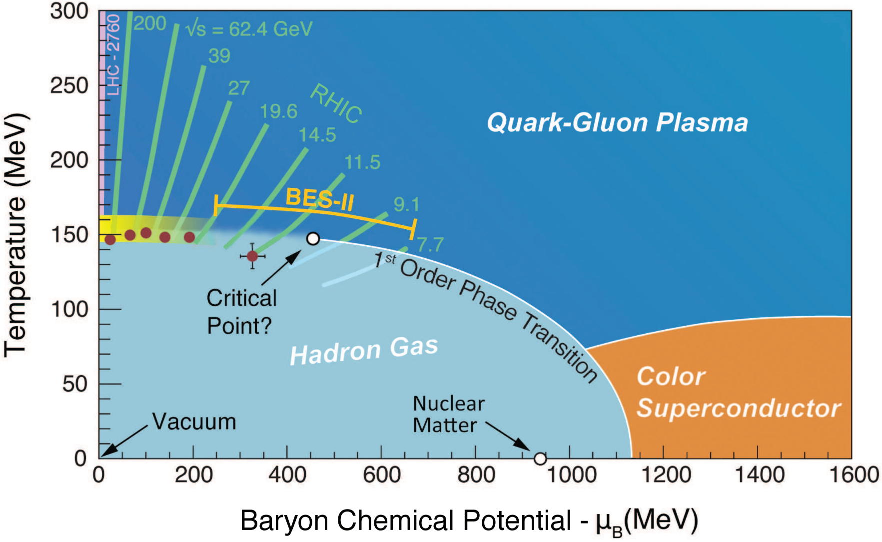
The QCD crossover
from Lattice QCD

May 16, 2018 | Patrick Steinbrecher

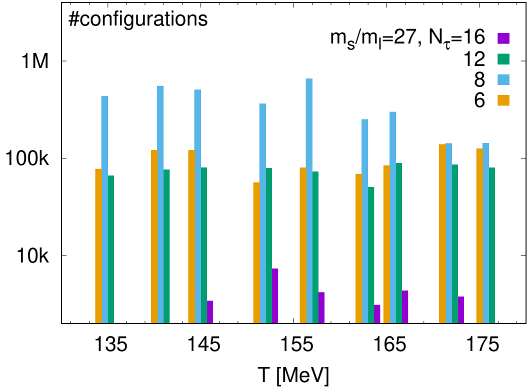
HotQCD collaboration

The QCD phase diagram



Quantum Chromodynamics
from first principles

- Lattice QCD
  - HISQ action
  - $N_\sigma = 4N_\tau$
  - sim. at $\mu = 0$
- physical quarks
  - 2 light quarks
  - 1 strange quark
  - $m_s/m_l = 27$
- $m_\pi \simeq 138\ \mathrm{MeV}$



everything continuum extrapolated

Chiral observables in two-flavor formulation

- subtracted condensate

$$\Sigma_{\mathrm{sub}} \equiv m_s(\Sigma_u + \Sigma_d) - (m_u + m_d)\Sigma_s$$

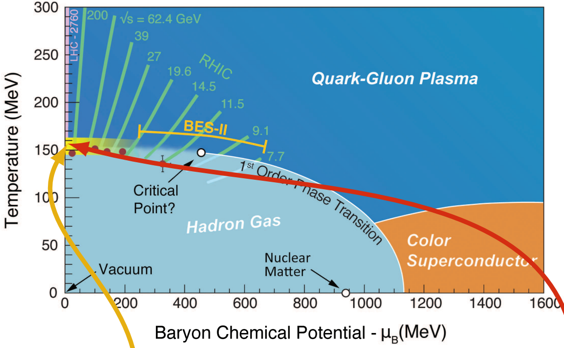
with $\qquad \Sigma_f = \frac{T}{V}\frac{\partial}{\partial m_f}\ln Z$

- subtracted susceptibility

$$\chi_{\mathrm{sub}} \equiv \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right)\Sigma_{\mathrm{sub}}$$

- χ_{disc} is defined as χ_{sub} without connected part

Start of the QCD crossover line: T_0



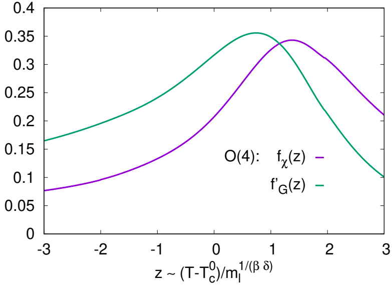
$$\frac{d^2}{dT^2}\frac{\Sigma_{\rm sub}}{f_K^4} \equiv 0 \qquad \text{and} \qquad \frac{d}{dT}\frac{\chi_{\rm sub}}{f_K^4} \equiv 0$$

two crossover temperatures: $T_0\left(\Sigma_{\rm sub}\right)$ and $T_0\left(\chi_{\rm sub}\right)$

Pseudo-critical temperatures

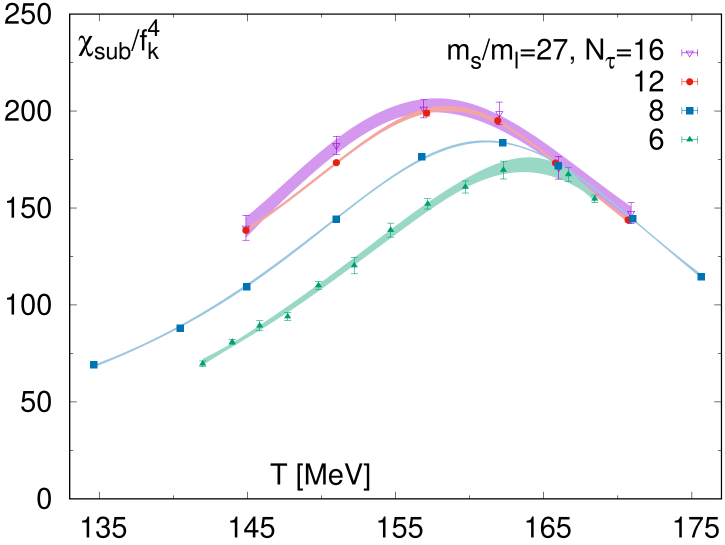
- for $m_l \to 0$: pseudo-critical temperatures converge to the chiral transition temperature T_c^0
- at finite quark mass it is given by maximum of $O(4)$ universal scaling functions (Tuesday talk, Anirban Lahiri, Chiral phase transition)

$$\chi_m = m_l^{1/\delta-1} \quad f_\chi(z) + \text{reg.}$$
$$\chi_t = m_l^{(\beta-1)/\beta\delta} f'_G(z) + \text{reg.}$$

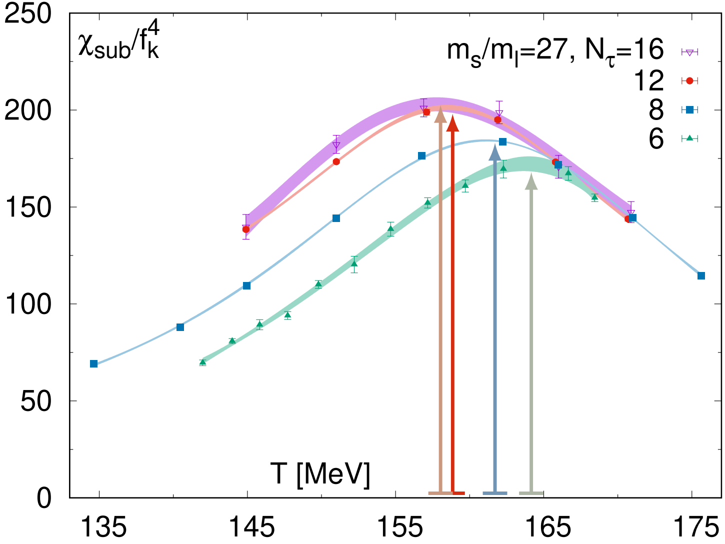


- for $m_l \to 0$
  - $\chi_t \sim \partial_T \Sigma_{\rm sub}$ and $\chi_t \sim \partial^2_{\mu_B} \Sigma_{\rm sub}$
  - $\chi_m \sim \chi_{\rm sub}$ and $\chi_m \sim \chi_{\rm disc}$

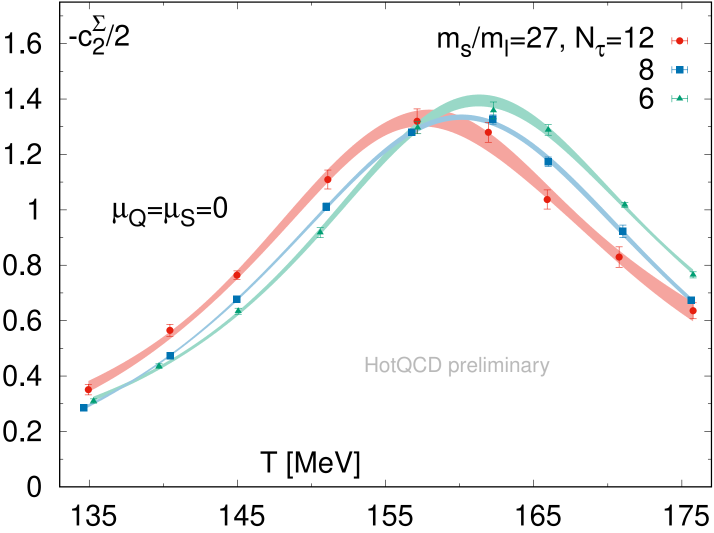
The subtracted chiral susceptibility



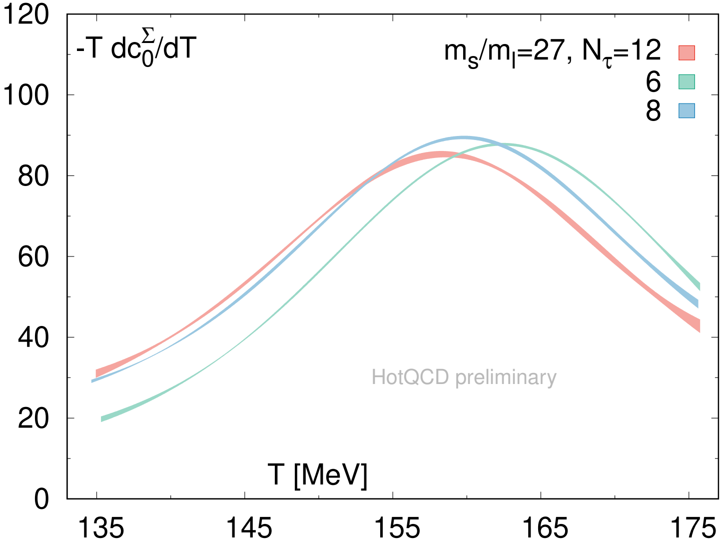
The subtracted chiral susceptibility



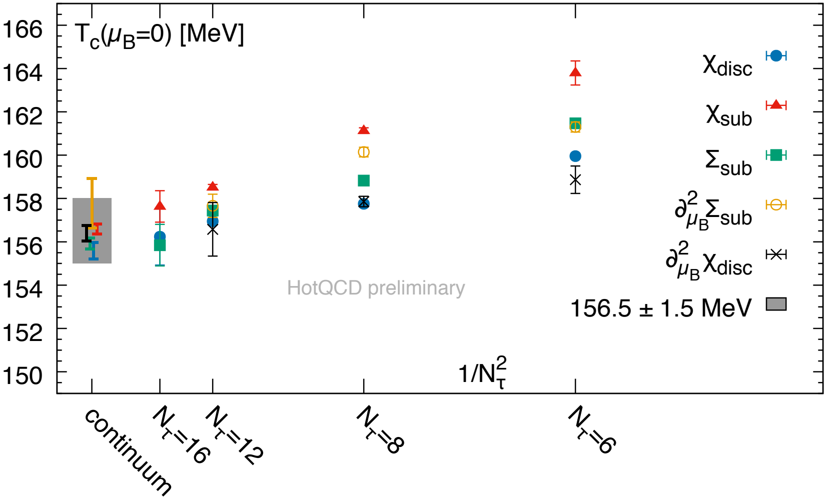
The 2nd μ_B derivative of chiral condensate $\Sigma_{\rm sub}$



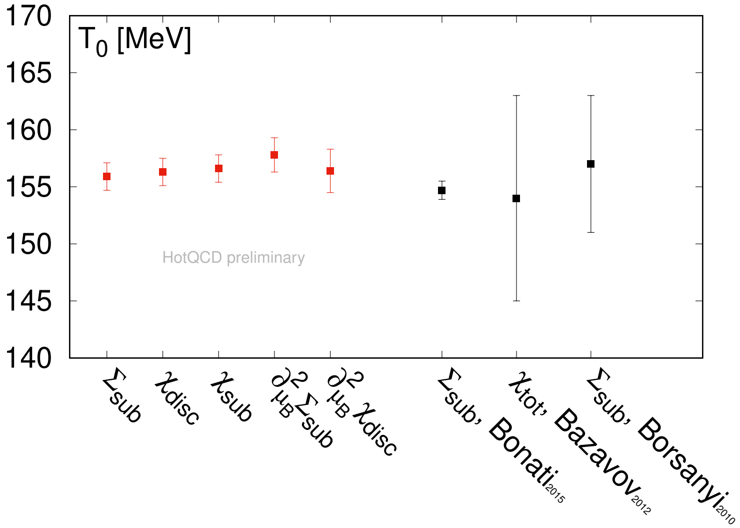
The 1st T derivative of chiral condensate Σ_{sub}



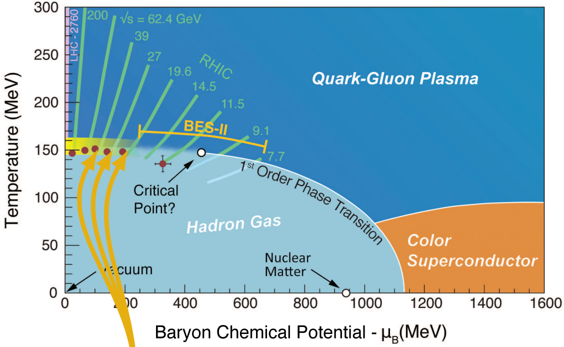
The T_0 continuum extrapolation



Crossover temperature T_0



The QCD crossover at $\mu \neq 0$



$$\frac{d^2}{dT^2}\frac{\Sigma_{\rm sub}(T,\mu_B)}{f_K^4} \equiv 0 \qquad \text{and} \qquad \frac{d}{dT}\frac{\chi_{\rm disc}(T,\mu_B)}{f_K^4} \equiv 0$$

$\Rightarrow$ need Taylor expansion in T and μ_B around $(T_0, 0)$

Taylor expansion in chemical potentials (just notation)

- simplest case $\mu_Q = \mu_S = 0$

- subtracted condensate

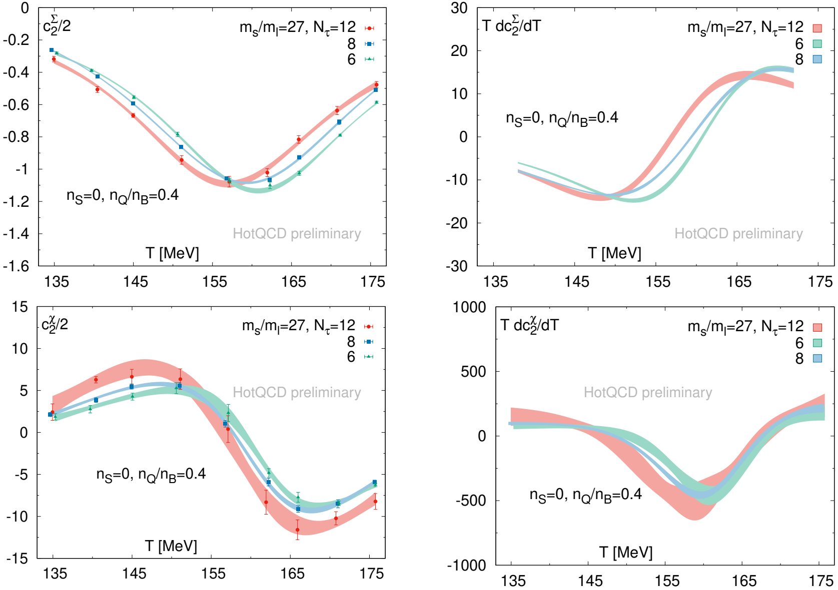
$$\frac{\Sigma_{\rm sub}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\Sigma}{n!} \hat{\mu}_B^n \qquad \text{with} \qquad c_n^\Sigma = \left. \frac{\partial \Sigma_{\rm sub}/f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0}$$

- disconnected susceptibility

$$\frac{\chi_{\rm disc}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\chi}{n!} \hat{\mu}_B^n \qquad \text{with} \qquad c_n^\chi = \left. \frac{\partial \chi_{\rm disc}/f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0}$$

- same notation for HIC: $n_S = 0, \quad \frac{n_Q}{n_B} = 0.4$

Coefficients for a strangeness neutral system



The curvature of the crossover line

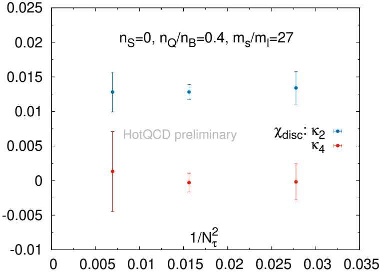
$$\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$$

- Taylor expansion in μ and T of:

$$\frac{d}{dT} \frac{\chi_{\rm disc}(T,\mu_B)}{f_K^4} = (\ldots)\mu_B^2 + (\ldots)\mu_B^4 + \ldots = \mathbf{0}$$

- has to be zero order by order

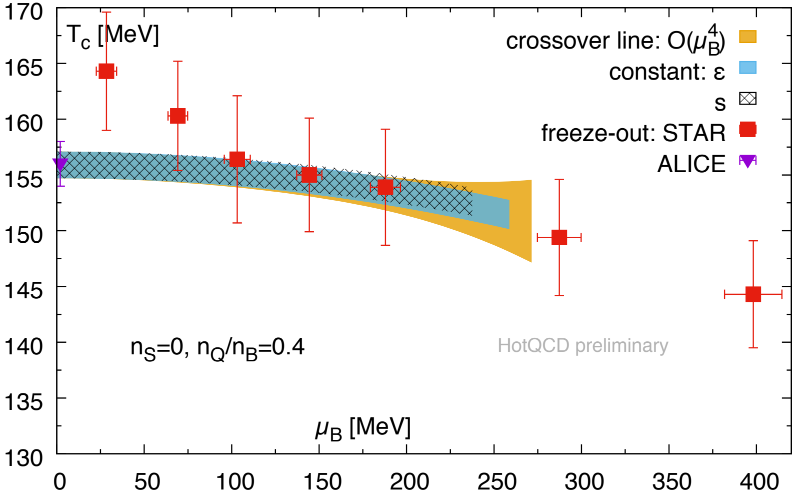
$$\kappa_2 = \frac{1}{2T_0^2} \frac{T_0 \left.\frac{\partial c_2^\chi}{\partial T}\right|_{(T_0,0)} - 2\, c_2^\chi\big|_{(T_0,0)}}{\left.\frac{\partial^2 c_0^\chi}{\partial T^2}\right|_{(T_0,0)}}$$



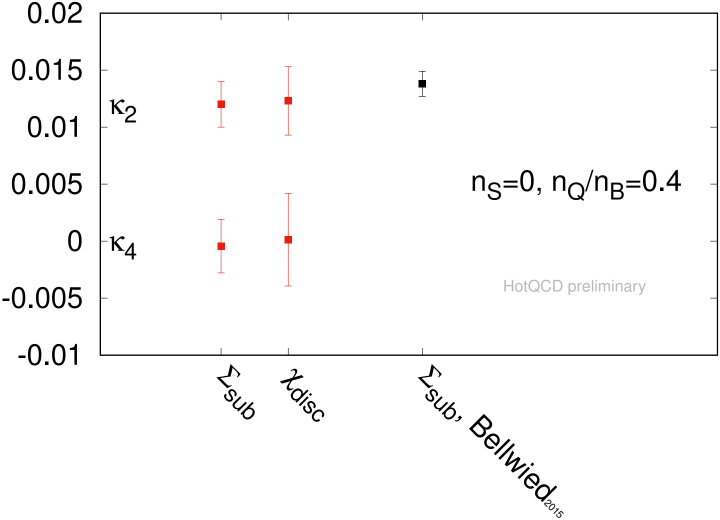
The QCD crossover line

STAR: arxiv:1701.07065

ALICE: arxiv:1408.6403



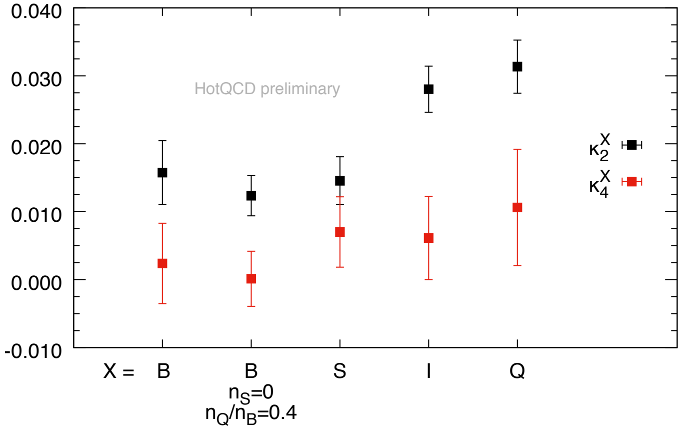
The curvature κ_n for strangeness neutral system



The crossover line

$$\frac{T_c(\mu_X)}{T_0} = 1 - \kappa_2^X \left(\frac{\mu_X}{T_0}\right)^2 - \kappa_4^X \left(\frac{\mu_X}{T_0}\right)^4 + \mathcal{O}(\mu_X^6)$$

Bonati 2018:
$X = B$, $\mu_S = 0$
$\kappa_2 = 0.0145(25)$



Fluctuations along the QCD crossover $T_c(\mu_B)$

- Baryon-number fluctuations

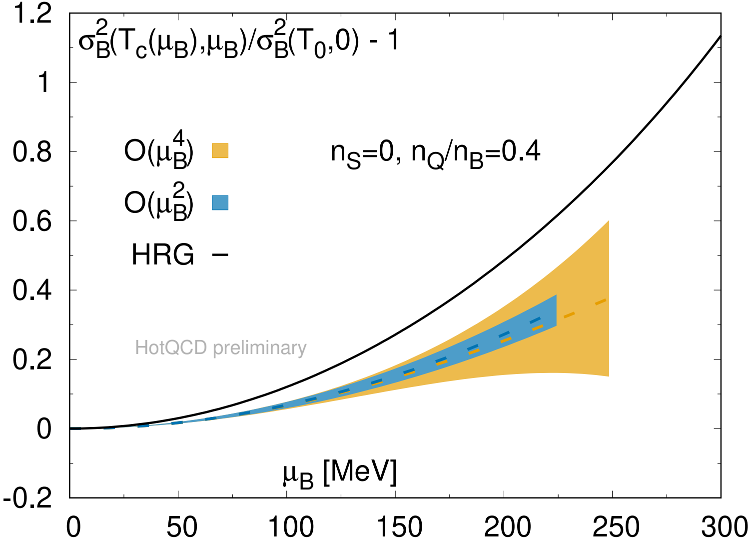
$$\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3}\frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!}\hat{\mu}_B^n \qquad \text{with} \qquad c_n^B = \frac{1}{Vf_K^3}\left.\frac{\partial \ln Z}{\partial \hat{\mu}_B^{n+2}}\right|_{\mu=0}$$

- σ_B^2 couples to condensate $\longrightarrow$ diverges at a critical point

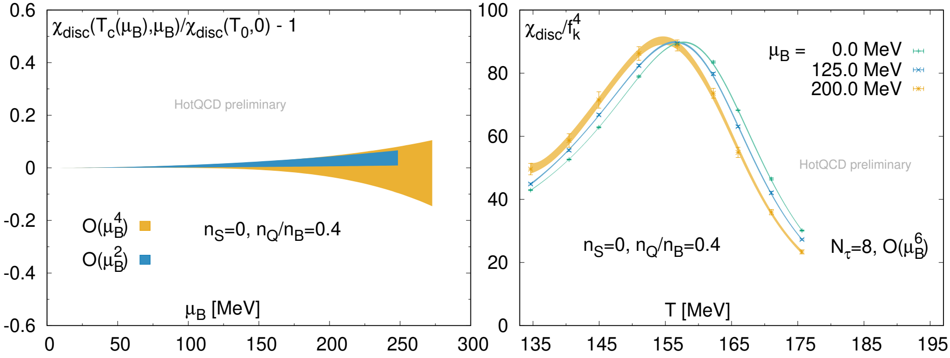
- study increase along the crossover line

$$\frac{\sigma_B^2(T_c(\mu_B),\mu_B) - \sigma_B^2(T_0,0)}{\sigma_B^2(T_0,0)} = \lambda_2\left(\frac{\mu_B}{T_0}\right)^2 + \lambda_4\left(\frac{\mu_B}{T_0}\right)^4 + \cdots$$

Baryon-number fluctuations $\rightsquigarrow$ along $T_c(\mu_B)$



Susceptibility fluctuations $\rightsquigarrow$ along $T_c(\mu_B)$



σ_B^2 and $\chi_{\rm disc}$ show no indication that crossover gets stronger

Summary

- crossover starts at $T_0 = 156.5 \pm 1.5\ \mathrm{MeV}$
- crossover curvature for strangeness neutral system
  - $\frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_0}\right)^4 + \mathcal{O}(\mu_B^6)$
  - $\kappa_2 = 0.0123 \pm 0.003$
  - $\kappa_4 = 0.000131 \pm 0.0041$
- for $\mu_B < 250\ \mathrm{MeV}$ and $n_s = 0, n_Q/n_B = 0.4$
  - crossover along const. entropy density and energy density
  - chemical freeze-out might be close to crossover
  - no indication for critical point

Thank you for your attention!