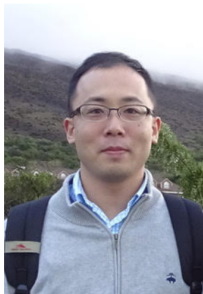


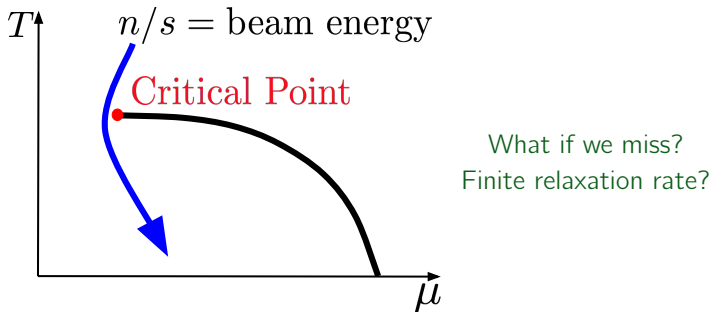
Transits of the QCD Critical Point

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Transits of the critical point: two parameters



- How does the finite expansion rate limit the critical fluctuations?

$$\epsilon \equiv \underbrace{\tau_o}_{\text{micro time}} \times \underbrace{\partial_\mu u^\mu}_{\text{expansion rate } 1/\tau_Q} = \frac{\tau_o}{\tau_Q}$$

- How does missing the critical point limit the critical fluctuations?

$$\Delta_s \equiv \frac{n_c}{s_c} \left(\frac{s}{n} - \frac{s_c}{n_c} \right)$$

What should we measure: baryon/entropy

see also Stephanov & Yin arXiv:1712.10305

$$C^{\hat{n}\hat{n}} \equiv \underbrace{\langle (\delta n - (n/s)\delta s)^2 \rangle}_{\text{flucts of } \delta \hat{n} \equiv s\delta(n/s)}$$

variance in n/s :
not baryon number

- Why? It is a hydro eigenmode and *always* diverges fast

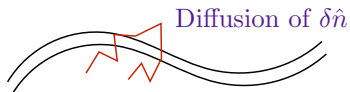
$$\underbrace{\langle (\delta n - (n/s)\delta s)^2 \rangle}_{\text{what we want}} \propto \underbrace{C_p}_{\text{specific heat}} \propto \underbrace{\chi_{\text{Ising}}}_{\text{Ising susceptibility}}$$

- Wavelength dependent model for the equilibrium correlator

$$C^{\hat{n}\hat{n}}(k, t) \equiv \langle \delta \hat{n}^*(k, t) \hat{n}(-k, t) \rangle \propto \underbrace{\frac{\chi_{\text{Ising}}}{1 + (k\xi)^{2-\eta}}}_{\text{Ising prediction}}$$

Question: What's the wavelength of fluctuations near the CP?

Estimate the wavelength of critical fluctuations



- The maximum wavelength that can be equilibrated by diffusion:

$$\underbrace{D_o}_{\text{diffusion coef}} \times \underbrace{\tau_Q}_{\text{the total time}} = \underbrace{\ell_{\max}^2}_{\text{the longest wavelength}}$$

- Here D_o is the (thermal) diffusion coefficient away from the CP:

$$D_o \sim \frac{\ell_o^2}{\tau_o} \Rightarrow \ell_{\max} \sim \ell_o \epsilon^{-1/2} \quad \ell_o \text{ is micro length}$$

The critical wavelength ℓ_{kz} must be in between ℓ_o and ℓ_{\max}

$$\underbrace{\ell_o}_{\text{microlength}} \ll \underbrace{\ell_{kz}}_{\text{typical critical wavelength}} \ll \underbrace{\ell_o \epsilon^{-1/2}}_{\ell_{\max}} \quad \epsilon \equiv \frac{\tau_o}{\tau_Q}$$

Slightly miss the critical point: constant n/s trajectories

- Ideal hydro conserves n/s

Entropy conservation: $u^\mu \partial_\mu s = -s \partial_\mu u^\mu \Rightarrow \partial_t s = -\frac{s}{\tau_Q}$

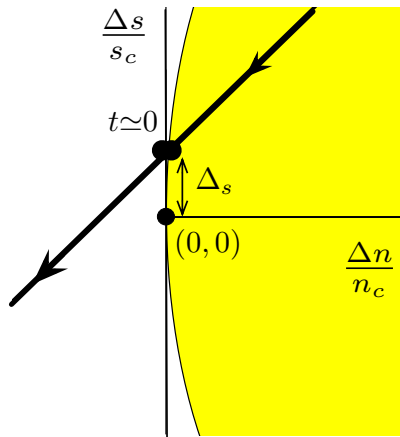
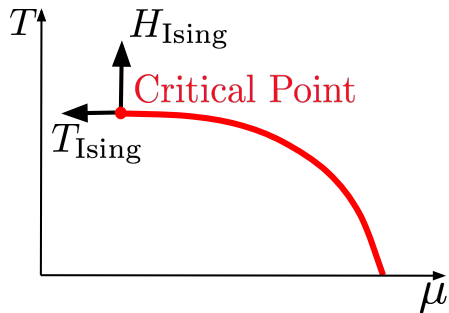
Baryon conservation: $u^\mu \partial_\mu n = -n \partial_\mu u^\mu \Rightarrow \partial_t n = -\frac{n}{\tau_Q}$

- Passing through critical point at $t = 0$ and expansion rate $1/\tau_Q$

$$\frac{\Delta n}{n_c} \approx -\frac{t}{\tau_Q} \quad \frac{\Delta s}{s_c} \approx \underbrace{\Delta_s}_{\text{detuning}} - \frac{t}{\tau_Q}$$

Near the CP, the trajectories are controlled by the detuning parameter Δ_s and the expansion rate $\partial_\mu u^\mu \equiv 1/\tau_Q$

Straight line trajectories in $s - n$ plane



$$-\Delta\mu_{\text{QCD}} \longleftrightarrow \Delta T_{\text{Ising}}$$

$$\Delta T_{\text{QCD}} \longleftrightarrow \Delta H_{\text{Ising}}$$

$$\Delta s \longleftrightarrow \Delta M_{\text{Ising}}$$

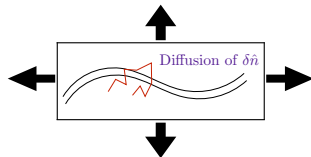
$$\Delta n \longleftrightarrow \Delta e_{\text{Ising}}$$

Hydrodynamic equation for $C^{\hat{n}\hat{n}}(k, t) = \langle \hat{n}(k, t) \hat{n}(-k, t) \rangle$

- Start from dissipative hydro with noise

$$\partial_\mu (T_{\text{ideal}}^{\mu\nu} + T_{\text{diss}}^{\mu\nu} + \xi^{\mu\nu}) = 0$$

$$\partial_\mu (j_{\text{ideal}}^\mu + j_{\text{diss}}^\mu + \xi^\mu) = 0$$



- Can derive time evolution equations for the correlators

$$C^{ee} = \langle \delta e^*(k, t) \delta e(-k, t) \rangle \quad C^{nn} = \langle \delta n^*(k, t) \delta n(-k, t) \rangle \quad , \text{ etc}$$

- From C^{nn} , C^{ee} derive an equation for $C^{\hat{n}\hat{n}} = \langle [\delta n - (n/s) \delta s]^2 \rangle$

$$\partial_t C^{\hat{n}\hat{n}} = - \underbrace{\frac{\lambda_{\text{eff}} k^2}{C_p}}_{\text{heat diffusion}} (C^{\hat{n}\hat{n}} - C_p)$$

From stochastic hydro find that $C^{\hat{n}\hat{n}}$ obeys a relaxation equation

Percentage changing rate of equilibrium

- ▶ The equilibrium correlator scales with correlation length

$$C_p \propto \chi_{\text{Ising}} \quad \chi_{\text{Ising}} \propto \xi^{2-\eta}$$

- ▶ The correlation scales with Ising energy

$$\xi \propto (\Delta e_{\text{Ising}})^{-a\nu} \quad a \equiv 1/(1-\alpha)$$

With the map $\Delta e_{\text{Ising}} \propto \Delta n/n_c = -t/\tau_Q$, we find time dependence:

$$\xi(t) = \ell_o \left(\frac{t}{\tau_Q} \right)^{-a\nu} \quad \chi_{\text{Ising}}(t) = \chi_o \left(\frac{\xi(t)}{\ell_o} \right)^{2-\eta}$$

- ▶ Percentage change per time in equilibrium

$$\left| \frac{\partial_t \xi}{\xi} \right| = \frac{a\nu}{t} \sim \frac{1}{t}$$

The equilibrium changes infinitely fast at the CP where $t = 0$

Relaxation rate of $C^{\hat{n}\hat{n}}$

- ▶ Substitute $\chi_{\text{Ising}} = \chi_o (\xi/\ell_o)^{2-\eta}$ into the relaxation equation

$$\partial_t C^{\hat{n}\hat{n}} = - \underbrace{\frac{\lambda_{\text{eff}}}{\chi_o \ell_o^2 (\xi/\ell_o)^{4-\eta}}}_{\text{relaxation rate } \Gamma} [C^{\hat{n}\hat{n}} - \chi(k, t)]$$

- ▶ Define the relaxation rate

$$\Gamma \equiv \underbrace{\frac{\lambda_{\text{eff}}}{\chi_o \ell_o^2}}_{1/\tau_o} \times \underbrace{\frac{1}{(\xi(t)/\ell_o)^{4-\eta}}}_{\text{goes to 0 at CP}}$$

- ▶ Simplify Γ with $\xi(t) = \ell_o (\frac{t}{\tau_Q})^{-a\nu}$

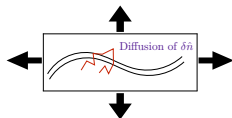
$$\Gamma = \frac{1}{\tau_o} \left(\frac{t}{\tau_Q} \right)^{a\nu z} \quad \text{where } z = 4 - \eta.$$

At CP, the hydro fluctuations relax infinitely slowly

Transiting the CP: Kibble-Zurek scales Berdnikov & Rajagopal; Mukherjee, Venugopalan, Yin

- When is this the changing rate of equilibrium comparable to the relaxation rate Γ ?

$$\left| \frac{\partial_t \xi}{\xi} \right| \sim \frac{1}{t} = \frac{(t/\tau_Q)^{a\nu z}}{\tau_Q} = \Gamma$$



- The solution is defined to be Kibble-Zurek time t_{kz}

$$t_{kz} = \epsilon^{1/(a\nu z+1)} \tau_Q = \epsilon^{0.26} \tau_Q$$

- Kibble-Zurek length is defined to be

$$\ell_{kz} = \xi(t_{kz}) = \ell_o \epsilon^{-a\nu/(a\nu z+1)} = \ell_o \epsilon^{-0.19}$$

$$\underbrace{\ell_o}_{\text{micro-length}} \ll \underbrace{\ell_o \epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \ll \underbrace{\ell_o \epsilon^{-0.5}}_{\text{cutoff } \ell_{\max}}$$

Transiting the critical point: Rescaled equation

- Measure t and k in KZ units

$$\bar{t} = t/t_{kz} \quad \bar{k} = k\ell_{kz}$$

- $C^{\hat{n}\hat{n}}$ is cut off at χ_{kz}

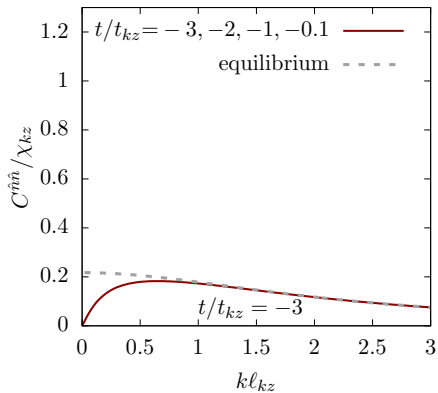
$$\bar{C}^{\hat{n}\hat{n}} = C^{\hat{n}\hat{n}}/\chi_{kz} \quad \chi_{kz} \equiv \chi_{\text{Is}}(t_{kz}) = \chi_o \epsilon^{-0.365}$$

- The rescaled equation becomes

$$\partial_{\bar{t}} \bar{C}^{\hat{n}\hat{n}} = -\frac{\bar{k}^2}{\bar{\chi}} (\bar{C}^{\hat{n}\hat{n}} - \bar{\chi}) \quad \bar{\chi} = \frac{\bar{\chi}_{\text{Ising}}}{1 + (\bar{k}\bar{\xi})^{2-\eta}}$$

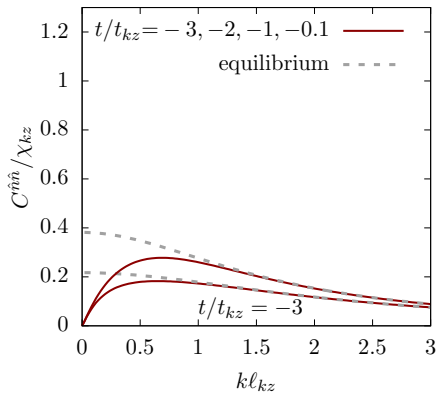
All dimensionful quantities are rescaled into the KZ-units

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



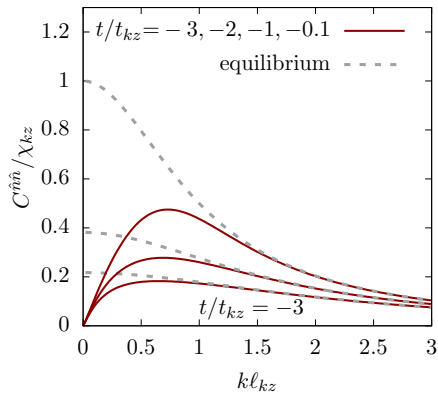
Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



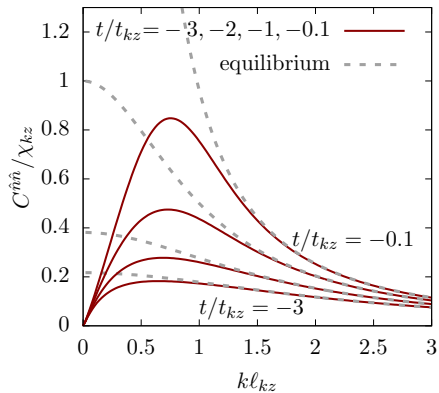
Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



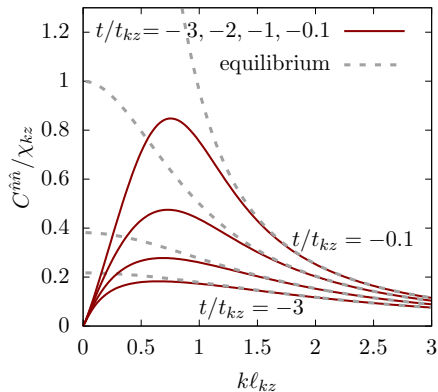
Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$

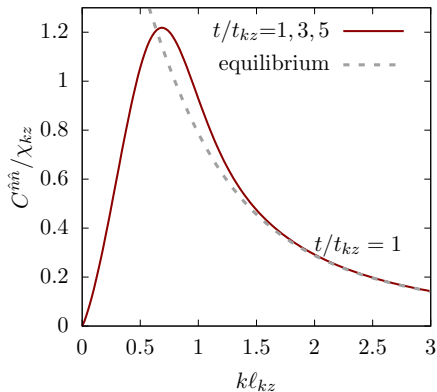


Before critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$

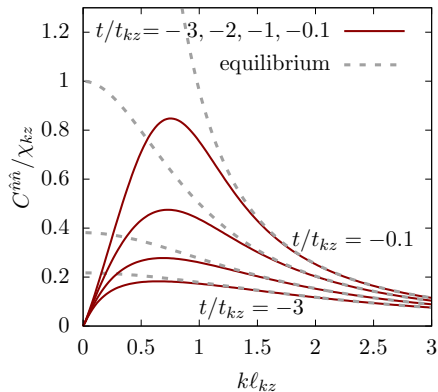


Before critical point

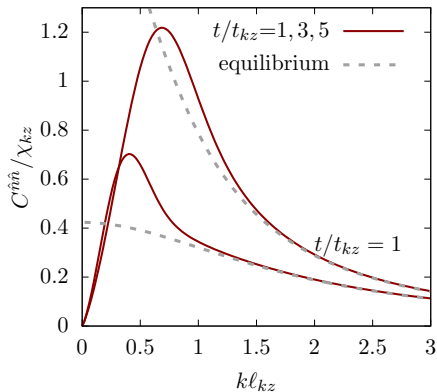


After critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$

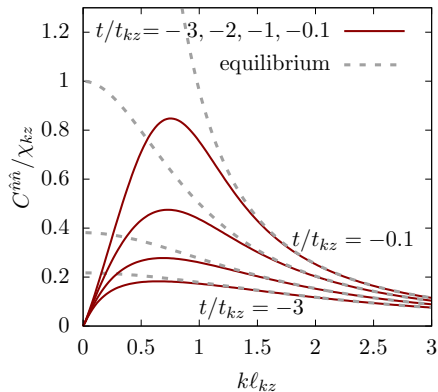


Before critical point

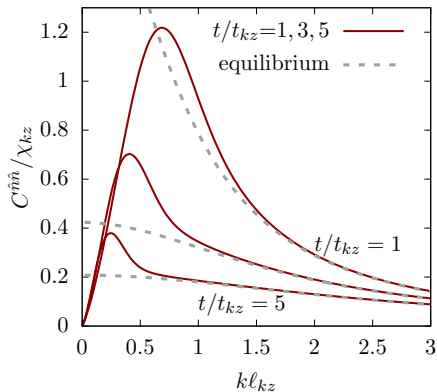


After critical point

Solutions for $C^{\hat{n}\hat{n}}/\chi_{kz}$



Before critical point



After critical point

What have we learnt so far?

- We have obtained order-1 plots after rescaling with KZ units

$$C^{\hat{n}\hat{n}}/\chi_{kz} \sim 1 \quad k l_{kz} \sim 1 \quad t/t_{kz} \sim 1$$

- The typical critical wavelength is ℓ_{kz}

$$\underbrace{\ell_0}_{\text{micro-length}} \ll \underbrace{\ell_0 \epsilon^{-0.19}}_{\text{kibble-zurek } \ell_{kz}} \ll \underbrace{\ell_0 \epsilon^{-0.5}}_{\text{cutoff } \ell_{\max}}$$

Numerically these evaluate to with $\epsilon = 1/5$ and $\ell_0 = 1.2$ fm

$$1.2 \text{ fm} \ll 1.6 \text{ fm} \ll 2.7 \text{ fm}$$

So the correlation is at most twice the interparticle spacing!

And the fluctuations are 80% larger than baseline:

$$\frac{C^{\hat{n}\hat{n}}}{\chi_0} \sim \left(\frac{\ell_{kz}}{\ell_0} \right)^{2-\eta} = \epsilon^{-0.365} \sim 1.8$$

$C^{\hat{n}\hat{n}}$ has length scale ℓ_{kz} and has limited growth of 80%

Slightly miss the critical point: scalings

- From Ising scaling, $\xi(\Delta e_{\text{Ising}}, \Delta M_{\text{Ising}})$, scales

$$\xi = \ell_o (\Delta e_{\text{Is}})^{-a\nu} f_{\xi} \underbrace{\left(\Delta e_{\text{Is}} / \Delta M_{\text{Is}}^{\frac{1-\alpha}{\beta}} \right)}_{\text{scaling var}}$$

- Translating to QCD

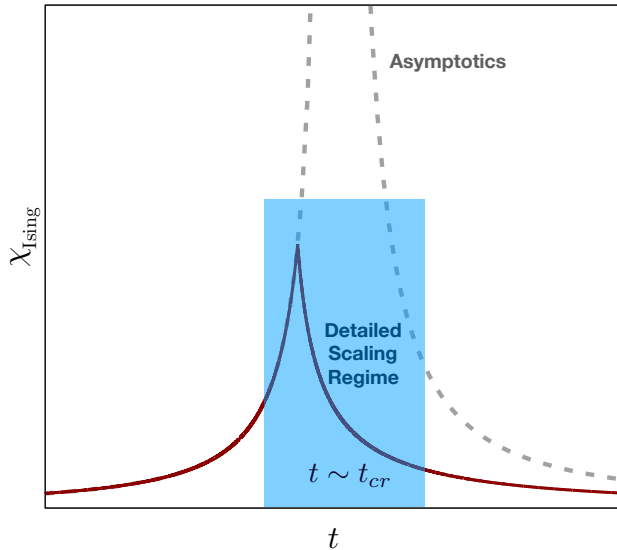
$$\Delta e_{\text{Ising}} \leftrightarrow \frac{\Delta n}{n_c} = -\frac{t}{\tau_Q} \qquad \Delta M_{\text{Ising}} \leftrightarrow \frac{\Delta s}{s_c} \sim \Delta_s$$

- The scaling of the Ising EOS implies a scaling in time

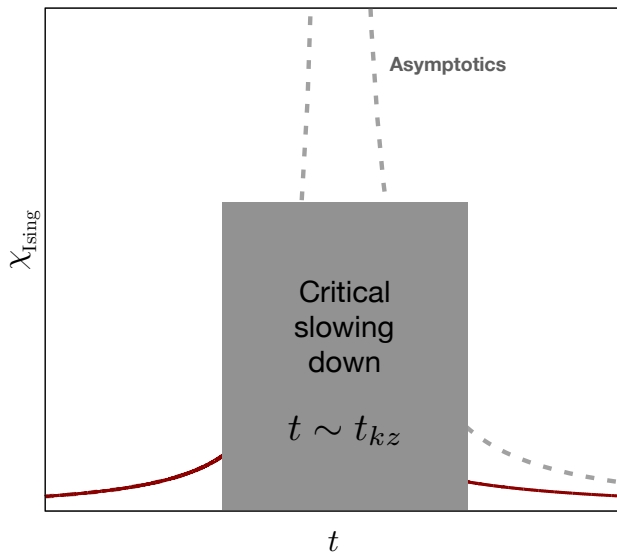
$$\xi = \ell_o \left(\frac{t}{\tau_Q} \right)^{-a\nu} \times \underbrace{f_{\xi}(t/t_{cr})}_{\text{scaling func}} \qquad t_{cr} \equiv \Delta_s^{\frac{1-\alpha}{\beta}} \tau_Q$$

t_{cr} is a new time scale that quantifies the missing of CP

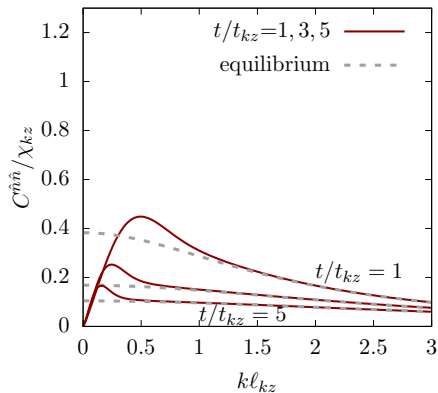
Detailed scaling regime happens when $t \sim t_{kz}$



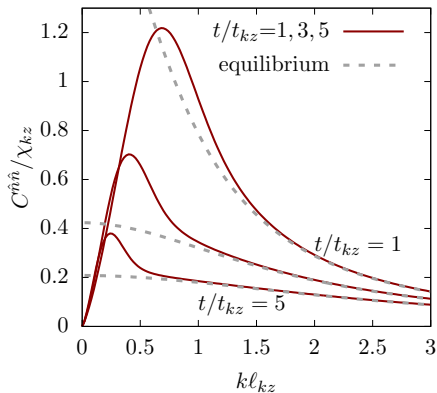
Detailed scaling regime is obscured by KZ dynamics if $t_{kz} \gg t_{cr}$



Missing the critical point further limits the fluctuations



$$t_{cr}/t_{kz} = 1$$



$$t_{cr}/t_{kz} = 0$$

Conclusions

- There are two scales t_{kz} and t_{cr} , they compete with each other

$$t_{kz} = \epsilon^{0.26} \tau_Q \quad \text{vs} \quad t_{cr} = \Delta_s^{2.72} \tau_Q$$

Numerically with $\tau_Q = 10$ fm, $\epsilon = 0.2$, $\Delta_s = 0.3$

$$t_{kz} = 6.58 \text{ fm} \quad \gg \quad t_{cr} = 0.38 \text{ fm}$$

So Kibble-Zurek dynamics is more important than detailed scaling

- $C^{\hat{n}\hat{n}}$ is a non-flow, and is quite local near CP

$$\underbrace{\ell_o}_{1.2 \text{ fm}} \ll \underbrace{\ell_o \epsilon^{-0.19}}_{1.6 \text{ fm}} \ll \underbrace{\ell_o \epsilon^{-0.5}}_{2.7 \text{ fm}} \ll \underbrace{R}_{15 \text{ fm}}$$

micro-length ℓ_o Kibble-Zurek ℓ_{kz} cutoff ℓ_{\max} nucleus size