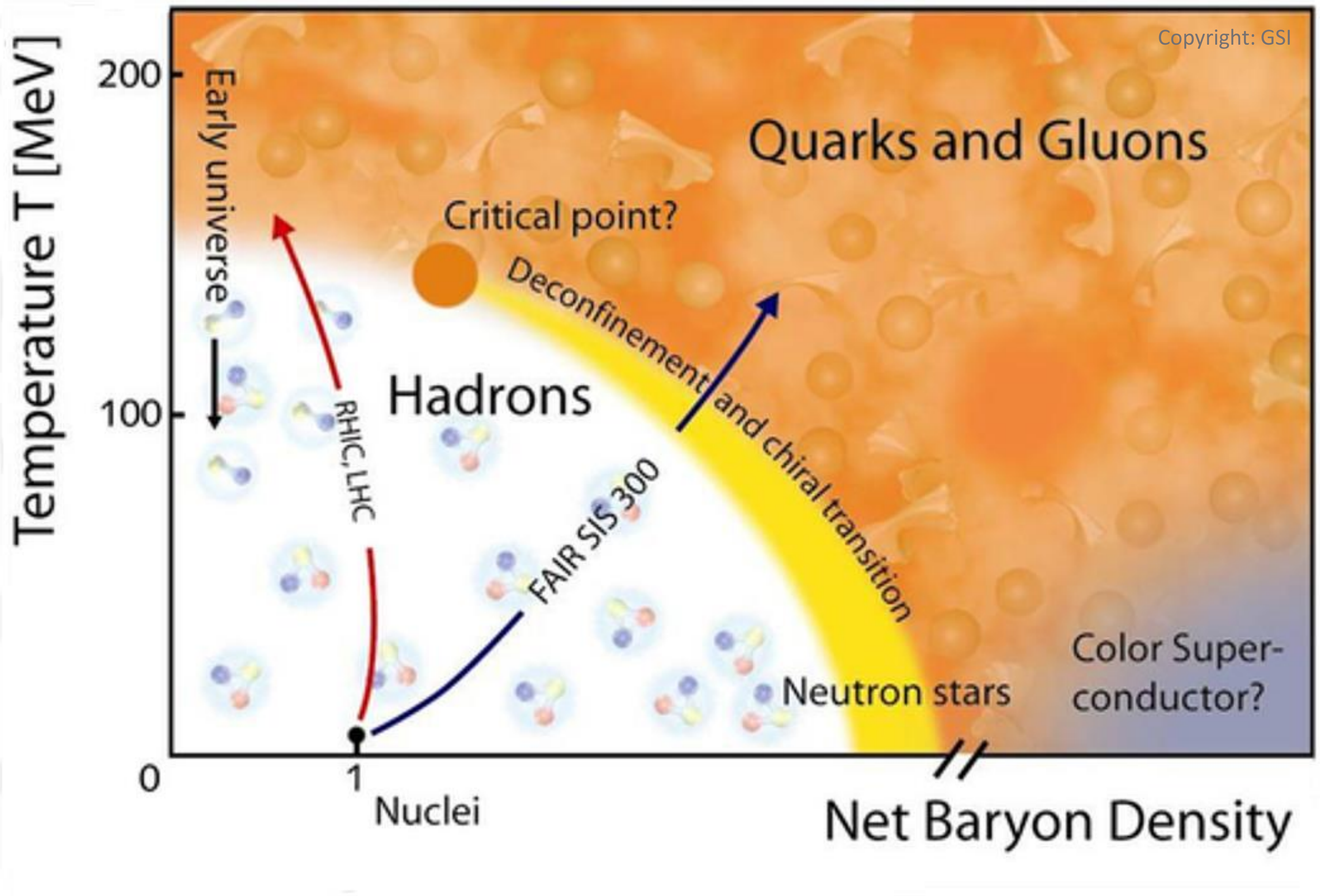
Time-evolution of fluctuations as signal of the phase transition dynamics in a QCD-assisted transport approach

Quark Matter 2018

Nicolas Wink

In collaboration with :

M. Bluhm, Y. Jiang, M. Mitter, M. Nahrgang, J. M. Pawlowski, F. Rennecke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Temperature T [MeV]
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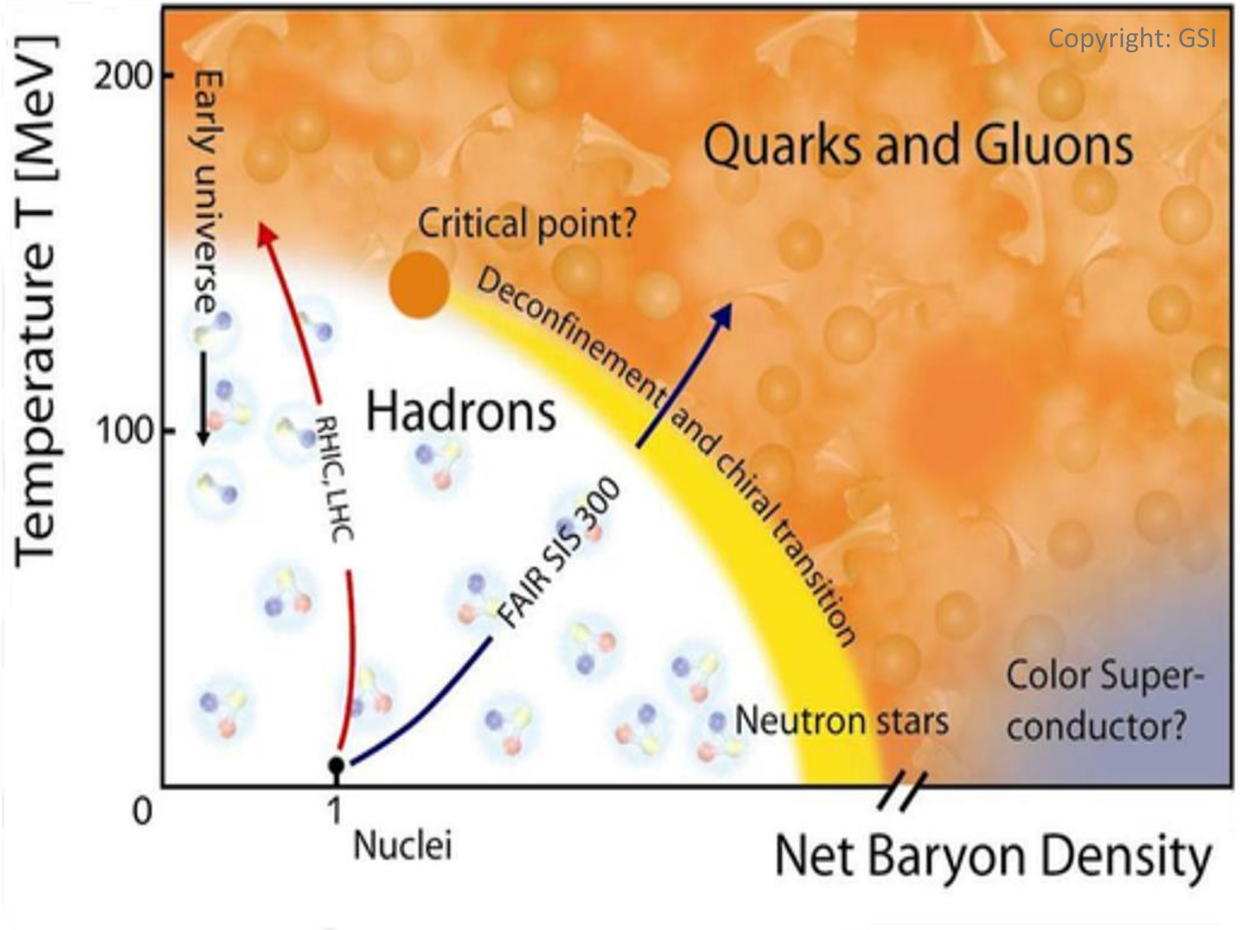
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
FAIR SIS 300
Hadrons
Critical point?
Quarks and Gluons
Deconfinement and chiral transition
Neutron stars
Color Super-
conductor?

Dynamical effects matter



- Is there a critical point?
- Connection between theoretical equilibrium results and Heavy Ion Collisions?

Realistic description of non-equilibrium dynamics is crucial!

see e.g. :

Stephanov, Rajagopal, Shuryak PRL81 (1998)
Nahrgang, Leupold, Herold, Bleicher PRC84 (2011)
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Nahrgang, Bluhm, Schäfer, Bass arXiv:1804.05728

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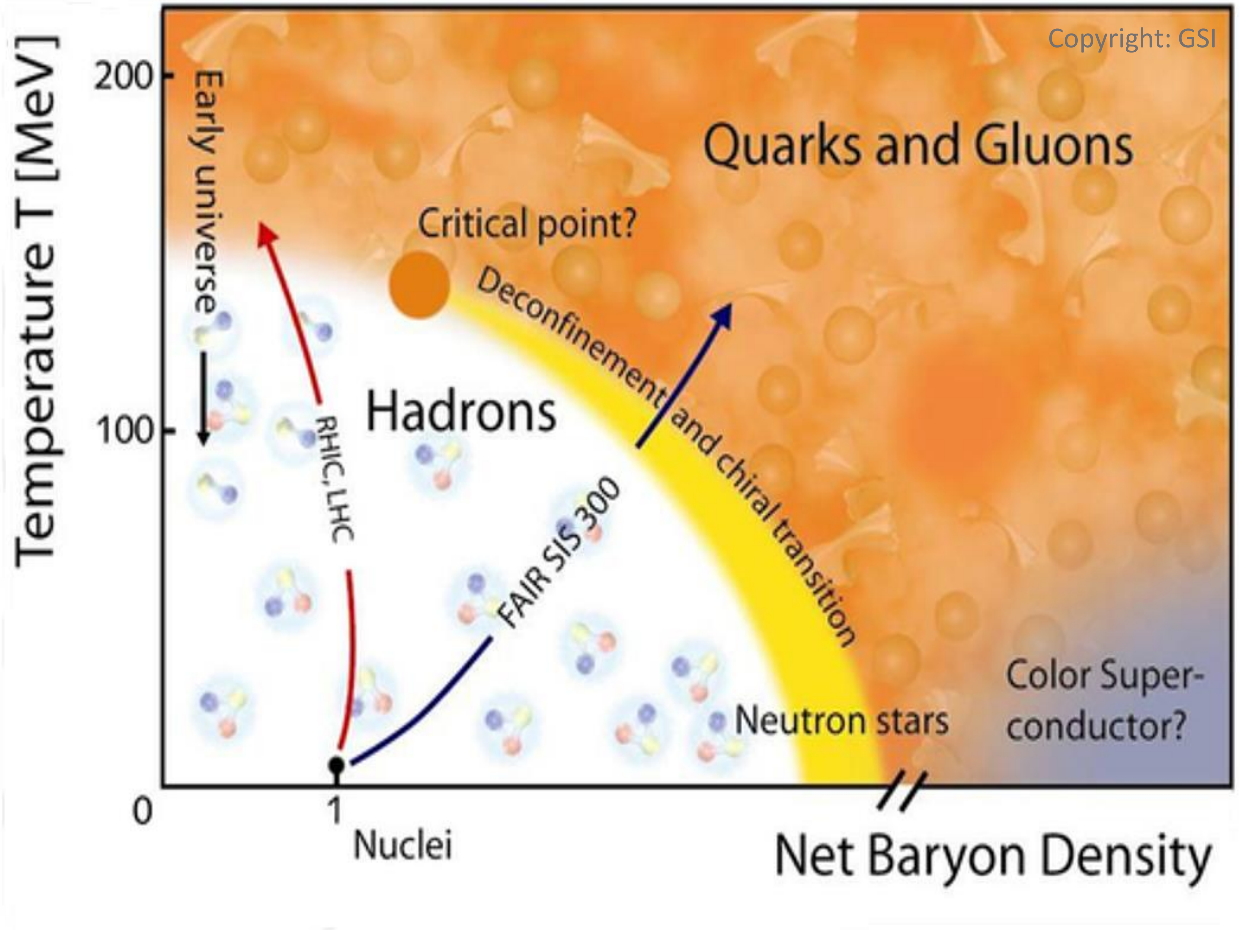

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In this talk:

Transport evolution of the critical mode
around the linear response regime of 2+1 flavor QCD

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Transport approach to QCD

Interested in the time evolution of the critical mode ⇒ Sigma meson

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Evolution governed by transport equation: $$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

with $$\left\{\mathrm{Re}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}),\ \mathrm{Im}\,\Gamma^{(2)}_\sigma(\omega,\vec{p}),\ U(\sigma)\right\} \in \Gamma$$

Equation of motion

Noise field
(Dissipation-Fluctuation)

Kinetic term

Diffusion term
➡ $\eta\,\partial_t\sigma$

Effective Potential

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

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Kinetic term

Effective Potential

Diffusion term
→ $\eta\, \partial_t \sigma$

→ Equation depends crucially on input from QCD

- Include effective potential beyond mean-field
- Include momentum dependent equilibrium sigma spectral function → Linear response regime of QCD

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Equilibrium QCD

Functional description of QCD

First principle calculations for the full phase diagram not yet available

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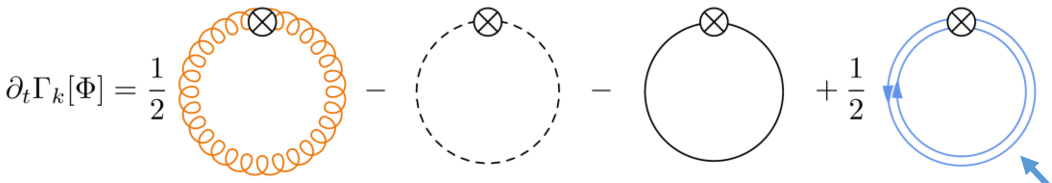
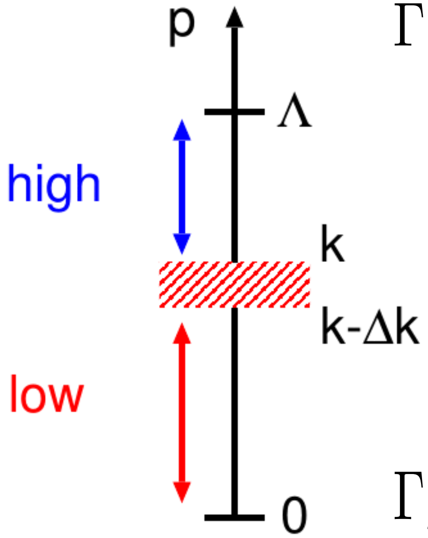
Flow equation for QCD



Bound states efficiently taken into account via Dynamical Hadronization

Functional Renormalization Group



$\Gamma_{k=\Lambda} = S$

- Non-perturbative first principle method
- Access to physical mechanisms
- No sign problem
  - Chemical potential
  - Real time

$\Gamma_{k=0} = \Gamma$

Functional description of QCD

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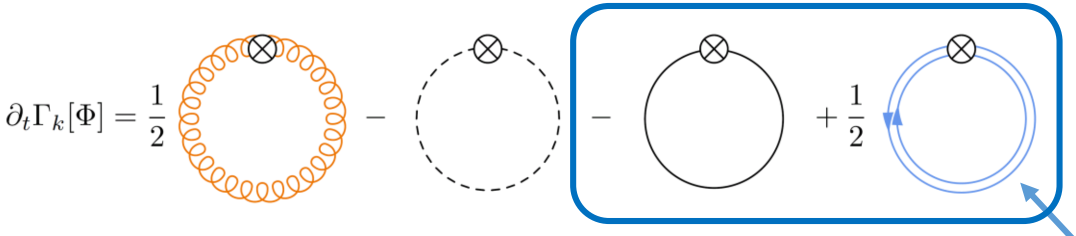

Bound states efficiently taken into account via Dynamical Hadronization

Effective description in terms of Quarks and Mesons

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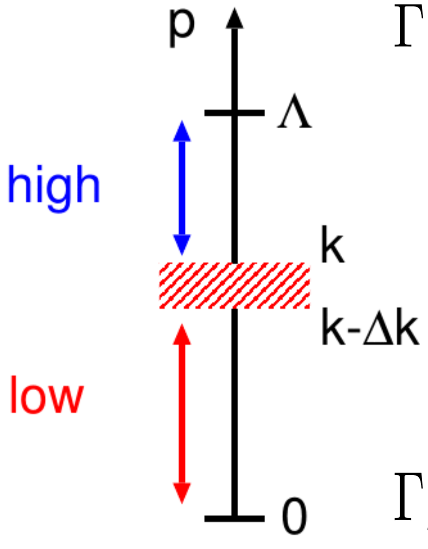

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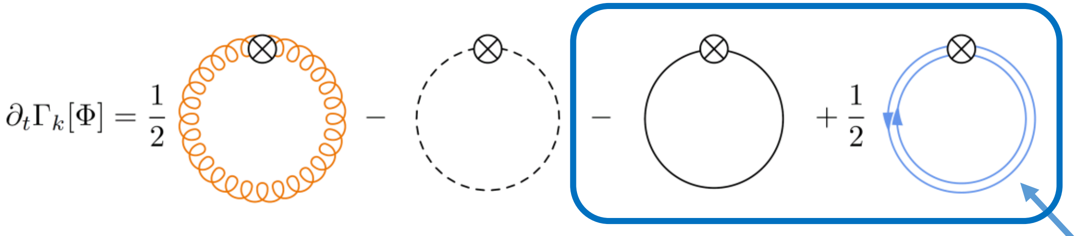

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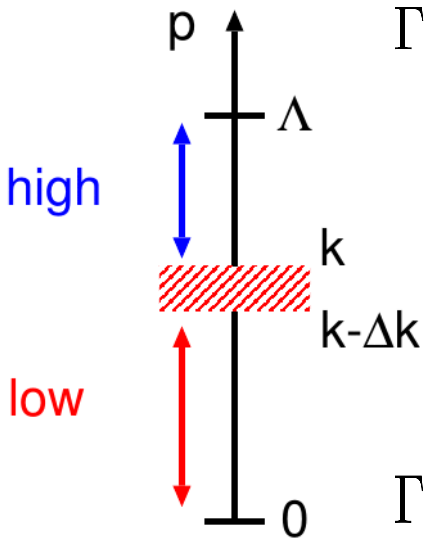

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First principle approach to QCD from the FRG
Cyrol, Mitter, Pawlowski, Strodthoff PRD97 (2018)

Poster (PHA-03): A.K. Cyrol, M. Mitter, J. M. Pawlowski, NW

Low-energy effective theory of QCD

Describes QCD at small chemical potentials and medium temperatures

Capture all dynamically relevant phenomena

→ 2+1 Quark Flavors

Remaining phase structure qualitatively similar to the conjectured QCD phase diagram

$$\Gamma_k = \int_x \left\{ i\bar{q} Z_{q,k} \left(\gamma_\mu \partial_\mu + \gamma_0 \mu\right) q + i\bar{q}\, h_{q,k} \cdot \Sigma_5 q + \mathrm{tr}\left(Z_{\Sigma,k} \partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger\right) + \tilde{U}_k(\Sigma, \Sigma^\dagger) \right\}$$

with $q = (u, d, s)$

Schäfer, Rennecke, PRD96 (2017)

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with $q = (u, d, s)$

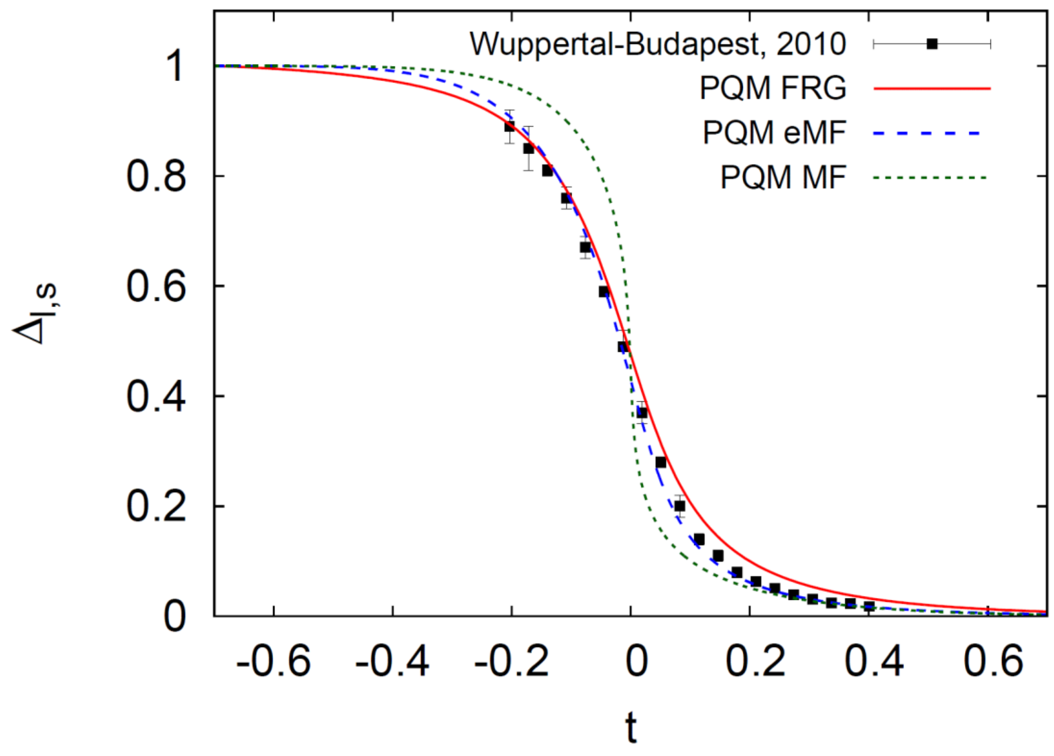
(Pseudo)scalar nonet field

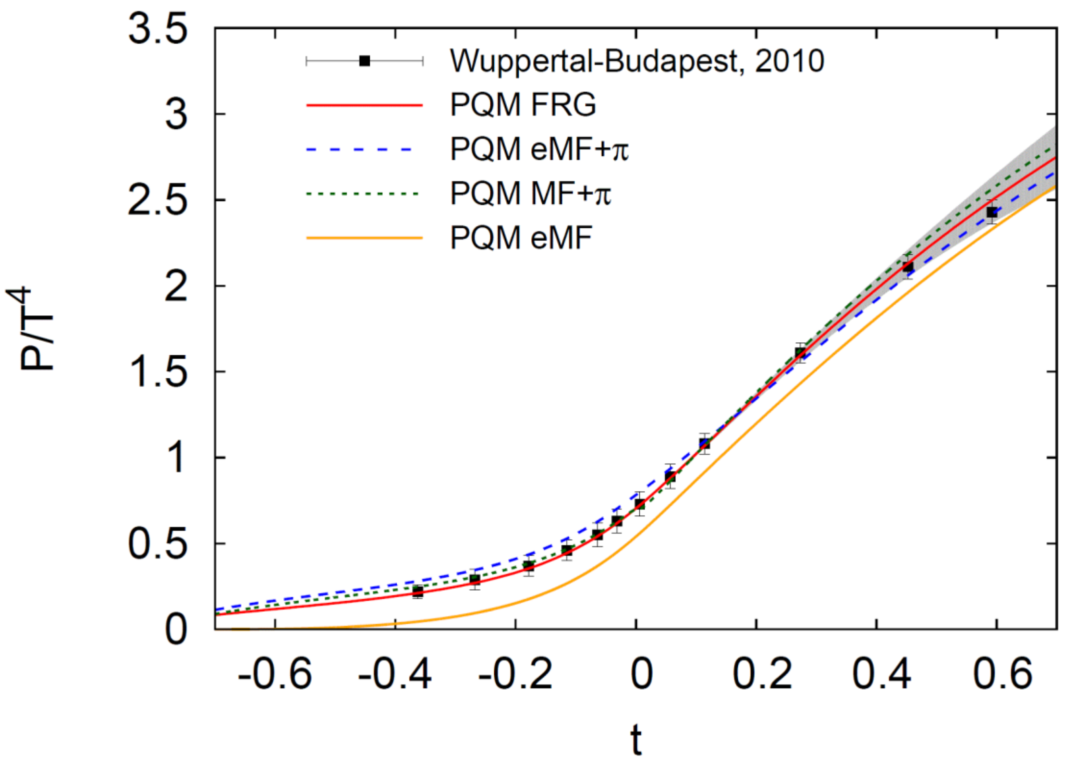
$$\Sigma = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\sigma_L + a_0^0 + i\eta_L + i\pi^0\right) & a_0^- + i\pi^- & \kappa^- + iK^- \\ a_0^+ + i\pi^+ & \frac{1}{\sqrt{2}}\left(\sigma_L - a_0^0 + i\eta_L - i\pi^0\right) & \kappa^0 + iK^0 \\ \kappa^+ + iK^+ & \bar{\kappa}^0 + i\bar{K}^0 & \frac{1}{\sqrt{2}}\left(\sigma_S + i\eta_S\right) \end{pmatrix}$$

Schäfer, Rennecke, PRD96 (2017)

Low-energy effective theory of QCD

Excellent description of phase structure at vanishing chemical potential

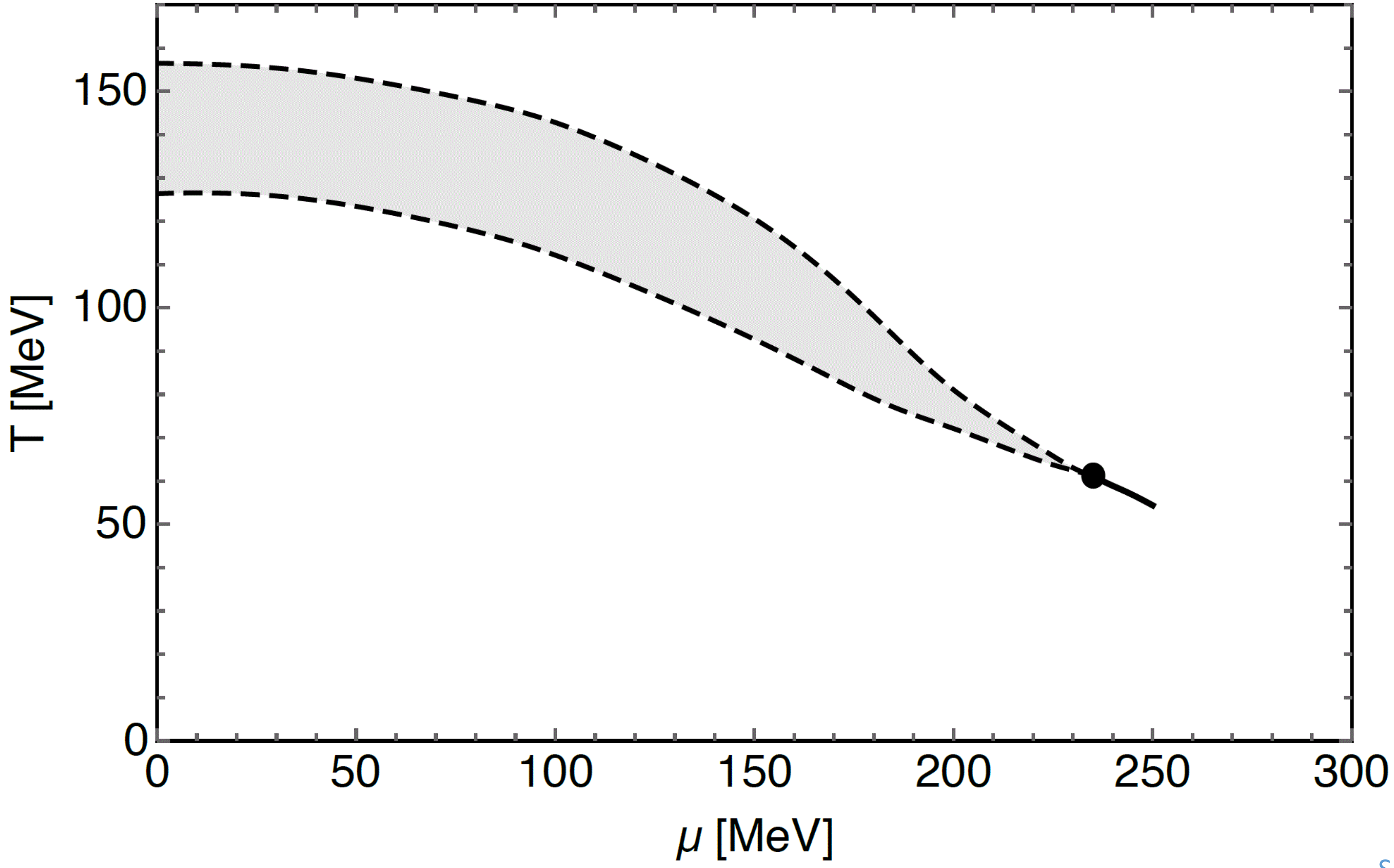




Herbst, Mitter, Pawlowski, Schäfer, Stiele, PLB731 (2014)

Low-energy effective theory of QCD

Phase structure contains a critical endpoint



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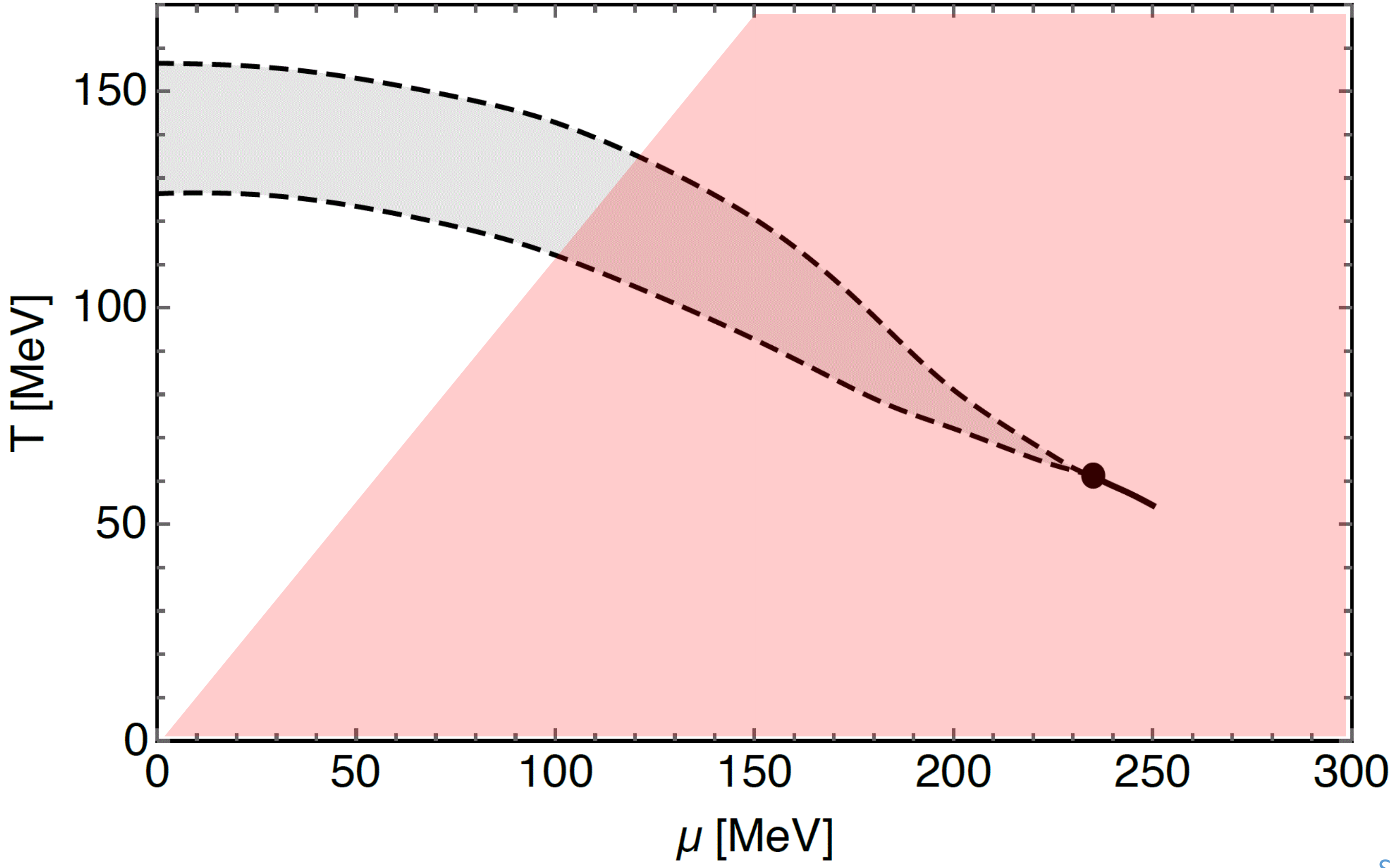

Schäfer, Rennecke, PRD96 (2017)

Towards non-equilibrium

Interested in the time evolution of the critical mode ⇨ Sigma meson

Pawlowski, Rennecke, NW, in prep.
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

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Linear response functions directly accessible via spectral functions in the FRG framework

Compute spectral functions from analytic continuation of the corresponding equations

for FRG see e.g.
Foerchinger, JHEP 1205 (2012) 021
Kamikado, Strodthoff, von Smekal, Wambach, EPJC74 (2014)
Pawlowski, Strodthoff, NW, arxiv:1711.07444

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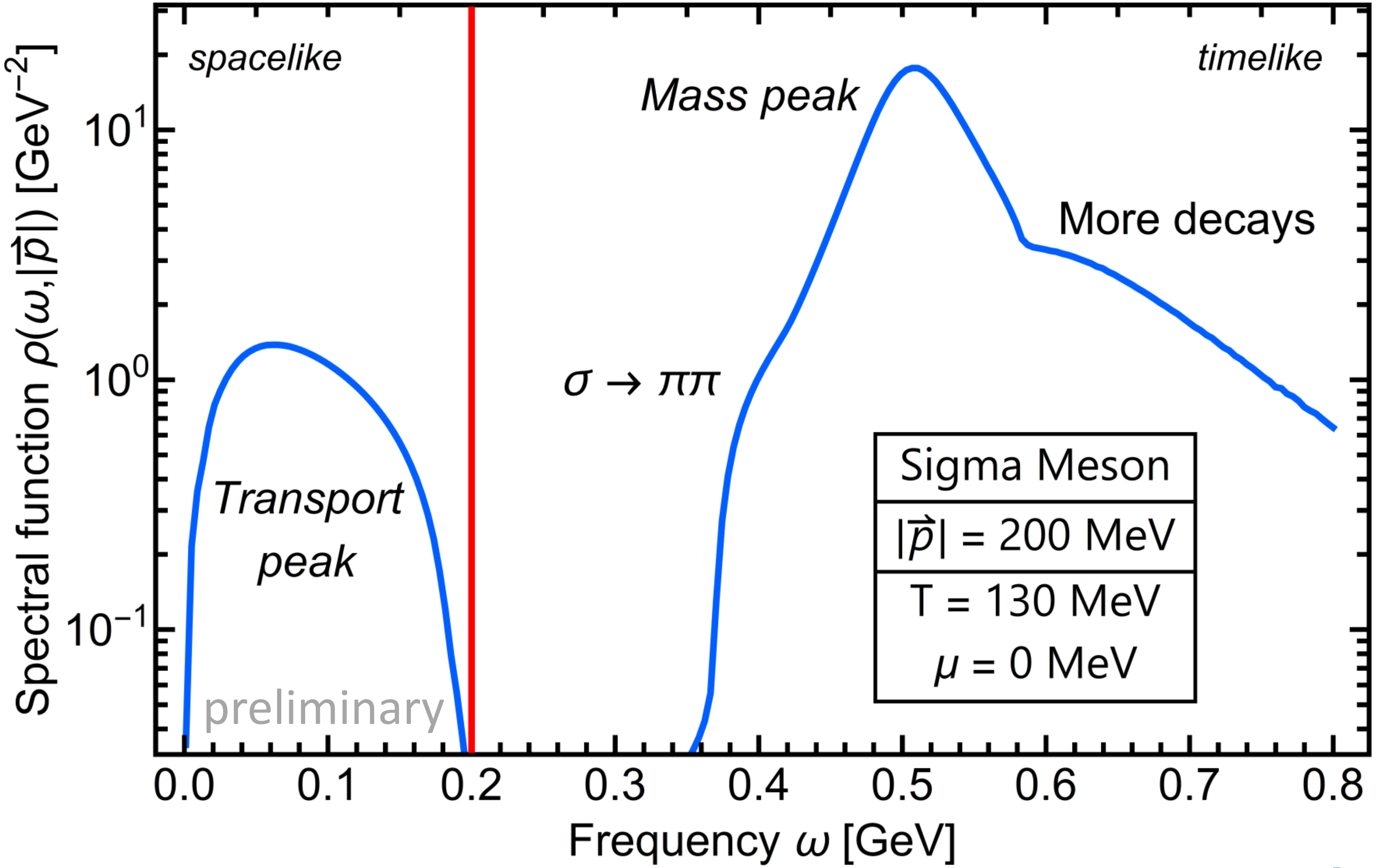
Use Effective Potential and Spectral Functions of the sigma meson as input

Pawlowski, Rennecke, NW, in prep.
Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Linear response function

Sigma meson spectral function at T = 130 MeV
and vanishing chemical potential



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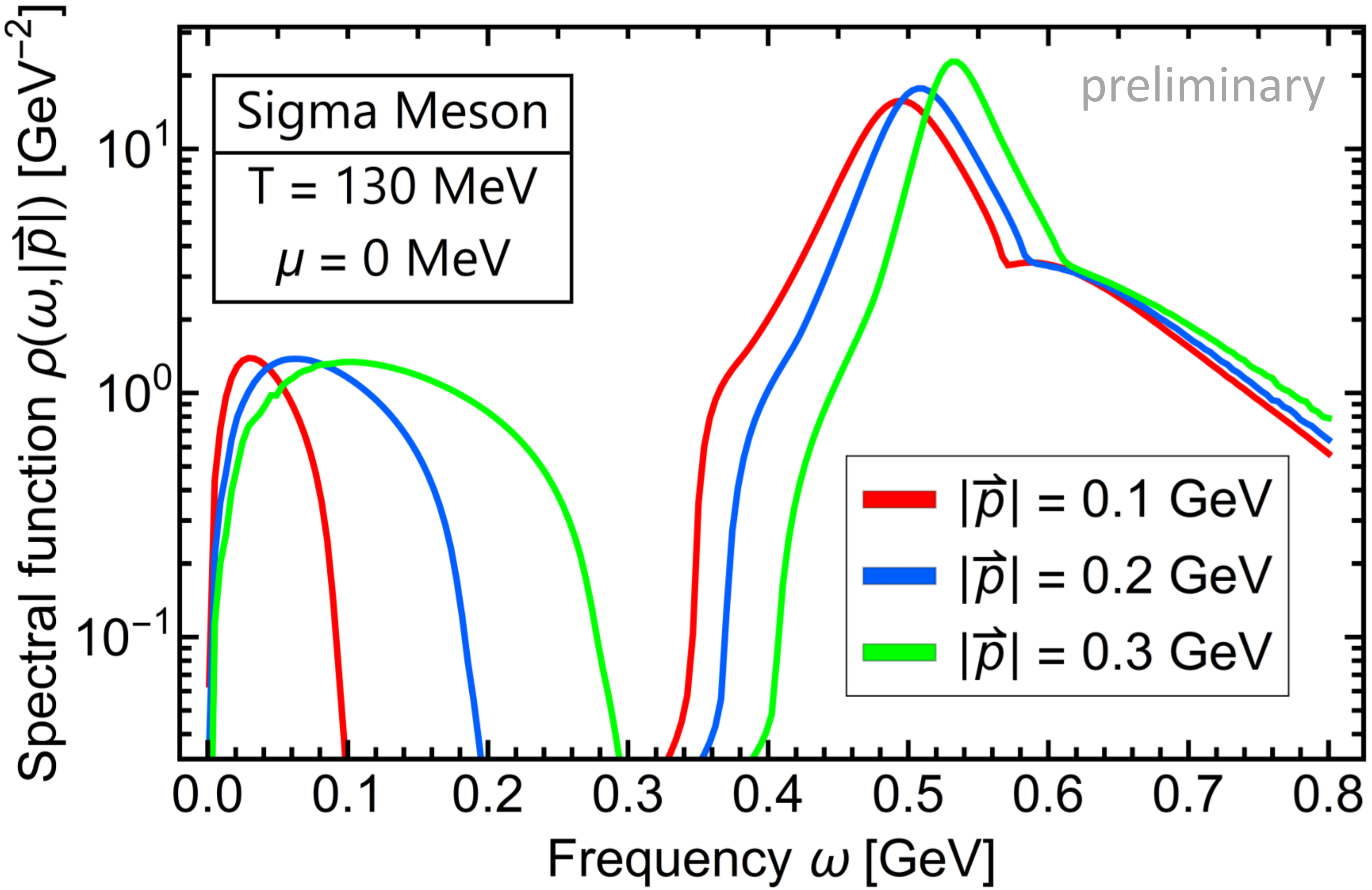

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QCD transport

Transport equation

Evolution governed by transport equation:

$$\frac{\delta \Gamma}{\delta \sigma} = \xi$$

with

$$\left\{ \mathrm{Re}\,\Gamma_\sigma^{(2)}(\omega, \vec{p}),\ \mathrm{Im}\,\Gamma_\sigma^{(2)}(\omega, \vec{p}),\ U(\sigma) \right\} \in \Gamma$$

$$\sigma(r,t) = \sigma_0 + \delta\sigma(r,t)$$

Split into equilibrium and fluctuation part

Transport equation

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Split into equilibrium and fluctuation part

White noise approximation:

$$\langle \xi(t) \rangle = 0$$
$$\langle \xi(t)\xi(t') \rangle = \frac{1}{V}\delta(t-t')m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

Spatial isotropy approximation:

$$\sigma(\vec{x}) = \sigma(r)$$

Initial conditions:

Quench from „high temperature“

$$\sigma(r) = 0 = \partial_t \sigma(r)$$

Transport equation

Calculate (standardized) cumulants

Evolution governed by transport equation:

$$\frac{\delta\Gamma}{\delta\sigma} = \xi$$

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$$\sigma(r,t) = \sigma_0 + \delta\sigma(r,t)$$

Split into equilibrium and fluctuation part

χ_n : nth central moment of the sigma field

Mean : $\sigma_0 = \langle\sigma\rangle$

Variance : $\sigma^2 = \left\langle(\sigma-\sigma_0)^2\right\rangle$

Standardized cumulants
0 for a Gaussian

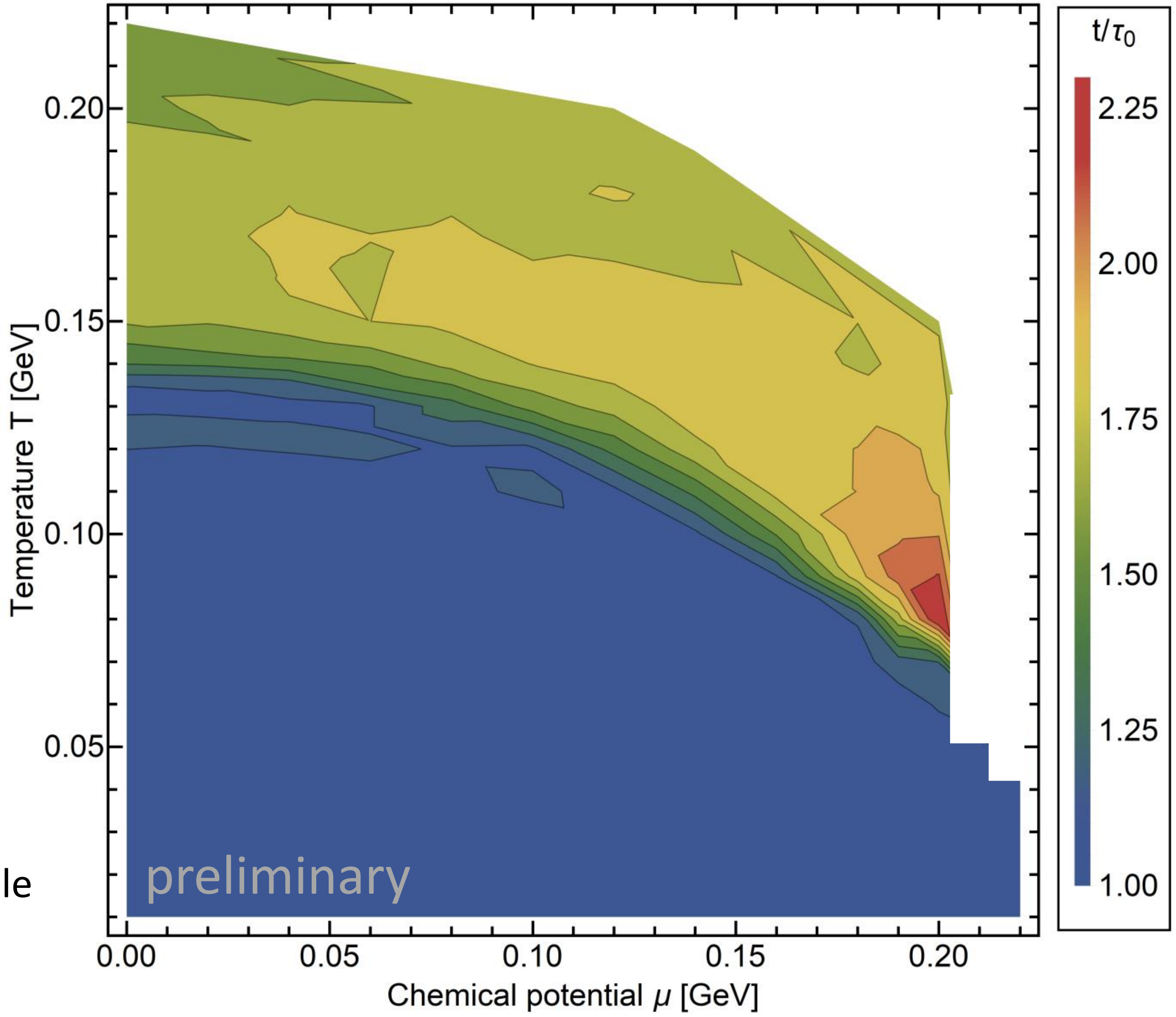
Skewness : $S = \dfrac{\chi_3}{\chi_2^{3/2}}$

Kurtosis : $\kappa = \dfrac{\chi_4}{\chi_2^2} - 3$

....

Equilibration time τ obtained from the Kurtosis

Equilibration time

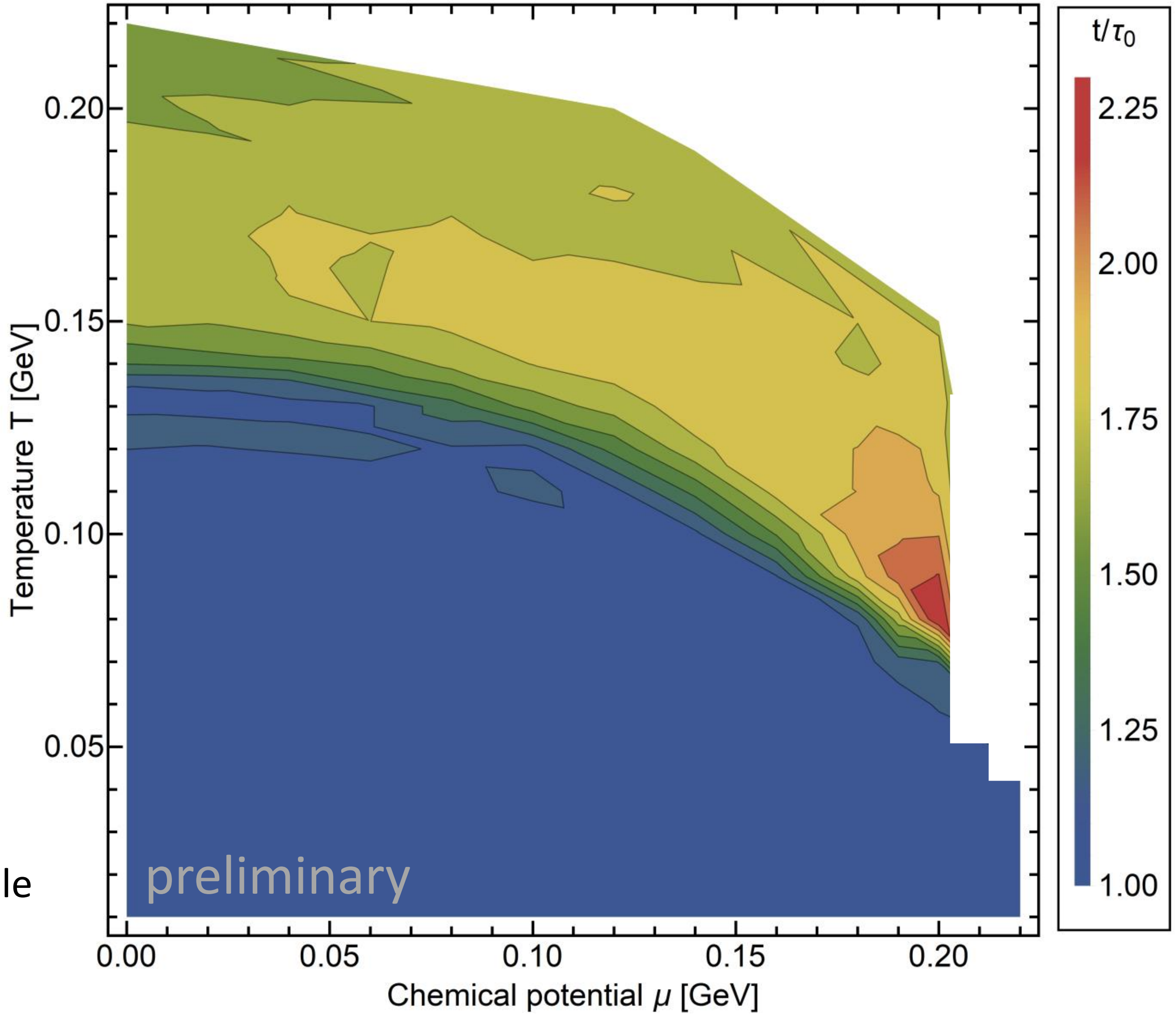


- Critical endpoint and phase boundary clearly identifiable
- Critical slowing down at the critical endpoint
- Impact on observables?

Reference time scale
$\tau_0 \approx 0.4$ fm/c

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Equilibration time



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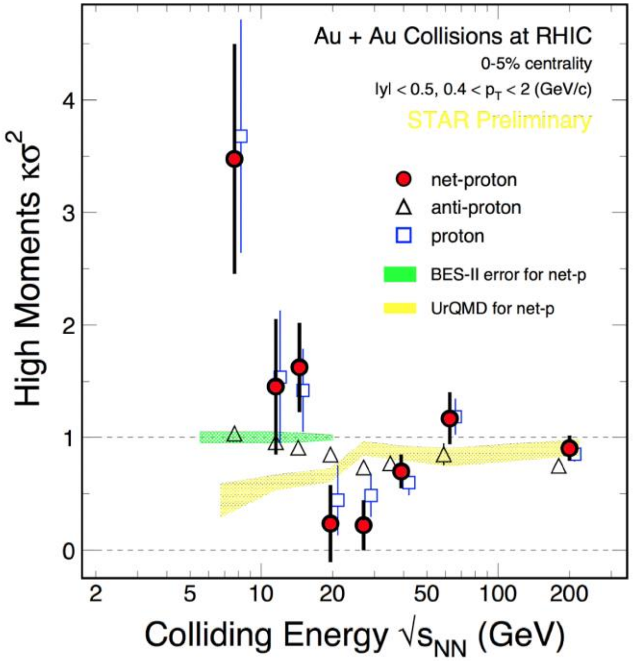
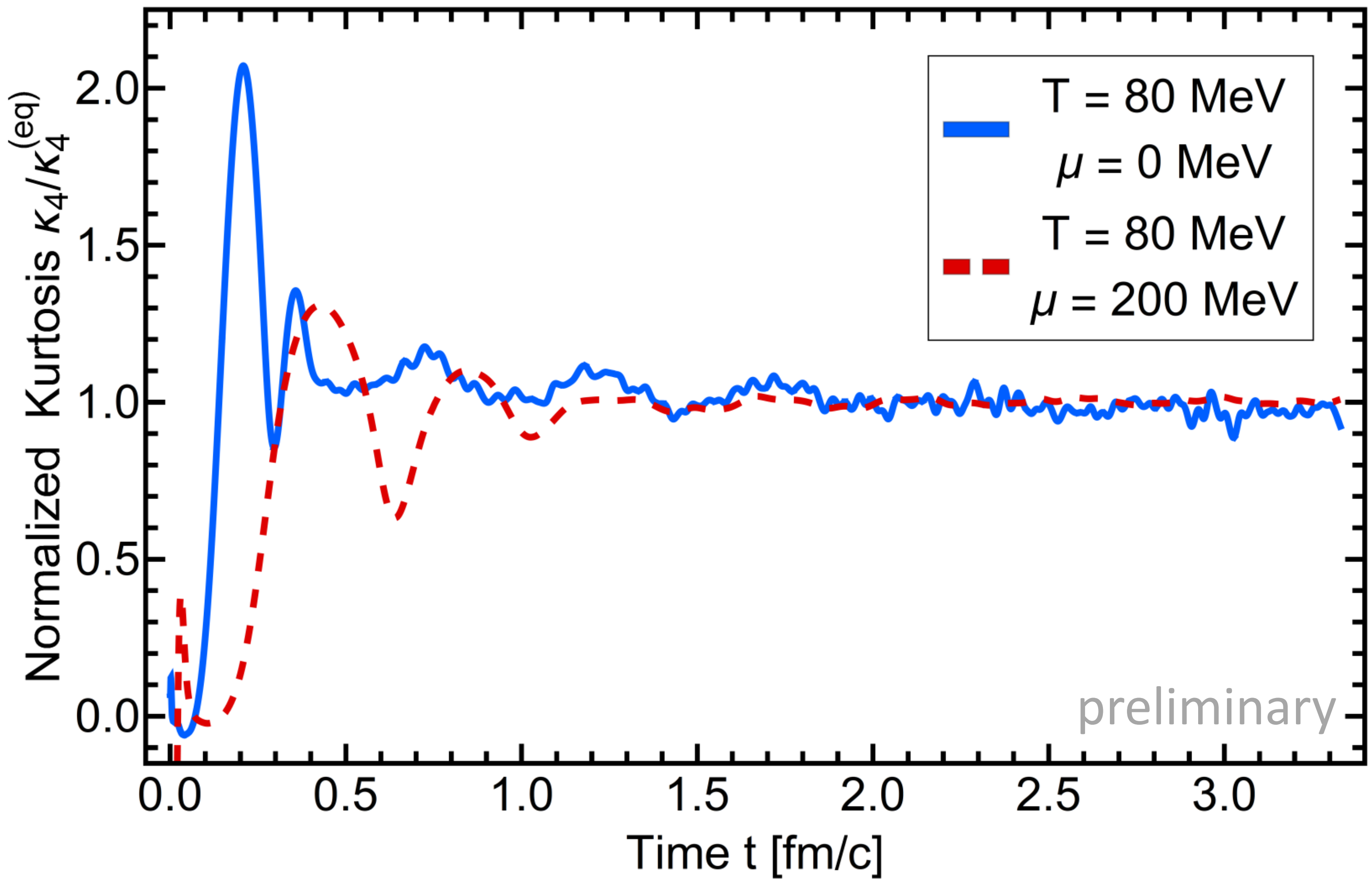


Luo, Xu, Nucl.Sci.Tech. 28 (2017)

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Time evolution of cumulants



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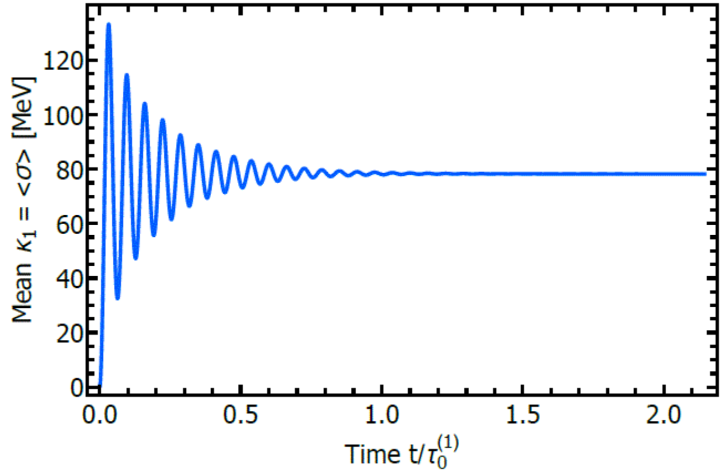
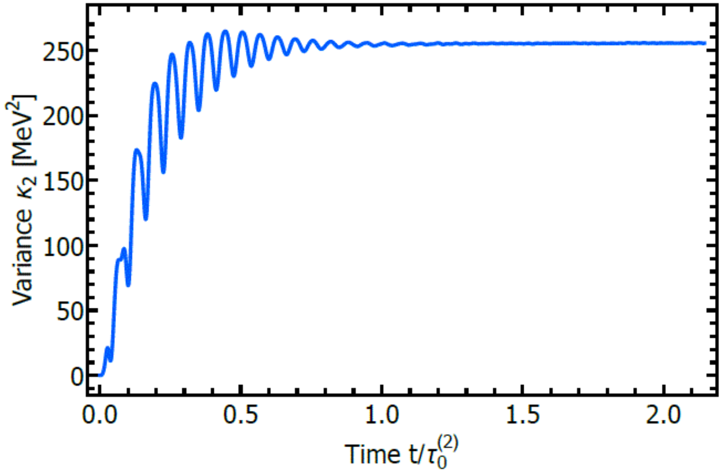
Time evolution of cumulants

Disclaimer: For the O(N)-model Qualitatively the same → Way more statistics

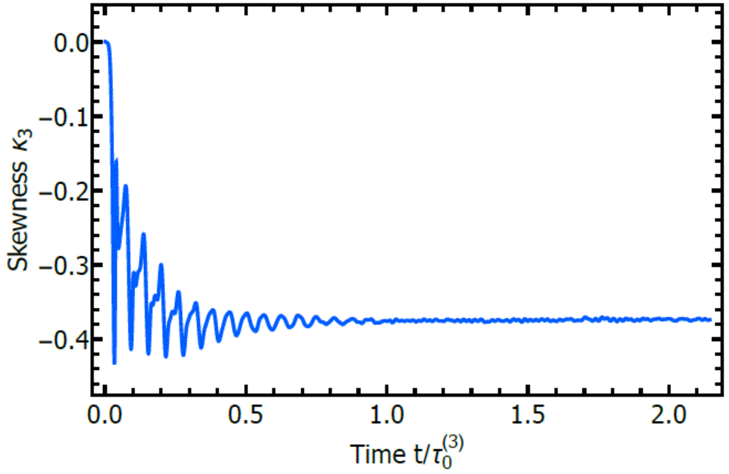
Mean



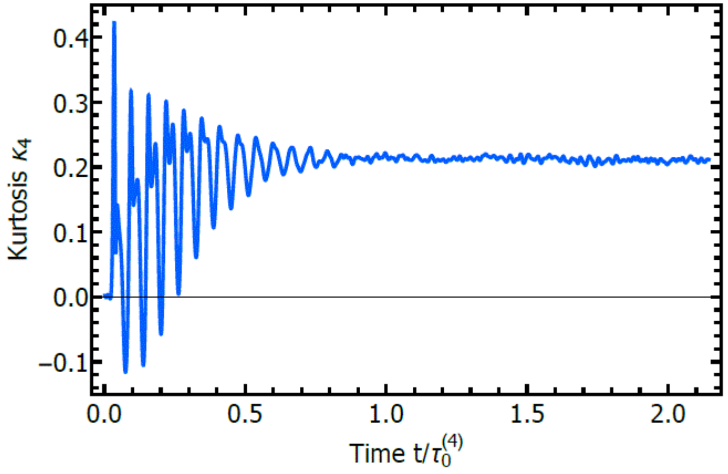
Variance



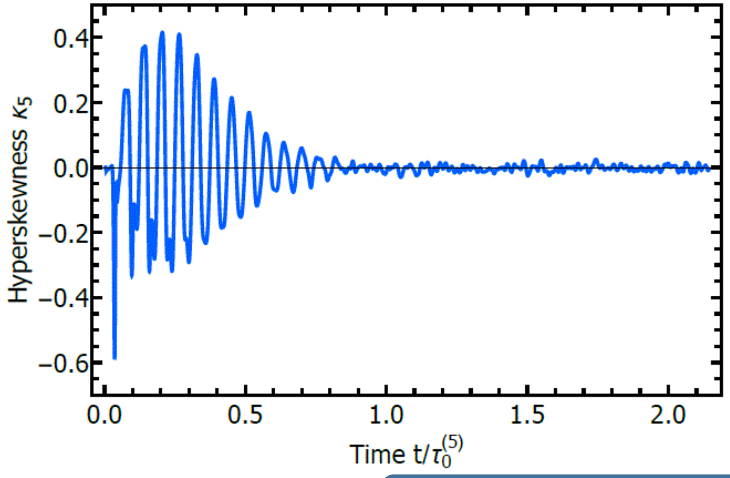
Skewness



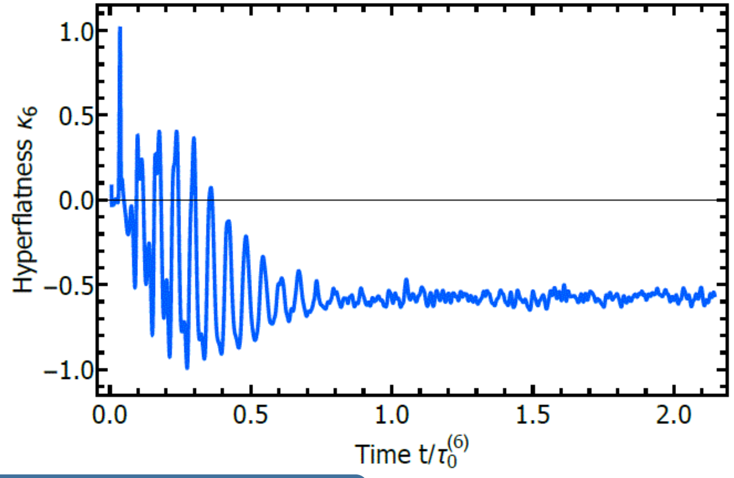
Kurtosis



Hyperskewness



Hyperflatness



Recovers correct equilibrium cumulants

preliminary

Bluhm, Jiang, Nahrgang, Pawlowski, Rennecke, NW, in prep.

Summary & Outlook

- QCD-assisted transport approach
- Spectral functions of low energy effective QCD
- Equilibration times across the QCD phase diagram
- Time evolution of cumulants

- Correlation functions
- Improved QCD input
- Colored noise, Conserved Baryon number, . . .

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Thank you for your attention!

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