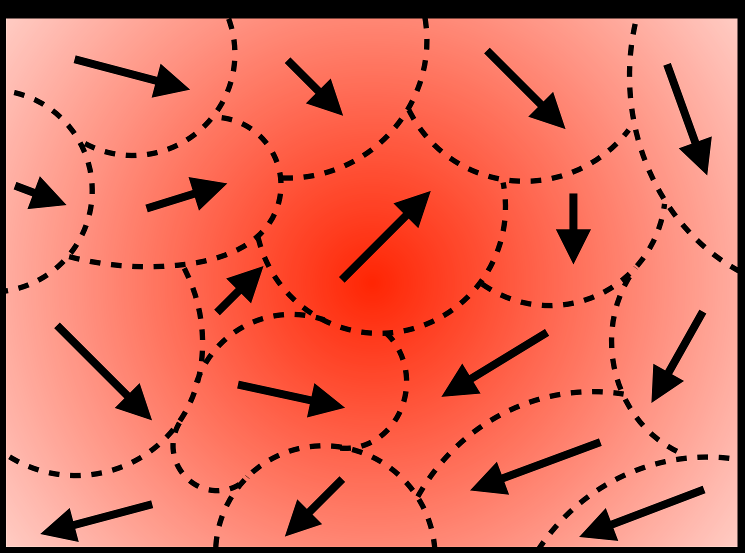
- **Effective theory** for **macroscopic dynamics**
- **Universal** description, not depending on details
- Only **conserved quantity ~ symmetry** of system

Quark-Gluon Plasma | **Hydro:** $\{\beta(x),\ \vec{v}(x)\}$ | **Neutron Star**



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

$\simeq$



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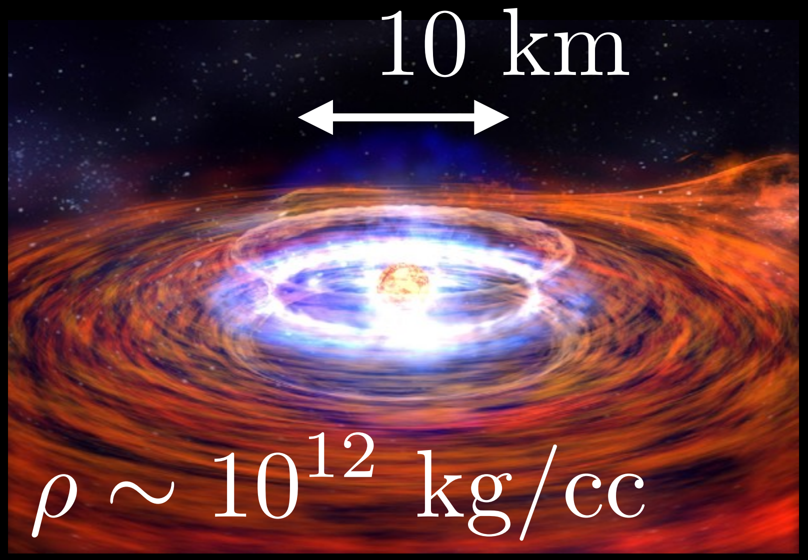

http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

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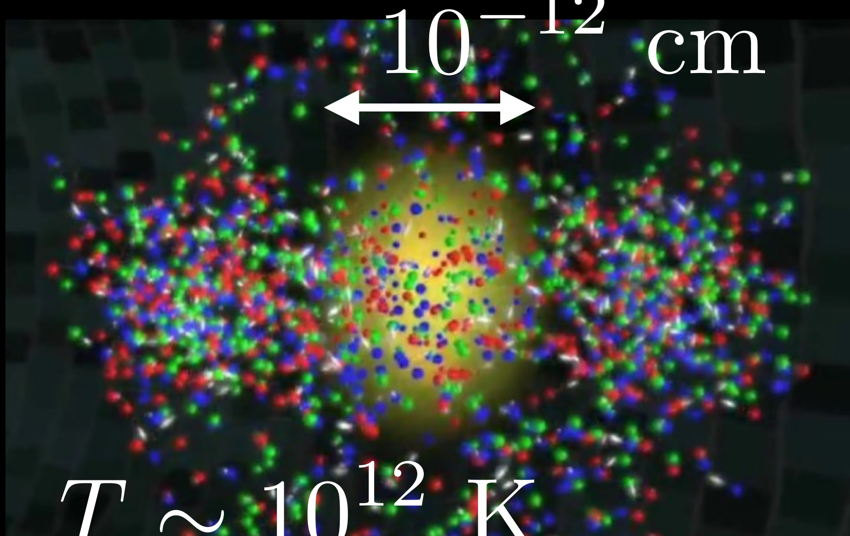

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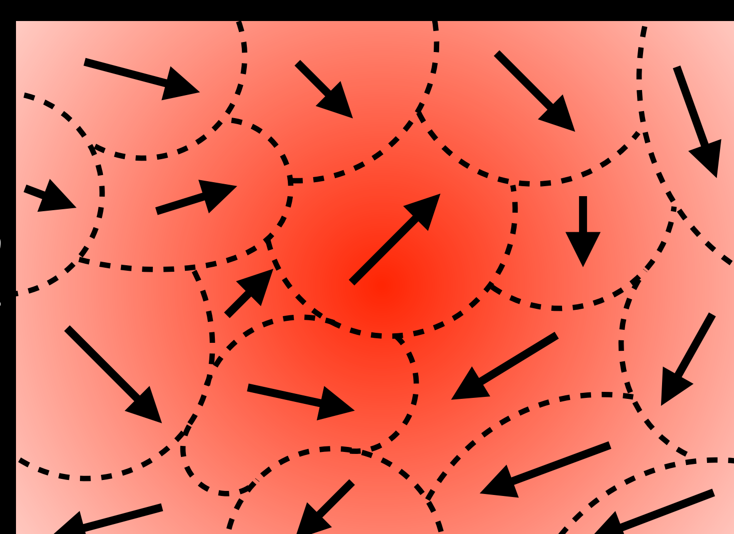

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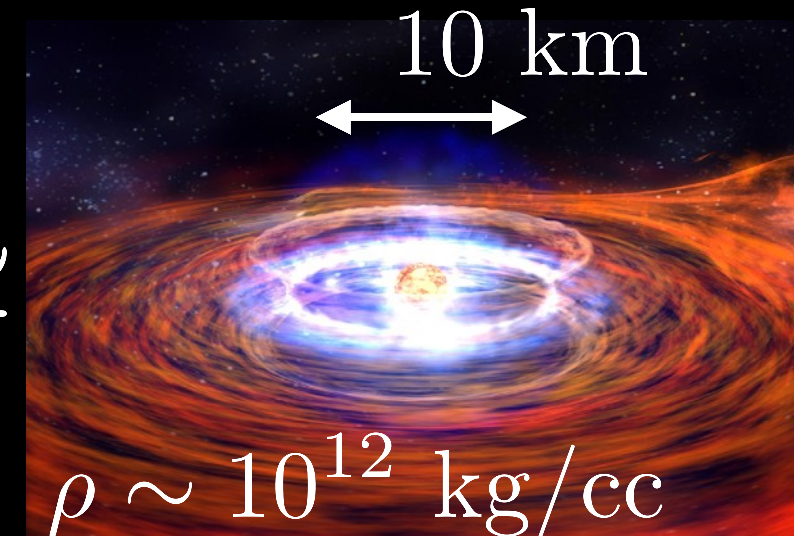
Neutron Star



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Symmetry breaking and Hydro

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- Spontaneous symmetry breaking

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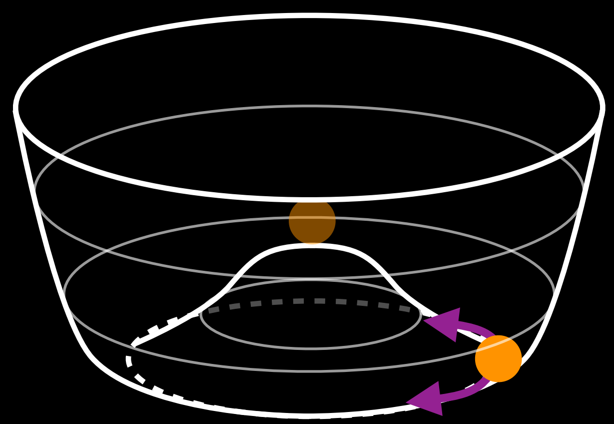
Micro：Selecting vacuum



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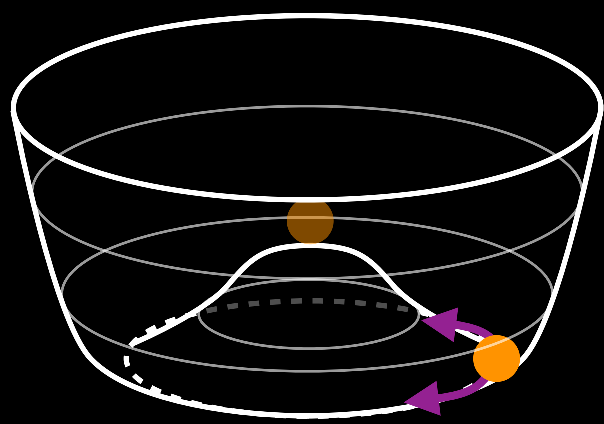
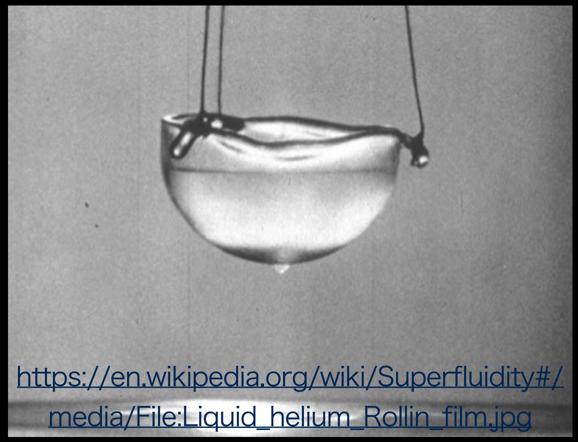


Macro：Superfluid



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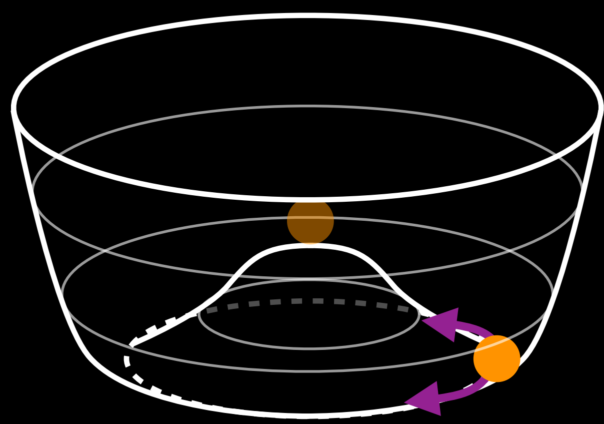

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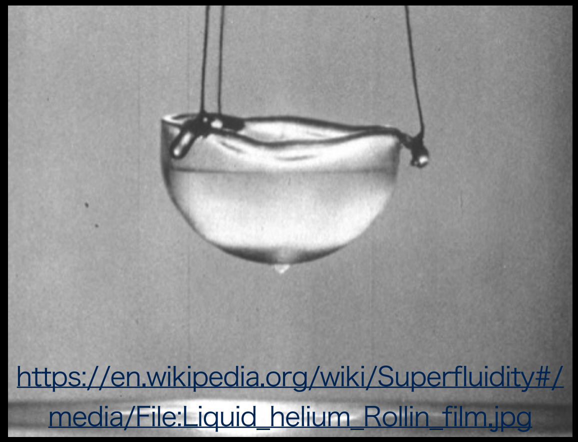

◆ Symmetry breaking by quantum anomaly

Symmetry breaking and Hydro

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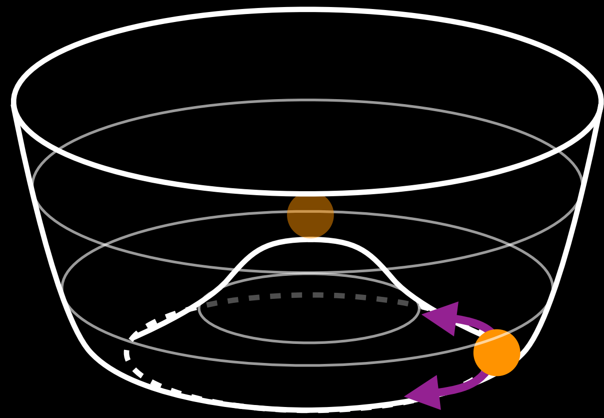

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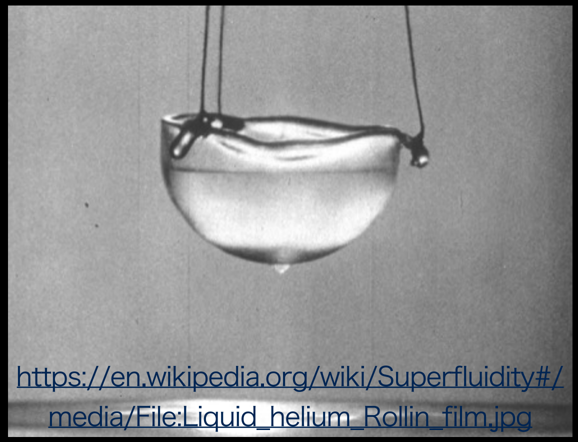

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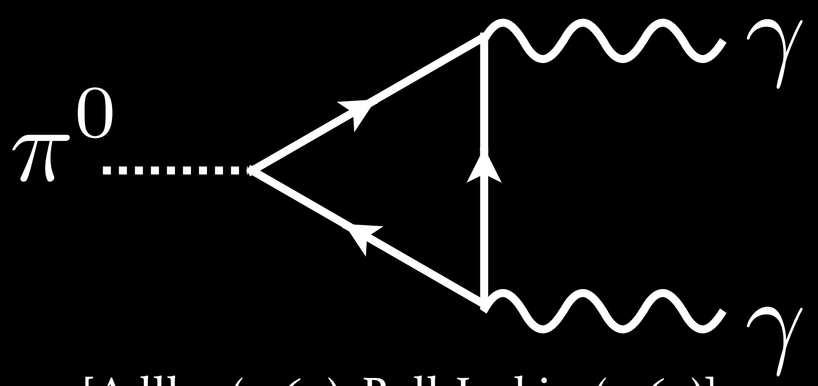
Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Symmetry breaking and Hydro

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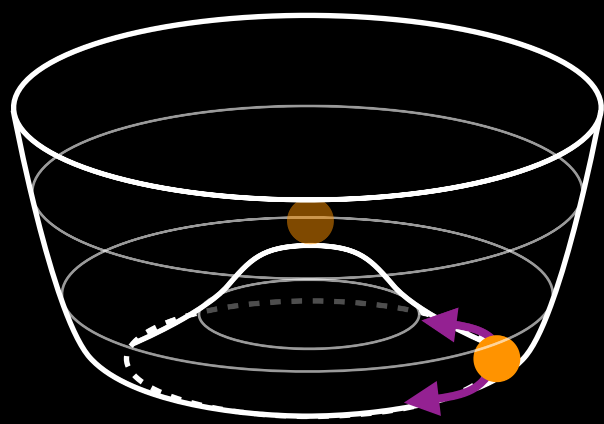

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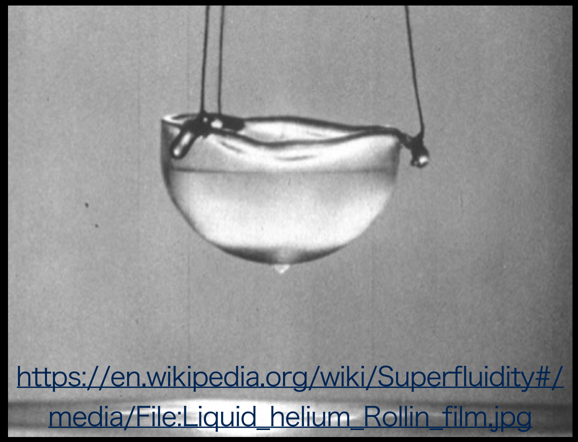

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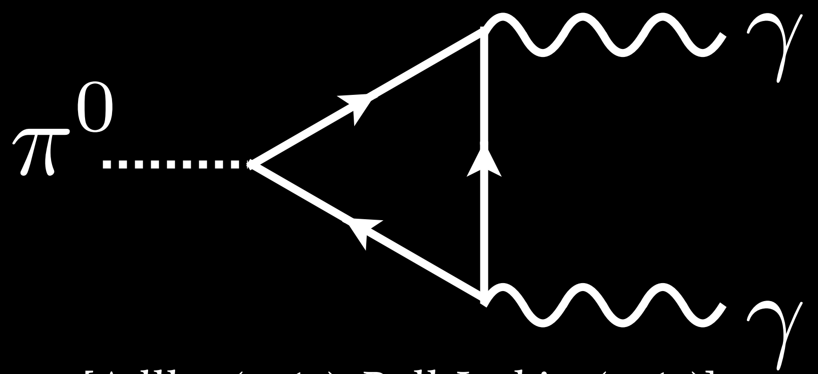

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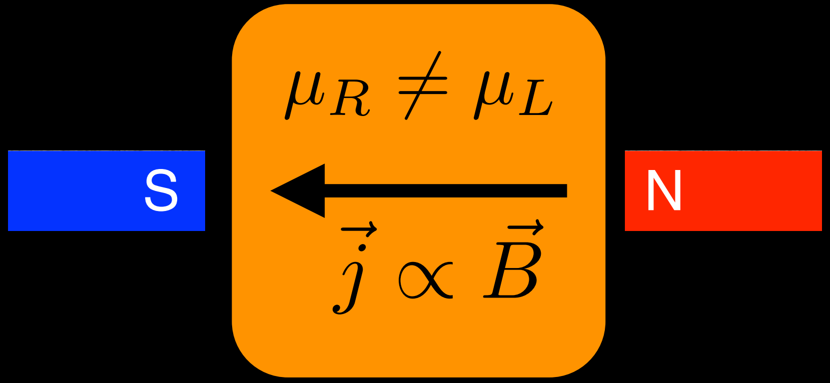
Macro：Anomalous transport



[Erdmenger et al. (2008), Son-Surowka (2009)]

Parity-violating chiral transport

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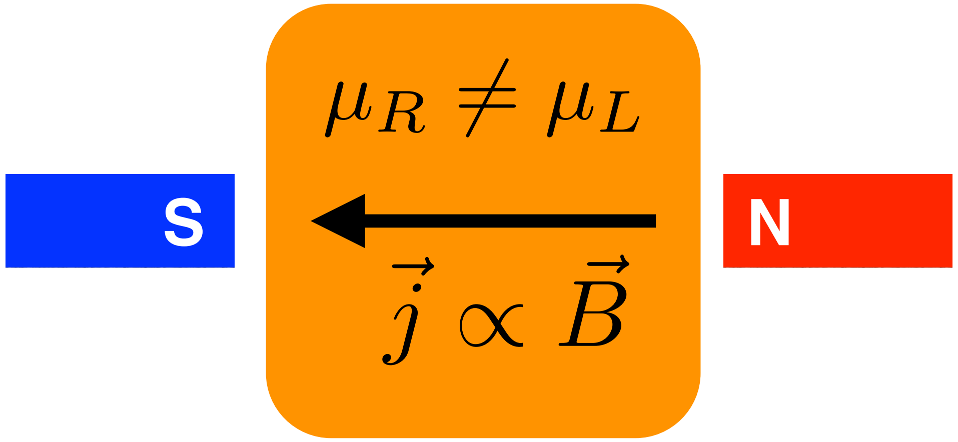
[Fukushima et al. (2008), Vilenkin (1980)]

◆**Chiral Magnetic Effect (CME)**

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



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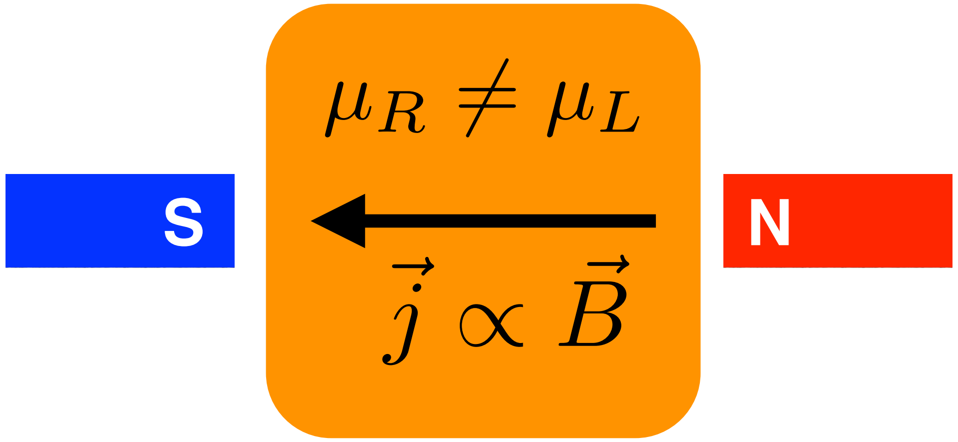

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◆ **Chiral Vortical Effect (CVE)**

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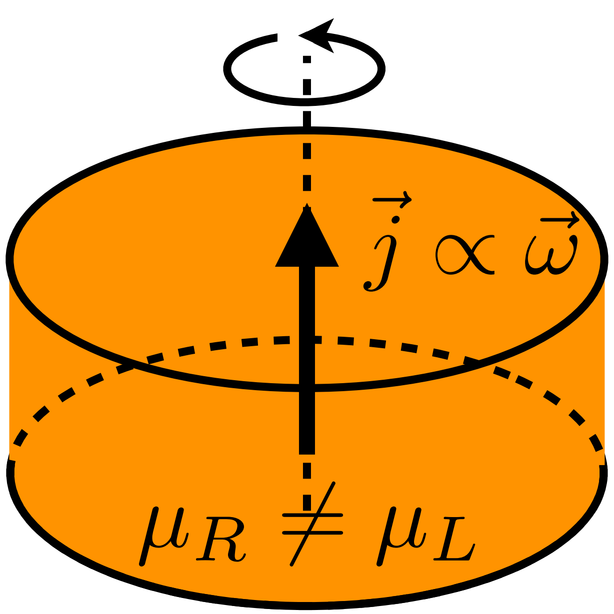

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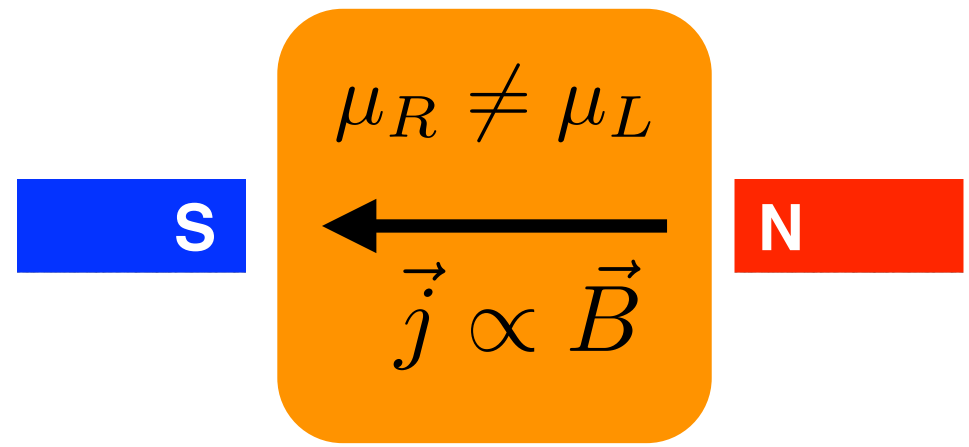

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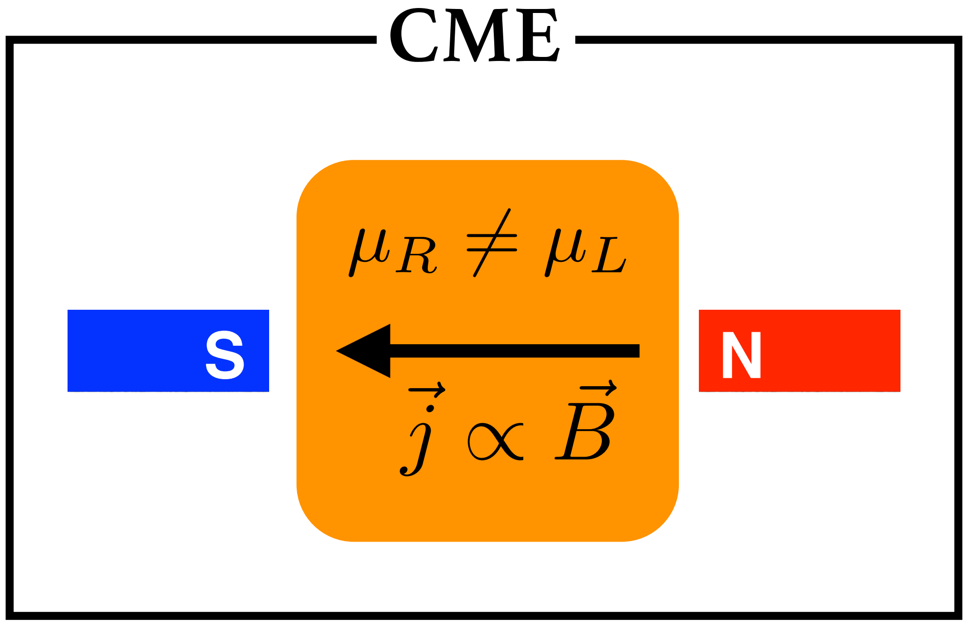
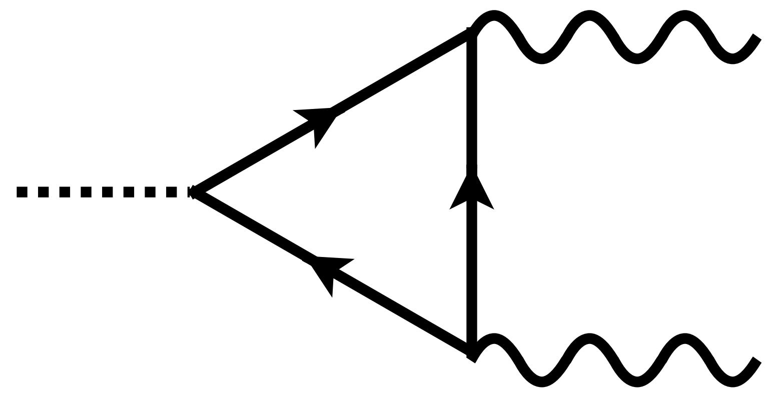
Anomaly and chiral transport



Anomaly and chiral transport

Quantum anomaly



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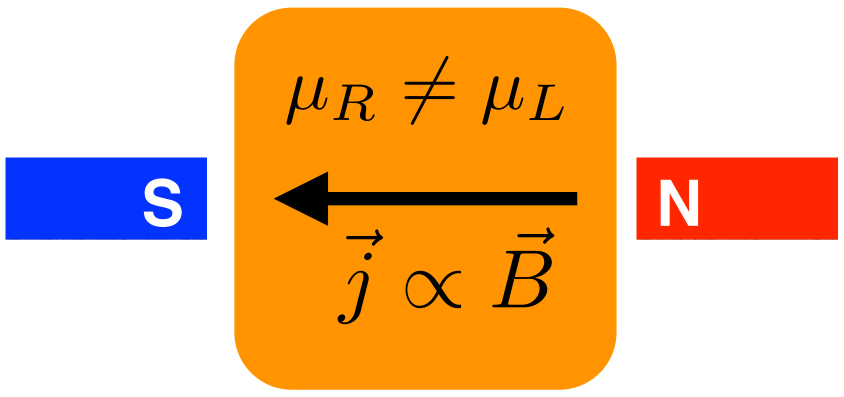

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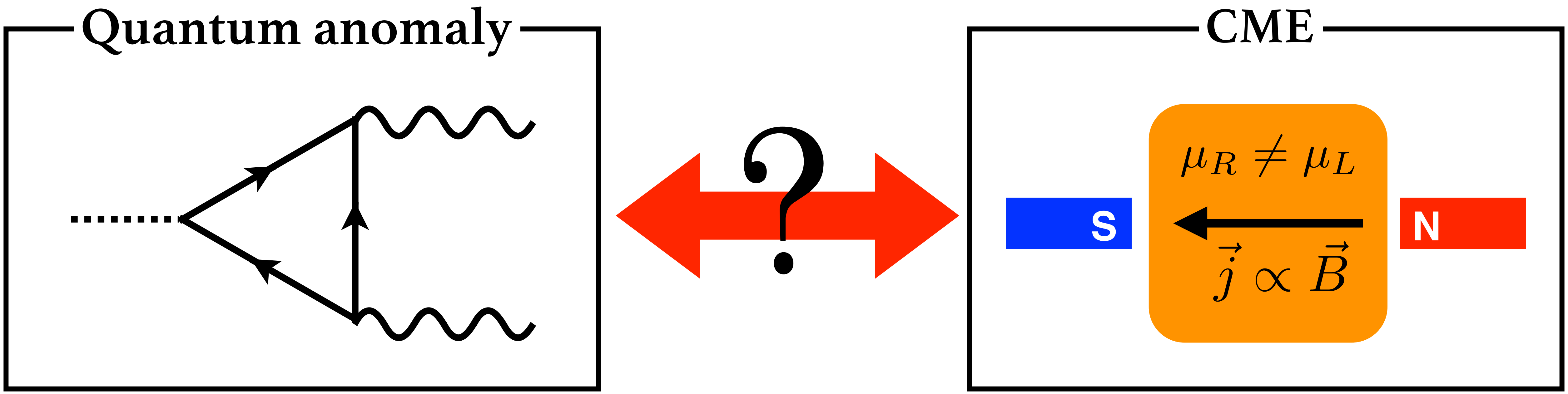

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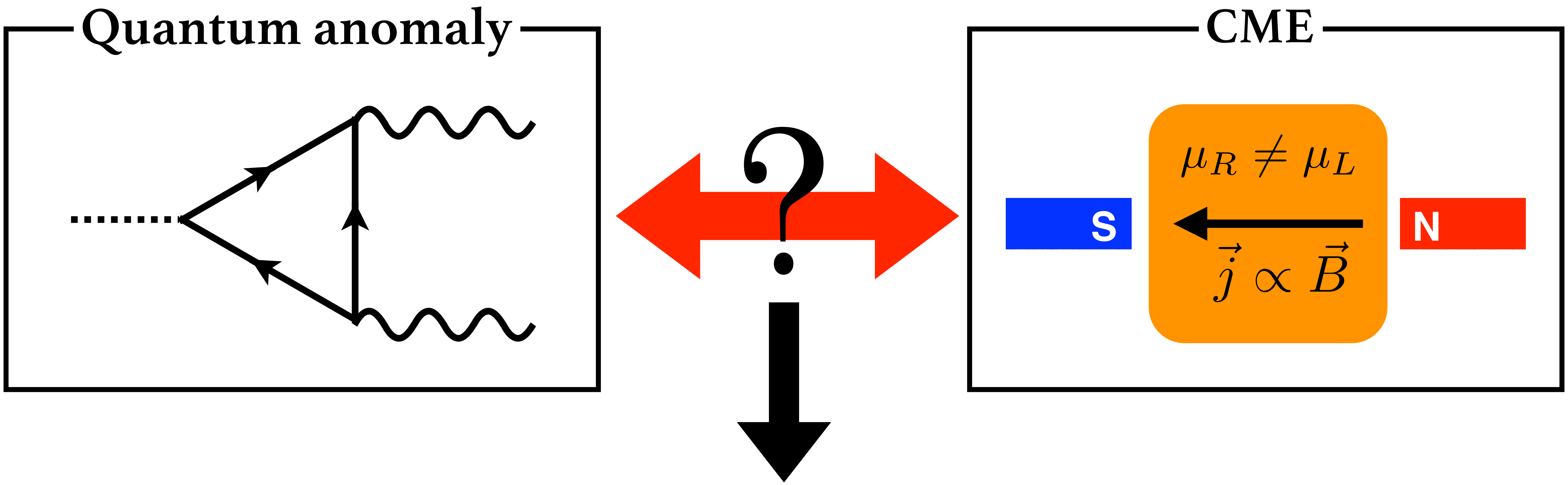
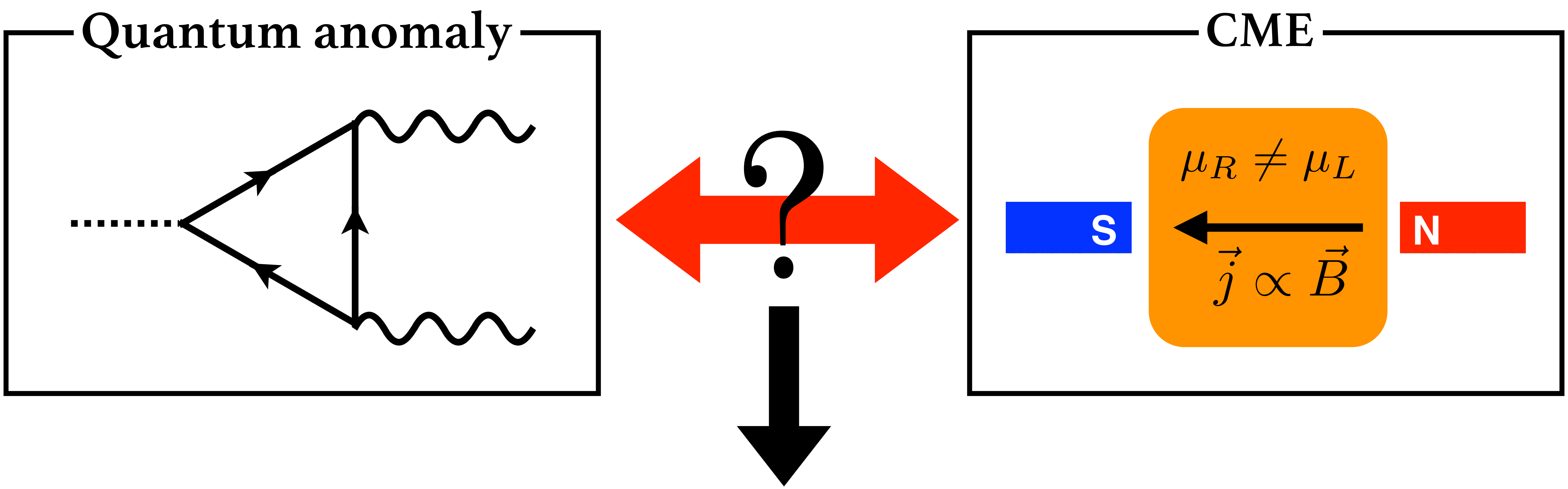


Can we understand this based on current algebra?

Anomaly and chiral transport



Can we understand this based on current algebra?

Problem.

Not vacuum physics as is the case for QCD!

→ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

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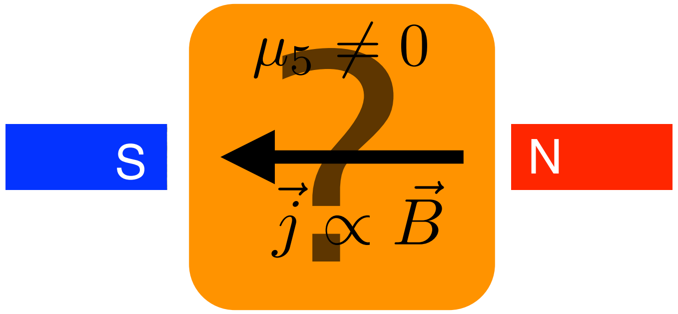

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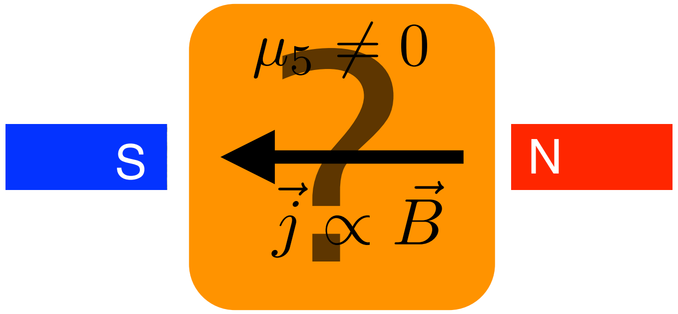

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◆ **Current algebra for** $SU(N)_R \times SU(N)_L$

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Low-energy theorem ⬇

- **Goldberger-Treiman relation**
- **Soft Pion theorem, ⋯**

Universal results for process with low-energy pion scattering!

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Universal results for process with low-energy pion scattering!

If current algebra satisfies the above relations,
it does not matter whether UV theory is QCD, NJL model, or anything!

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for $U(1)_V \times U(1)_A$**

Current algebra and chiral anomaly

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Definition of Noether current gives

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we can directly show the above current algebraic structure!

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Sketch of Proof.

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Sketch of Proof.

Ward-Takahashi identity is not $\langle \partial_\mu J^\mu_5(x) \rangle_A = 0$ but

$$\langle \partial_\mu J^\mu_5(x) \rangle_A = C \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x)$$

Current algebra and chiral anomaly

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$$[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = [\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})] = 0$$

$$[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y}) \partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$$

Sketch of Proof.

Ward-Takahashi identity is not $\langle \partial_\mu J^\mu_5(x) \rangle_A = 0$ but

$$\langle \partial_\mu J^\mu_5(x) \rangle_A = C\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x) \sim C\, dA\, dA$$

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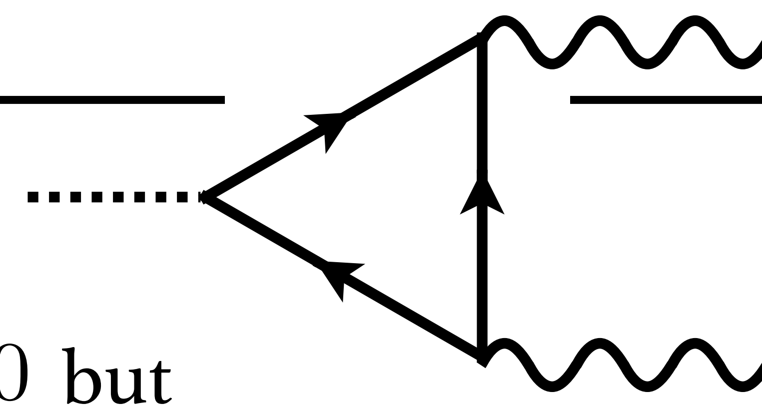

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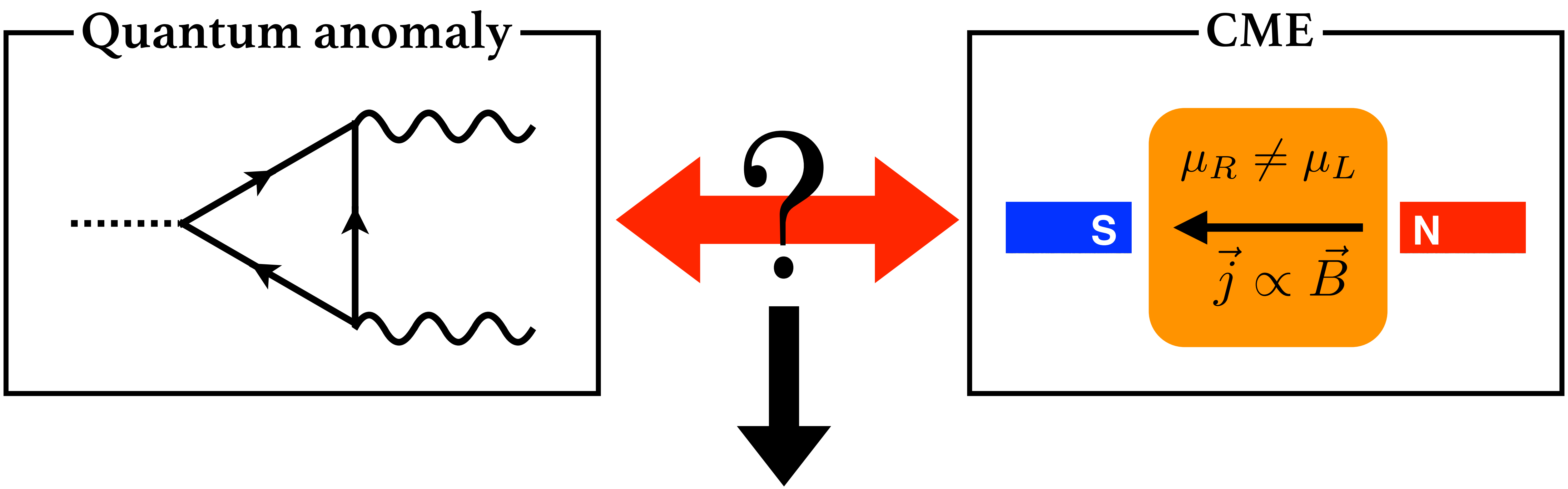
Anomaly and chiral transport



Can we understand this based on current algebra?

Problem.

Not vacuum physics as is the case for QCD!

→ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

Mori's projection operator method

[Mori (1965)]

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A method to write down

Equation of Motion (EoM)

only focusing on $\hat{A}_n(t)$

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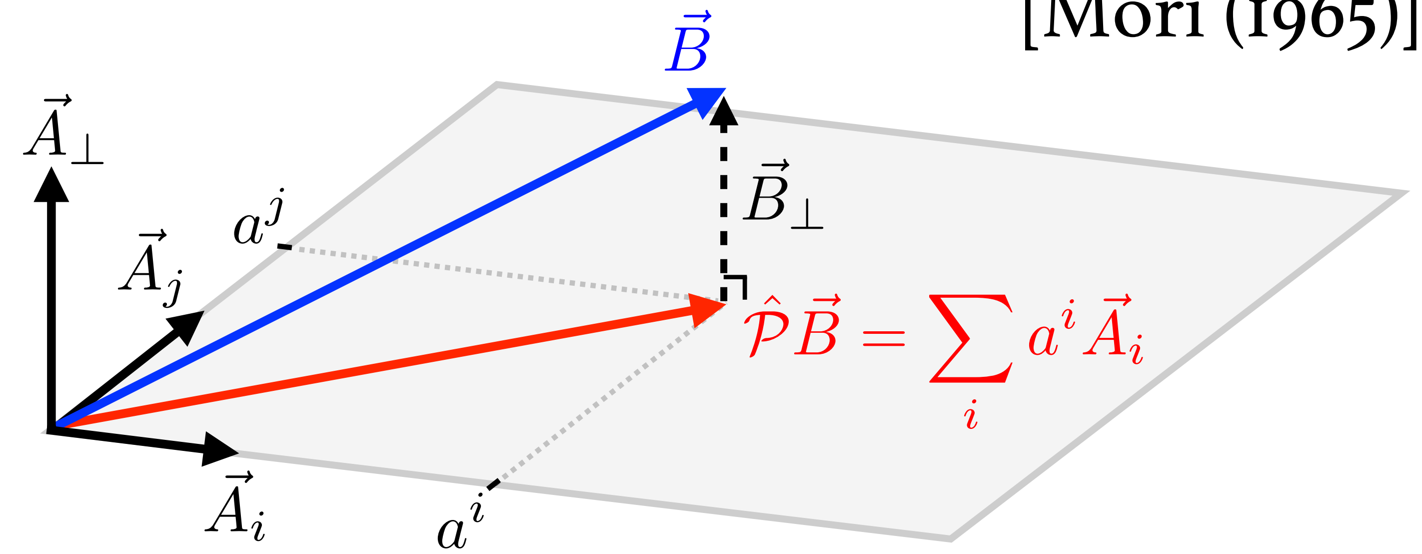

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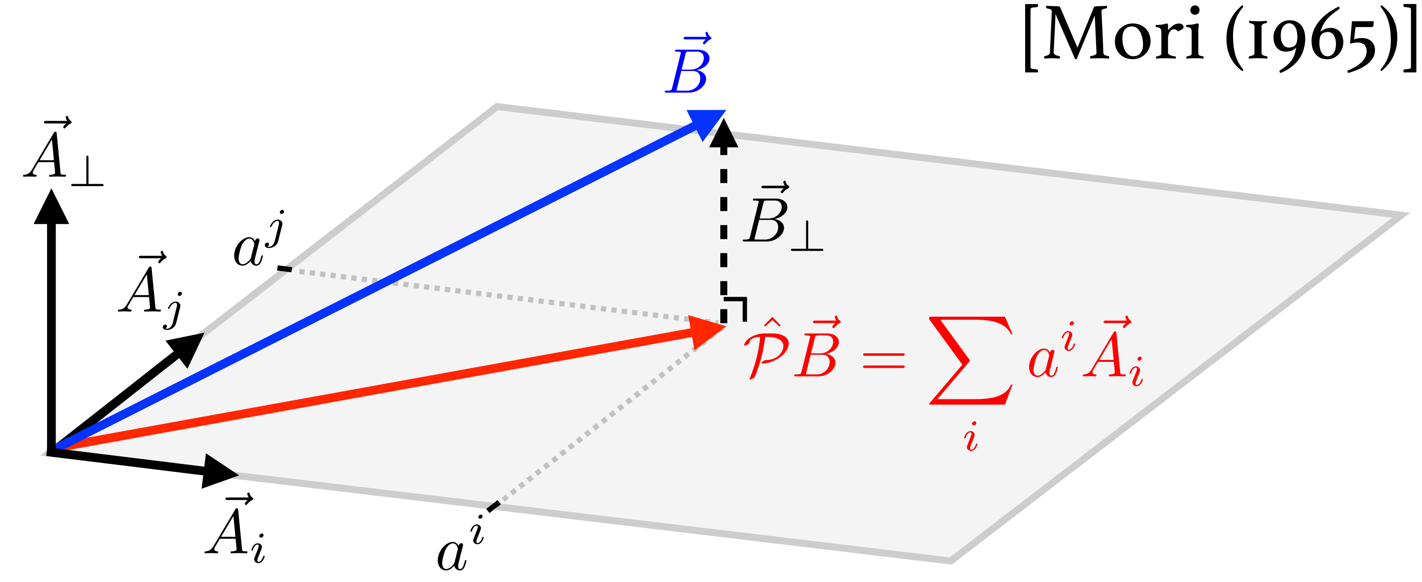

◆ EoM given by Mori's projection operator method

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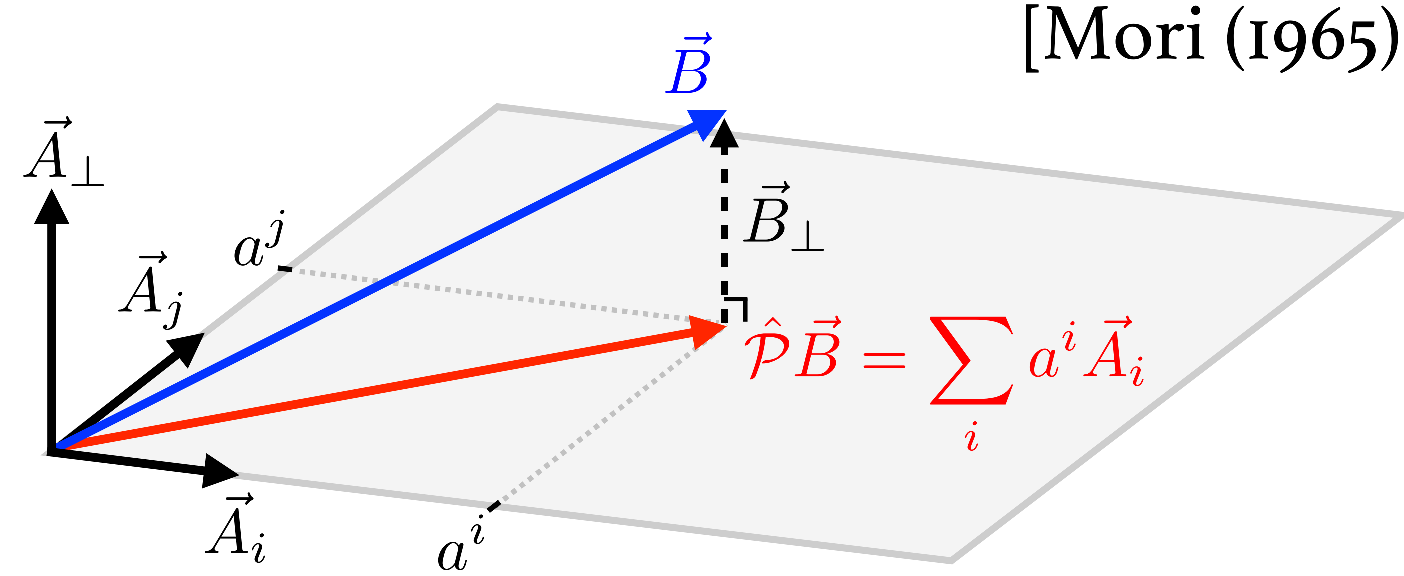

◆ **EoM given by Mori's projection operator method**

$$\partial_0 \hat{A}_n(t) = i\Omega_n{}^m \hat{A}_m(t) - \int_0^t ds \Phi_n{}^m(t-s)\hat{A}_m(s, \boldsymbol{y}) + \hat{R}_n(t)$$

Mori's projection operator method

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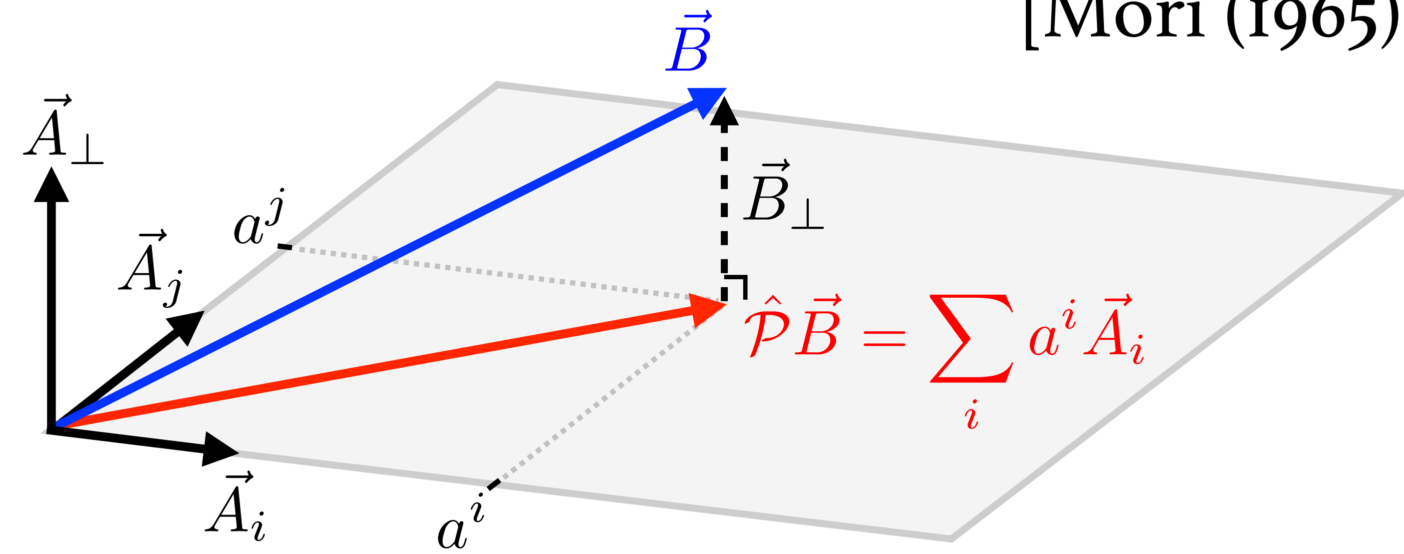

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Reversible

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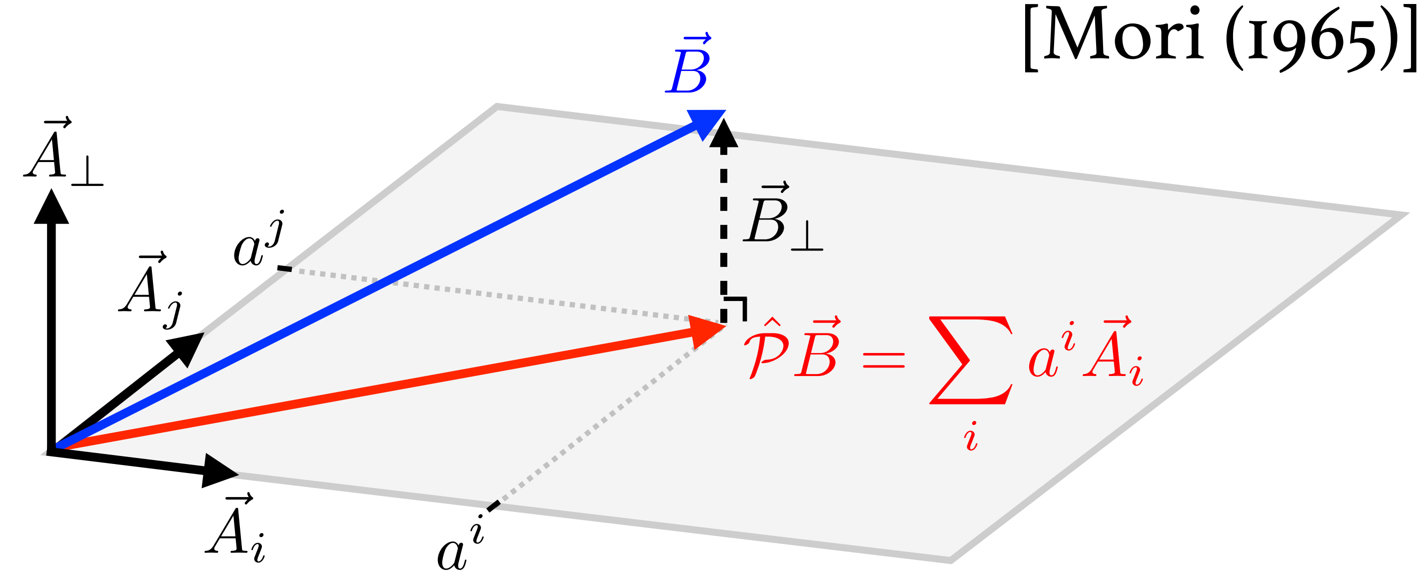

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Reversible

$$\begin{cases} i\Omega_n{}^m = -\dfrac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu\big([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0)\big) \\ \end{cases}$$

Mori's projection operator method

[Mori (1965)]

A method to write down

Equation of Motion (EoM)

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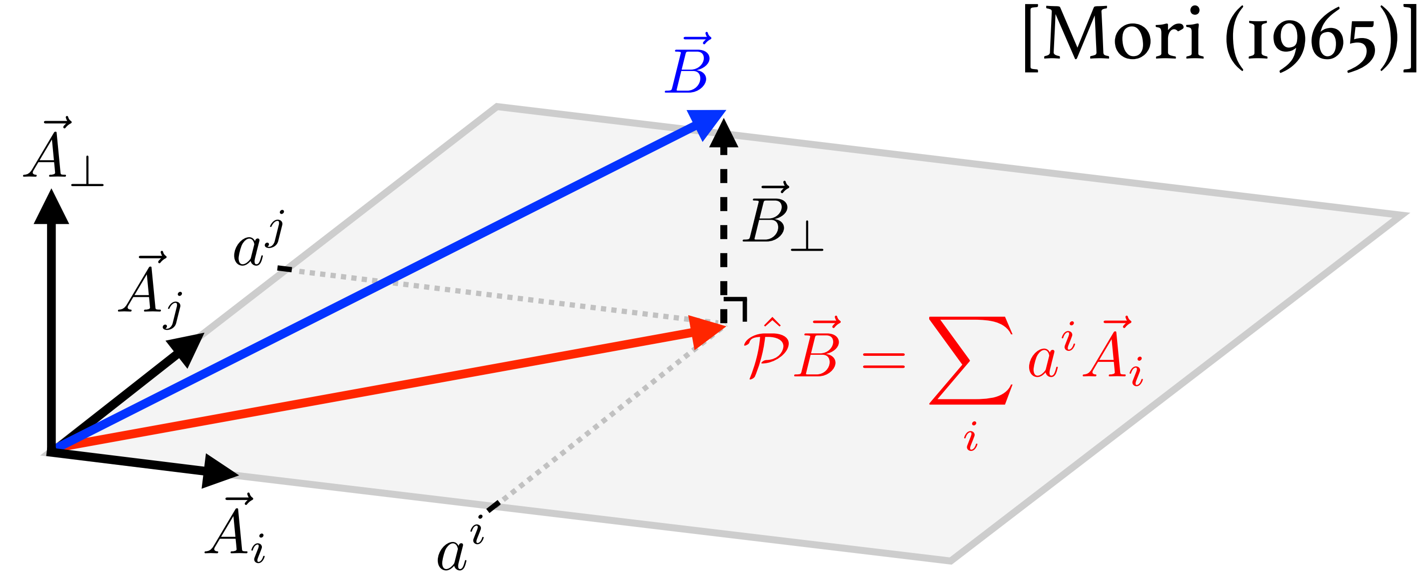

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Reversible　　Dissipative　　Noise

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Fluctuation Dissipation relation: $\Phi_n^m(t-s) = \big(\hat{R}_n(t-s), \hat{R}^m(0)\big)$

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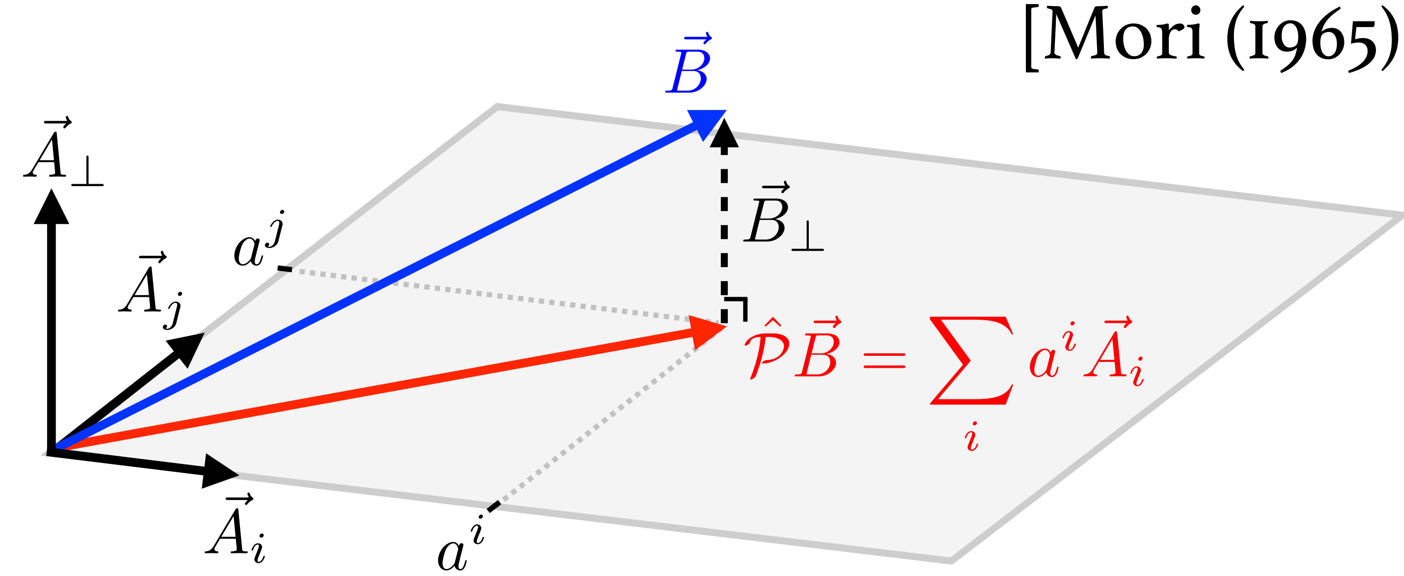

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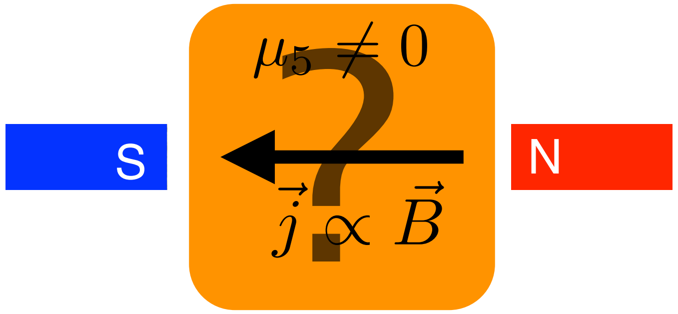
Outline

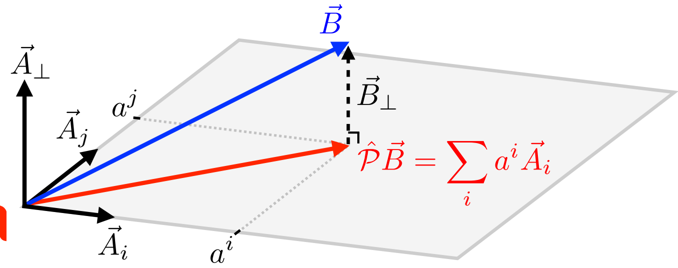
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

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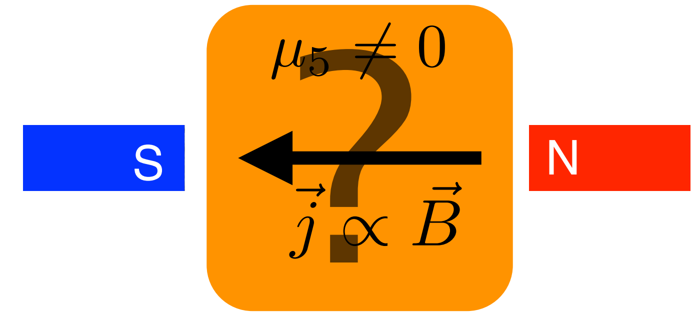

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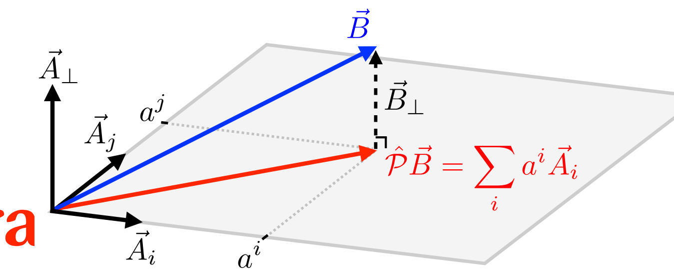

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Mori's method and current algebra

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◆ **Leading Order term in EOM**

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$$ (χ^{lm} : inv. suscep.)

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◆ **Current algebra related to relativistic hydrodynamics**

Mori's method and current algebra

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➡ **EoM(LO) is controlled by energy-momentum density algebra!**

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Current algebra

$$\begin{cases} [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_0(t,\boldsymbol{y})] = -i\big(\hat{T}^k{}_0(t,\boldsymbol{x}) + \hat{T}^k{}_0(t,\boldsymbol{y})\big)\partial_k\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{y})] = -i\big(\hat{T}^j{}_i(t,\boldsymbol{x})\partial_j - \hat{T}^0{}_0(t,\boldsymbol{y})\partial_i\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{T}^0{}_j(t,\boldsymbol{y})] = -i\big(\hat{T}^0{}_j(t,\boldsymbol{x})\partial_i + \hat{T}^0{}_i(t,\boldsymbol{y})\partial_j\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \end{cases}$$

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➡ EoM for perfect fluid (Sound wave) is derived!!

Perfect fluid from Mori's method

Perfect fluid from Mori's method

◆ **Relativistic hydrodynamic from Mori's method**

[Minami-Hidaka (2013)]

$$\partial_0 \delta \hat{T}^0{}_0 = -ik^i \delta \hat{T}^0{}_i$$

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◆ **Green-Kubo formula for transport coefficients (viscosity)**

$$\zeta = \beta_{\mathrm{eq}} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{0}))$$

$$\eta = \frac{\beta_{\mathrm{eq}}}{(d+1)(d-2)} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{\pi}_{ik}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{\pi}_{jl}(0, \boldsymbol{0})) \Delta^{ij} \Delta^{kl}$$

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➡ **Reversible → Sound wave / Dissipative → Diffusion mode**

CME from anomalous commutation

CME from anomalous commutation

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x}), J^0(t,\boldsymbol{x}), J^0_5(t,\boldsymbol{x})\}$

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Current algebra

with

anomalous

commutation rel.

$$\begin{cases} \left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -i\hat{J}^0(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = -i\hat{J}^0_5(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2}B^i(t,\boldsymbol{y})\partial^x_i\delta(\boldsymbol{x}-\boldsymbol{y}) \\ \left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0 \end{cases}$$

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Current algebra with anomalous commutation rel.

$$\left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -i\hat{J}^0(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\left[\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = -i\hat{J}^0_5(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2}B^i(t,\boldsymbol{y})\partial^x_i\delta(\boldsymbol{x}-\boldsymbol{y})$$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0$$

◆ **EoM for** $\hat{J}^0(t,\boldsymbol{x})$

$$\partial_0 \hat{J}^0(x) + \partial^x_i\left[\frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2}B^i(x)\right] + \cdots = 0$$

CME from anomalous commutation

$$\partial_0 \hat{J}^0(x) + \partial_i^x \left[\frac{\chi^{nn_5} \hat{J}_5^0(x)}{2\pi^2} B^i(x) \right] + \cdots = 0$$

CME from anomalous commutation

$$\partial_0 \hat{J}^0(x) + \partial_i^x \left[\underbrace{\frac{\chi^{nn_5} \hat{J}_5^0(x)}{2\pi^2} B^i(x)}_{= \hat{J}^i(x)} \right] + \cdots = 0$$

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◆ **Summary of result**

- **Conservation law:** $\partial_\mu \hat{J}^\mu(x) = 0$

- **Const. relation:** $\hat{J}^i(x) = \frac{\chi^{nn_5} \hat{J}_5^0(x)}{2\pi^2} B^i(x)$

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◆ **Summary of result**

- Conservation law: $\partial_\mu \hat{J}^\mu(x) = 0$
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Chiral Magnetic Wave (CMW)

Chiral Magnetic Effect

$$\hat{J}^i(x) = \frac{\chi^{nn_5}\,\hat{J}^0_5(x)}{2\pi^2} B^i(x)$$

Chiral Magnetic Wave (CMW)

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Ex.

Chiral Magnetic Wave (CMW)

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$$\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}_5^0(x)}{2\pi^2}B^i(x)$$

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$$\hat{J}_5^i(x) = \frac{\chi^{n_5n}\hat{J}^0(x)}{2\pi^2}B^i(x)$$

Ex.

$$\mu > 0$$

$$\mu_5 = 0$$

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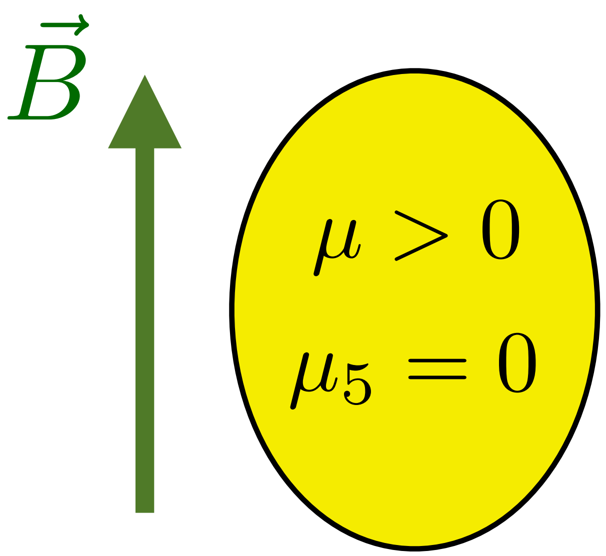

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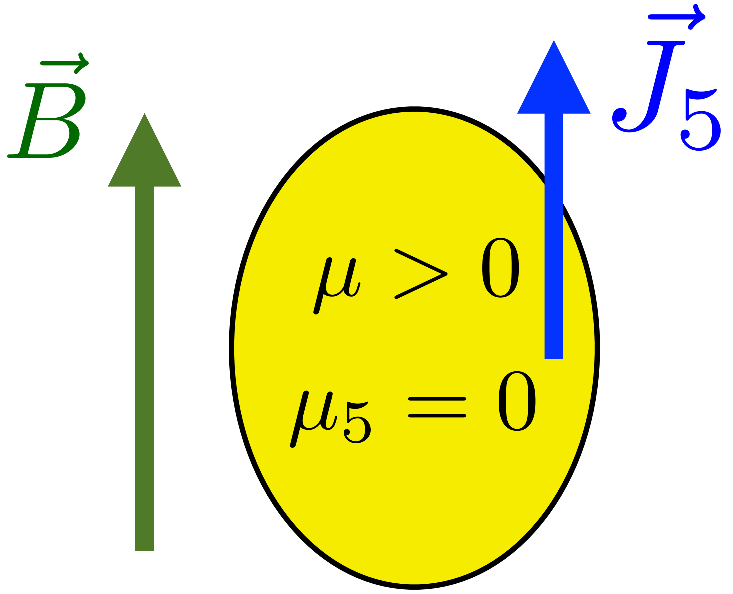

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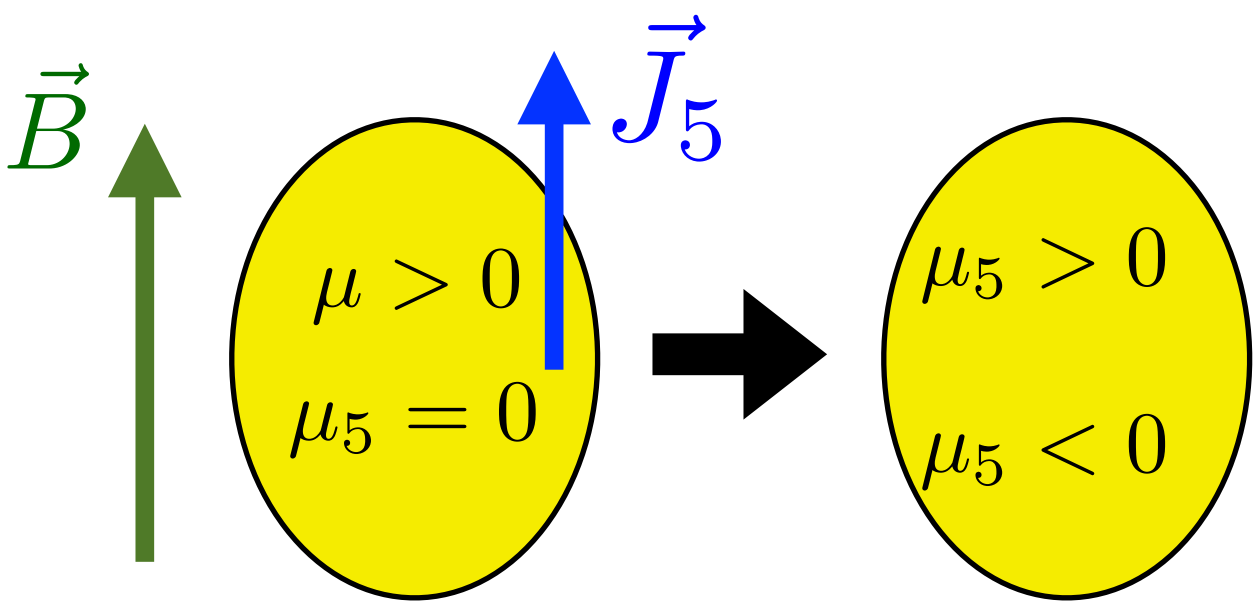

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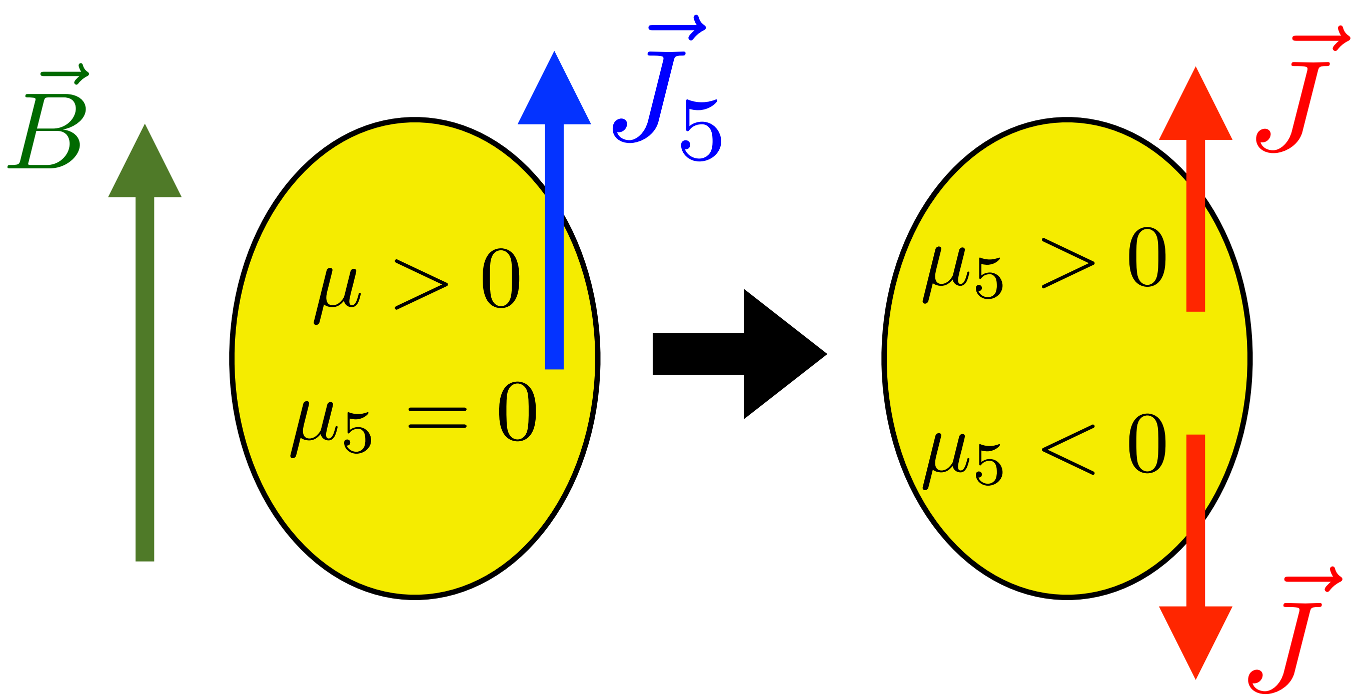

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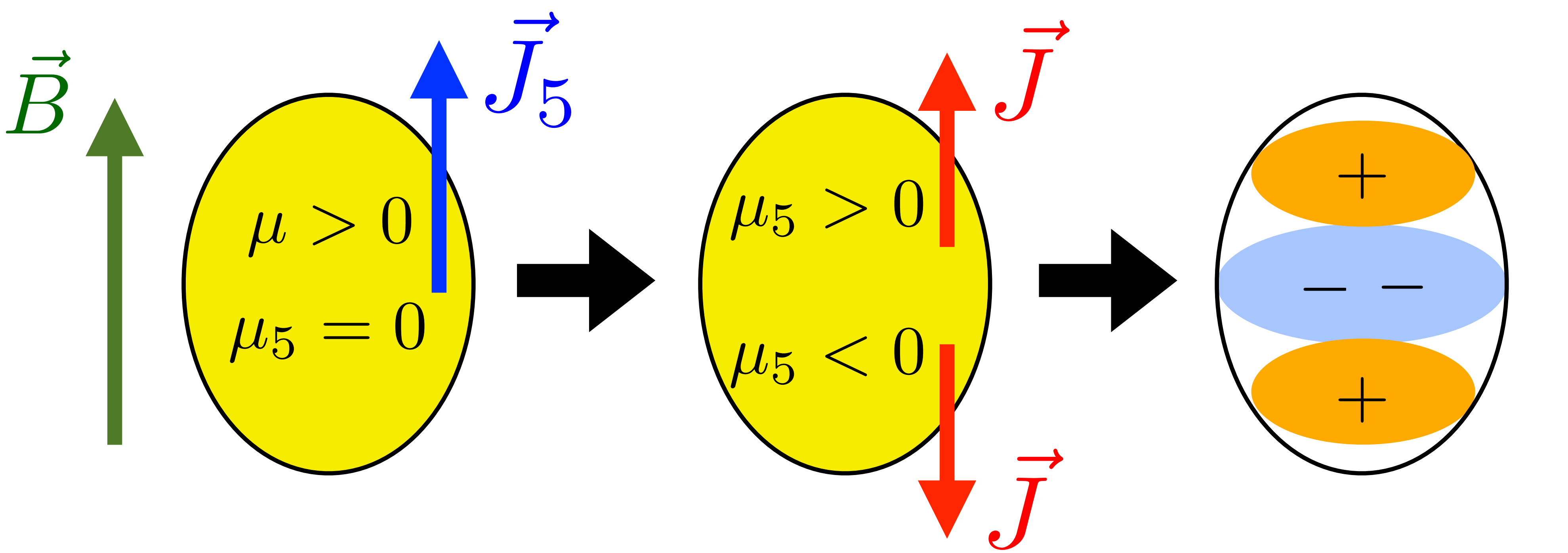

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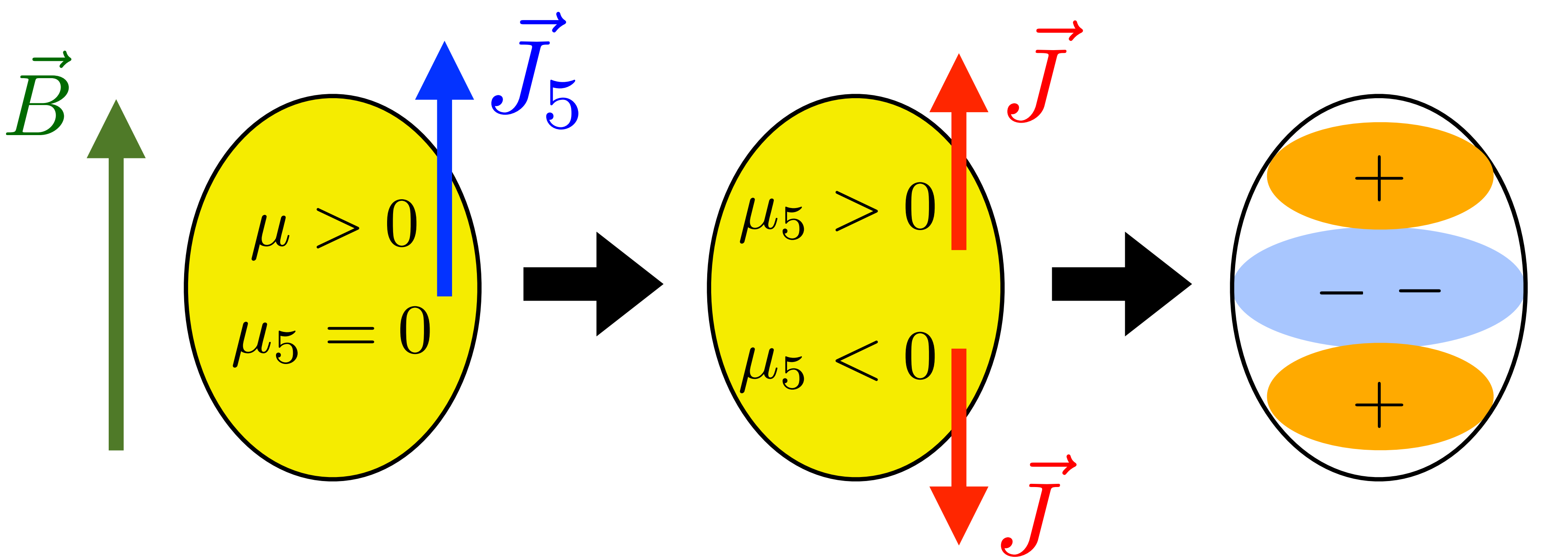

Charge propagation along B

||

Collective excitation

Chiral Magnetic Wave (CMW)

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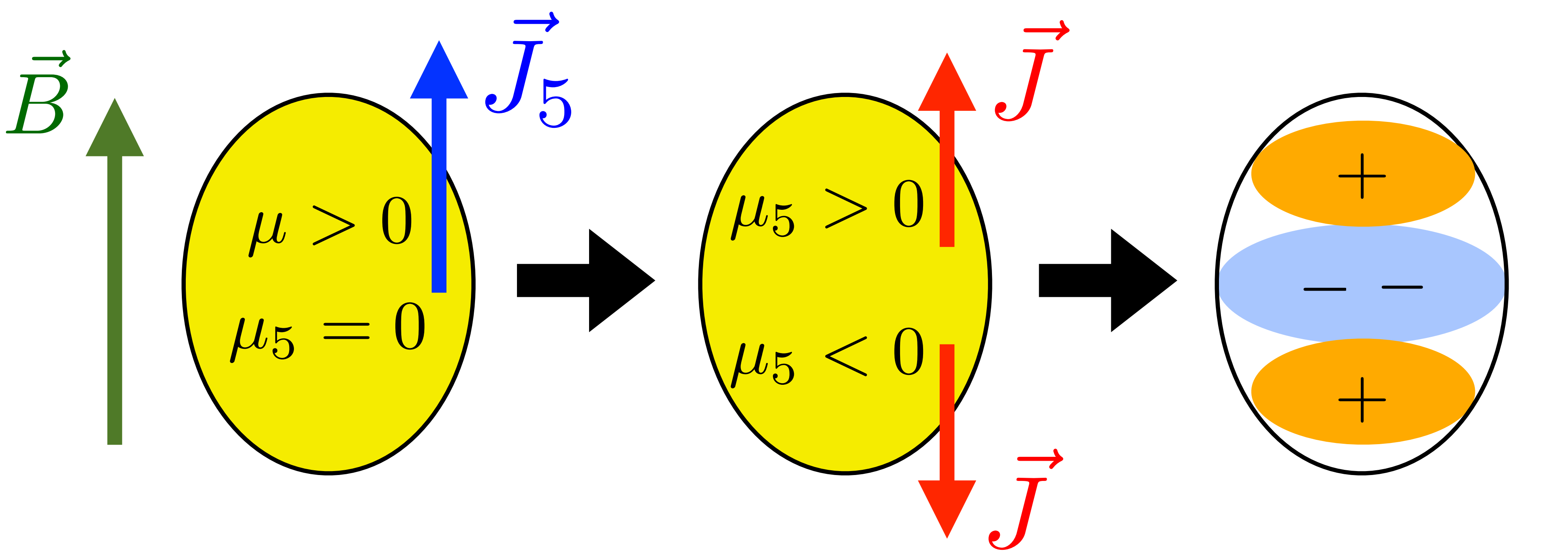

Charge propagation along B

||

Collective excitation

→ **Chiral Magnetic Wave**

[Kharzeev, Yee, (2011)]

Chiral Magnetic **Wave** (CMW)

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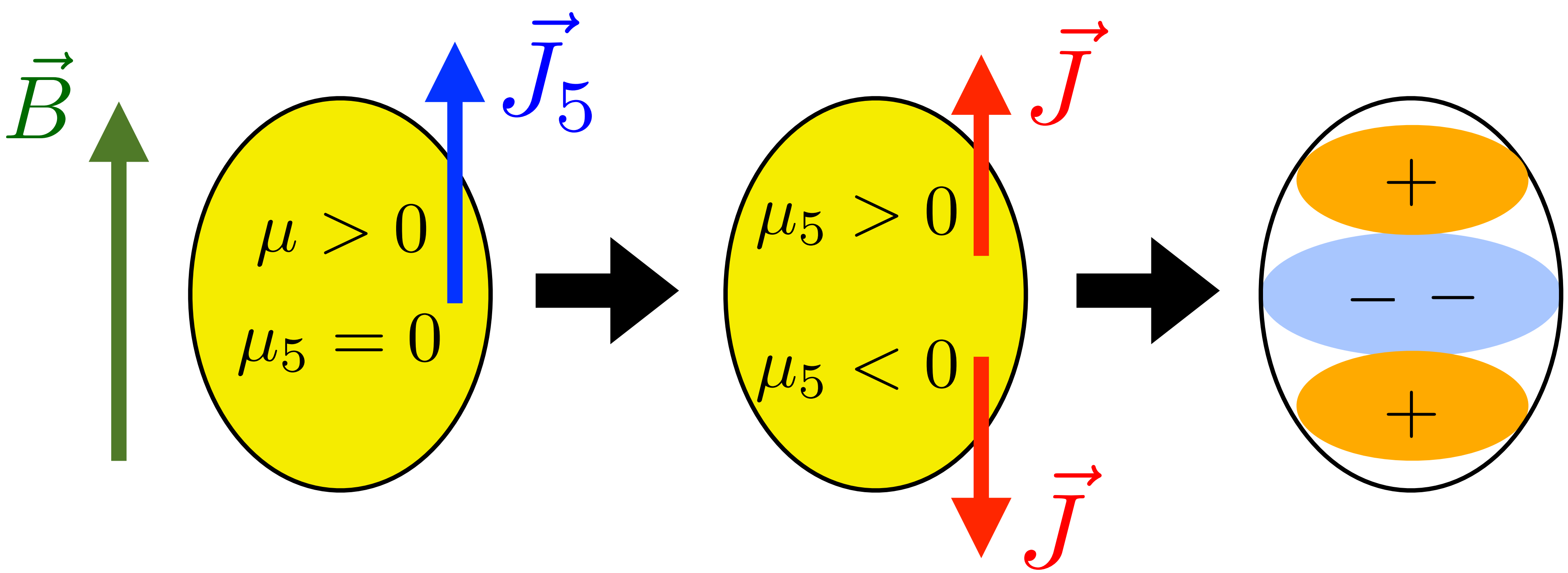


Charge propagation along B
||
Collective excitation

Chiral Magnetic Wave = Analogue of Charge density wave in Tomonaga-Luttinger liquid

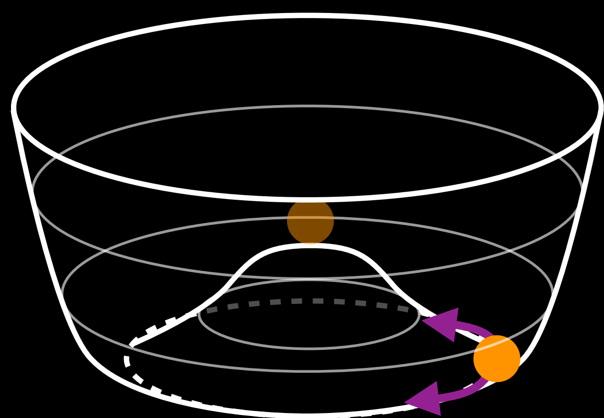
[Kharzeev, Yee, (2011)]

$$[\hat{J}^0_R(t,x), \hat{J}^0_R(t,y)] = -iC\partial_x\delta(x-y)$$

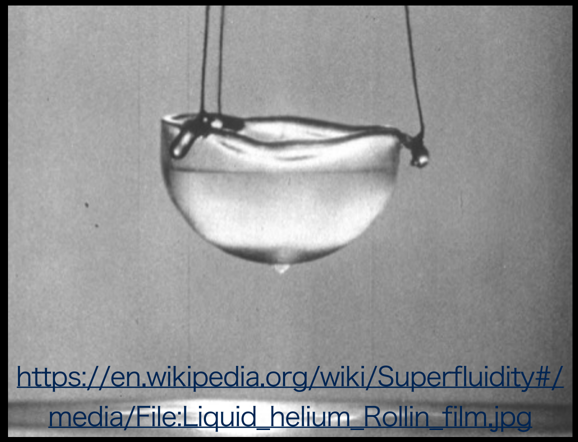
Anomalous superfluid?

- Spontaneous symmetry breaking

Micro：Selecting vacuum

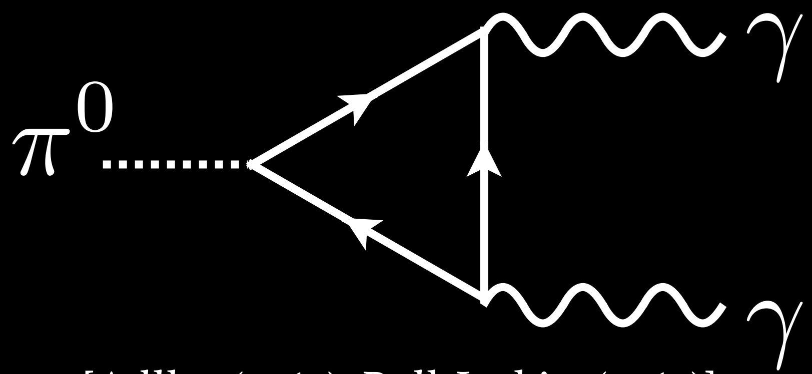


Macro：Superfluid



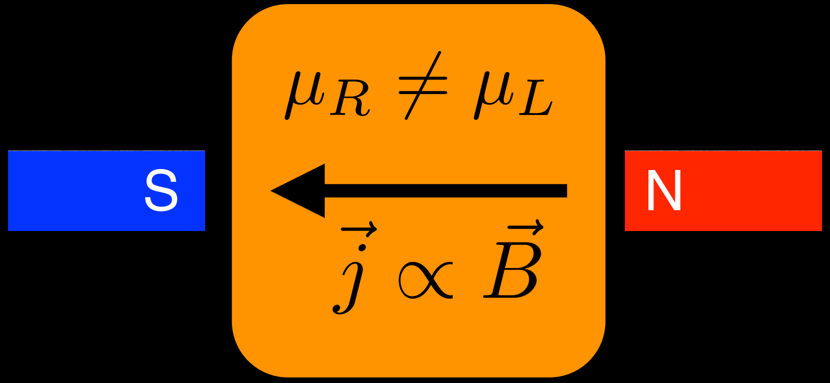
- **Symmetry breaking by quantum anomaly**

Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport



[Erdmenger et al. (2008), Son-Surowka (2009)]

Anomalous superfluid?

- Spontaneous symmetry breaking

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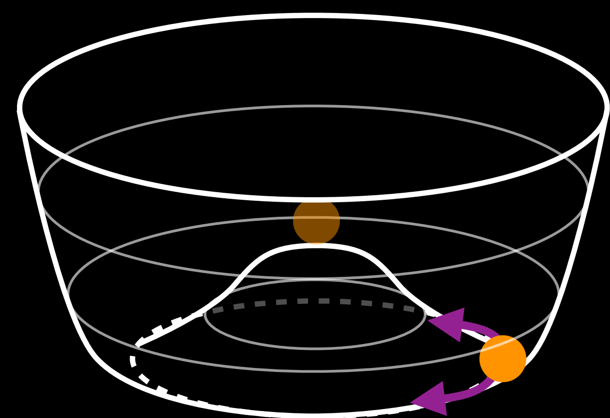
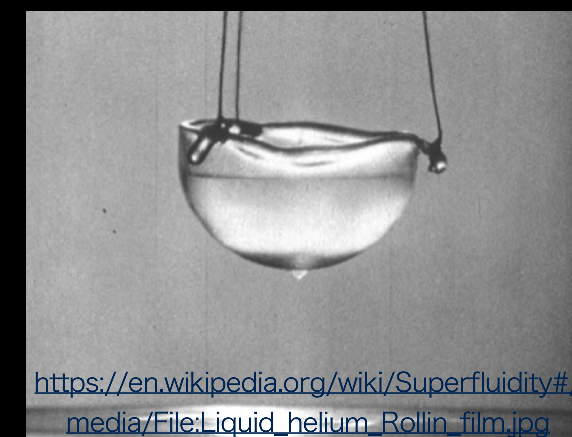


Macro : Superfluid



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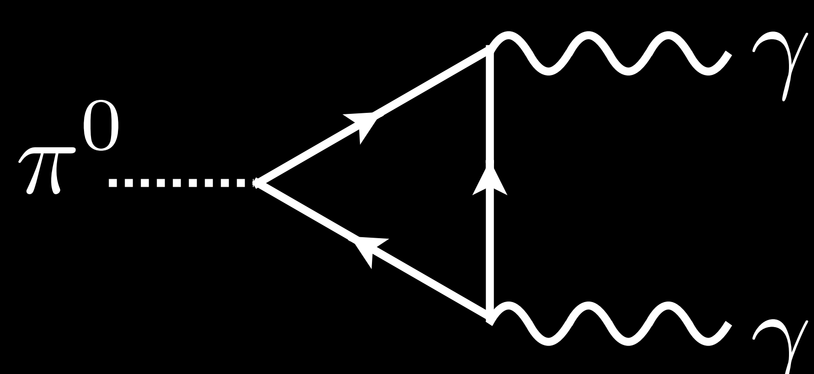

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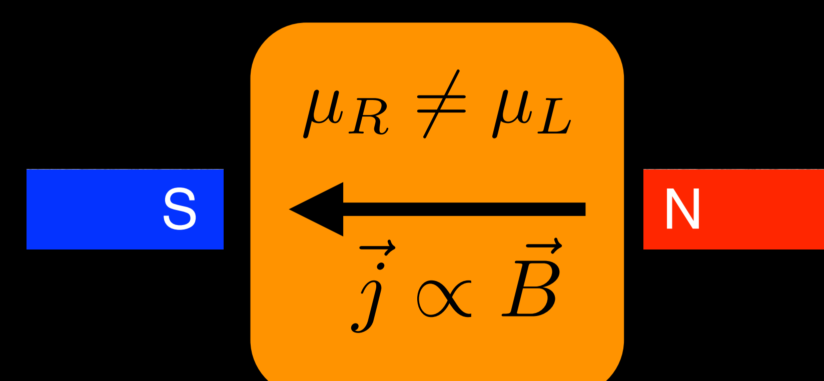

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Symmetry of QCD under B

- **Symmetry structure of chiral limit QCD under magnetic field**

Symmetry of QCD under B

◆ **Symmetry structure of chiral limit QCD under magnetic field**

$$SU(2)_R \times SU(2)_L \to U(1)_V^3 \times U(1)_A^3 \to U(1)_V^3$$

Original symmetry of QCD

Symmetry of QCD under B

◆ **Symmetry structure of chiral limit QCD under magnetic field**

Explicitly broken by magnetic field

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Symmetry of QCD under B

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Original symmetry of QCD

➡ **1 SSB = 1 NG mode = π_0 (=Neutral pion) appears!**

Symmetry of QCD under B

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Explicitly broken by magnetic field / Spontaneously broken

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cf. Spontaneous symmetry breaking & Nambu-Goldstone mode

For some conserved charge $\hat{Q}_a$

$$\exists\, \hat{\pi}_i(x) \text{ such that } \langle [i\hat{Q}_a, \hat{\pi}_i(x)] \rangle = \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a, \hat{\pi}_i(x)]\big) \neq 0$$

Spontaneous Symmetry Breaking (SSB)

Symmetry of QCD under B

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Spontaneous Symmetry Breaking (SSB)

➡ **This can be also captured by the projection operator method!**

Towards anomalous superfluid

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm} \langle [\hat{A}_n(0), \hat{A}_m(0)] \rangle \hat{A}_l(t) + \cdots$

Towards anomalous superfluid

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Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(x), \hat{T}^0{}_i(x), \hat{J}^0(x), \hat{J}^0_5(x), \hat{\pi}(x)\}$

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Current algebra with Anomaly & SSB

$$\chi^{\pi\pi}(\boldsymbol{k}) = \boldsymbol{k}^2 + O(\boldsymbol{k}^4)$$

$$\langle[\hat{J}^0_5(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})]\rangle = \sigma_0\delta(\boldsymbol{x}-\boldsymbol{y})$$

Others: the same as before

Towards anomalous superfluid

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◆ **EoM for** $\{\hat{J}^0(x), \hat{J}^0_5(x), \hat{\Phi}(x)\}$

$$\partial_0 \hat{J}^0_V(x) + \partial_i\left(C\chi^{AA}B^i\hat{J}_A\right) + \cdots = 0$$

$$\partial_0 \hat{J}^0_A(x) + \partial_i\left(C\chi^{VV}\hat{J}_V(x)B^i(x) - \sigma_0\partial_i\hat{\pi}(x)\right) + \cdots = 0$$

$$\partial_0\hat{\pi}(x) + \chi^{AA}\sigma_0\hat{J}_A + \cdots = 0$$

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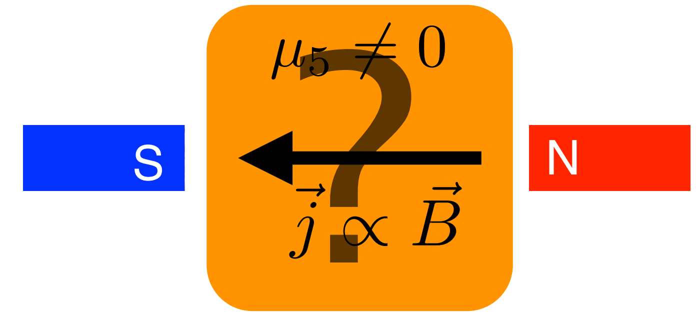
$$\partial_0\hat{\pi}(x) + \chi^{AA}\sigma_0\hat{J}_A + \cdots = 0$$

➡ $\omega^2 = \sigma_0^2\chi^{AA}\boldsymbol{k}^2 + C^2\chi^{VV}\chi^{AA}(\boldsymbol{k}\cdot\boldsymbol{B})^2$ **: Mixing CMW and NG mode!**

Summary

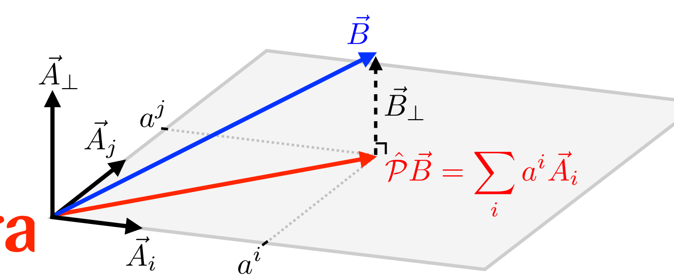
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism: $\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)$

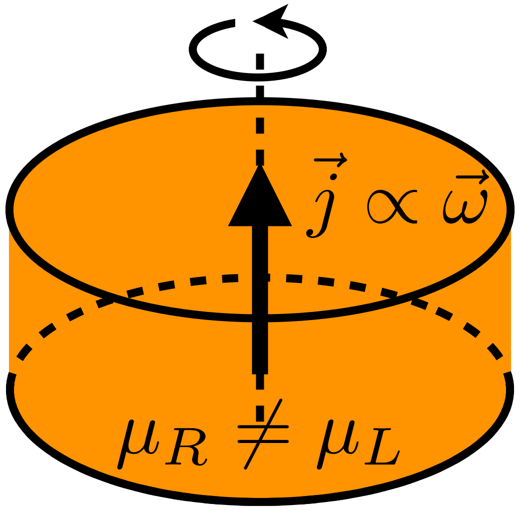
Anomalous superfluid: mixing between CMW and NG mode

Outlook

Outlook

◆ Chiral vortical effect

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\omega}$$



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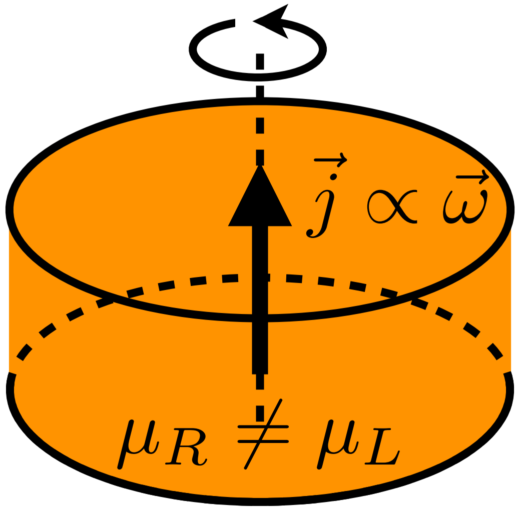
Originated from chiral anomaly



Outlook

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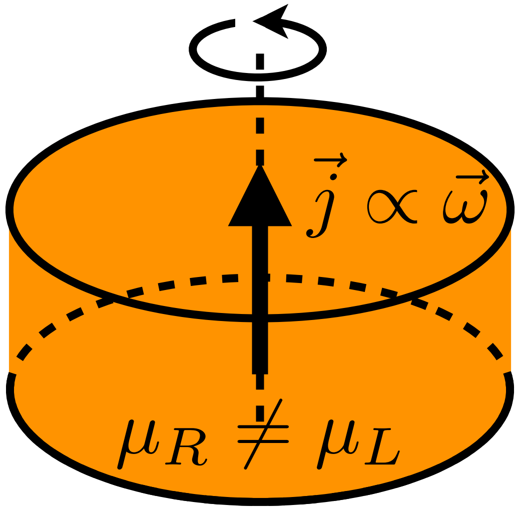
Originated from chiral anomaly

Origin of this term?



Outlook

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Originated from chiral anomaly

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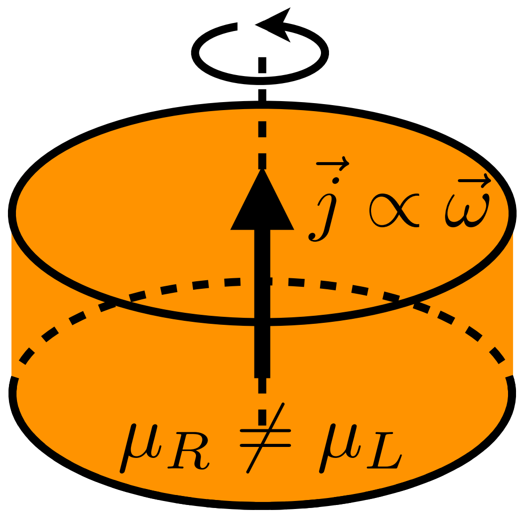


◆ Path-integral treatment?

	Operator formalism	Path-integral formalism
QCD	CA w/ Anomalous CR	Chiral pert. w/ Wess-Zumino term
Hydro	Mori's projection w/ Anomalous CR	MSRJD effective lagrangian w/ ? ? [Crossley et al. (2015)]

Back up

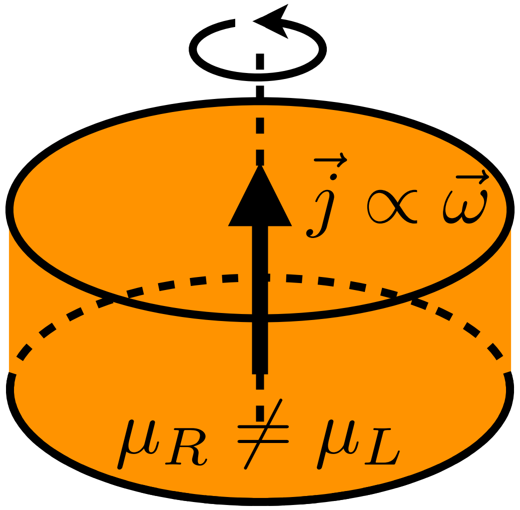
Outlook

◆ Chiral vortical effect

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \left(\frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\right)\vec{\omega}$$

Originated from chiral anomaly

Origin of this term?



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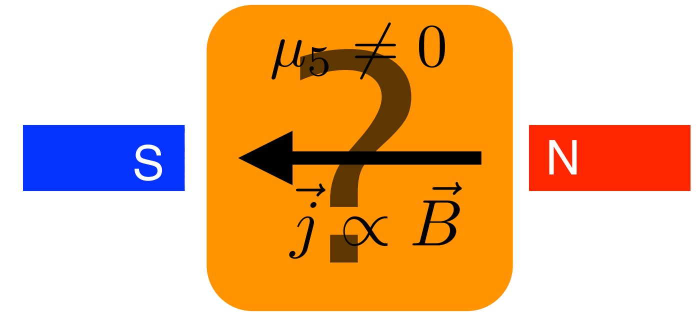
	Operator formalism	Path-integral formalism
QCD	CA w/ Anomalous CR	Chiral pert. w/ Wess-Zumino term
Hydro	Mori's projection w/ Anomalous CR	MSRJD effective lagrangian w/ ? ? [Crossley et al. (2015)]

Summary

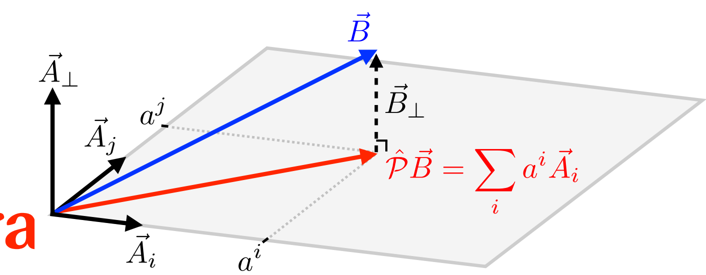
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism: $\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2} B^i(x)$

Anomalous superfluid: mixing between CMW and NG mode

Towards anomalous superfluid

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(x), \hat{T}^0{}_i(x), \hat{J}^0(x), \hat{J}^0_5(x), \hat{\pi}(x)\}$

Current algebra with Anomaly & SSB

- $\chi^{\pi\pi}(\boldsymbol{k}) = \boldsymbol{k}^2 + O(\boldsymbol{k}^4)$
- $\langle[\hat{J}^0_5(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})]\rangle = \sigma_0\delta(\boldsymbol{x}-\boldsymbol{y})$
- **Others: the same as before**

◆ **EoM for** $\{\hat{J}^0(x), \hat{J}^0_5(x), \hat{\Phi}(x)\}$

$$\partial_0 \hat{J}^0_V(x) + \partial_i\left(C\chi^{AA}B^i\hat{J}_A\right) + \cdots = 0$$

$$\partial_0 \hat{J}^0_A(x) + \partial_i\left(C\chi^{VV}\hat{J}_V(x)B^i(x) - \sigma_0\partial_i\hat{\pi}(x)\right) + \cdots = 0$$

$$\partial_0\hat{\pi}(x) + \chi^{AA}\sigma_0\hat{J}_A + \cdots = 0$$

➡ $\omega^2 = \sigma_0^2\chi^{AA}\boldsymbol{k}^2 + C^2\chi^{VV}\chi^{AA}(\boldsymbol{k}\cdot\boldsymbol{B})^2$ **: Mixing CMW and NG mode!**

Symmetry of QCD under B

◆ **Symmetry structure of chiral limit QCD under magnetic field**

Explicitly broken by magnetic field	Spontaneously broken	
$SU(2)_R \times SU(2)_L \rightarrow$	$U(1)^3_V \times U(1)^3_A$	$\rightarrow U(1)^3_V$

Original symmetry of QCD

➡ **1 SSB = 1 NG mode = π_0 (=Neutral pion) appears!**

cf. Spontaneous symmetry breaking & Nambu-Goldstone mode

For some conserved charge $\hat{Q}_a$

$$\exists\, \hat{\pi}_i(x) \text{ such that } \langle [i\hat{Q}_a, \hat{\pi}_i(x)] \rangle = \mathrm{Tr}\big(\hat{\rho}[i\hat{Q}_a, \hat{\pi}_i(x)]\big) \neq 0$$

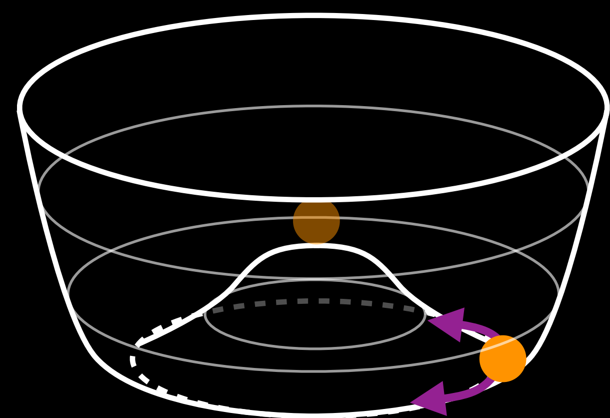
Spontaneous Symmetry Breaking (SSB)

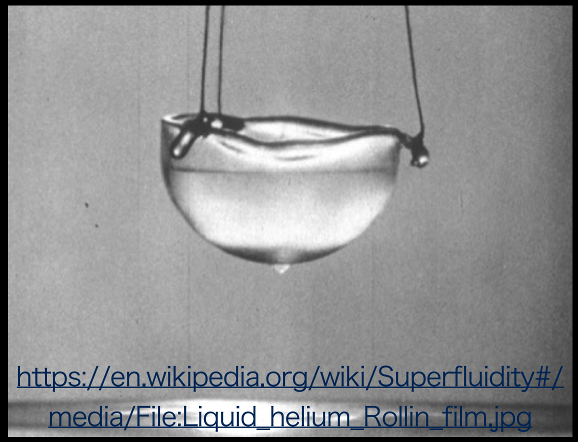
➡ **This can be also captured by the projection operator method!**

Anomalous superfluid?

- **Spontaneous symmetry breaking**

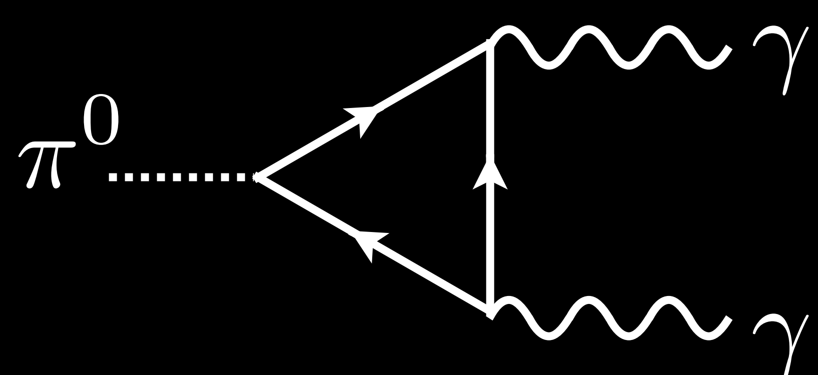
Micro：Selecting vacuum | **Macro：Superfluid**



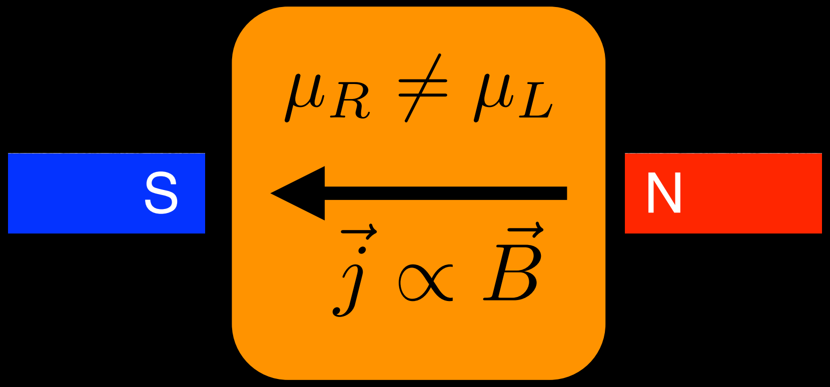


- **Symmetry breaking by quantum anomaly**

Micro：π^0 decay | **Macro：Anomalous transport**



[Adller (1969), Bell-Jackiw (1969)]



[Erdmenger et al. (2008), Son-Surowka (2009)]

Chiral Magnetic **Wave** (CMW)

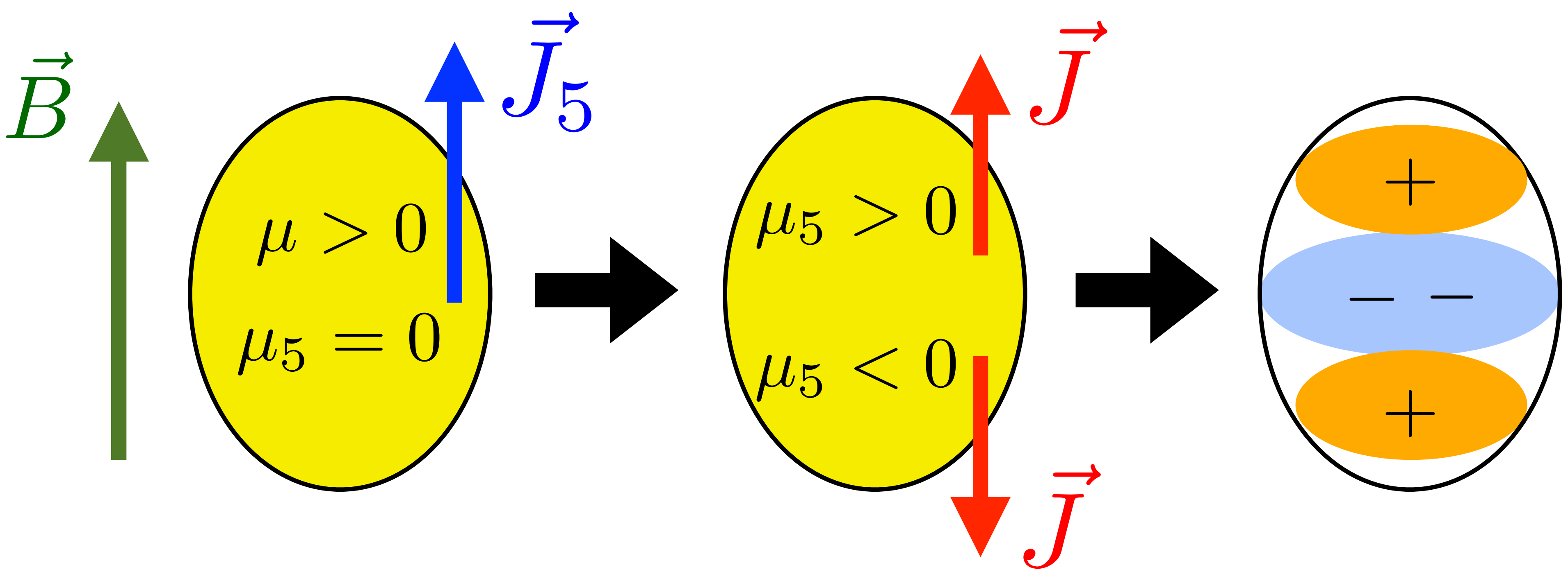
Chiral Magnetic Effect

$$\hat{J}^i(x) = \frac{\chi^{nn_5}\hat{J}_5^0(x)}{2\pi^2}B^i(x)$$

Chiral Separation Effect

$$\hat{J}_5^i(x) = \frac{\chi^{n_5 n}\hat{J}^0(x)}{2\pi^2}B^i(x)$$

Ex.



Charge propagation along B

||

Collective excitation

→ **Chiral Magnetic Wave** = Analogue of Charge density wave In Tomonaga-Luttinger liquid

[Kharzeev, Yee, (2011)]

$$[\hat{J}_R^0(t,x), \hat{J}_R^0(t,y)] = -iC\partial_x\delta(x-y)$$

CME from anomalous commutation

$$\partial_0 \hat{J}^0(x) + \partial_i^x \underbrace{\left[\frac{\chi^{nn_5} \hat{J}_5^0(x)}{2\pi^2} B^i(x) \right]}_{= \hat{J}^i(x)} + \cdots = 0$$

◆ **Summary of result**

- **Conservation law:** $\partial_\mu \hat{J}^\mu(x) = 0$
- **Const. relation:** $\hat{J}^i(x) = \dfrac{\chi^{nn_5} \hat{J}_5^0(x)}{2\pi^2} B^i(x)$ ⬅ **Chiral Magnetic Effect (CME)**

CME from anomalous commutation

For EoM: $\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$

Choose $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x}), J^0(t,\boldsymbol{x}), J^0_5(t,\boldsymbol{x})\}$

Current algebra with anomalous commutation rel.

$$\begin{cases} [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -i\hat{J}^0(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})] = -i\hat{J}^0_5(t,\boldsymbol{x})\partial_j\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = -\dfrac{i}{2\pi^2}B^i(t,\boldsymbol{y})\partial^x_i\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})] = [\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})] = 0 \end{cases}$$

◆ **EoM for** $\hat{J}^0(t,\boldsymbol{x})$

$$\partial_0\hat{J}^0(x) + \partial^x_i\left[\frac{\chi^{nn_5}\hat{J}^0_5(x)}{2\pi^2}B^i(x)\right] + \cdots = 0$$

Perfect fluid from Mori's method

◆ Relativistic hydrodynamic from Mori's method

[Minami-Hidaka (2013)]

$$\partial_0 \delta \hat{T}^0{}_0 = -ik^i \delta \hat{T}^0{}_i$$

$$\partial_0 \delta \hat{T}^0{}_i = -ik_i h_{\mathrm{eq}} \chi^{ee} \delta \hat{T}^0{}_0 - \left[k_i k^k \left(\frac{\zeta}{h_{\mathrm{eq}}} + \frac{d-3}{d-1} \frac{\eta}{h_{\mathrm{eq}}} \right) + \boldsymbol{k}^2 \delta_i^k \frac{\eta}{h_{\mathrm{eq}}} \right] \delta \hat{T}^0{}_k + \hat{R}_{\pi_i}$$

◆ Green-Kubo formula for transport coefficients (viscosity)

$$\zeta = \beta_{\mathrm{eq}} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{p}(0, \boldsymbol{0}))$$

$$\eta = \frac{\beta_{\mathrm{eq}}}{(d+1)(d-2)} \int_0^\infty dt \int d^{d-1}x (e^{\hat{\mathcal{Q}} i \hat{\mathcal{L}} t} \hat{\mathcal{Q}} \delta \hat{\pi}_{ik}(0, \boldsymbol{x}), \hat{\mathcal{Q}} \delta \hat{\pi}_{jl}(0, \boldsymbol{0})) \Delta^{ij} \Delta^{kl}$$

➡ **Reversible → Sound wave / Dissipative → Diffusion mode**

Mori's method and current algebra

◆ **Leading Order term in EOM**

$$\partial_0 \hat{A}_n(t) = -i\chi^{lm}\langle[\hat{A}_n(0), \hat{A}_m(0)]\rangle \hat{A}_l(t) + \cdots$$ (χ^{lm} : inv. suscep.)

◆ **Current algebra related to relativistic hydrodynamics**

Choose $\hat{A}_n(t)$ **as conserve charges:** $\hat{A}_n(t) = \{\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{x})\}$

➡ **EoM(LO) is controlled by energy-momentum density algebra!**

Current algebra

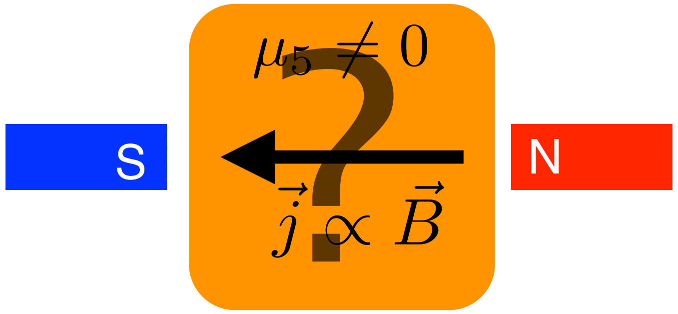
$$\begin{cases} [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_0(t,\boldsymbol{y})] = -i\big(\hat{T}^k{}_0(t,\boldsymbol{x}) + \hat{T}^k{}_0(t,\boldsymbol{y})\big)\partial_k\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_0(t,\boldsymbol{x}), \hat{T}^0{}_i(t,\boldsymbol{y})] = -i\big(\hat{T}^j{}_i(t,\boldsymbol{x})\partial_j - \hat{T}^0{}_0(t,\boldsymbol{y})\partial_i\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \\ [\hat{T}^0{}_i(t,\boldsymbol{x}), \hat{T}^0{}_j(t,\boldsymbol{y})] = -i\big(\hat{T}^0{}_j(t,\boldsymbol{x})\partial_i + \hat{T}^0{}_i(t,\boldsymbol{y})\partial_j\big)\delta(\boldsymbol{x}-\boldsymbol{y}) \end{cases}$$

➡ **EoM for perfect fluid (Sound wave) is derived!!**

Outline

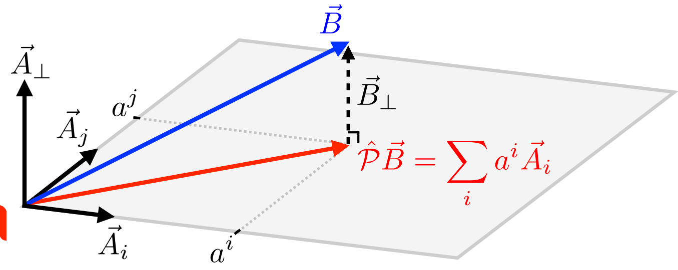
MOTIVATION:

Origin of chiral transport (Chiral Magnetic Effect)?



APPROACH:

Mori's method as a generalization of current algebra



Anomalous commutation: $\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y}) \partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$

RESULT:

Chiral Magnetic Effect in operator formalism:

Anomalous superfluid

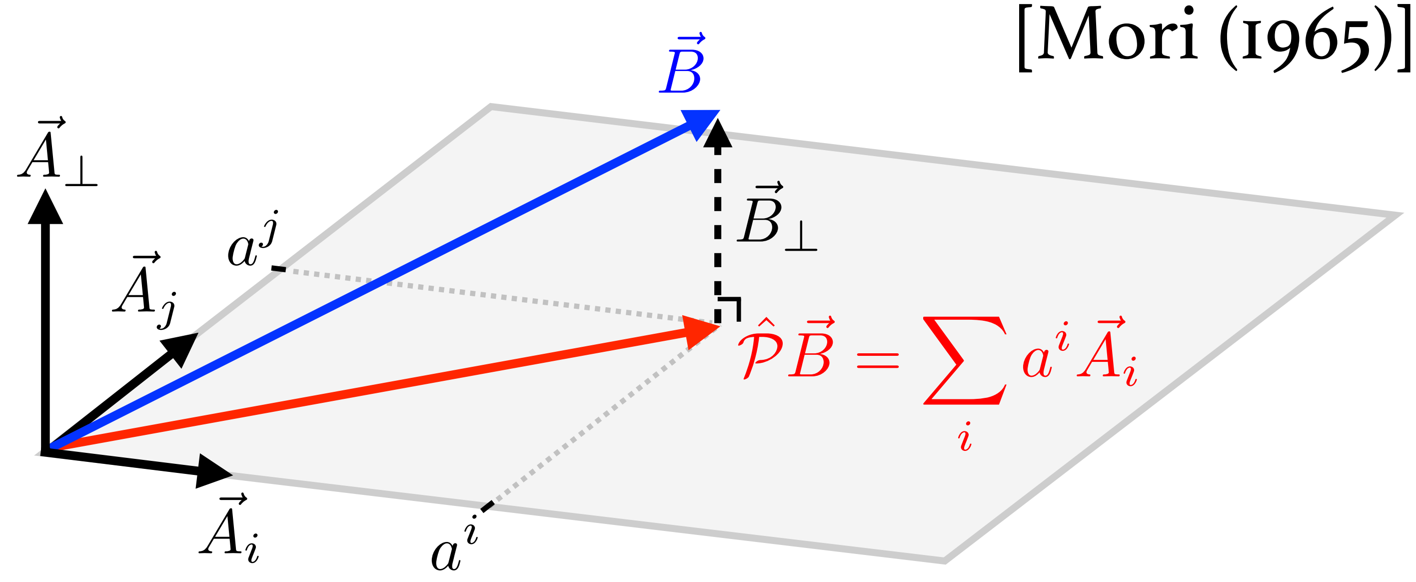
Mori's projection operator method

[Mori (1965)]

A method to write down

Equation of Motion (EoM)

only focusing on $\hat{A}_n(t)$



◆ **EoM given by Mori's projection operator method**

$$\partial_0 \hat{A}_n(t) = i\Omega_n{}^m \hat{A}_m(t) - \int_0^t ds \Phi_n{}^m(t-s)\hat{A}_m(s,\boldsymbol{y}) + \hat{R}_n(t)$$

Reversible **Dissipative** **Noise**

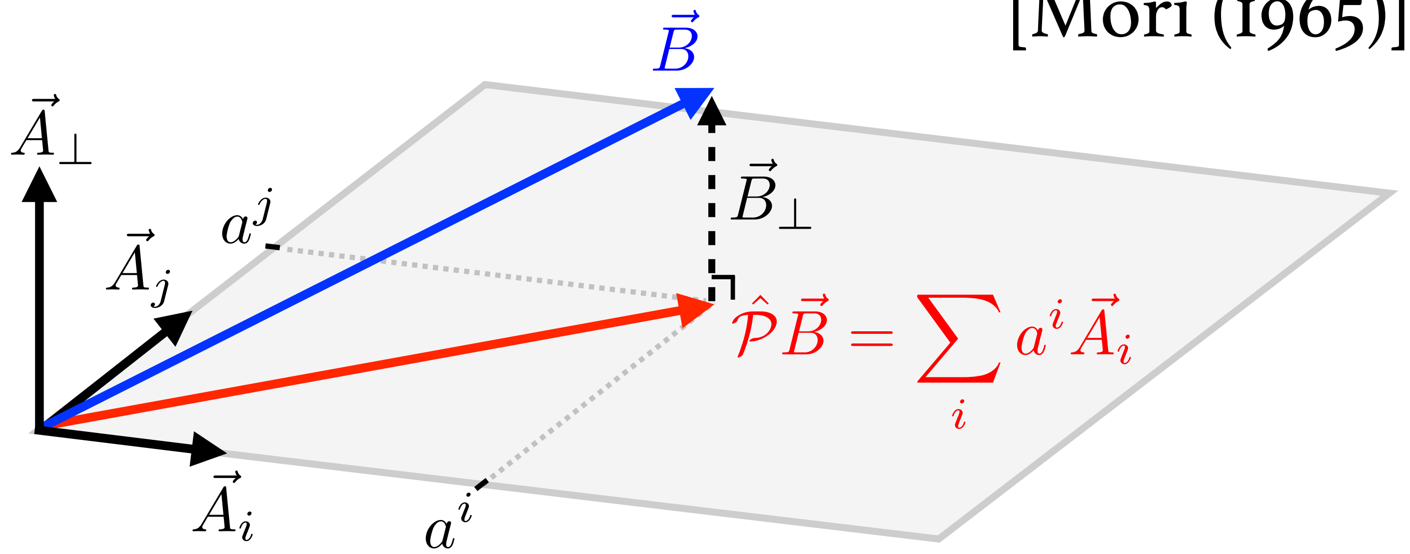
$$i\Omega_n{}^m = -\frac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0))$$

Fluctuation Dissipation relation: $\Phi_n^m(t-s) = (\hat{R}_n(t-s), \hat{R}^m(0))$

Mori's projection operator method

[Mori (1965)]

A method to write down
Equation of Motion (EoM)
only focusing on $\hat{A}_n(t)$



◆ EoM given by Mori's projection operator method

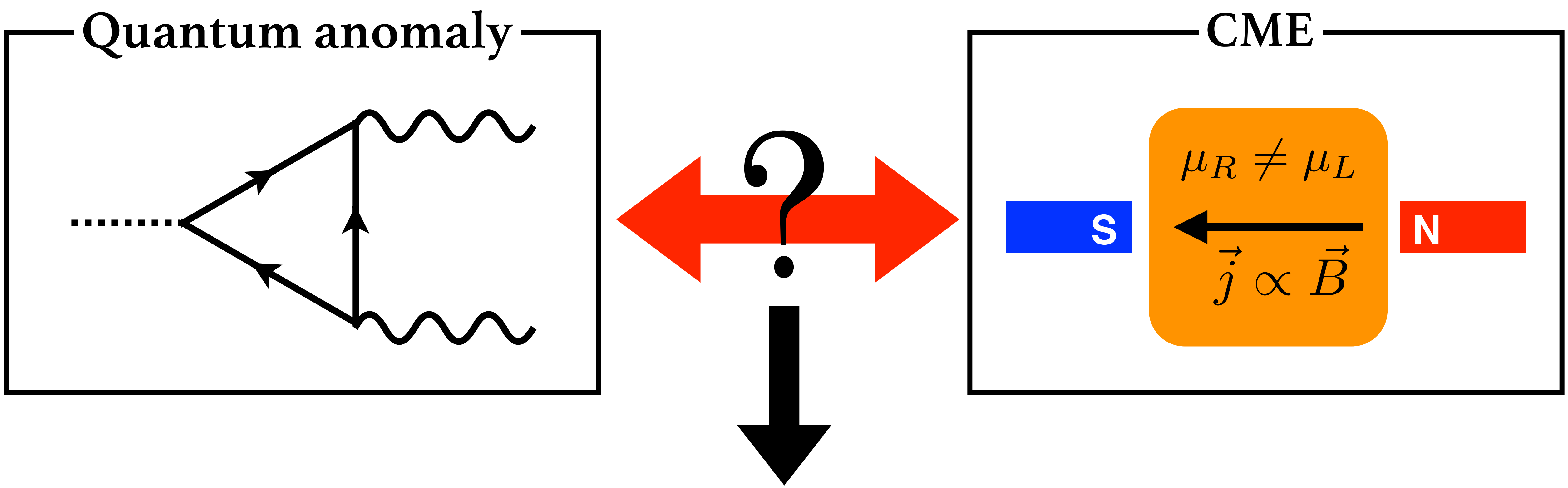
$$\partial_0 \hat{A}_n(t) = i\Omega_n{}^m \hat{A}_m(t) - \int_0^t ds \Phi_n{}^m(t-s)\hat{A}_m(s,\boldsymbol{y}) + \hat{R}_n(t)$$

Reversible — Dissipative — Noise

$$i\Omega_n{}^m = -\frac{i}{\beta}\langle[\hat{A}_n(0), \hat{A}^m(0)]\rangle + i\mu\big([\hat{N}, \hat{A}_n(0)], \hat{A}^m(0)\big)$$

Fluctuation Dissipation relation: $\Phi_n^m(t-s) = \big(\hat{R}_n(t-s), \hat{R}^m(0)\big)$

Anomaly and chiral transport



Can we understand this based on current algebra?

Problem.

Not vacuum physics as is the case for QCD!

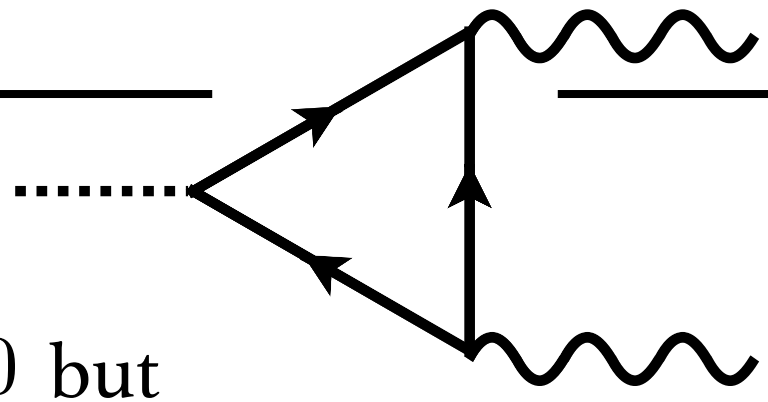
$\rightarrow$ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0$$

$$\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = -\frac{i}{2\pi^2} B^i(t,\boldsymbol{y})\partial^x_i \delta(\boldsymbol{x}-\boldsymbol{y})$$



__Sketch of Proof.__

Ward-Takahashi identity is not $\langle \partial_\mu J^\mu_5(x)\rangle_A = 0$ but

$$\langle \partial_\mu J^\mu_5(x)\rangle_A = C\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x) \sim CdAdA$$

Variation w.r.t A_0 **gives** $\partial_\mu \langle J^\mu_5(x) J^0(y)\rangle_A \sim CddA \sim CdB$

"Corr. function = T-product in operator formalism" gives the above □

Current algebra and chiral anomaly

◆ **Current algebra in external EM fields for** $U(1)_V \times U(1)_A$

$$\left[\hat{J}^0(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = \left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0_5(t,\boldsymbol{y})\right] = 0$$

$$\left[\hat{J}^0_5(t,\boldsymbol{x}), \hat{J}^0(t,\boldsymbol{y})\right] = 0$$

Proof.

Definition of Noether current gives

$$\hat{J}^0(x) = -\frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} i\hat{\phi} = -i\hat{\pi}(x)\hat{\phi}(x), \quad \hat{J}^0_5(x) = -i\hat{\pi}(x)\gamma_5\hat{\phi}(x)$$

Using canonical commutation relation $\left[\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{y})\right] = i\delta(\boldsymbol{x}-\boldsymbol{y})$

we can directly show the above current algebraic structure! □

Review: Current algebra for QCD

◆ **Current algebra for** $SU(N)_R \times SU(N)_L$

$$\left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{L,b}(x)\right] = i f^{c}{}_{ab} \hat{J}^{\mu}_{L,c}(x), \quad \left[\hat{Q}_{L,a}, \hat{J}^{\mu}_{R,b}(x)\right] = 0$$

$$\left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{R,b}(x)\right] = i f^{c}{}_{ab} \hat{J}^{\mu}_{R,c}(x), \quad \left[\hat{Q}_{R,a}, \hat{J}^{\mu}_{L,b}(x)\right] = 0$$

Low-energy theorem

- Goldberger-Treiman relation
- Soft Pion theorem, $\cdots$

Universal results for process with low-energy pion scattering!

If current algebra satisfies the above relations,

it does not matter whether UV theory is QCD, NJL model, or anything!

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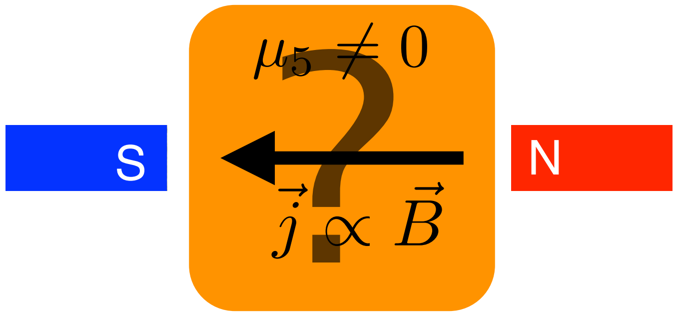

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RESULT:

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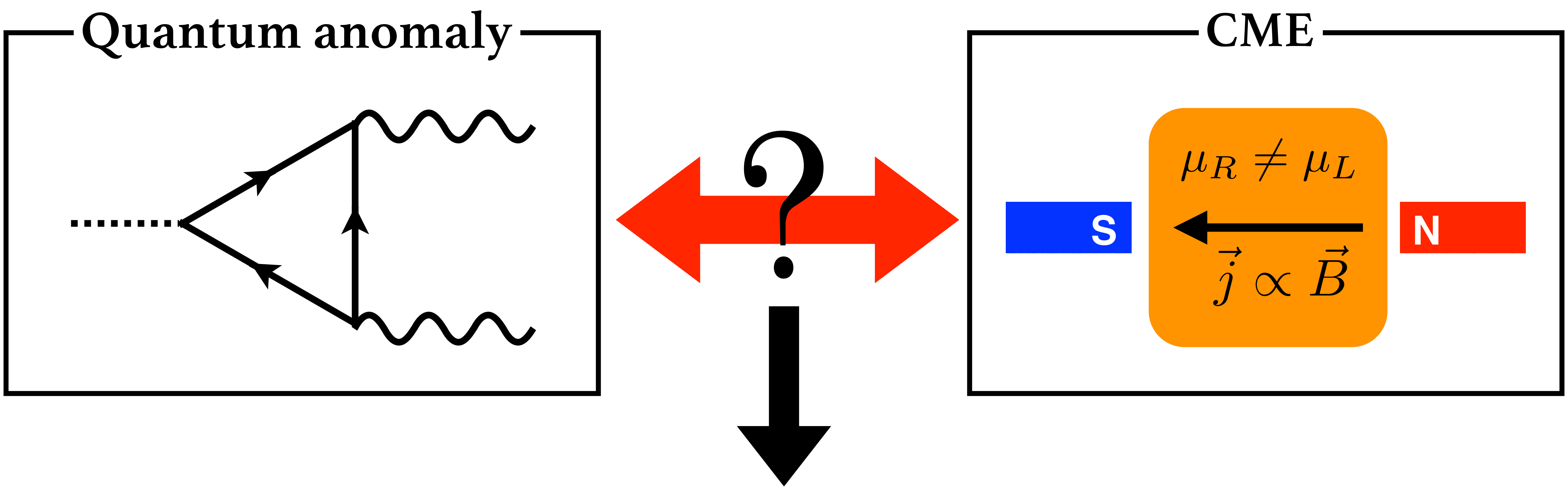
Anomaly and chiral transport



Can we understand this based on current algebra?

Problem.

Not vacuum physics as is the case for QCD!

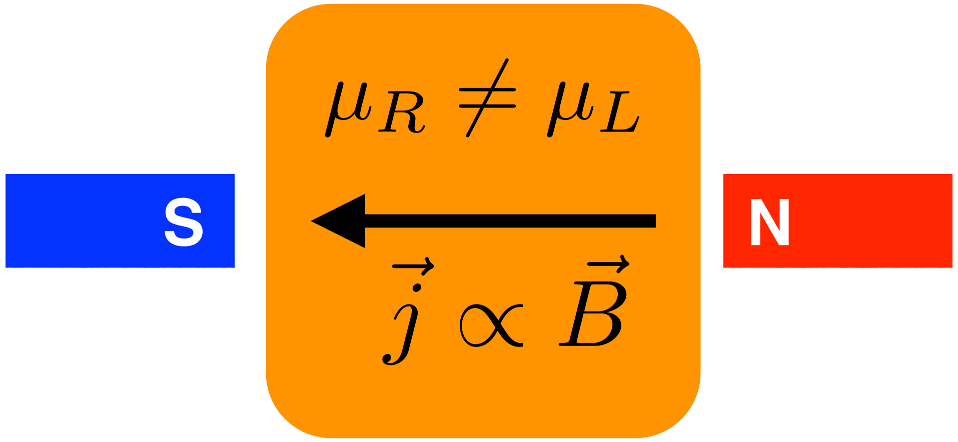
$\rightarrow$ We have to generalize current algebra for $T \neq 0, \mu \neq 0$

Parity-violating chiral transport

[Fukushima et al. (2008), Vilenkin (1980)]

◆Chiral Magnetic Effect (CME)

$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$



[Erdmenger et al. (2008), Son-Surowka (2009)]

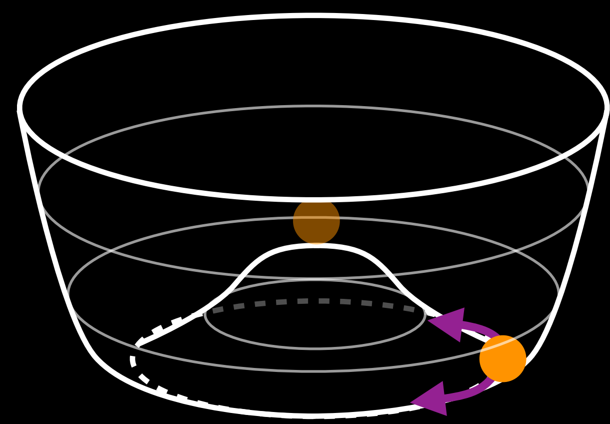
◆ Chiral Vortical Effect (CVE)

$$\vec{J} = \frac{\mu\mu_5}{2\pi^2}\vec{\omega}$$

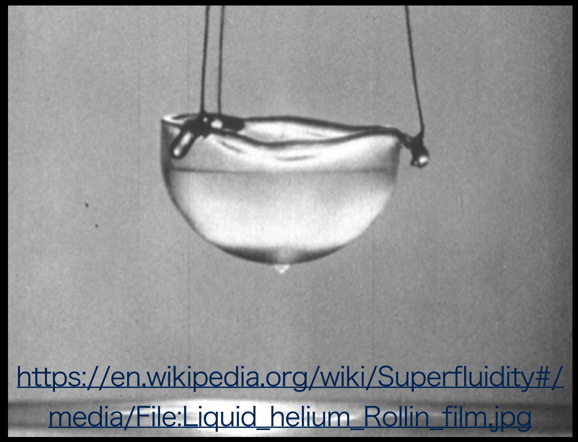
Symmetry breaking and Hydro

- Spontaneous symmetry breaking

Micro：Selecting vacuum

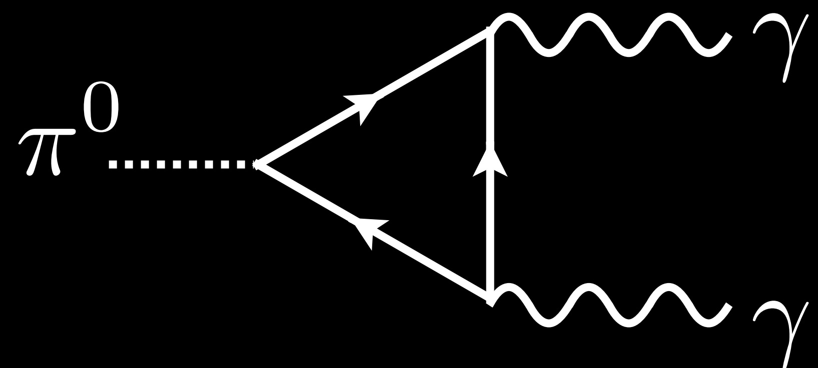


Macro：Superfluid



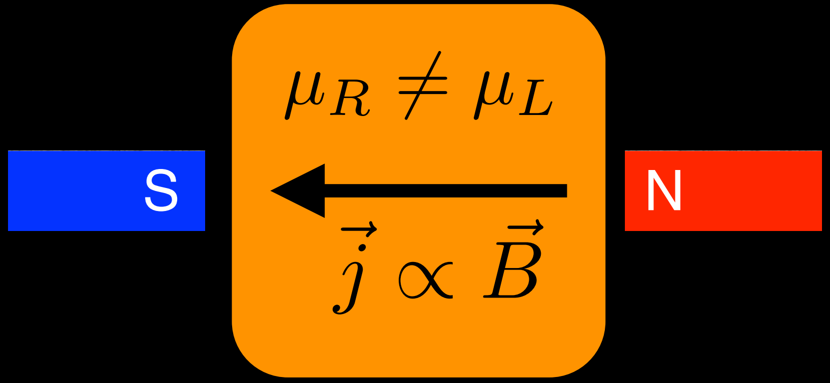
- Symmetry breaking by quantum anomaly

Micro：π^0 decay



[Adller (1969), Bell-Jackiw (1969)]

Macro：Anomalous transport

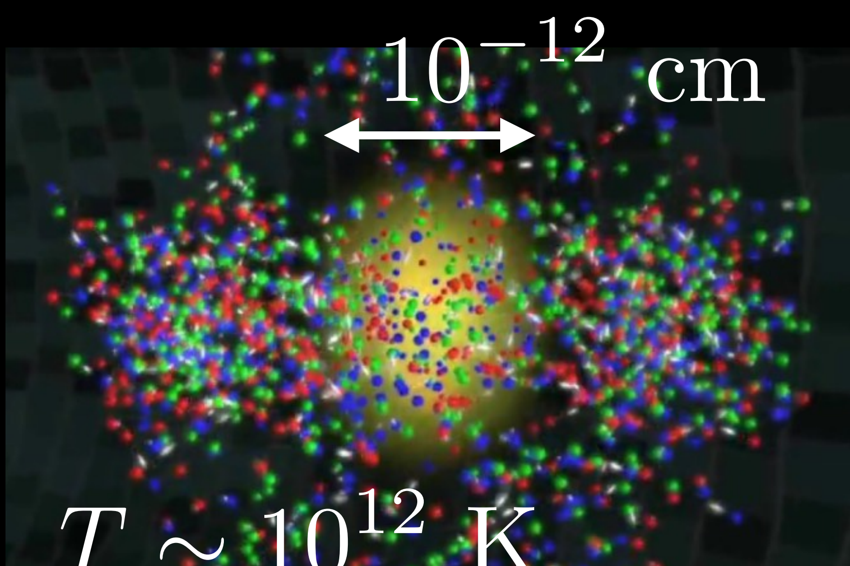


[Erdmenger et al. (2008), Son-Surowka (2009)]

Hydrodynamics is

- **Effective theory** for **macroscopic dynamics**
- **Universal** description, not depending on details
- Only **conserved quantity ~ ~~symmetry~~** of system

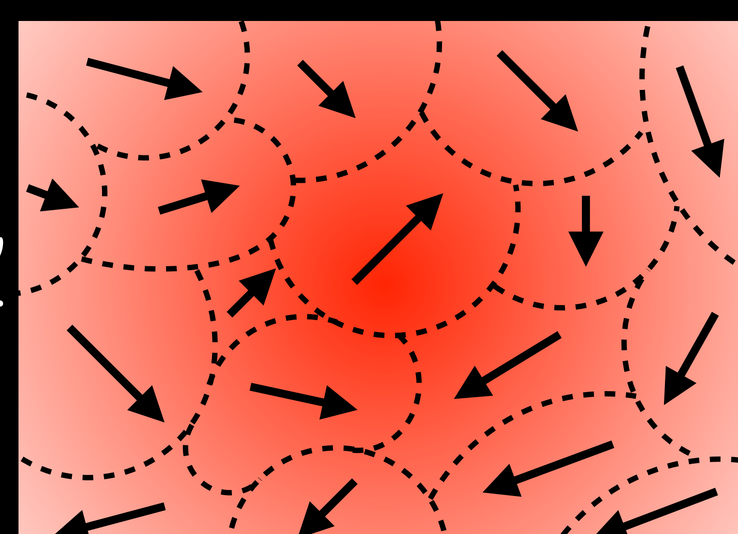
Quark-Gluon Plasma



http://www.bnl.gov/rhic/news2/news.asp?a=1403&t=pr

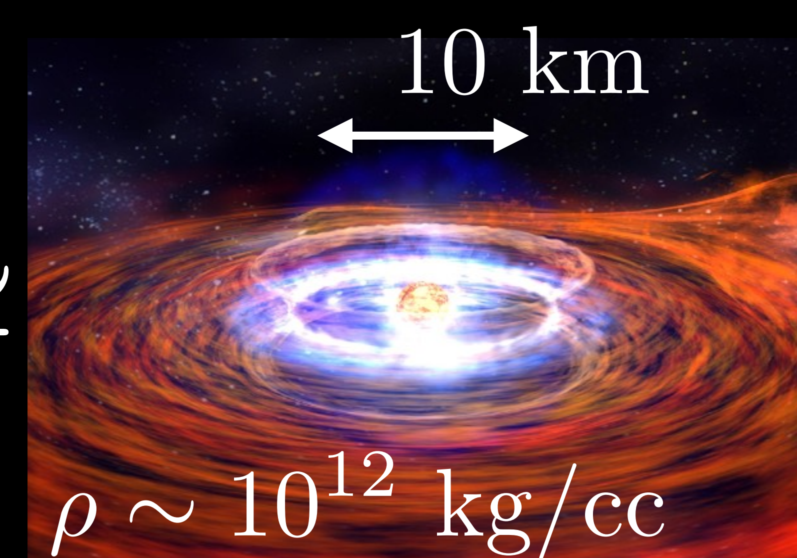
$\simeq$

Hydro: $\{\beta(x),\ \vec{v}(x)\}$



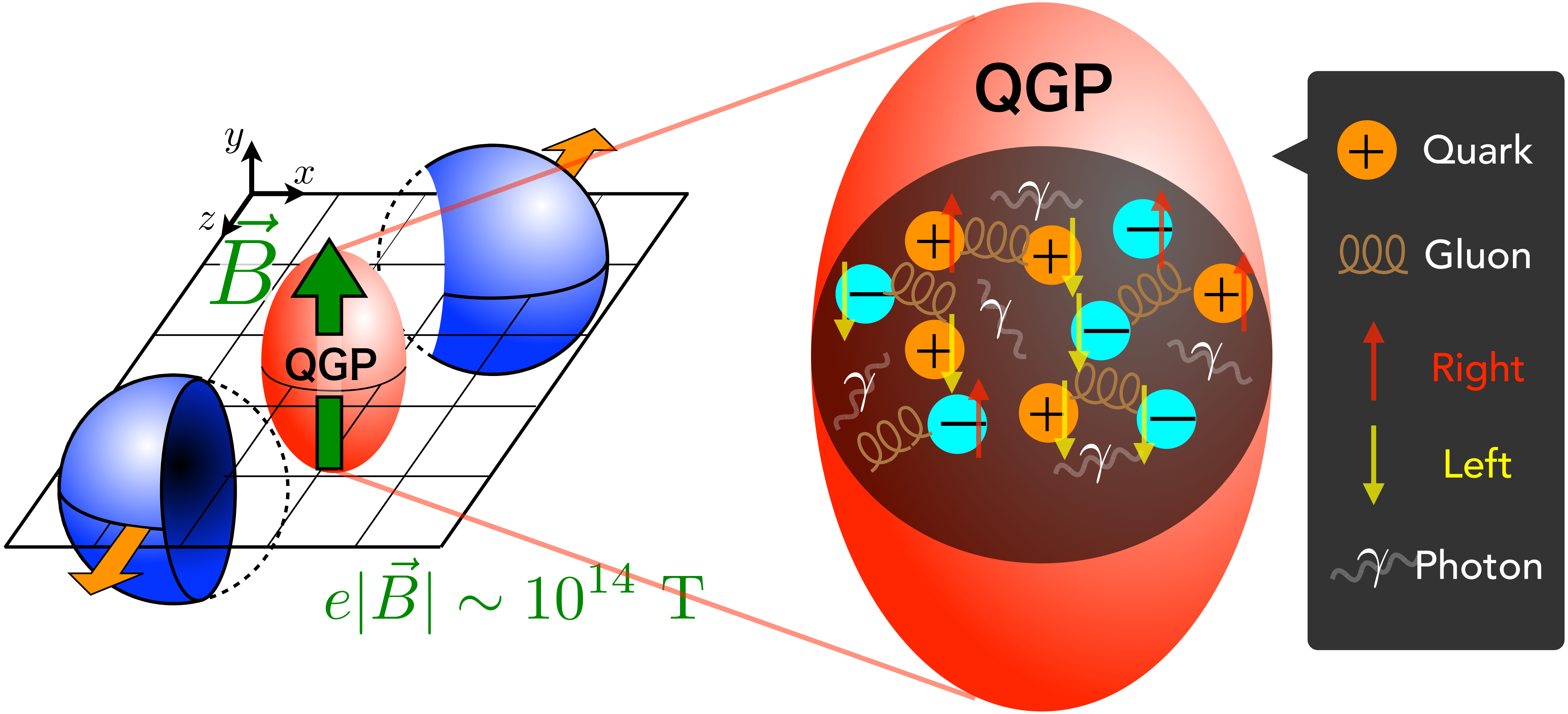
$\simeq$

Neutron Star



http://newsoffice.mjitugenn.edu/2012/model-bursting-star-0302

QGP as Chiral fluid



- Existence of extremely strong magnetic field
- Chirality drastically affect hydrodynamic transport

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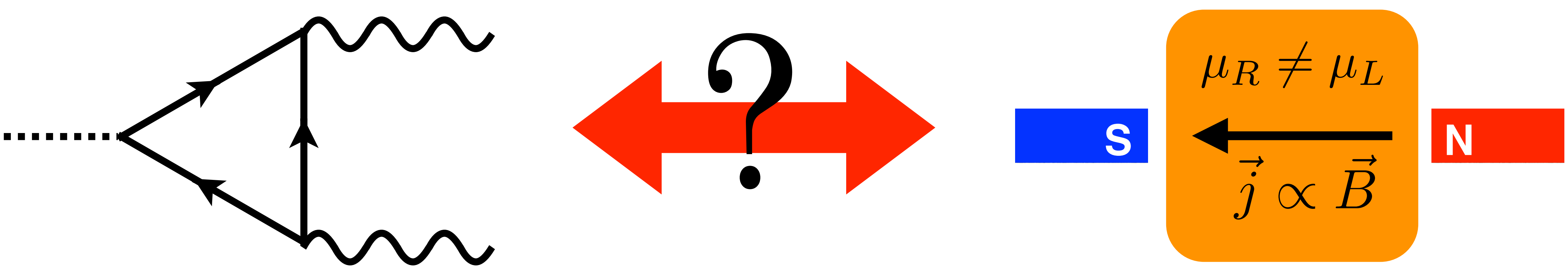

Masaru Hongo (RIKEN, iTHMES)

Quark Matter 2018 The 27th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions, 2018 May 16th, Lido

Collaborators: Noriyuki Sogabe, Naoki Yamamoto