Hydrodynamization and Attractors at Intermediate Coupling

Ben Meiring
Jorge Casalderrey-Solana
Nikola Gushterov

University of Oxford
ben.meiring@physics.ox.ac.uk

May 2018

Motivation & Questions

- Why does Hydrodynamics work outside it's regime of applicability?

Motivation & Questions

- Why does Hydrodynamics work outside it's regime of applicability?
- What qualitative changes do we observe as we interpolate between gauge theories at infinite and finite coupling?

Motivation & Questions

- Why does Hydrodynamics work outside it's regime of applicability?
- What qualitative changes do we observe as we interpolate between gauge theories at infinite and finite coupling?

We will be studying the $T_{\mu\nu}$ of strongly coupled, conformal, boost invariant field theories that relax to Bjorken Flow at late times.
Our Paper: [1712.02772]

Motivation & Questions

- Why does Hydrodynamics work outside it's regime of applicability?
- What qualitative changes do we observe as we interpolate between gauge theories at infinite and finite coupling?

We will be studying the $T_{\mu\nu}$ of strongly coupled, conformal, boost invariant field theories that relax to Bjorken Flow at late times. Our Paper: [1712.02772]

This amounts to studying the Anisotropy $R(w)$ as a function of the dimensionless parameter $w = \tau T$.

$$R(w) = \frac{P_T - P_L}{P_{\text{Tot}}}. \tag{1}$$

Cartoon of Heavy Ion Collision: Scenario 1

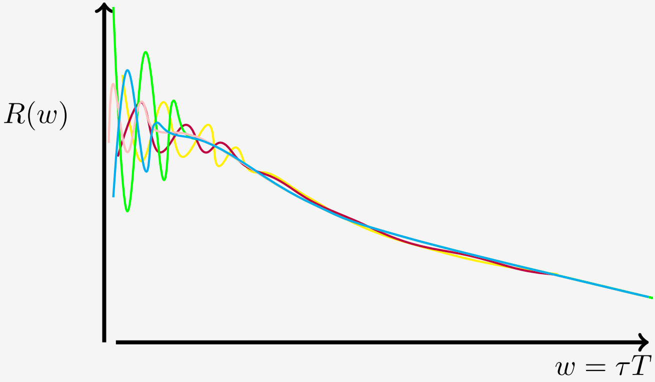


Figure: Early system dynamics has a microscopic description that relaxes to a common curve, called the Attractor. The late time dynamics should be described by Hydrodynamics when gradients are small ($w = \tau T > 1$).

Cartoon of Heavy Ion Collision: Scenario 1

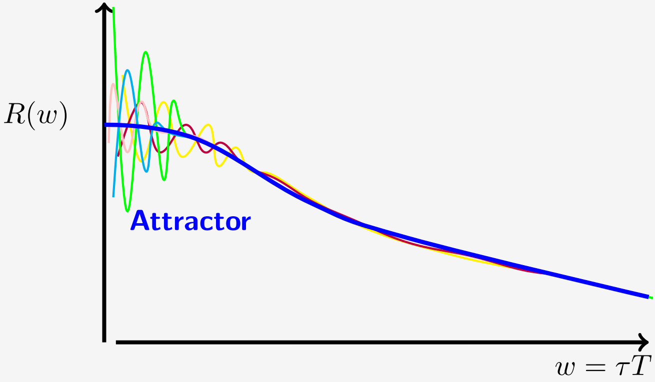


Figure: Early system dynamics has a microscopic description that relaxes to a common curve, called the Attractor. The late time dynamics should be described by Hydrodynamics when gradients are small ($w = \tau T > 1$).

Cartoon of Heavy Ion Collision: Scenario 1

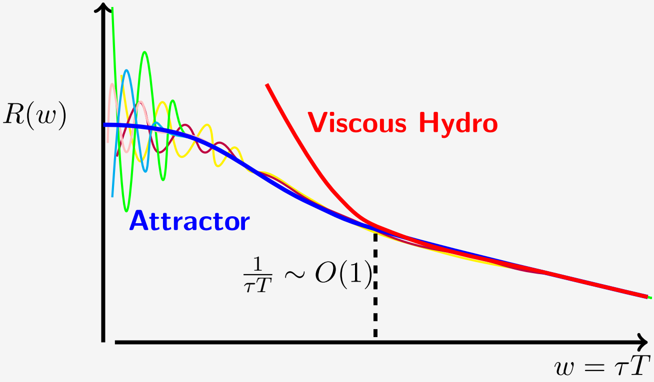


Figure: Early system dynamics has a microscopic description that relaxes to a common curve, called the Attractor. The late time dynamics should be described by Hydrodynamics when gradients are small ($w = \tau T > 1$).

Cartoon of Heavy Ion Collision: Scenario 2

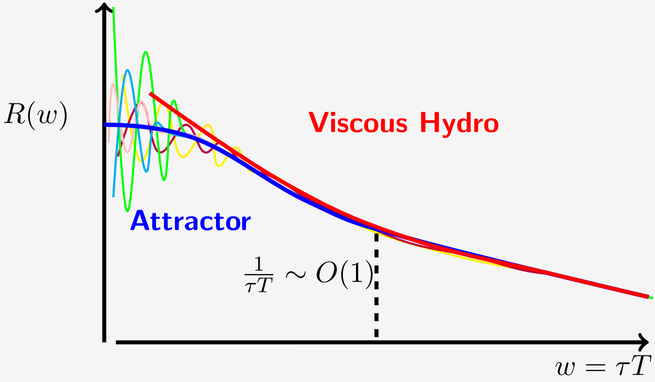


Figure: Early system dynamics has a microscopic description that relaxes to a common curve, called the Attractor. The late time dynamics should be described by Hydrodynamics when gradients are small ($w = \tau T > 1$).

Problem Set-up

- We will use Gauss-Bonnet Gravity (a higher derivative theory of gravity) to study gauge theories at intermediate coupling through the Gauge-Gravity duality.

Problem Set-up

- We will use Gauss-Bonnet Gravity (a higher derivative theory of gravity) to study gauge theories at intermediate coupling through the Gauge-Gravity duality.
- One can find,

$$\frac{\eta}{s} = \frac{1 - \lambda_{GB}}{4\pi} \tag{2}$$

  where λ_{GB} is a parameter of the theory we can tune to pick out our preferred coupling.
  The case $\lambda_{GB} = 0$ is Einstein Gravity ($\mathcal{N} = 4$ SYM) and was studied by M. Heller, R. Janik, P. Witaszczyk. 2013.

Problem Set-up

- We will use Gauss-Bonnet Gravity (a higher derivative theory of gravity) to study gauge theories at intermediate coupling through the Gauge-Gravity duality.
- One can find,
$$\frac{\eta}{s} = \frac{1-\lambda_{GB}}{4\pi} \tag{2}$$
where λ_{GB} is a parameter of the theory we can tune to pick out our preferred coupling.
The case $\lambda_{GB} = 0$ is Einstein Gravity ($\mathcal{N} = 4$ SYM) and was studied by M. Heller, R. Janik, P. Witaszczyk. 2013.
- We calculate the hydrodynamic expansion for the Anisotropy $R(w)$, where powers of $w^{-1} = \frac{1}{\tau T}$ corresponds to orders in gradients:
$$R(w) = \underbrace{r_1 w^{-1}}_{\substack{1^{\text{st}}\text{ order}\\ \text{viscous hydro}}} + \underbrace{r_2 w^{-2} + r_3 w^{-3} + ...}_{\text{Further gradient corrections}} \tag{3}$$

Gravity Solution & Hydrodynamics

The Action for Gauss-Bonnet Gravity is given by,

$$S = \int d^5x \sqrt{-g} \left(R + 12 + \frac{\lambda_{GB}}{2} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right),$$

the solution takes the form of a power series in $\tau^{-2/3} \sim w$:

$$ds^2 = A(r,\tau) d\tau^2 + 2 d\tau dr + B(r,\tau) d\eta^2 + C(r,\tau) dx_\perp^2 \tag{4}$$

where

$$\begin{aligned} A(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3} i} A_i(s) \quad , \; s = r^{-1} \tau^{-1/3} \\ B(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3} i} B_i(s) \\ C(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3} i} C_i(s) \end{aligned}$$

Gravity Solution & Hydrodynamics

The Action for Gauss-Bonnet Gravity is given by,

$$S = \int d^5x \sqrt{-g} \left(R + 12 + \frac{\lambda_{GB}}{2} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right),$$

the solution takes the form of a power series in $\tau^{-2/3} \sim w$:

$$ds^2 = A(r,\tau) d\tau^2 + 2 d\tau dr + B(r,\tau) d\eta^2 + C(r,\tau) dx_\perp^2 \qquad (4)$$

where

$$\begin{aligned} A(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} A_i(s) \quad , \; s = r^{-1}\tau^{-1/3} \\ B(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} B_i(s) \\ C(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} C_i(s) \end{aligned}$$

and reproduces the desired form of the Anisotropy

$$R(w) = r_1 w + r_2 w^2 + r_3 w^3 + \ldots$$

Gravity Solution & Hydrodynamics

The Action for Gauss-Bonnet Gravity is given by,

$$S = \int d^5x \sqrt{-g} \left(R + 12 + \frac{\lambda_{GB}}{2} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right),$$

the solution takes the form of a power series in $\tau^{-2/3} \sim w$:

$$ds^2 = A(r,\tau) d\tau^2 + 2 d\tau dr + B(r,\tau) d\eta^2 + C(r,\tau) dx_\perp^2 \qquad (5)$$

where

$$\begin{aligned} A(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} A_i(s) \quad , s = r^{-1}\tau^{-1/3} \\ B(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} B_i(s) \\ C(\tau,r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} C_i(s) \end{aligned}$$

and reproduces the desired form of the Anisotropy

$$R(w) = r_1 w + r_2 w^2 + r_3 w^3 + \ldots$$

Gravity Solution & Hydrodynamics

The Action for Gauss-Bonnet Gravity is given by,

$$S = \int d^5x \sqrt{-g} \left(R + 12 + \frac{\lambda_{GB}}{2} \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right),$$

the solution takes the form of a power series in $\tau^{-2/3} \sim w$:

$$ds^2 = A(r,\tau) d\tau^2 + 2 d\tau dr + B(r,\tau) d\eta^2 + C(r,\tau) dx_\perp^2 \tag{6}$$

where

$$\begin{aligned} A(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} A_i(s) \quad , \; s = r^{-1} \tau^{-1/3} \\ B(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} B_i(s) \\ C(\tau, r) &\sim \sum_{i=0} \tau^{-\frac{2}{3}i} C_i(s) \end{aligned}$$

and reproduces the desired form of the Anisotropy

$$R(w) = r_1 w + r_2 w^2 + r_3 w^3 + ...$$

Resurgence

The coefficients r_n calculated from our gravity solution shows that each Hydrodynamic series does not converge.

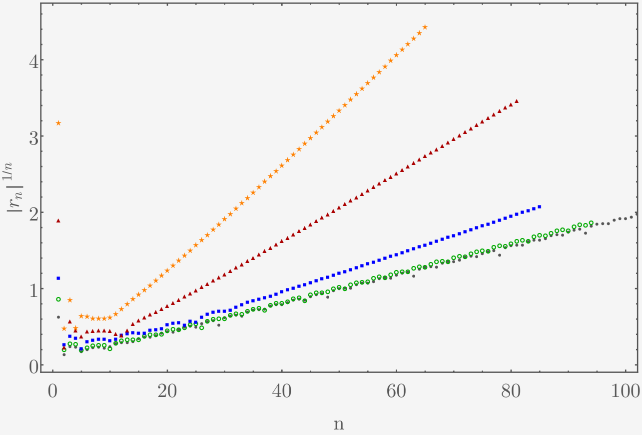


Figure: Anisotropy co-efficients showing $r_n \sim n!$ as a function of order n. r_n are displayed for $\lambda_{GB} = 0$, -0.1, -0.2, -0.5, and -1.

Resurgence

We rewrite the divergent series a Laplace Transform,

$$R(w) = r_1 w^{-1} + r_2 w^{-2} + r_3 w^{-3} + \ldots$$

$$= w \int_0^{\infty e^{i\theta}} du\, e^{-uw} \underbrace{\left(\frac{r_1}{1!} w^{-1} + \frac{r_2}{2!} w^{-2} + \frac{r_3}{3!} w^{-3} + \ldots\right)}_{\text{The Borel Transform } R_B(u)}$$

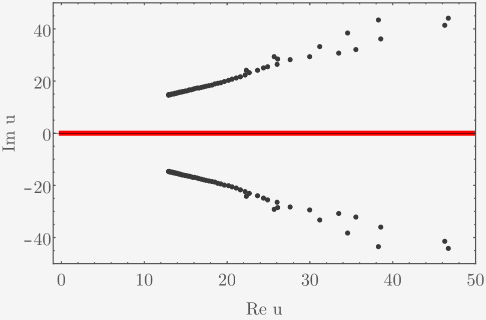
Resurgence

We rewrite the divergent series a Laplace Transform,

$$R(w) = r_1 w^{-1} + r_2 w^{-2} + r_3 w^{-3} + \ldots$$

$$= w \int_0^{\infty e^{i\theta}} du\, e^{-uw} \underbrace{\left(\frac{r_1}{1!} w^{-1} + \frac{r_2}{2!} w^{-2} + \frac{r_3}{3!} w^{-3} + \ldots \right)}_{\text{The Borel Transform } R_B(u)}$$

Any Laplace transform of the Pade Approximant of $R_B(w)$ is meant to be a valid solution, which means that to the linearized level the difference between two contours will also be a solution.



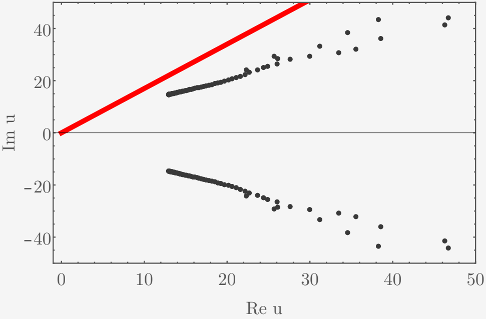
Resurgence

We rewrite the divergent series a Laplace Transform,

$$R(w) = r_1 w^{-1} + r_2 w^{-2} + r_3 w^{-3} + \dots$$

$$= w \int_0^{\infty e^{i\theta}} du\, e^{-uw} \underbrace{\left(\frac{r_1}{1!} w^{-1} + \frac{r_2}{2!} w^{-2} + \frac{r_3}{3!} w^{-3} + \dots\right)}_{\text{The Borel Transform } R_B(u)}$$

Any Laplace transform of the Pade Approximant of $R_B(w)$ is meant to be a valid solution, which means that to the linearized level the difference between two contours will also be a solution.



Resurgence

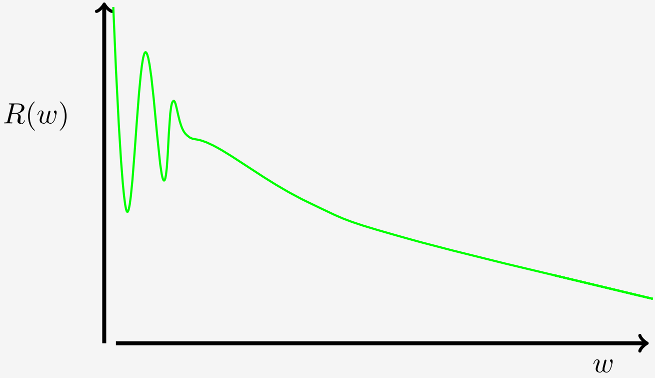


Figure: Choosing one contour leads to one particular evolution. Including linear combinations of the non-hydrodynamic solutions leads to a characteristic spread of solutions. These different choices amount to assigning different initial data to the system.

Resurgence

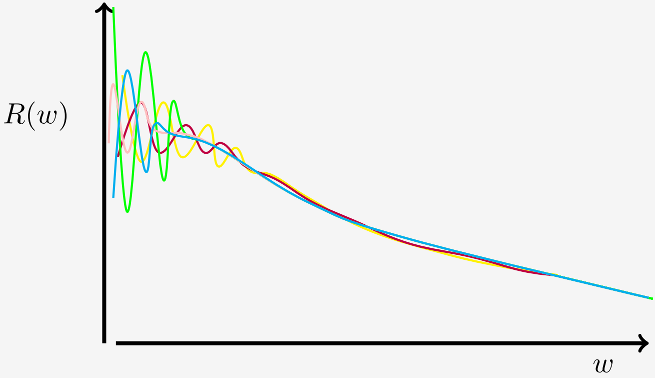


Figure: Choosing one contour leads to one particular evolution. Including linear combinations of the non-hydrodynamic solutions leads to a characteristic spread of solutions. These different choices amount to assigning different initial data to the system.

Resurgence

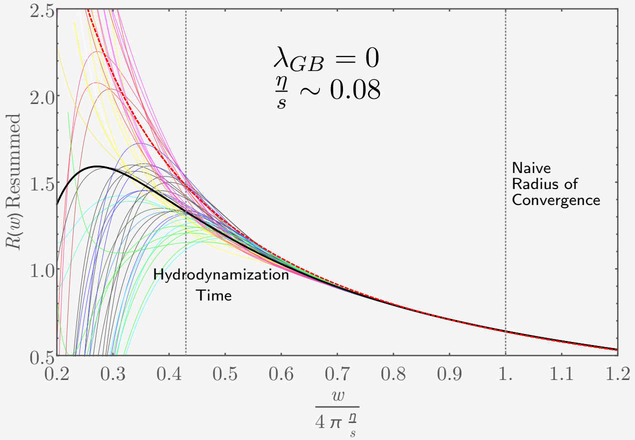


Figure: Resummed $R(w)$ plus non-hydrodynamic solutions with varied initial conditions. We define the Hydrodynamization Time as the w where $R(w)$ deviates from it's first order truncation by 10%. 1^{st} order hydro is given by the red dashed curve.

What are these fluctuations?

What are these fluctuations?

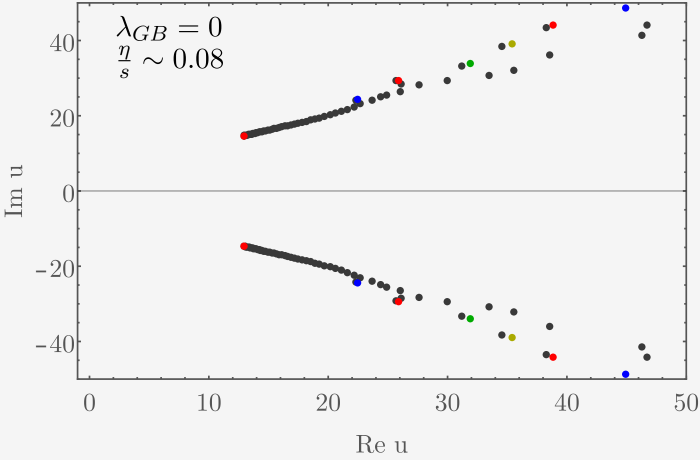


Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = 0$, M. Heller, R. Janik, P. Witaszczyk. 2013. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by A. Starinets. 2002.

Intermediate Coupling Effects

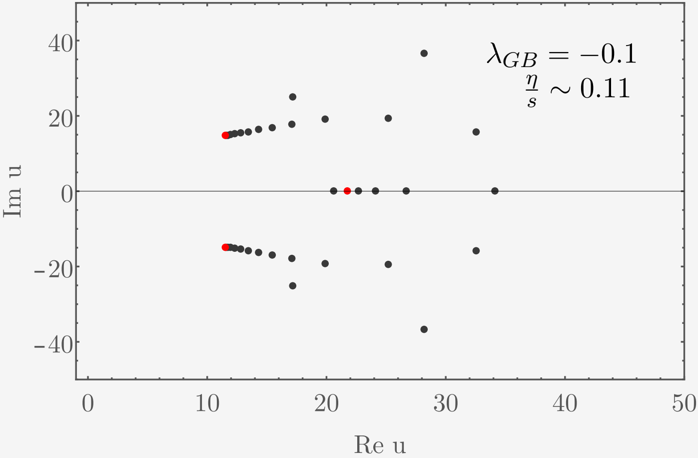


Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -0.1$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

Intermediate Coupling Effects

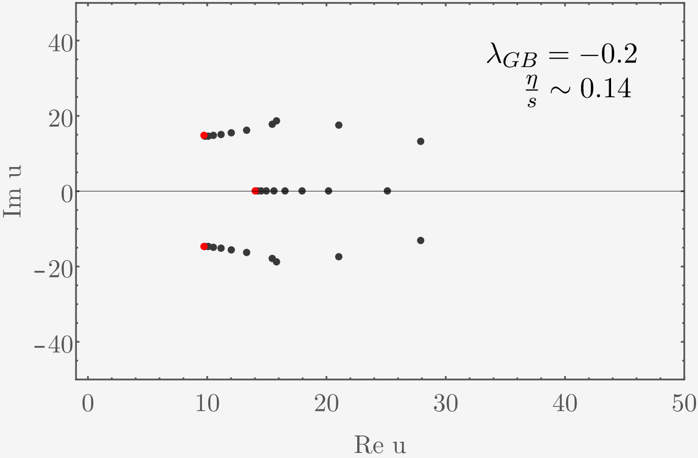


Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -0.2$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

Intermediate Coupling Effects

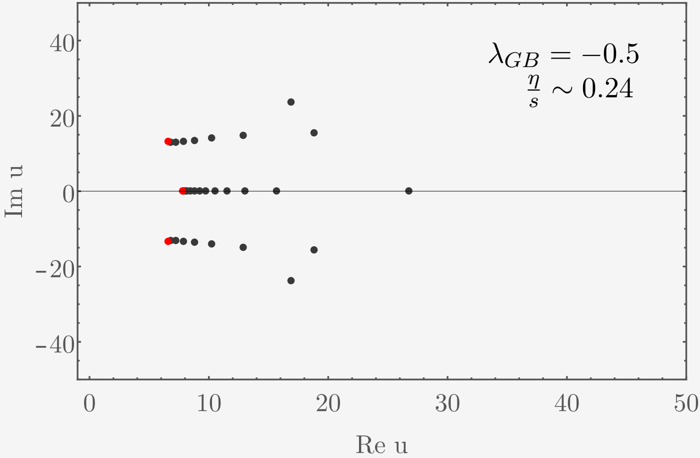


Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -0.5$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

Intermediate Coupling Effects

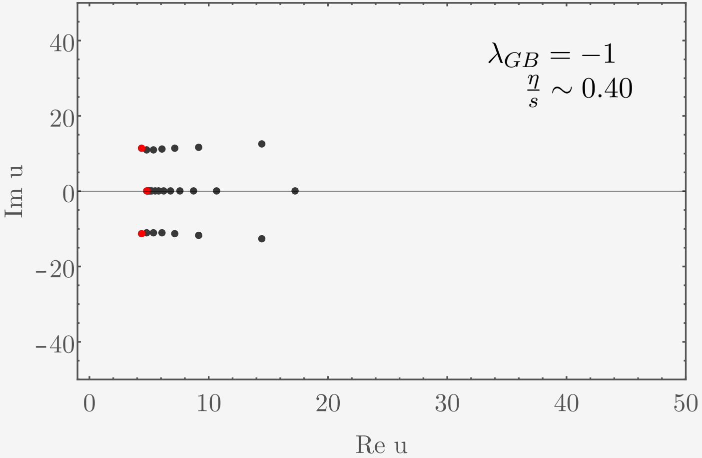


Figure: Non-hydrodynamic information is encoded in the poles (grey dots) of $R_B(u)$ for $\lambda_{GB} = -1$. We can identify QNM's (colourful dots) as the non-hydrodynamic modes of the microscopic theory, computed by S. Grozdanov, N. Kaplis, A. O. Starinets. 2016.

Intermediate Coupling Effects

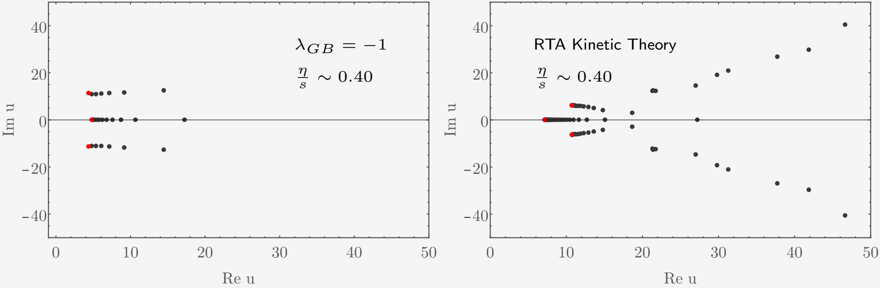


Figure: There are qualitative similarities with the $R_B(u)$ for Conformal RTA Kinetic theory. M. Heller, A. Kurkela, M. Spalinski, V. Svensson. 2016.

Intermediate Coupling Effects

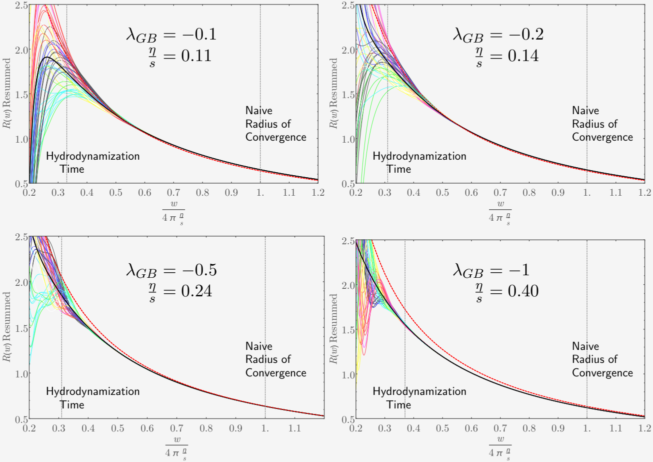


Figure: Applying similar methodology in the case of finite coupling, we can estimate the characteristic spread of solutions as they decay to the attractor. In all cases we study the result is well approximated by 1^{st} order hydrodynamics (red dashed line).

Conclusion

- Why does Hydrodynamics work outside it's regime of applicability?
  - Our estimate of this regime relies on the series converging, which it does not.
  - All you can do to justify hydrodynamics is to resum the series and compare to truncations.
- What qualitative changes do we observe as we interpolate between gauge theories at infinite and finite coupling?
  - At finite coupling our microscopic theory gains a dissipative mode, compatible with kinetic theory.
  - Comparing the full resummation to the truncated series, 1^{st} order viscous hydro works very well in all cases.

Back up slides

Hydrodynamics in 3+1 Dimensions

The equation of motion for Hydrodynamics is the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{7}$$

where $T^{\mu\nu} = T^{\mu\nu}(\epsilon, P, u^\mu)$ with ϵ the energy density, P the Pressure, and u^μ the fluid velocity.

Hydrodynamics in 3+1 Dimensions

The equation of motion for Hydrodynamics is the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{7}$$

where $T^{\mu\nu} = T^{\mu\nu}(\epsilon, P, u^\mu)$ with ϵ the energy density, P the Pressure, and u^μ the fluid velocity. For a perfect fluid

$$T^{\mu\nu}_{ideal} = (\epsilon_0 + P_0)u^\mu u^\nu - P_0 \eta^{\mu\nu}. \tag{8}$$

Hydrodynamics in 3+1 Dimensions

The equation of motion for Hydrodynamics is the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{7}$$

where $T^{\mu\nu} = T^{\mu\nu}(\epsilon, P, u^\mu)$ with ϵ the energy density, P the Pressure, and u^μ the fluid velocity. For a perfect fluid

$$T^{\mu\nu}_{\textit{ideal}} = (\epsilon_0 + P_0)u^\mu u^\nu - P_0 \eta^{\mu\nu}. \tag{8}$$

For a non-ideal fluid, we include every possible tensor combination of ∂^μ, u^μ and $\eta^{\mu\nu}$ with co-efficients c_i.

Hydrodynamics in 3+1 Dimensions

The equation of motion for Hydrodynamics is the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{7}$$

where $T^{\mu\nu} = T^{\mu\nu}(\epsilon, P, u^\mu)$ with ϵ the energy density, P the Pressure, and u^μ the fluid velocity. For a perfect fluid

$$T^{\mu\nu}_{\textit{ideal}} = (\epsilon_0 + P_0)u^\mu u^\nu - P_0 \eta^{\mu\nu}. \tag{8}$$

For a non-ideal fluid, we include every possible tensor combination of ∂^μ, u^μ and $\eta^{\mu\nu}$ with co-efficients c_i.

$$T^{\mu\nu} = T^{\mu\nu}_{\textit{ideal}} + c_1 \partial^\mu u^\nu + c_2 \partial^\nu u^\mu + c_3 \eta^{\mu\nu} \partial_\alpha u^\alpha + c_4 u^\mu u^\nu \partial_\alpha u^\alpha + \ldots \tag{9}$$

Hydrodynamics in 3+1 Dimensions

When ∂u is small we can order the series in derivatives of u^μ

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + O(\sim \partial^\mu u^\nu) + O(\sim (\partial^\mu u^\nu)^2) + ... \qquad (10)$$

- This series is known as the Gradient Expansion.

Hydrodynamics in 3+1 Dimensions

When ∂u is small we can order the series in derivatives of u^μ

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + O(\sim \partial^\mu u^\nu) + O(\sim (\partial^\mu u^\nu)^2) + ... \tag{10}$$

- This series is known as the Gradient Expansion.
- The co-efficients c_i are known as transport co-efficients and uniquely specify our theory.

The Fluid-Gravity correspondence

We can perform classical gravity calculations to find strongly coupled QFT results.

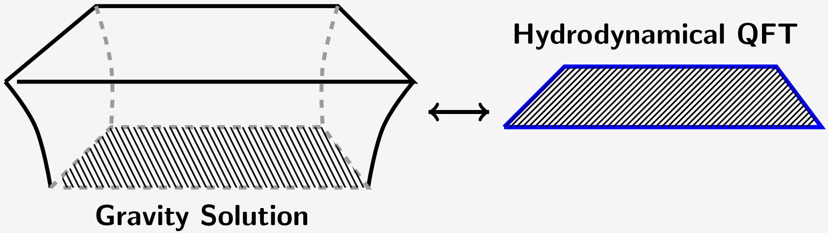


Figure: A full microscopic description would .

The Fluid-Gravity correspondence

We can construct a dynamical gravity solution which will be dual to Bjorken Flow for $\mathcal{N} = 4$ SYM:

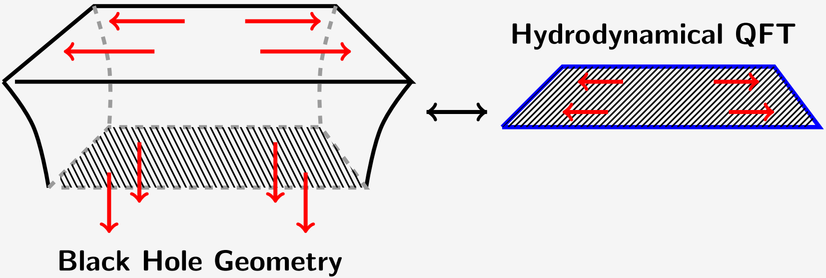


Figure: Some Gauge theories and Gravity theories are conjectured to be the same theory under a field redefinition.