

Development of heavy-flavour flow-harmonics in high-energy nuclear collisions

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Istituto Nazionale di Fisica Nucleare

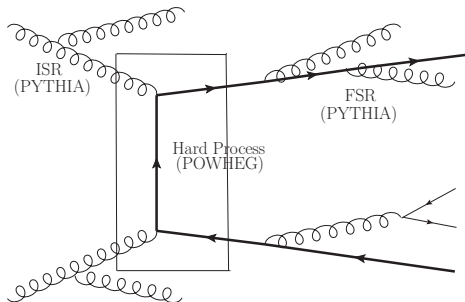
Outline

- The POWLANG transport setup (hard event + transport + in-medium hadronization)
- Event-by event fluctuations and development of HF azimuthal anisotropies (v_2 and v_3) in heavy-ion collisions
- Event-shape engineering and HF observables: preliminary studies

The POWLANG setup

- $Q\bar{Q}$ production;
- HQ transport;
- HQ hadronization.

HQ production: NLO calculation + Parton Shower



- A convenient automated tool to simulate the initial $Q\bar{Q}$ production (the POWHEG-BOX package) interfaces the output of a **NLO event-generator** for the **hard process** with a **parton-shower** describing the **Initial State Radiation** and modeling other *non-perturbative processes* (intrinsic k_T , MPI, **hadronization**)
- This provides a *fully exclusive information on the final state*

Transport theory: the Boltzmann equation

Time evolution of HQ phase-space distribution $f_Q(t, \mathbf{x}, \mathbf{p})$ ¹:

$$\frac{d}{dt} f_Q(t, \mathbf{x}, \mathbf{p}) = C[f_Q]$$

- **Total derivative** along particle trajectory

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}}$$

Neglecting \mathbf{x} -dependence and mean fields: $\partial_t f_Q(t, \mathbf{p}) = C[f_Q]$

- **Collision integral**:

$$C[f_Q] = \int d\mathbf{k} \left[\underbrace{w(\mathbf{p} + \mathbf{k}, \mathbf{k}) f_Q(\mathbf{p} + \mathbf{k})}_{\text{gain term}} - \underbrace{w(\mathbf{p}, \mathbf{k}) f_Q(\mathbf{p})}_{\text{loss term}} \right]$$

$w(\mathbf{p}, \mathbf{k})$: HQ transition rate $\mathbf{p} \rightarrow \mathbf{p} - \mathbf{k}$

¹Approach adopted by [Catania](#), [Nantes](#), [Frankfurt](#), [LBL groups](#)

From Boltzmann to Fokker-Planck

Expanding the collision integral for *small momentum exchange*² (Landau)

$$C[f_Q] \approx \int dk \left[k^i \frac{\partial}{\partial p^i} + \frac{1}{2} k^i k^j \frac{\partial^2}{\partial p^i \partial p^j} \right] [w(\mathbf{p}, \mathbf{k}) f_Q(t, \mathbf{p})]$$

²B. Svetitsky, PRD 37, 2484 (1988)

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The **Boltzmann** equation **reduces** to the **Fokker-Planck** equation

$$\frac{\partial}{\partial t} f_Q(t, \mathbf{p}) = \frac{\partial}{\partial p^i} \left\{ A^i(\mathbf{p}) f_Q(t, \mathbf{p}) + \frac{\partial}{\partial p^j} [B^{ij}(\mathbf{p}) f_Q(t, \mathbf{p})] \right\}$$

where

$$A^i(\mathbf{p}) = \int d\mathbf{k} k^i w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{A^i(\mathbf{p}) = A(\mathbf{p}) p^i}_{\text{friction}}$$

$$B^{ij}(\mathbf{p}) = \frac{1}{2} \int d\mathbf{k} k^i k^j w(\mathbf{p}, \mathbf{k}) \longrightarrow \underbrace{B^{ij}(\mathbf{p}) = (\delta^{ij} - \hat{p}^i \hat{p}^j) B_0(\mathbf{p}) + \hat{p}^i \hat{p}^j B_1(\mathbf{p})}_{\text{momentum broadening}}$$

Problem reduced to the *evaluation of three transport coefficients*,
directly derived from the scattering matrix

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The relativistic Langevin equation

The Fokker-Planck equation can be recast into a form suitable to follow the dynamics of each individual quark arising from the pQCD Monte Carlo simulation of the initial $Q\bar{Q}$ production: the **Langevin equation**

$$\frac{\Delta p^i}{\Delta t} = - \underbrace{\eta_D(\mathbf{p}) p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}},$$

with the properties of the *white* ($\sim \delta_{tt'}$), *multiplicative* noise encoded in

$$\langle \xi^i(\mathbf{p}_t) \rangle = 0 \quad \langle \xi^i(\mathbf{p}_t) \xi^j(\mathbf{p}_{t'}) \rangle = b^{ij}(\mathbf{p}) \frac{\delta_{tt'}}{\Delta t} \quad b^{ij}(\mathbf{p}) \equiv \kappa_L(\mathbf{p}) \hat{p}^i \hat{p}^j + \kappa_T(\mathbf{p}) (\delta^{ij} - \hat{p}^i \hat{p}^j)$$

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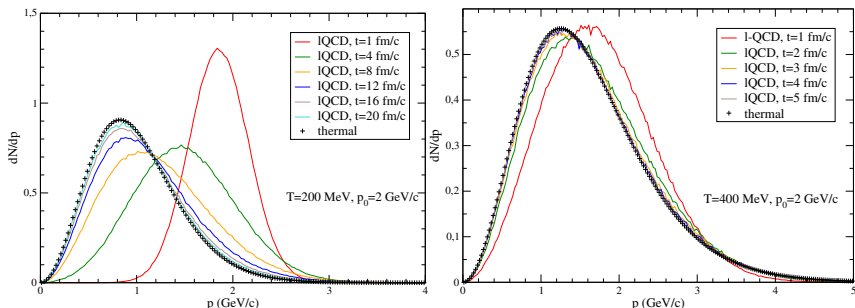
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Transport coefficients related to the FP ones:

- **Momentum diffusion**: $\kappa_T(\mathbf{p}) = 2B_0(\mathbf{p})$ and $\kappa_L(\mathbf{p}) = 2B_1(\mathbf{p})$
- **Friction** term, in the *Ito pre-point discretization scheme*, fixed by the Einstein *fluctuation-dissipation* relation

$$\eta_D^{\text{Ito}}(\mathbf{p}) = A(\mathbf{p}) = \frac{B_1(\mathbf{p})}{TE_p} - \left[\frac{1}{p} \frac{\partial B_1(\mathbf{p})}{\partial p} + \frac{d-1}{p^2} (B_1(\mathbf{p}) - B_0(\mathbf{p})) \right]$$

A first check: thermalization in a static medium



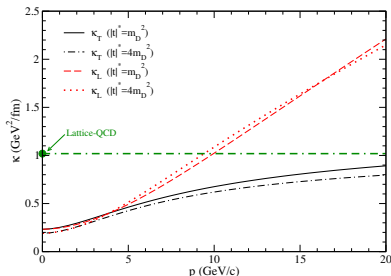
If the Einstein relation is imposed, for $t \gg 1/\eta_D$ HQ's approach kinetic equilibrium, with momenta described by a **Maxwell-Jüttner distribution**

$$f_{MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T K_2(M/T)}, \quad \text{with } \int d^3p f_{MJ}(p) = 1$$

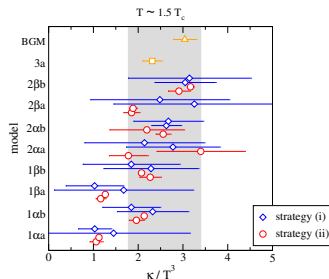
The larger κ ($\kappa \sim T^3$), the faster the approach to thermalization.

Transport coefficients: weak-coupling vs I-QCD

Weak-coupling (beauty shown)



Lattice QCD ($M = \infty$)



Obtained accounting for $Qq \rightarrow Qq$ and $Qg \rightarrow Qg$ scattering, with resummation of medium effects for soft ($|t| < |t^*|$) collisions (Hard Thermal Loop approximation)

$$\kappa = \frac{1}{3} \int_{-\infty}^{+\infty} dt \langle \xi^i(t) \xi^i(0) \rangle_{\text{HQ}}$$

given by *electric-field correlator*, available only for *imaginary times*

From quarks to hadrons

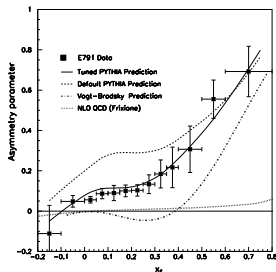
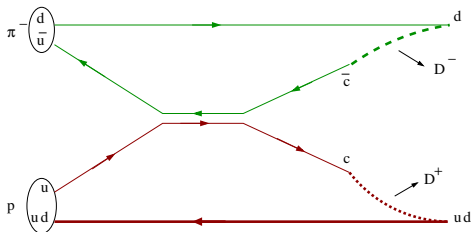
In the presence of a medium, rather than fragmenting like in the vacuum (e.g. $c \rightarrow cg \rightarrow c\bar{q}q$), HQ's can hadronize by recombining with light thermal partons from the medium.

In-medium hadronization may affect the R_{AA} and v_2 of final D-mesons due to the **collective flow of light quarks**. We tried to estimate the effect through this **model** interfaced to our **POWLANG transport code**:

- At T_{dec} **c-quarks coupled to light \bar{q} 's** from a local *thermal distribution*, eventually *boosted* ($u_{\text{fluid}}^\mu \neq 0$) to the lab frame;
- **Strings are formed** and given to PYTHIA 6.4 to simulate their fragmentation and produce the final hadrons ($D + \pi + \dots$)

From quarks to hadrons

Breaking of factorized description of hadronization $d\sigma^h = d\sigma_f \otimes D_{f \rightarrow h}$ in terms of independent fragmentation functions already observed in hadronic collisions at Fermilab and at SPS



Second endpoint boosts the string along the direction of the beam-remnant (*beam-drag effect*), leading to an asymmetry in the rapidity distribution of D^+/D^- mesons

$$A = \frac{\sigma_{D^-} - \sigma_{D^+}}{\sigma_{D^-} + \sigma_{D^+}}$$

New results at 5.02 TeV: D -meson v_2 and v_3 in Pb-Pb

The study of **higher flow-harmonics** in AA collisions requires a **modeling of initial-state event-by-event fluctuations**. We perform a Glauber-MC sampling of the initial conditions, each one characterized by a *complex eccentricity*

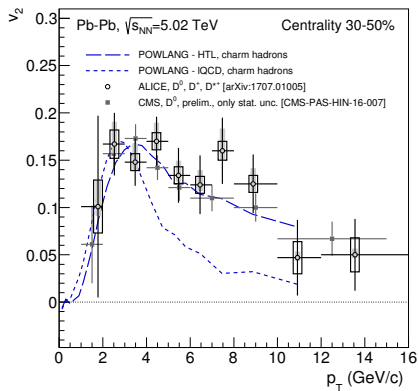
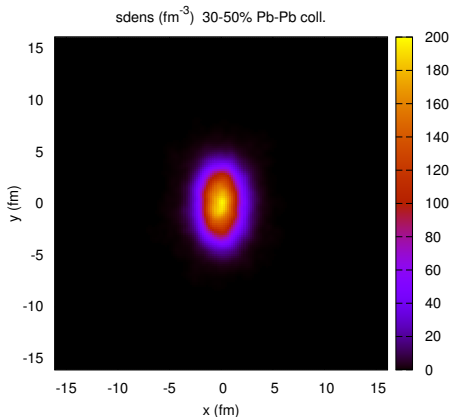
$$s(\mathbf{x}) = \frac{K}{2\pi\sigma^2} \sum_{i=1}^{N_{\text{coll}}} \exp \left[-\frac{(\mathbf{x} - \mathbf{x}_i)^2}{2\sigma^2} \right] \longrightarrow \epsilon_m e^{im\Psi_m} \equiv -\frac{\{r^2 e^{im\phi}\}}{\{r^2\}}$$

with **orientation** and **modulus** given by

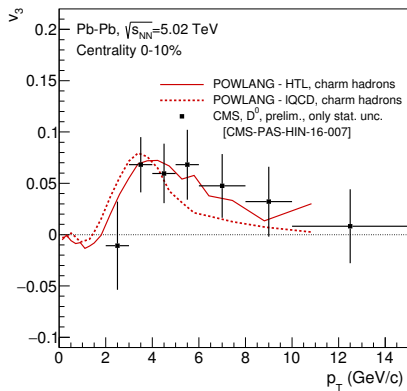
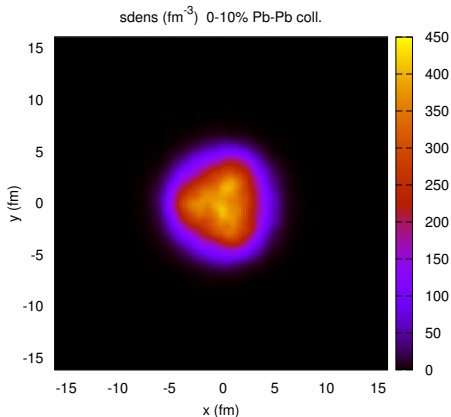
$$\Psi_m = \frac{1}{m} \text{atan2} \left(-\{r^2 \sin(m\phi)\}, -\{r^2 \cos(m\phi)\} \right)$$

$$\epsilon_m = \frac{\sqrt{\{r_{\perp}^2 \cos(m\phi)\}^2 + \{r_{\perp}^2 \sin(m\phi)\}^2}}{\{r_{\perp}^2\}} = -\frac{\{r^2 \cos[m(\phi - \Psi_m)]\}}{\{r^2\}}$$

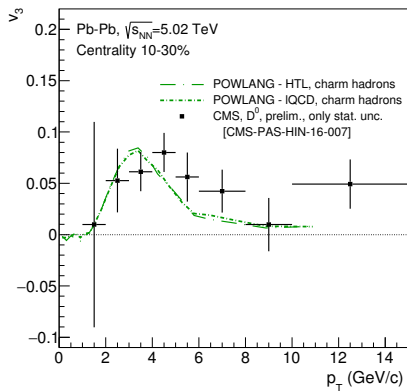
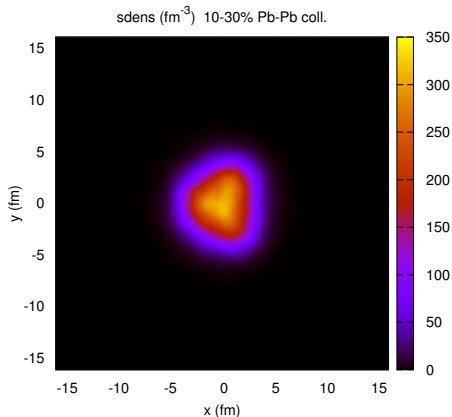
Exploiting the fact that, on an event-by-event basis, for $m = 2, 3$ $v_m \sim \epsilon_m$ we consider an **average background** obtained **summing** all the **events** of a given centrality class, each one **rotated by its event-plane angle ψ_m** , depending on the harmonics one is considering.

New results at 5.02 TeV: D -meson v_2 and v_3 in Pb-Pb

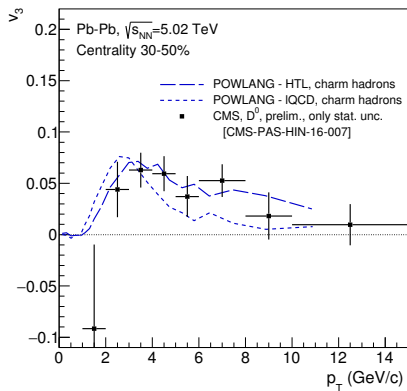
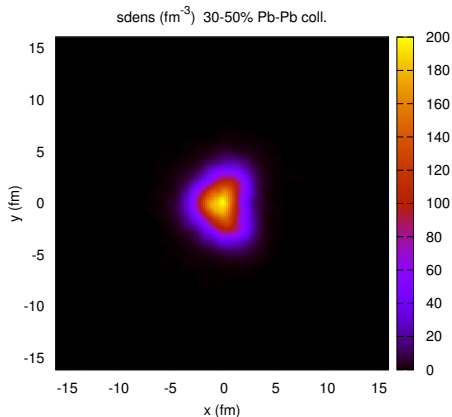
Transport calculations carried out in [JHEP 1802 \(2018\) 043](#), with hydro background calculated via the [ECHO-QGP code \(EPJC 73 \(2013\) 2524\)](#)

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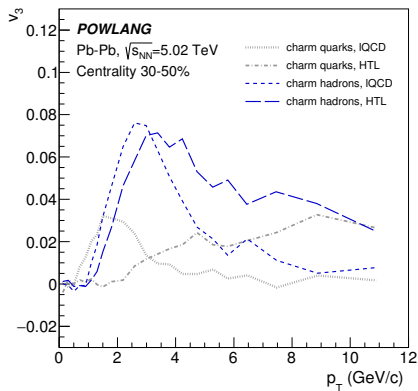
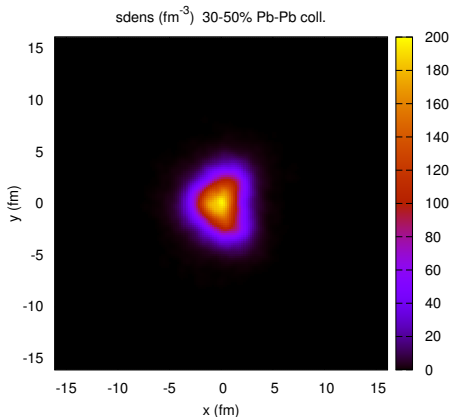
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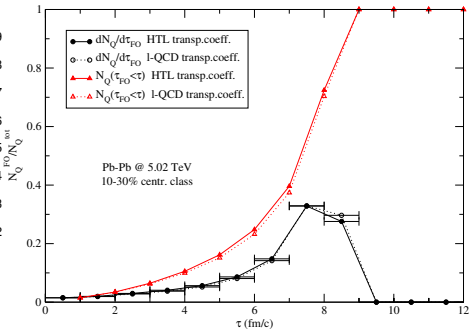
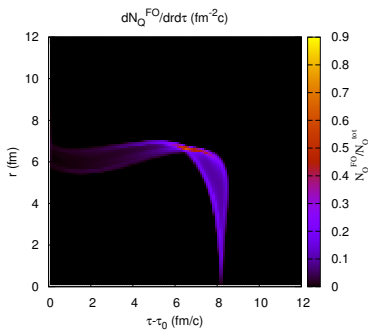
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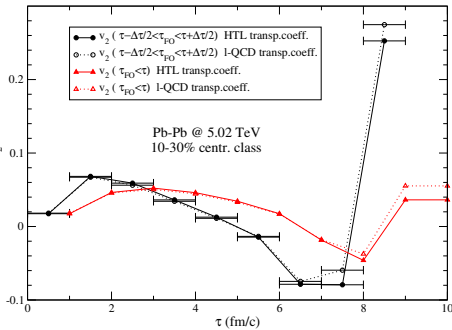
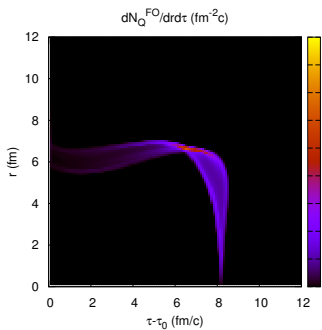
- CMS (and ALICE) data for D -meson $v_{2,3}$ satisfactory described;
- Recombination with light quarks provides a relevant contribution.

Time development of azimuthal anisotropies



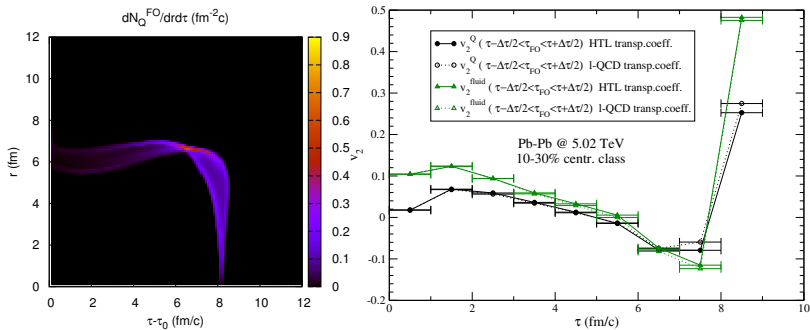
- Most of the HQ's decouple quite late ($\sim 50\%$ after 8 fm/c);

Time development of azimuthal anisotropies



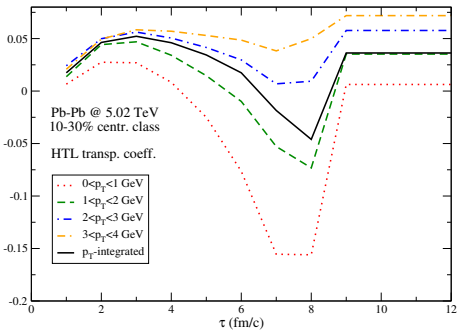
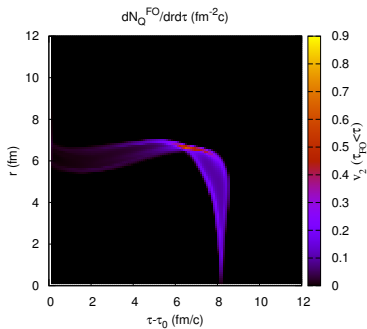
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- Final elliptic flow from a complex interplay of contributions from the whole medium history;

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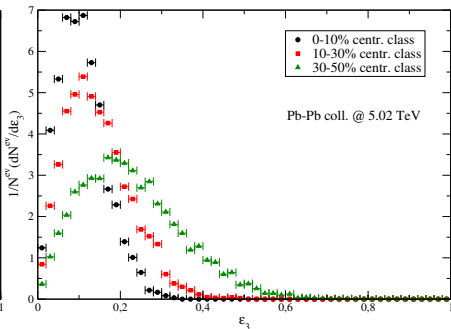
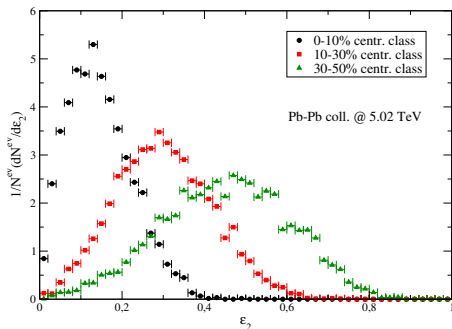
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- HQ v_2 correlated with the one of the fluid cell;

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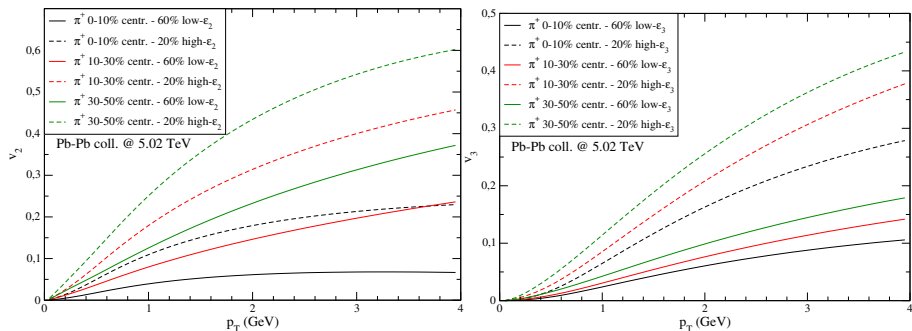
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- supplementary information from p_T -differential analysis;

Eccentricity fluctuations



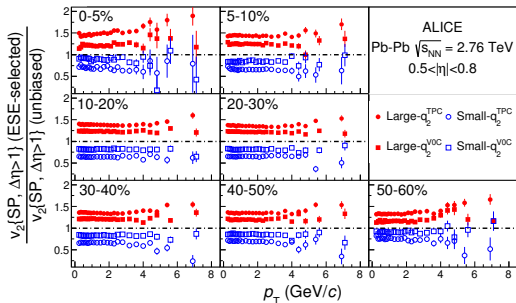
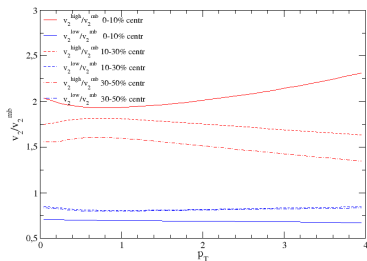
Events within the same *centrality class* can be characterized by very **different eccentricities**. Eccentricity distributions of different classes display a significant overlap, in particular for ϵ_3 , which is less correlated with the impact parameter. It is interesting to study in each class the **0-20% high- ϵ_n** and the **0-60% low- ϵ_n** events. **We want to extend this program**, usually carried out for soft-hadrons (ALICE Coll. PRC 93 (2016), 034916), **to HF particles**.

Eccentricity fluctuations



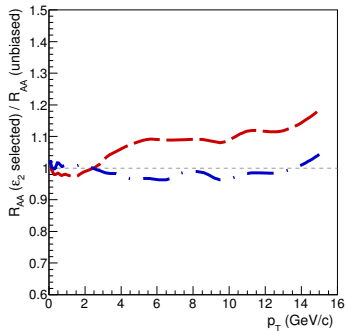
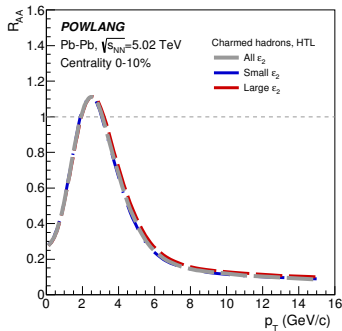
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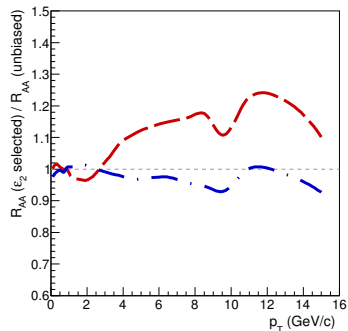
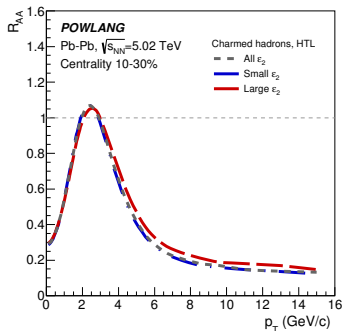
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ESE and HF observables: nuclear modification factor



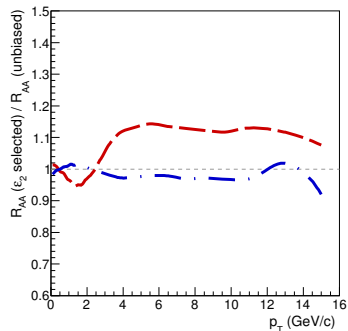
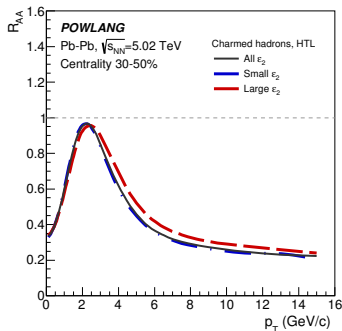
The nuclear modification factor of charmed hadrons, within a given centrality class, displays only a *mild dependence on the initial eccentricity*. This holds for both choices of transport coefficients.

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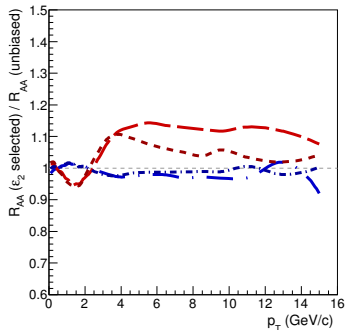
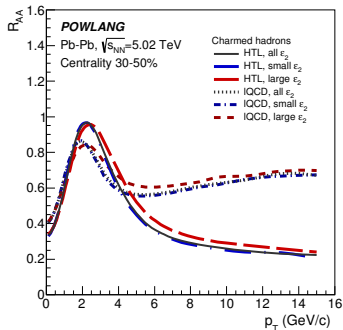
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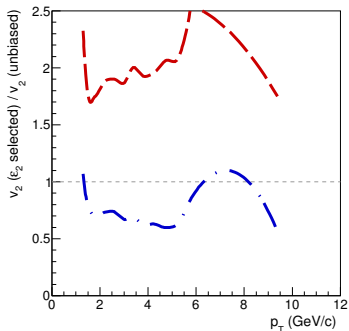
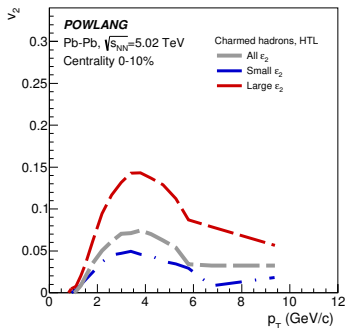
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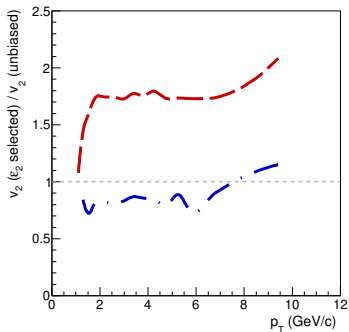
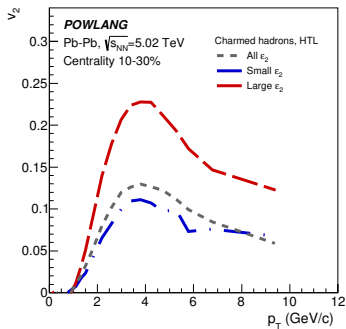
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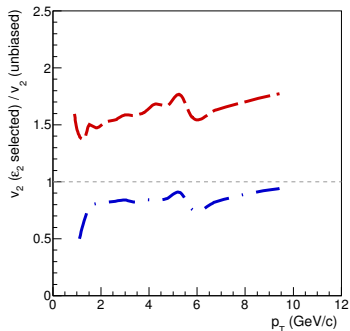
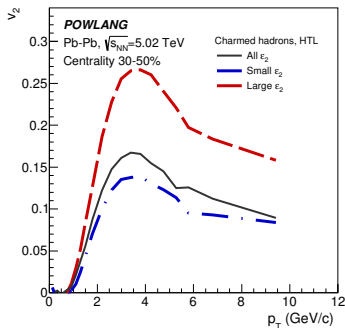
Charmed hadron v_2 is strongly affected by eccentricity fluctuations within a given centrality class. The size of the effect looks independent from the choice of the transport coefficients.

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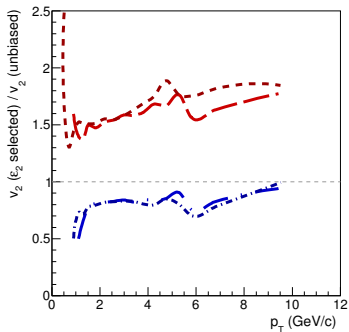
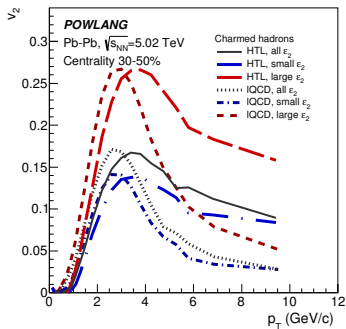
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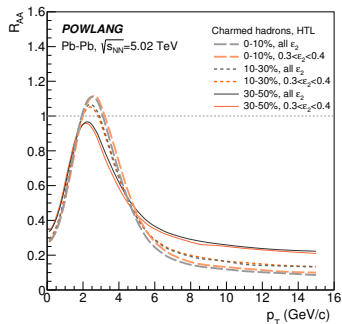
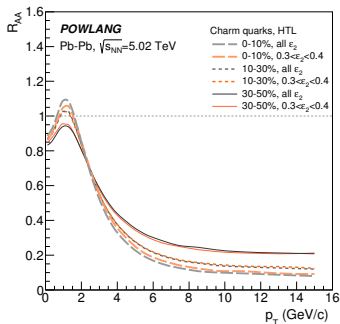
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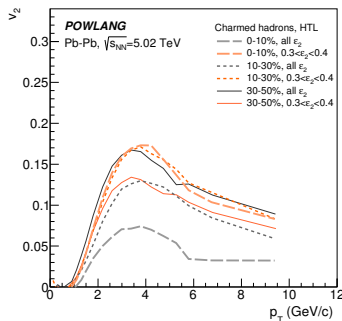
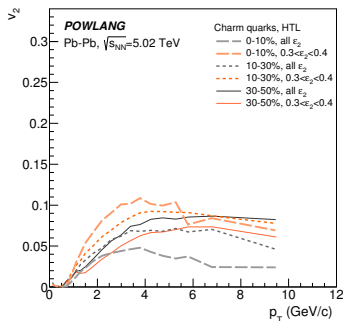
Selecting events at fixed eccentricity



One can adopt a different strategy and **select events of a given eccentricity** (e.g. $0.3 < \epsilon_2 < 0.4$) in the different centrality classes.

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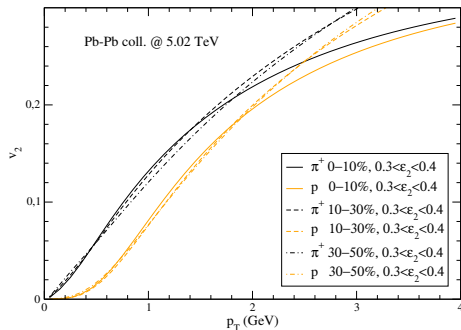
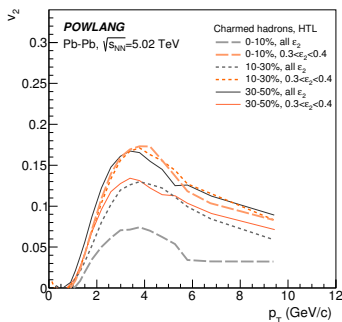
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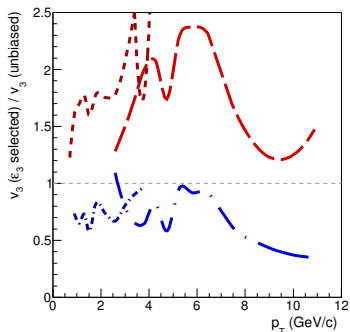
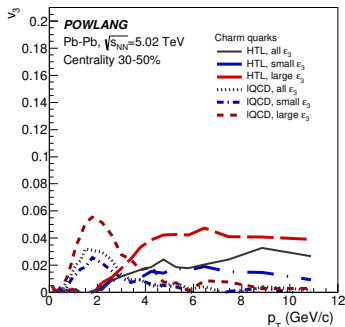
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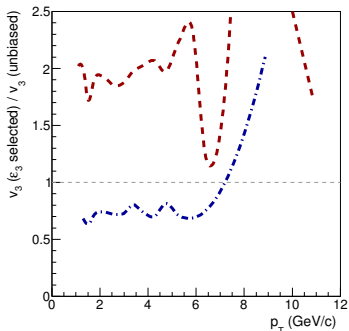
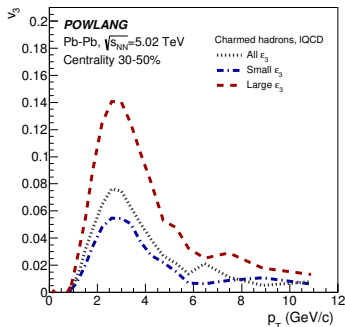
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Conclusions

- With initial conditions accounting for **event-by-event fluctuations** it is possible to provide a consistent description of **even and odd harmonics** of HF azimuthal anisotropies (v_2 and v_3);
- The way is open to perform the study of HF observables within an **Event-Shape-Engineering analysis**: the potential to extract **information on the initial conditions and on the transport coefficients of the medium** has to be explored;
- More results going to become available soon.