The QCD crossover from Lattice QCD
The QCD phase diagram
Quantum Chromodynamics
from first principles

- Lattice QCD
  - HISQ action
  - $N_\sigma = 4N_\tau$
  - sim. at $\mu = 0$

- physical quarks
  - 2 light quarks
  - 1 strange quark
  - $m_s/m_l = 27$

- $m_\pi \simeq 138$ MeV

![Graph showing #configurations vs T [MeV] with lines indicating m_s/m_l=27, N_\tau=16, 12, 8, 6 at different temperatures. The graph indicates everything continuum extrapolated.]
Chiral observables in two-flavor formulation

- subtracted condensate

\[ \Sigma_{\text{sub}} \equiv m_s (\Sigma_u + \Sigma_d) - (m_u + m_d) \Sigma_s \]

with \[ \Sigma_f = \frac{T}{V} \frac{\partial}{\partial m_f} \ln Z \]

- subtracted susceptibility

\[ \chi_{\text{sub}} \equiv \frac{T}{V} m_s \left( \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) \Sigma_{\text{sub}} \]

- \( \chi_{\text{disc}} \) is defined as \( \chi_{\text{sub}} \) without connected part
Start of the QCD crossover line: $T_0$

\[
\frac{d^2}{dT^2} \frac{\Sigma_{\text{sub}}}{f_K^4} \equiv 0 \quad \text{and} \quad \frac{d}{dT} \frac{\chi_{\text{sub}}}{f_K^4} \equiv 0
\]

two crossover temperatures: $T_0(\Sigma_{\text{sub}})$ and $T_0(\chi_{\text{sub}})$
Pseudo-critical temperatures

- for $m_l \to 0$: pseudo-critical temperatures converge to the chiral transition temperature $T_c^0$

- at finite quark mass it is given by maximum of $O(4)$ universal scaling functions (Tuesday talk, Anirban Lahiri, Chiral phase transition)

\[
\begin{align*}
\chi_m &= m_l^{1/\delta - 1} f_\chi(z) + \text{reg.} \\
\chi_t &= m_l^{(\beta - 1)/\beta \delta} f'_G(z) + \text{reg.}
\end{align*}
\]

- for $m_l \to 0$
  \[\begin{align*}
  \chi_t &\sim \partial_T \Sigma_{\text{sub}} \quad \text{and} \quad \chi_t \sim \partial_{\mu B}^2 \Sigma_{\text{sub}} \\
  \chi_m &\sim \chi_{\text{sub}} \quad \text{and} \quad \chi_m \sim \chi_{\text{disc}}
  \end{align*}\]
The subtracted chiral susceptibility

\[
\chi_{\text{sub}}/f_k^4
\]

\[m_s/m_l=27, \ N_\tau=16\]

\[T [\text{MeV}]\]

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The subtracted chiral susceptibility

\[ \chi_{\text{sub}} / f_k^4 \]

\[ m_s / m_l = 27, \ N_\tau = 16 \]

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The 2nd $\mu_B$ derivative of chiral condensate $\Sigma_{\text{sub}}$

$m_s/m_l=27$, $N_\tau=12$

$\mu_Q=\mu_S=0$

HotQCD preliminary
The 1st $T$ derivative of chiral condensate $\Sigma_{\text{sub}}$

$-T \frac{d\Sigma}{dT}$

$m_s/m_l=27$, $N_\tau=12$

$\tau = 6$, $\tau = 8$

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The $T_0$ continuum extrapolation

$T_c(\mu_B=0)$ [MeV]

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$156.5 \pm 1.5$ MeV
Crossover temperature $T_0$

![Graph showing crossover temperature $T_0$](image_url)

- $T_0$ [MeV]

- Symbols represent different calculations:
  - $\Sigma_{sub}$
  - $\chi_{disc}$
  - $\chi_{sub}$
  - $\partial^2 \Sigma_{sub}$
  - $\partial^2 \chi_{disc}$
  - $\Sigma_{sub}$, Bonati 2015
  - $\chi_{tot}$, Bazavov 2012
  - $\Sigma_{sub}$, Borsanyi 2010

- HotQCD preliminary
The QCD crossover at $\mu \neq 0$

$$\frac{d^2}{dT^2} \frac{\Sigma_{\text{sub}}(T, \mu_B)}{f_K^4} \equiv 0$$

and

$$\frac{d}{dT} \frac{\chi_{\text{disc}}(T, \mu_B)}{f_K^4} \equiv 0$$

need Taylor expansion in $T$ and $\mu_B$ around $(T_0, 0)$
Taylor expansion in chemical potentials (just notation)

- simplest case $\mu_Q = \mu_S = 0$

- subtracted condensate

\[ \frac{\Sigma_{\text{sub}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\Sigma}{n!} \hat{\mu}_B \]

with

\[ c_n^\Sigma = \left. \frac{\partial \Sigma_{\text{sub}}/f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0} \]

- disconnected susceptibility

\[ \frac{\chi_{\text{disc}}}{f_K^4} = \sum_{n=0}^{\infty} \frac{c_n^\chi}{n!} \hat{\mu}_B^n \]

with

\[ c_n^\chi = \left. \frac{\partial \chi_{\text{disc}}/f_K^4}{\partial \hat{\mu}_B^n} \right|_{\mu=0} \]

- same notation for HIC: $n_S = 0$, $\frac{n_Q}{n_B} = 0.4$
Coefficients for a strangeness neutral system

$-1.6 \quad -1.4 \quad -1.2 \quad -1.0 \quad -0.8 \quad -0.6 \quad -0.4 \quad -0.2 \quad 0$

$135 \quad 145 \quad 155 \quad 165 \quad 175$

$n_S=0, n_Q/n_B=0.4$

$T \text{[MeV]}$

$T_{dc^2/2}/dT$

$m_s/m_l=27, N_t=12$

$T_{dc^2/2}/dT$

$n_S=0, n_Q/n_B=0.4$

$T \text{[MeV]}$

$H\text{otQCD preliminary}$

$T_{dc^2/2}/dT$

$m_s/m_l=27, N_t=12$

$T \text{[MeV]}$

$H\text{otQCD preliminary}$

$H\text{otQCD preliminary}$

$H\text{otQCD preliminary}$
The curvature of the crossover line

\[ \frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left( \frac{\mu_B}{T_0} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T_0} \right)^4 + \mathcal{O}(\mu_B^6) \]

- Taylor expansion in \( \mu \) and \( T \) of:

\[ \frac{d}{dT} \frac{\chi_{\text{disc}}(T, \mu_B)}{f_K^4} = (\ldots) \mu_B^2 + (\ldots) \mu_B^4 + \ldots = 0 \]

- has to be zero order by order

\[ \kappa_2 = \frac{1}{2 T_0^2} \left[ T_0 \left. \frac{\partial c_{2}^\chi}{\partial T} \right|_{(T_0,0)} - 2 \left. c_{2}^\chi \right|_{(T_0,0)} \right] \]
The QCD crossover line

$n_S=0$, $n_Q/n_B=0.4$

crossover line: $O(\mu_B^4)$
constant: $\epsilon$
freeze-out: STAR
ALICE

HotQCD preliminary

STAR: arxiv:1701.07065
ALICE: arxiv:1408.6403
The curvature $\kappa_n$ for strangeness neutral system

$\Sigma_{\text{sub}}, \chi_{\text{disc}}, \Sigma_{\text{sub, Bellwied}}$

$n_S=0$, $n_Q/n_B=0.4$

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The crossover line

\[
\frac{T_c(\mu X)}{T_0} = 1 - \kappa_2^X \left( \frac{\mu X}{T_0} \right)^2 - \kappa_4^X \left( \frac{\mu X}{T_0} \right)^4 + \mathcal{O}(\mu_X^6)
\]

Bonati 2018:
\[X = B, \mu_S = 0\]
\[\kappa_2 = 0.0145(25)\]
Fluctuations along the QCD crossover $T_c(\mu_B)$

- Baryon-number fluctuations

\[
\frac{\sigma_B^2}{Vf_K^3} = \frac{1}{Vf_K^3} \frac{\partial \ln Z}{\partial \hat{\mu}_B^2} = \sum_{n=0}^{\infty} \frac{c_n^B}{n!} \hat{\mu}_B^n \\
\text{with} \quad c_n^B = \left. \frac{1}{Vf_K^3} \frac{\partial \ln Z}{\partial \hat{\mu}_B^{n+2}} \right|_{\mu=0}
\]

- $\sigma_B^2$ couples to condensate $\rightarrow$ diverges at a critical point

- study increase along the crossover line

\[
\frac{\sigma_B^2(T_{c}(\mu_B), \mu_B) - \sigma_B^2(T_0, 0)}{\sigma_B^2(T_0, 0)} = \lambda_2 \left( \frac{\mu_B}{T_0} \right)^2 + \lambda_4 \left( \frac{\mu_B}{T_0} \right)^4 + \ldots
\]
Baryon-number fluctuations $\sim$ along $T_c(\mu_B)$

$$\frac{\sigma_B^2(T_c(\mu_B),\mu_B)}{\sigma_B^2(T_0,0)} - 1$$

$O(\mu_B^4)$

$O(\mu_B^2)$

$\sigma_B^2(T_c(\mu_B),\mu_B)/\sigma_B^2(T_0,0) - 1$

$n_S=0$, $n_Q/n_B=0.4$

HRG

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$\mu_B$ [MeV]

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Susceptibility fluctuations along $T_c(\mu_B)$

$\sigma_B^2$ and $\chi_{\text{disc}}$ show no indication that crossover gets stronger
Summary

- crossover starts at $T_0 = 156.5 \pm 1.5$ MeV

- crossover curvature for strangeness neutral system
  \[ \frac{T_c(\mu_B)}{T_0} = 1 - \kappa_2 \left( \frac{\mu_B}{T_0} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T_0} \right)^4 + O(\mu_B^6) \]
  \[ \kappa_2 = 0.0123 \pm 0.003 \]
  \[ \kappa_4 = 0.000131 \pm 0.0041 \]

- for $\mu_B < 250$ MeV and $n_s = 0$, $n_Q/n_B = 0.4$
  - crossover along const. entropy density and energy density
  - chemical freeze-out might be close to crossover
  - no indication for critical point
Thank you for your attention!