Mass hierarchy of parton energy loss in heavy-ion collisions

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Mass effects in heavy quark energy loss

- Produced early and probe the full QGP history
- Provide unique opportunity to study mass effects on energy loss

$R_{AA} (b\rightarrow e) > R_{AA} (c\rightarrow e)$ at low $p_T$

Comparable $R_{AA}$ between light hadron, $B$ and $D$ at high $p_T$

This motivates a detailed theoretical investigation of the mass hierarchy of parton energy loss, especially its $p_T$ dependence.
Outline

• A linear Boltzmann transport model (LBT) for heavy vs. light parton energy loss in QGP

• A multi-stage approach for heavy quark energy loss

• Effects of jet-induced medium excitation
Part I: A linear Boltzmann transport model

( In collaboration with T. Luo, G.-Y. Qin and X.-N. Wang, PRC 94 and PLB 777 )

Boltzmann equation for parton “1” distribution:

\[ p_1 \cdot \partial f_1(x_1, p_1) = E_1 C[f_1] \]

The collision term:

Transition rate from \( p_1 \) to \( p_1 - k \)

\[ C[f_1] \equiv \int d^3k \begin{bmatrix} w(\vec{p}_1 + \vec{k}, \vec{k}) f_1(\vec{p}_1 + \vec{k}) - w(\vec{p}_1, \vec{k}) f_1(\vec{p}_1) \end{bmatrix} \]

Elastic scattering (2->2 process)

\[ w(\vec{p}_1, \vec{k}) \equiv \sum_{2,3,4} w_{12\rightarrow34}(\vec{p}_1, \vec{k}) \]

\[ w_{12\rightarrow34}(\vec{p}_1, \vec{k}) = \gamma_2 \int \frac{d^3p_2}{(2\pi)^3} f_2(\vec{p}_2) \begin{bmatrix} 1 \pm f_3(\vec{p}_1 - \vec{k}) \end{bmatrix} \begin{bmatrix} 1 \pm f_4(\vec{p}_2 + \vec{k}) \end{bmatrix} \]

\[ \times v_{\text{rel}} d\sigma_{12\rightarrow34}(\vec{p}_1, \vec{p}_2 \rightarrow \vec{p}_1 - \vec{k}, \vec{p}_2 + \vec{k}) \]

Microscopic cross section of 12->34
Inelastic scattering (2->2+n process)

Inelastic scattering rate (average gluon number per $\Delta t$):

$$\Gamma_{\text{inel}} = \langle N_g \rangle (E, T, t, \Delta t)/\Delta t = \int dxdk_\perp^2 \frac{dN_g}{dxdk_\perp^2 dt}$$

Medium-induced gluon spectrum

$$\frac{dN_g}{dxdk_\perp^2 dt} = \frac{2\alpha_s C_A P(x)}{\pi k_\perp^4} \hat{q} \left( \frac{k_\perp^2}{k_\perp^2 + x^2 M^2} \right)^4 \sin^2 \left( \frac{t - t_i}{2\tau_f} \right)$$


$\hat{q} : dp_\perp^2/dt$ of quark/gluon due to 2->2 scatterings
Framework overview

(Parton evolution inside the QGP)

• Generation of QGP medium: hydrodynamic model
• Initialization of hard partons: MC-Glauber for position space and pQCD calculation for momentum space (PDF: CTEQ5+EPS09)
• Simulation of parton evolution: the Boltzmann transport model in the local rest frame of the medium
• Hadronization: fragmentation + recombination model

outside the medium (below $T_c$), converted into hadrons
Heavy vs. light hadron suppression

- $u/d/s$ are slightly more suppressed than $c$ quark, $g$ is significantly more suppressed

- Due to different fragmentation function, $\pi$ from light quark has similar $R_{AA}$ to $D$, $\pi$ from gluon is still more suppressed
Simultaneous description of $D$ and light hadron $R_{AA}$

- LBT treats heavy and light parton evolution on the same footing and simultaneously describes $R_{AA}$ of $D$ and light flavor hadrons.
- Predictions for the Xe-Xe collisions: Poster by G.-Y. Qin
Part II: Effects of multi-stage evolution

( In collaboration with G.-Y. Qin, C. Shen and A. Majumder, arXiv:1711.09053 )

Particles in current transport models are usually on-shell

\[ p^2 - m^2 = E^2 - \bar{p}^2 - m^2 = 0 \]

Quantum fluctuation allows particles to be off-shell (virtual)

\[ Q^2 = p^2 - m^2 \neq 0 \]

within a finite splitting time

\[ \tau_f \sim \gamma \frac{1}{Q} \sim \frac{E}{Q^2} \]

Partons produced in energetic collisions usually start with high \( Q \) and then evolve back to shell after several splittings

Parton evolution at high \( Q \) is usually treated by vacuum showers (Pythia) in transport without considering medium modification

Develop a combined approach that includes medium modification in different stages
A multi-stage approach

Stage 1 \((Q>>M_{\text{HM}})\) – rare scattering multiple emission

- HQ fragmentation function is treated with DGLAP equation

\[
\frac{\partial}{\partial Q^2} D \left( z, E, Q^2 \right) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} \int_{z}^{1} \frac{dy}{y} P(y) D \left( \frac{z}{y}, E, Q^2 \right)
\]

- Splitting function:

\[
P(y) = P_{\text{vac}}(y) + P_{\text{med}}(y)
\]

\[
P_{\text{med}}(y, k_{\perp}^2) = \frac{2C_A \alpha_s}{\pi k_{\perp}^4} P_{\text{vac}}(y) \int_{t_i}^{t_{\text{max}}} dt \hat{q}(t) \sin^2 \left( \frac{t - t_i}{2\tau_f} \right)
\]


- Input fragmentation function \(D \left( z, E, Q_0^2 \right)\)

Extracted from transport model (in stage 2) – medium-modified fragmentation function at \(Q_0 \sim M_{\text{HM}}\)
A multi-stage approach

Stage 2 ($Q \sim M_{HM}$) – single scattering induced emission

- Transport model (elastic + inelastic processes)
  Inelastic scattering rate

$$\Gamma_{rad}(\zeta^-) = \int dy \int d\ell_{\perp}^2 \frac{dN}{d\ell_{\perp}^2 d\zeta^-}$$

- Medium-induced gluon spectra (contribution from longitudinal drag and diffusion to slowly moving heavy quark)

$$\frac{dN}{dy d\ell_{\perp}^2 d\zeta^-} = \frac{2C_F \alpha_S}{2\pi} \frac{P(y)}{(\ell_{\perp}^2 + y^2 M^2)^2} 2 \sin^2 \left( \frac{\ell_{\perp}^2 + y^2 M^2}{4l^- y(1 - y) \zeta^-} \right)$$

$$\times \left[ \hat{q} \left( 1 - \frac{y}{2} - \frac{y^2 M^2}{\ell_{\perp}^2} \right) + \hat{e} \frac{y^2 M^2}{l^-} + \hat{e}_2 \frac{y^2 M^2}{2(l^-)^2} \right].$$

- Assume Einstein relation:  \( \hat{e} = \hat{e}_2/(2T), \hat{q} = 2\hat{e}_2 \)

- Extract medium-modified \( D(z, E, Q_{0}^2) \) for DGLAP in stage 1

Abir, Kaur, Majumder, PRD 90 (2014) 114026
Abir, Majumder, PRC 94 (2016) 054902
Evolution of $b$-quark fragmentation function

- vac $Q_0 = 5$ GeV
- vac $Q = 20$ GeV
- med $Q_0 = 5$ GeV
- med $Q = 20$ GeV
- $b$ quark
  - $E = 20$ GeV
  - $Q_0 = 5$ GeV

**Evolution of $b$-quark fragmentation function in medium transport**

**Vacuum DGLAP**

**Medium-modified DGLAP**
Heavy meson spectra

- Convolute vacuum/medium-modified fragmentation function with $c/b$ initial spectra

- Vacuum fragmentation function $\rightarrow D/B$ spectra consistent with p-p data
Contributions from $\hat{e}$ and $\hat{e}_2$ terms

- More important to $b$-quark than $c$-quark (mass hierarchy)
- Non-negligible at low $p_T$ but diminish at large $p_T$
- Decrease $B$ meson $R_{AA}$ and narrow the difference between $D$ and $B$
Part III: Effects of jet-induced medium excitation

(In collaboration with Y. Tachibana, in progress)

- “Linear” Boltzmann: only consider medium modification of jet
- Jet-medium interaction also includes modification of the medium

Medium background

\[(T, u^\nu) \quad \frac{1}{p.u} p^\mu \partial_\mu f = C\]

Hard parton

\[(j^\nu) \quad \partial_\mu T^{\mu \nu} = j^\nu\]

Energy-momentum deposition

- Important effects in jet observables
- e.g. enhancement of soft hadron production in $\gamma$-jet [W. Chen, S. Cao, T. Luo, L.-G. Pang, X.-N Wang, PLB 777 (2018) 86-90]
Energy deposition of heavy quarks into QGP

\[ p = 2 \text{ GeV} \ c\text{-quark} \ (E = 2.4 \text{ GeV}) \]

\[ p = 2 \text{ GeV} \ b\text{-quark} \ (E = 4.7 \text{ GeV}) \]

- Energy deposition from a slowly moving heavy quark can travel faster than the heavy quark affects its own subsequent motion
Summary

• Discussed the mass/flavor hierarchy of parton energy loss from different perspectives

• Presented a linear Boltzmann transport model (LBT) that treats heavy and light parton evolution on the same footing and simultaneously describes the nuclear modification of charm and light hadrons

• Introduced a multi-stage approach of heavy quark energy loss that further narrows down the difference between $D$ and $B$ meson $R_{AA}$

• Discussed possible mass/velocity dependences of medium response to heavy quark energy loss
Thank you!
MATTER vs. vacuum shower

- Energy loss in MATTER is stronger when $Q_0$ is smaller.
- Energy loss in MATTER disappears when $Q_0^2 > \hat{Q} \tau_f$ (medium scale).
- $\hat{Q} \tau_f$ increases with the jet energy; MATTER evolution is more important to high energy partons than to low energy ones.
LBT vs. MARTINI at different times

- Parton energy loss in LBT (HT) is weaker than in MARTINI (AMY) at early $t$, but catches up and becomes stronger when $t$ is large.
- At $t$ around the QGP lifetime, LBT and MARTINI are indistinguishable.
- Need to explore observables beyond $R_{AA}$ to distinguish them.
Flavor hierarchy of parton energy loss

- Consistency between semi-analytical calculation and MC simulation with the assumption of fixed medium $T$ and fixed parton $E$
- Clear flavor hierarchy within transport model: $\Delta E_g > \Delta E_q > \Delta E_c > \Delta E_b$
Energy deposition of heavy quarks into QGP

5 GeV charm quark

Energy density (GeV/fm$^3$) at $\eta_s = 0$

5 GeV beauty quark

Energy density (GeV/fm$^3$) at $\eta_s = 0$
Elastic vs. Inelastic Energy Loss

Divide scattering probability of jet parton into two regions:
1. Pure elastic scattering without radiated gluons: \( P_{\text{el}}(1 - P_{\text{inel}}) \)
2. Inelastic scattering: \( P_{\text{inel}} \)

Total probability: \( P_{\text{tot}} = P_{\text{el}} + P_{\text{inel}} - P_{\text{el}}P_{\text{inel}} \)

In model calculation:
1. Use \( P_{\text{tot}} \) to determine whether the jet parton scatter with the thermal medium
2. If so, we then determine whether this scattering is pure elastic or inelastic
3. Simulate the 2->2 or 2->2+n process

HQ energy loss due to elastic and inelastic processes are comparable at early time, but is dominated by the inelastic process at large \( t \).
Simultaneous Description of $D$ and $\pi$ $R_{AA}$ in 5.02 TeV Pb-Pb Collisions

With a delicate treatment of heavy and light parton in-medium evolution and their hadronization, one may provide reasonable description of heavy and light hadron suppression simultaneously.
The extracted $\hat{q}$ from model to data comparison within our LBT framework is consistent with the value constrained by the earlier work by the JET Collaboration [Phys. Rev. C90, 014909 (2014)].
Anisotropic Flow ($v_2$ and $v_3$) of $D$ Mesons

- Predictions of $v_2$ and $v_3$ are consistent with CMS data at 5.02 TeV.
- Strong $v_2$ is observed for the full $p_T$ range.
- Strong $v_3$ is observed at low $p_T$, but it is consistent with 0 at high $p_T$. 