Relating the Lyapunov exponents to transport coefficients in kinetic theory

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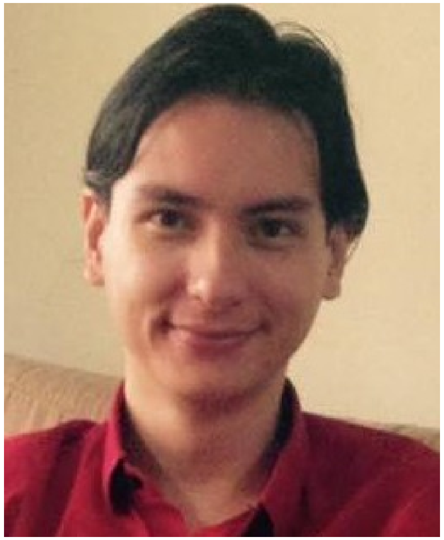


C. N. Cruz-Camacho

S. Kamata

Towards far-from-equilibrium hydro

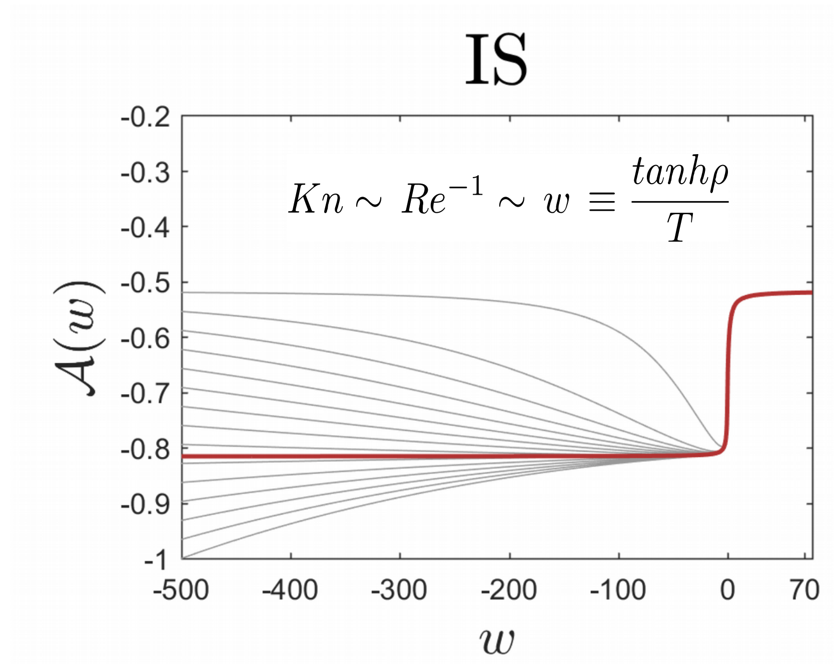
- **Hydro works fairly well in Pb-Pb, pA and p-p collisions (Weller & Romatschke, Werner et. al., Bozek)**
- **Different theoretical models indicate hydro work even when the pressure anisotropy $\mathcal{P}_L / \mathcal{P}_T \sim 0.3$**

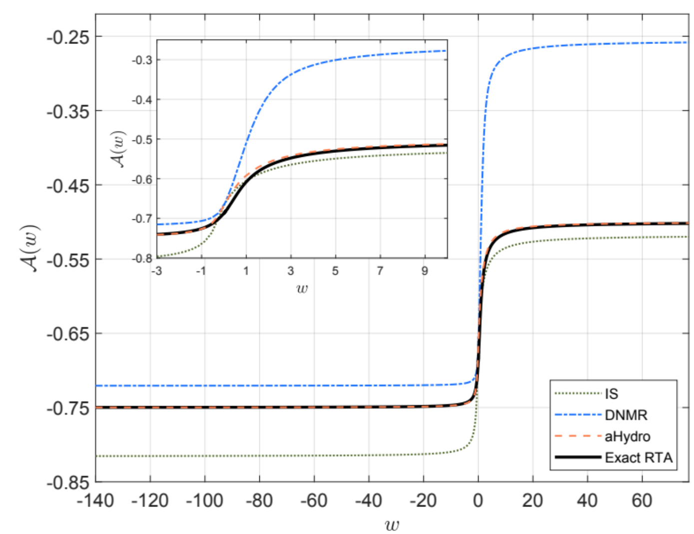
  ⇛ Fluid dynamics can be valid in far-from-equilibrium situations (Heller, Spalinski, Strickland, Martinez, Ryblewski, Florkowski, Romatschke, Casalderrey, Noronha, Denicol…...)

- **The onset of hydrodynamics is determined by the decay of non-hydrodynamic modes**

  ⇛ Hierarchy of microscopic/macroscopic scales seems to be unnecessary

- **Each hydrodynamic theory has its own attractor**





A. Behtash, CN Cruz, M. Martinez
PRD 97, 044041 (2018)
See C.N. Cruz and G. Denicol's posters

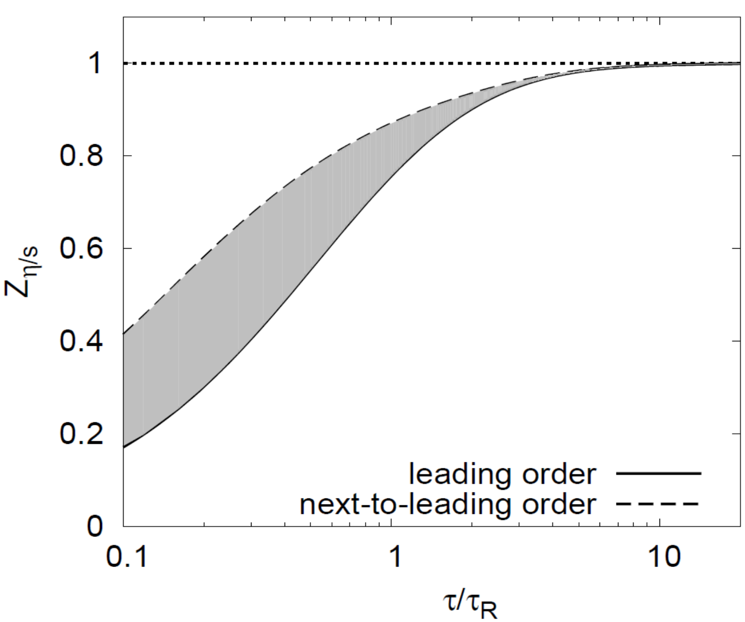
Motivation: Transport coefficients in far-from-equilibrium regimes

- **Can we infer new transport properties in far-from-equilibrium fluid dynamics?**

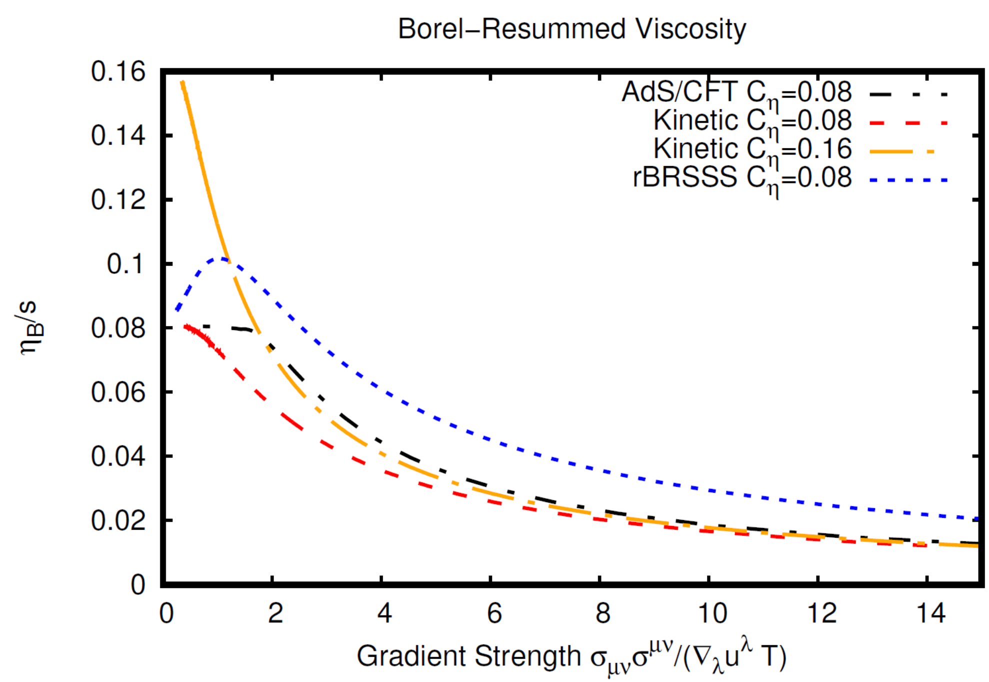
- **New ideas related on renormalized/effective transport coefficients**
  Yan, Blaizot, Romatschke

See L. Yan's talk

L. Yan & J. P. Blaizot
Phys. Lett. B 780, 283 (2018)



P. Romatschke
PRL 120, 012301 (2018)



Our idea

- **Develop a new truncation scheme which captures some of the main features of far-from-equilibrium fluids (e.g. non-hydrodynamical modes) while being simple enough to perform concrete calculations**

$$\tau_\pi \, D_\tau \pi^{\mu\nu} + \pi^{\mu\nu} = \eta \sigma^{\mu\nu}$$

$$\tau_\pi(\sigma^{\mu\nu}) \, D_\tau \pi^{\mu\nu} + \pi^{\mu\nu} = \eta(\sigma^{\mu\nu}) \sigma^{\mu\nu}$$

Keep track of the deformation history of the fluid

$\Rrightarrow$ Study its rheological properties

Kinetic theory model

Boltzmann Equation in Relaxation Time Approximation (RTA)

$$\partial_\tau f = -\frac{1}{\tau_r(\tau)}\left(f - f_{eq.}\right) \qquad \tau_r = \theta_0/T(\tau)$$

By considering the following ansatz

$$f\left(\tau, p_T, p_\varsigma\right) = f_{eq.}\left(\frac{p^\tau}{T}\right)\left[\sum_{n=0}^{N_n}\sum_{l=0}^{N_l} c_{nl}(\tau)\,\mathcal{P}_{2l}\left(\frac{p_\varsigma}{\tau\, p^\tau}\right)\mathcal{L}_n^{(3)}\left(\frac{p^\tau}{T}\right)\right]$$

The components of energy-momentum tensor are

$$\epsilon = \langle (u\cdot p)^2\rangle = \frac{3}{\pi^2}T^4\,,$$

$$P_T = \langle p_T^2\rangle = \epsilon\left(\tfrac{1}{3} - \tfrac{1}{15}c_1\right)\,,$$

$$P_L = \langle (p_\varsigma/\tau)^2\rangle = \epsilon\left(\tfrac{1}{3} + \tfrac{2}{15}c_1\right)\,.$$

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Moments equations

The mathematical problem of solving the RTA Boltzmann Eq. is mapped into solving equations for the moments (Grad, 1949)

The equations of the moments written in terms of w =(τ T)$^{-1}$

$$\tilde{\mathbf{c}} = U\mathbf{c}$$

$$\left(1 - \frac{c_1}{20}\right)\frac{d\tilde{\mathbf{c}}}{dw} + \hat{\Lambda}\tilde{\mathbf{c}} + \frac{1}{w}\hat{\mathcal{B}}_D\tilde{\mathbf{c}} - \frac{c_1}{5w}\tilde{\mathbf{c}} + \frac{3}{2w}\tilde{\gamma} = 0$$

Elements of Λ are the Lyapunov exponents

Nonlinearities due to mode to mode coupling

Couplings with moments of different order

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Moments equations: resurgent properties

$$\left(1-\frac{c_1}{20}\right)\frac{d\tilde{\mathbf{c}}}{dw}+\hat{\Lambda}\tilde{\mathbf{c}}+\frac{1}{w}\hat{\mathcal{B}}_D\tilde{\mathbf{c}}-\frac{c_1}{5w}\tilde{\mathbf{c}}+\frac{3}{2w}\tilde{\gamma}=0$$

Moments equations admit the following trans-series solution (O. Costin, 1998). For L=1

$$\tilde{c}_1=\sum_n \tilde{u}_{l,k}^{(0)}w^{-k}+\sigma\, e^{-s_1 w}w^{b_1}\sum_n \tilde{u}_{l,k}^{(1)}w^{-k}+\cdots$$

- **Late time behaviour is 'perturbative' and it does not depend on the initial conditions**
- **The real part of the parameter σ is the freedom associated with the initial conditions**
- **Information about the non-hydrodynamic transient modes is encoded in the exponentially suppressed terms**
- **The exponential decay of the non-hydrodynamic modes keep track of how the system forgets the initial conditions**
- **Important: non-hydrodynamic series are related among them via differential recursive relations**

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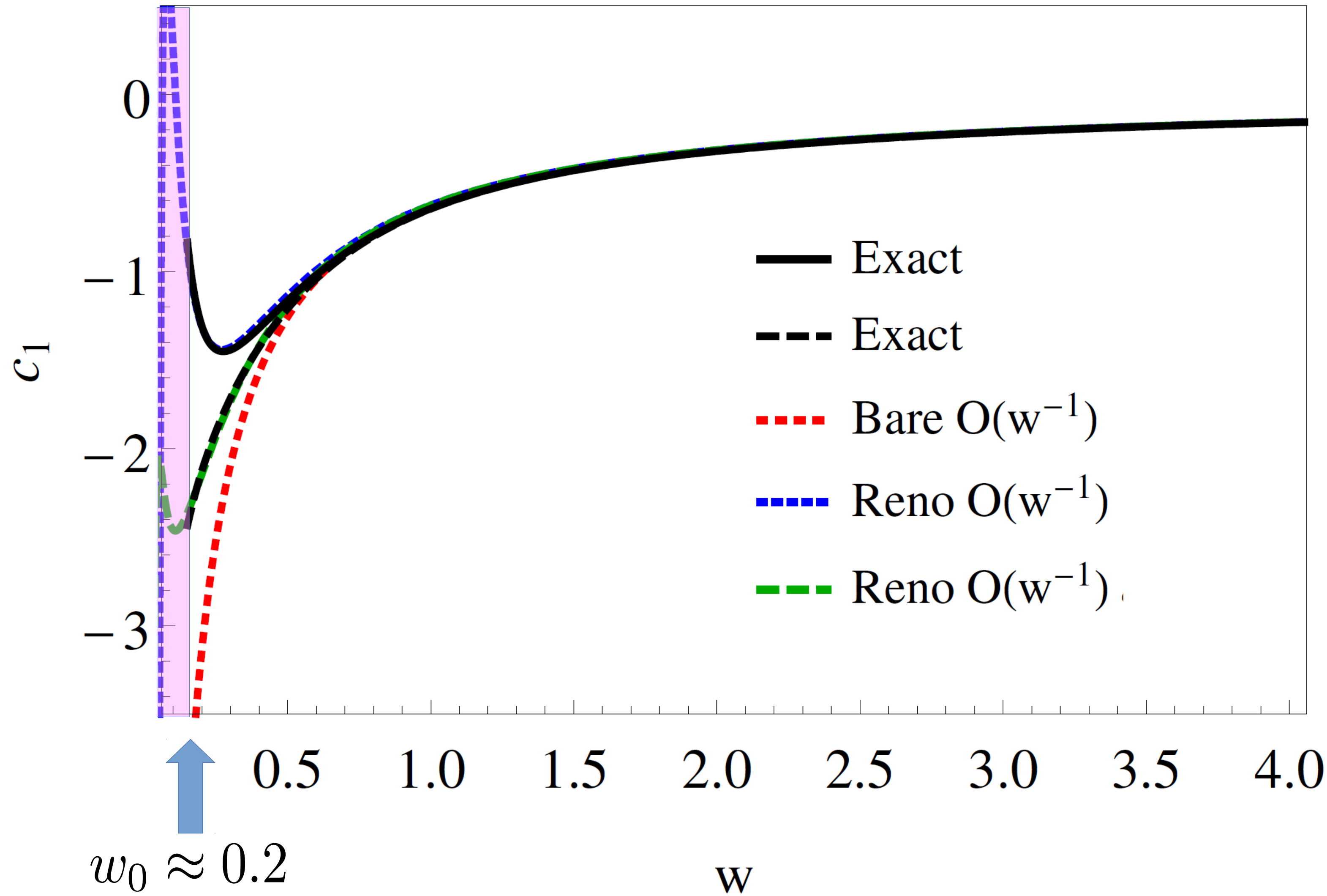
Numerical results: Evolution of c_1

Asymptotically each solution merges to the attractor



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Effective η/s as a non-hydrodynamical series

At $\mathcal{O}(w^{-1})$ the dominant term of the trans-series is

$$c_1 = \frac{\sum_l U^{-1}_{1l}\tilde{u}^{(0)}_{l,1}}{w}$$

On the other hand, Chapman-Enskog expansion gives the asymptotic behavior of c_1

$$c_1 = -\frac{40}{3}\frac{1}{w}\left(\frac{\eta}{s}\right)_0$$

$$\Rightarrow \left(\frac{\eta}{s}\right)_0 = -\frac{3}{40}\sum_l U^{-1}_{1l}\tilde{u}^{(0)}_{l,1}$$

- **Effective η/s is the asymptotic limit of a trans-series**
- **We can study its rheology by following the 'history' of the corresponding trans-series**

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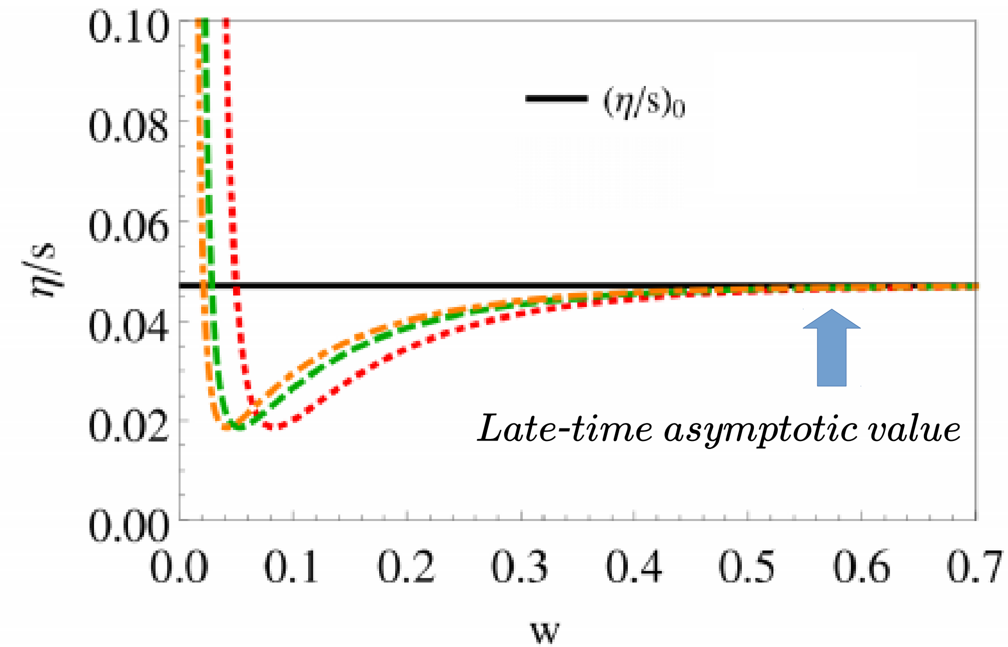
Effective η/s as a non-hydrodynamic series

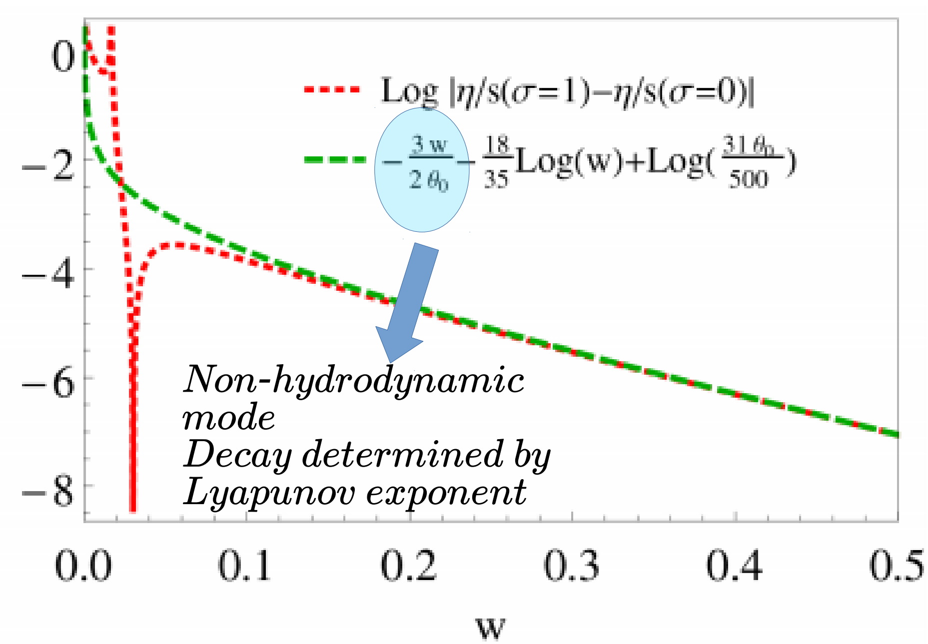
Thus effective η/s is

$$\left(\frac{\eta}{s}\right)_R = -\frac{3}{40}\sum_l U_{1l}^{-1}\tilde{C}_{l,1}(\sigma e^{-Sw} w^{\tilde{b}})$$

Its RG flow evolution is one of the differential recursive relation of the corresponding trans-series

$$\frac{d}{dw}\left(\frac{\eta}{s}\right)_R = -\frac{3}{40}\sum_l U_{1l}^{-1}\frac{d}{dw}\tilde{C}_{l,1}(\sigma e^{-Sw} w^{\tilde{b}})$$





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Conclusions

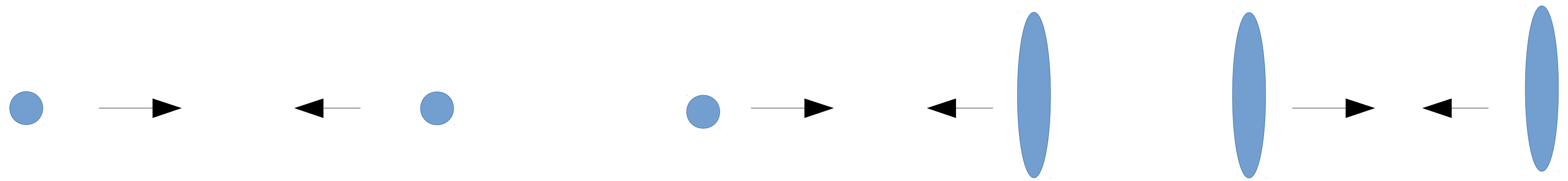
- We propose a new way to study hydrodynamization and relaxation processes of the Boltzmann equation
  $\Rightarrow$ Summation of non-hydrodynamic series
- Prior to the thermal equilibrium η/s receives non-hydrodynamic contributions which depend on the deformation history of the fluid
  $\Rightarrow$ η/s features a transition to its equilibrium fixed value which is a diagnostic of Non-newtonian transient behaviour
- The transient rheological behavior of the fluid might explain the unreasonable success of applying hydrodynamics in far-from-equilibrium situations

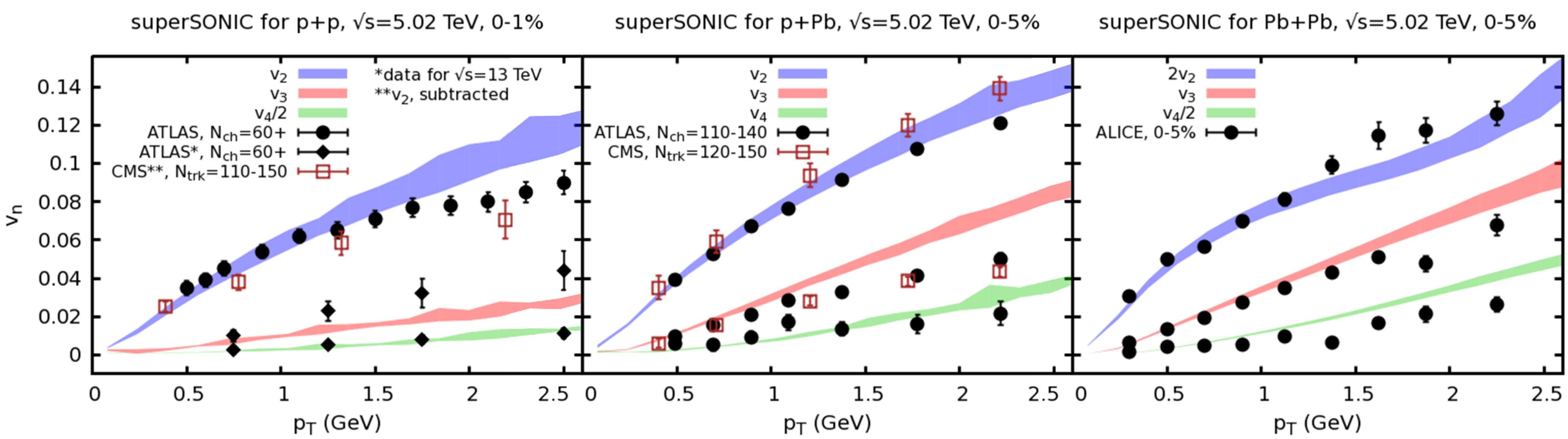
- **Outlook**
  1. Non-conformal systems
  2. Finite chemical potential

- **Challenges:**
  1. How to generalize to arbitrarily expanding geometries in kinetic theory?
  2. Phase transitions?

Backup slides

Motivation: Unreasonable success of hydro





*Weller and Romatschke, Phys. Lett. B*774 (2017) 351-356

Hydrodynamics provide a fairly good description of small, medium and large size systems