

**Particle production
in
high energy collisions:
from high to low p_t and back**

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pQCD: the standard paradigm

ENERGY

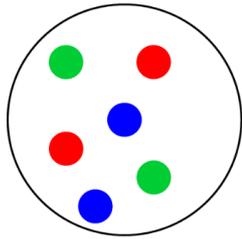
Non-perturbative

$$E \frac{d\sigma}{d^3p} \sim f_1(x) \otimes f_2(x) \otimes \frac{d\sigma}{dt} \otimes D(z)$$

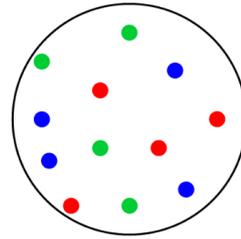
$$x \sim \frac{p_t}{\sqrt{s}} e^{-y}$$

smaller x

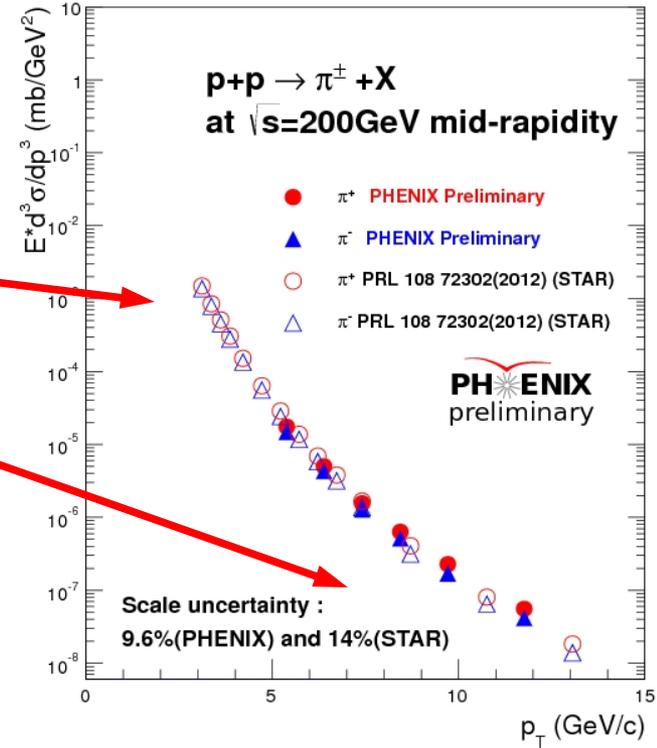
high x



pQCD (DGLAP)



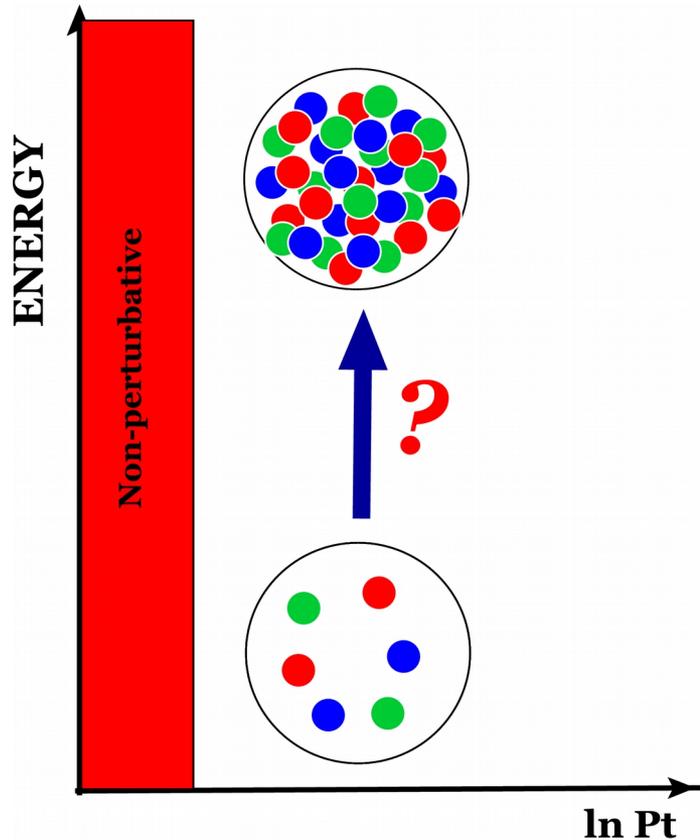
p_t



bulk of QCD phenomena happens at low p_t (small x)



A hadron/nucleus at high energy: gluon saturation



$$\frac{\partial W_E[A]}{\partial E} = \mathcal{H} W_E[A]$$

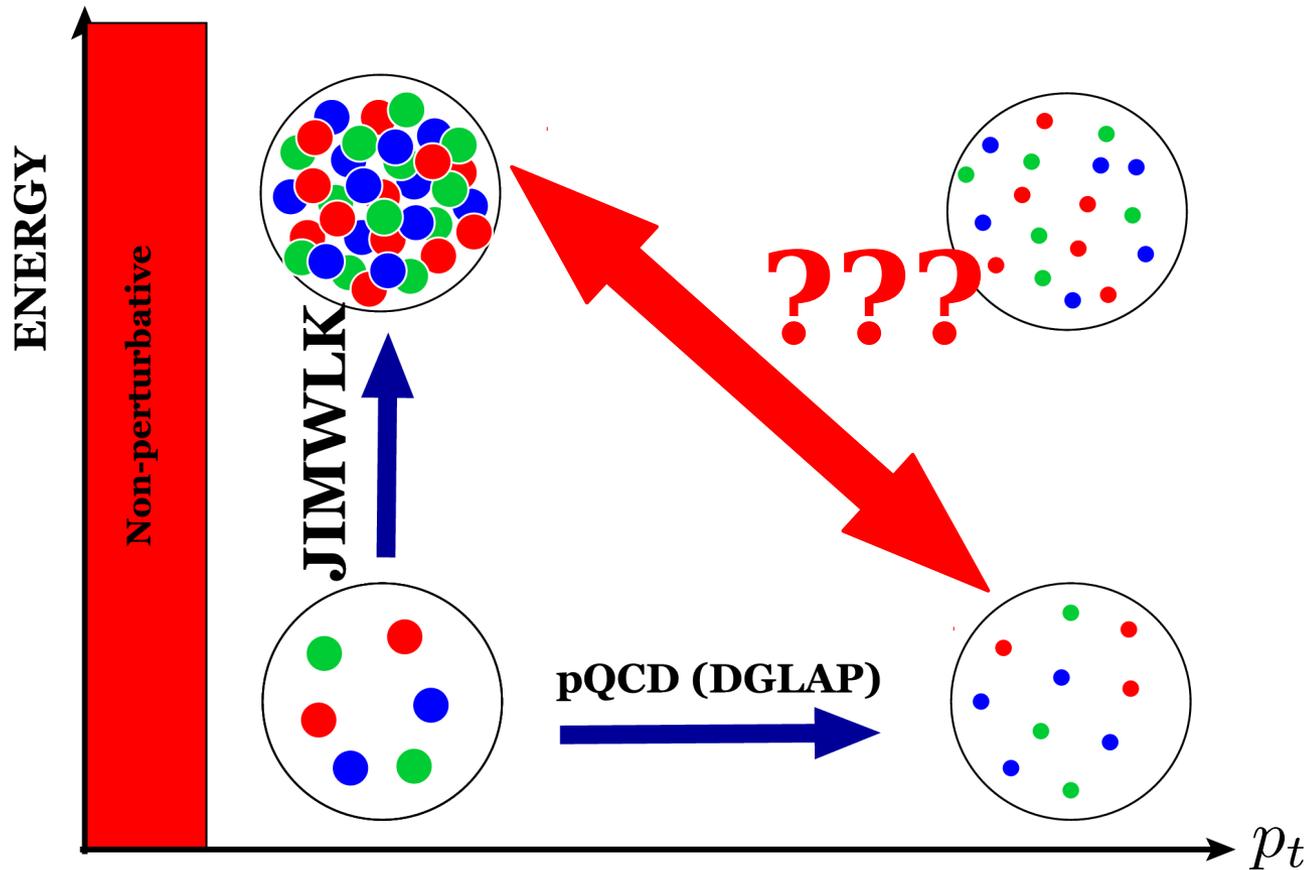
JIMWLK eq.

dynamics of universal gluonic matter

a framework for multi-particle production in QCD

not applicable at high x /high p_t

QCD kinematic phase space



connecting JIMWLK with pQCD?

jet physics

partially/fully coherent energy loss

interactions of UHE neutrinos, ...

dense target (proton/nucleus) as a background color field

$$J_a^\mu \simeq \delta^{\mu-} \rho_a$$

$$D_\mu J^\mu = D_- J^- = 0$$

$$\partial_- J^- = 0 \quad (\text{in } A_+ = 0 \text{ gauge})$$

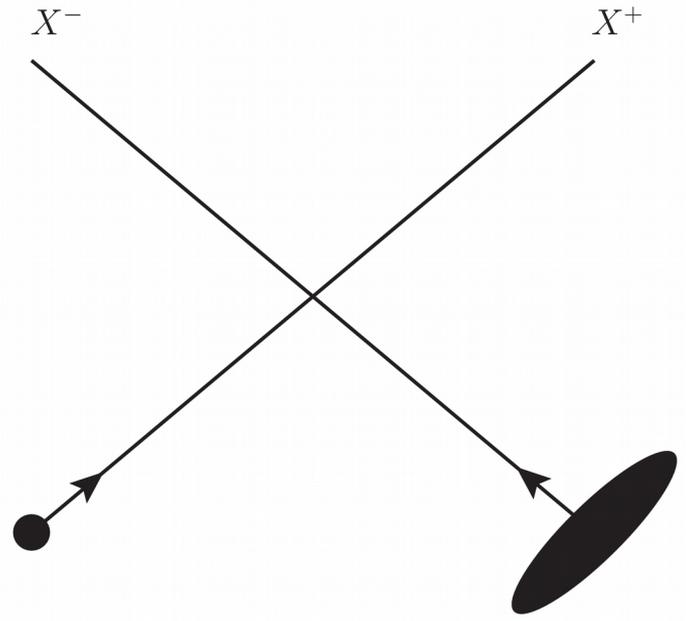
does not depend on x^-

solution to
classical
EOM:

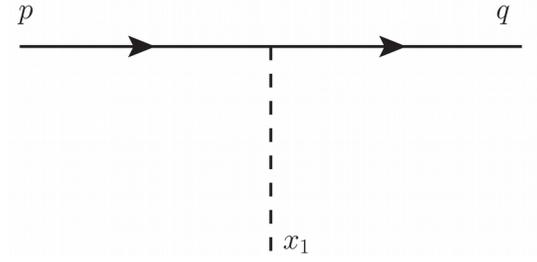
$$A_a^-(x^+, x_t) \equiv n^- S_a(x^+, x_t)$$

scattering of a quark from background color field

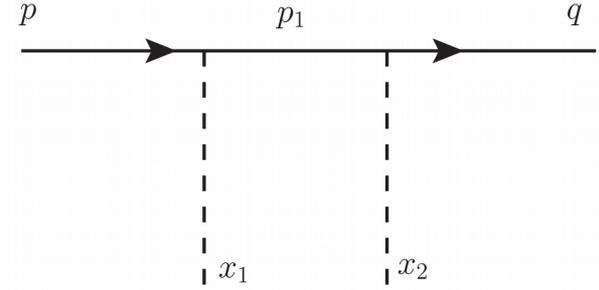
$$A_a^-(x^+, x_t)$$



$$\begin{aligned}
i\mathcal{M}_1 &= (ig) \int d^4x_1 e^{i(q-p)x_1} \bar{u}(q) [\not{\epsilon} S(x_1)] u(p) \\
&= (ig)(2\pi)\delta(p^+ - q^+) \int d^2x_{1t} dx_1^+ e^{i(q^- - p^-)x_1^+} e^{-i(q_t - p_t)x_{1t}} \\
&\quad \bar{u}(q) [\not{\epsilon} S(x_1^+, x_{1t})] u(p)
\end{aligned}$$



$$\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 \int d^4x_1 d^4x_2 \int \frac{d^4p_1}{(2\pi)^4} e^{i(p_1 - p)x_1} e^{i(q - p_1)x_2} \\
&\quad \bar{u}(q) \left[\not{\epsilon} S(x_2) \frac{i\not{p}_1}{p_1^2 + i\epsilon} \not{\epsilon} S(x_1) \right] u(p)
\end{aligned}$$

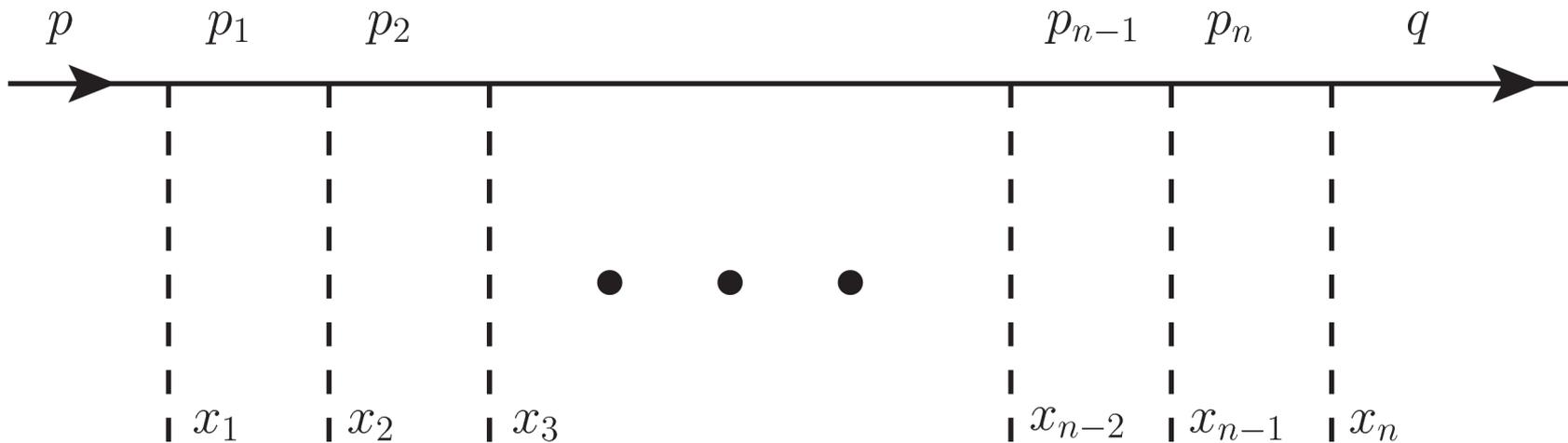


$$\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+ \left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+} \right]} = \frac{-i}{2p^+} \theta(x_2^+ - x_1^+) e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

contour integration over the pole leads to time ordering of scattering

ignore all terms: $O\left(\frac{p_t}{p^+}, \frac{q_t}{q^+}\right)$ and use $\not{\epsilon} \frac{\not{p}_1}{2n \cdot p} \not{\epsilon} = \not{\epsilon}$

$$\begin{aligned}
i\mathcal{M}_2 &= (ig)^2 (-i)(i) 2\pi\delta(p^+ - q^+) \int dx_1^+ dx_2^+ \theta(x_2^+ - x_1^+) \int d^2x_{1t} e^{-i(q_t - p_t) \cdot x_{1t}} \\
&\quad \bar{u}(q) [S(x_2^+, x_{1t}) \not{\epsilon} S(x_1^+, x_{1t})] u(p)
\end{aligned}$$



$$\begin{aligned}
 i\mathcal{M}_n &= 2\pi\delta(p^+ - q^+) \bar{u}(q) \not{n} \int d^2x_t e^{-i(q_t - p_t) \cdot x_t} \\
 &\left\{ (ig)^n (-i)^n (i)^n \int dx_1^+ dx_2^+ \cdots dx_n^+ \theta(x_n^+ - x_{n-1}^+) \cdots \theta(x_2^+ - x_1^+) \right. \\
 &\left. [S(x_n^+, x_t) S(x_{n-1}^+, x_t) \cdots S(x_2^+, x_t) S(x_1^+, x_t)] \right\} u(p)
 \end{aligned}$$

sum over all scatterings gives (**Eikonal approximation**)

$$i\mathcal{M}(p, q) = 2\pi\delta(p^+ - q^+) \bar{u}(q) \not{n} \int d^2x_t e^{-i(q_t - p_t) \cdot x_t} [V(x_t) - 1] u(p)$$

with $V(x_t) \equiv \hat{P} \exp \left\{ ig \int_{-\infty}^{+\infty} dx^+ n^- S_a(x^+, x_t) t_a \right\}$

including
propagation
"backward" gives V^\dagger

toward unifying small and large x (multiple scattering)

scattering from small x modes of the target field S involves only small transverse momenta exchange (small angle deflection)

$$p^\mu = (p^+ \sim \sqrt{s}, p^- = 0, p_t = 0)$$

$$S = S(p^+ \sim 0, p^- / P^- \ll 1, p_t)$$

allow hard scattering by including one all x field during which there is large momenta exchanged and **quark can get deflected by a large angle.**

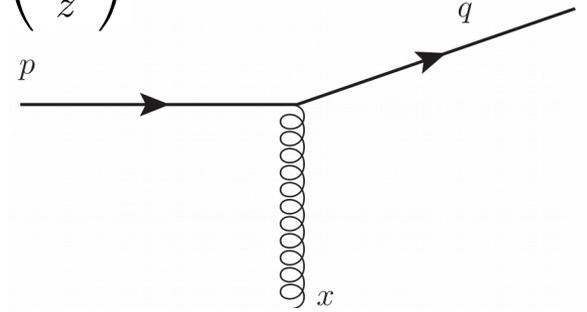
$$A_a^\mu(x^+, x^-, x_t)$$

include Eikonal multiple scattering before and after (along a different direction) the hard scattering

hard scattering: large deflection

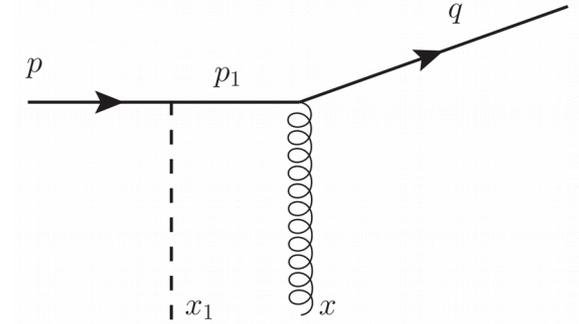
scattered quark travels in the new "z" direction: \bar{z}

$$\begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} = \mathcal{O} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

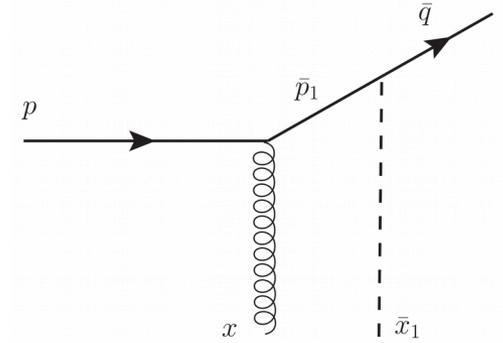


$$i\mathcal{M}_1 = (ig) \int d^4x e^{i(\bar{q}-p)x} \bar{u}(\bar{q}) [A(x)] u(p)$$

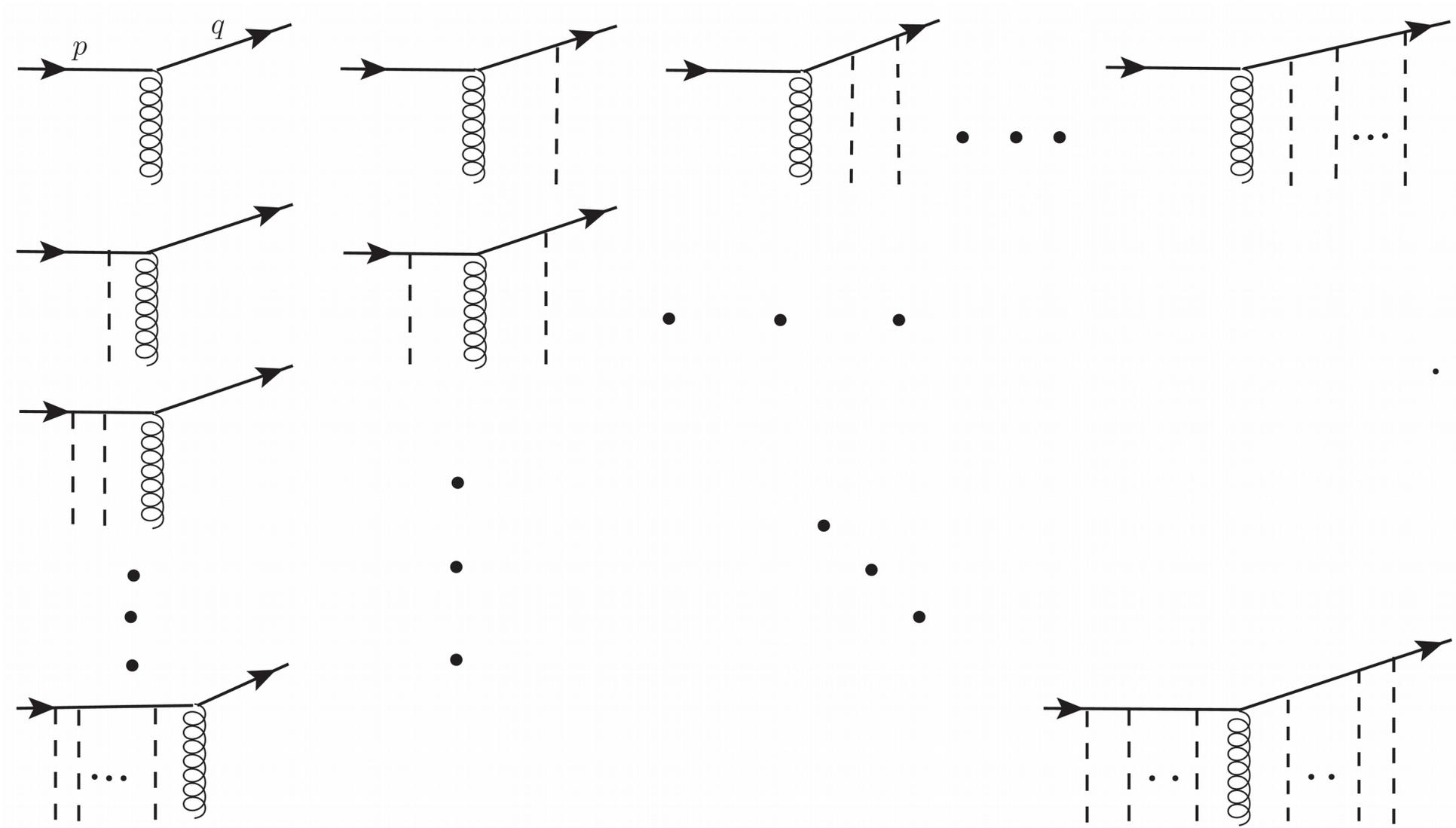
$$i\mathcal{M}_2 = (ig)^2 \int d^4x d^4x_1 \int \frac{d^4p_1}{(2\pi)^4} e^{i(p_1-p)x_1} e^{i(\bar{q}-p_1)x} \bar{u}(\bar{q}) \left[A(x) \frac{i\not{p}_1}{p_1^2 + i\epsilon} \not{n} S(x_1) \right] u(p)$$



$$i\mathcal{M}_2 = (ig)^2 \int d^4x d^4\bar{x}_1 \int \frac{d^4\bar{p}_1}{(2\pi)^4} e^{i(\bar{p}_1-p)x} e^{i(\bar{q}-\bar{p}_1)\bar{x}_1} \bar{u}(\bar{q}) \left[\not{n} \bar{S}(\bar{x}_1) \frac{i\not{\bar{p}}_1}{\bar{p}_1^2 + i\epsilon} A(x) \right] u(p)$$



with $\bar{S}^\mu = \Lambda^\mu_\nu S^\nu$



$$S_F(p, \bar{q}) = (2\pi)^4 \delta^4(p - \bar{q}) S_F^0(p) + S_F^0(p) \tau_{hard}(p, \bar{q}) S_F^0(\bar{q})$$

$$\begin{aligned} \tau_{hard}(p, \bar{q}) \equiv & (ig) \int d^4x \int \frac{d^2 k_t}{(2\pi)^2} \frac{d^2 \bar{k}_t}{(2\pi)^2} d^2 z_t d^2 \bar{z}_t e^{i(\bar{k}-k)x} e^{-i(\bar{q}_t - \bar{k}_t) \cdot \bar{z}_t} e^{-i(k_t - p_t) \cdot z_t} \\ & \left\{ \theta(p^+) \theta(\bar{q}^+) V(z_t, x^+) \not{k} \frac{\not{k}}{2k^+} A(x) \frac{\not{\bar{k}}}{2\bar{k}^+} \not{n} \bar{V}(x^+, \bar{z}_t) - \right. \\ & \left. \theta(-p^+) \theta(-\bar{q}^+) V^\dagger(z_t, x^+) \not{k} \frac{\not{k}}{2k^+} A(x) \frac{\not{\bar{k}}}{2\bar{k}^+} \not{n} \bar{V}^\dagger(x^+, \bar{z}_t) \right\} \end{aligned}$$

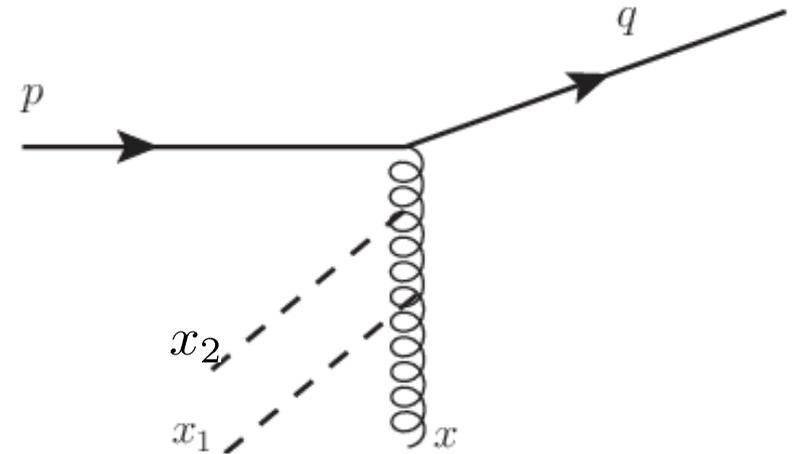
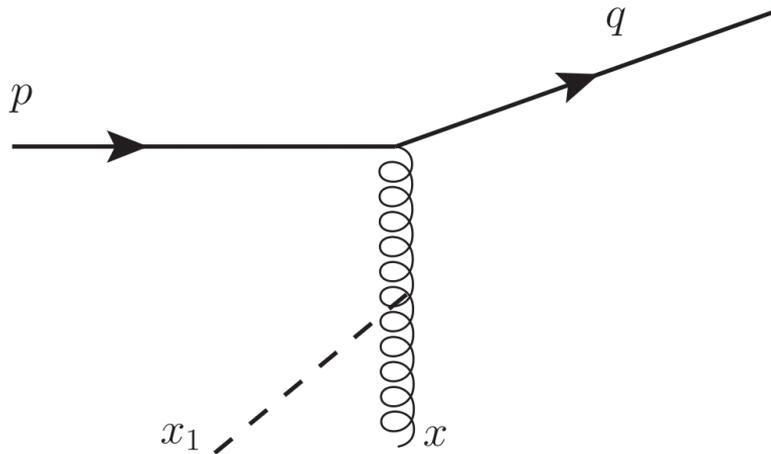
with

$$\bar{V}(x^+, \bar{z}_t) \equiv \hat{P} \exp \left\{ ig \int_{x^+}^{+\infty} d\bar{z}^+ \bar{S}_a(\bar{z}^+, \bar{z}_t) t_a \right\}$$

all “bar-ed” quantities are in a rotated frame where quark’s new direction of propagation (after a hard scattering) is \bar{z}

this quark propagator is the building block for DIS structure functions, single inclusive particle production in pA,....

but there is more to do:
interactions of large and small x modes



in progress

SUMMARY

CGC is a systematic approach to high energy collisions

CGC breaks down at large x (high p_t)

Toward a unified formalism:

quark propagator in the background of small and large x fields

QCD structure functions at both small and large x

particle production in pp , pA in both small and large p_t regions