Baryons, chiral symmetry and in-medium effects: Results from lattice QCD

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FASTSUM Collaboration

Quark Matter 2018, Venezia
Mesons in a medium

Mesons in a medium very well studied

- hadronic phase: thermal broadening, mass shift
- QGP: deconfinement/dissolution/melting
- quarkonia survival as thermometer
- transport: conductivity/dileptons from vector current
- chiral symmetry restoration

Relatively easy on the lattice

- high-precision correlators

What about baryons?
Baryons in a medium

Lattice studies of baryons at finite temperature very limited

- screening masses De Tar and Kogut 1987
- ... with a small chemical potential QCD-TARO: Pushkina, de Forcrand, Kim, Nakamura, Stamatescu et al 2005
- temporal correlators Datta, Gupta, Mathur et al 2013

Not much more ...

- effective models, mostly at $T \sim 0$ and nuclear density $\Rightarrow$ parity doubling models De Tar & Kunihiro 89
  Mukherjee, Schramm, Steinheimer & Dexheimer, Sasaki 17
Baryons in a medium

But understanding of in-medium effects highly relevant for

- hadron resonance gas descriptions in confined phase
- benchmarking models for dense QCD
- extensions into QCD phase diagram
- ...

...
Outline

Baryons across the deconfinement transition:

► baryon correlators

► FASTSUM collaboration

► in-medium effects below $T_c$

► parity doubling above $T_c$

► spectral functions

+ in preparation
Baryons

- correlators
  \[ G^{\alpha\alpha'}(x) = \langle O^{\alpha}(x) \overline{O}^{\alpha'}(0) \rangle \]

- examples: \( N, \Delta, \Omega \) baryons

\[
O_N^\alpha(x) = \epsilon_{abc} u_a^\alpha(x) \left( d_b^T(x) C \gamma_5 u_c(x) \right)
\]

\[
O_{\Delta, i}^\alpha(x) = \epsilon_{abc} \left[ 2u_a^\alpha(x) \left( d_b^T(x) C \gamma_i u_c(x) \right) + d_a^\alpha(x) \left( u_b^T(x) C \gamma_i u_c(x) \right) \right]
\]

\[
O_{\Omega, i}^\alpha(x) = \epsilon_{abc} s_a^\alpha(x) \left( s_b^T(x) C \gamma_i s_c(x) \right)
\]

- essential difference with mesons: role of parity

\[
P O(\tau, x) P^{-1} = \gamma_4 O(\tau, -x)
\]

- positive/negative parity operators

\[
O^\pm(x) = P^\pm O(x) \quad P^\pm = \frac{1}{2} (1 \pm \gamma_4)
\]
Baryons

- positive/negative parity operators

\[ O_\pm(x) = P_\pm O(x) \quad P_\pm = \frac{1}{2}(1 \pm \gamma_4) \]

- no parity doubling in Nature: nucleon ground state

positive parity: \[ m_+ = m_N = 0.939 \text{ GeV} \]

negative parity: \[ m_- = m_{N^*} = 1.535 \text{ GeV} \]

- thread: what happens as temperature increases?

How are \( \pm \) parity states encoded in correlators?

\[ G_\pm(x-x') = \langle \text{tr}P_\pm O(x)\overline{O}(x') \rangle \quad \rho_\pm(x-x') = \langle \text{tr}P_\pm \{O(x),\overline{O}(x')\} \rangle \]
Charge conjugation

Charge conjugation symmetry (at vanishing density):

\[ G_{\pm}(\tau, p) = -G_{\mp}(1/T - \tau, p) \quad \rho_{\pm}(-\omega, p) = -\rho_{\mp}(\omega, p) \]

- relates \( \pm \) parity channels

Using \( G_{+}(\tau, p) \) and \( \rho_{+}(\omega, p) \)

- positive- (negative-) parity states propagate forward (backward) in euclidean time

- negative part of spectrum of \( \rho_{+} \leftrightarrow \) positive part of \( \rho_{-} \)

example: single state

\[ G_{+}(\tau) = A_{+} e^{-m_{+} \tau} + A_{-} e^{-m_{-}(1/T - \tau)} \]

\[ \rho_{+}(\omega)/(2\pi) = A_{+} \delta(\omega - m_{+}) + A_{-} \delta(\omega + m_{-}) \]
Chiral symmetry

- propagator

\[ G(x) = \sum_\mu \gamma_\mu G_\mu(x) + \gamma_5 G_m(x) \]

- chiral symmetry \( \{ \gamma_5, G \} = 0 \implies G_m = 0 \)

- hence

\[ G_+(\tau, p) = -G_-(\tau, p) = G_+(1/T - \tau, p) = 2G_4(\tau, p) \]

Degeneracy of ± parity channels

\[ \rho_+(p) = -\rho_-(p) = \rho_+(-p) = 2\rho_4(p) \]

- parity doubling

- in Nature at \( T = 0 \): no chiral symmetry/parity doubling
Parity and chiral symmetry

However, if chiral symmetry is unbroken ($m_q = 0$ and no SSB)

- degeneracy between $\pm$ parity channels already at the level of the correlators

What happens at the confinement/deconfinement transition?

- SU(2)$_A$ chiral symmetry restored
- expect degeneracies to emerge
- how does this affect mass spectrum?
- role of $m_s > m_{u,d}$?
Anisotropic $N_f = 2 + 1$ Wilson-clover ensembles

**FASTSUM** collaboration

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This work

Gert Aarts, CA, Simon Hands, Kristi Praki, Jonivar Skullerud

Davide de Boni, Benjamin Jäger

in preparation
FASTSUM set up

- $N_f = 2 + 1$ dynamical quark flavours
- $M_\pi = 384(4)$ MeV
- anisotropic lattices $a_\tau < a_s$
  - allowing better resolution, particularly at finite temperatures
  - since
  $$ T = \frac{1}{L_\tau} = \frac{1}{N_\tau a_\tau} $$

- moving towards continuum, infinite volume, realistic light quark masses
Physics/lattice parameters

2nd Generation
2+1 flavours

larger volume: \((3\text{fm})^3 - (4\text{fm})^3\)
finer lattices: \(a_s = 0.123\text{ fm}\)
quark mass: \(M_\pi = 384(4)\text{ MeV}\)
temporal cut-off: \(a_s \sim 3.5 a_\tau\text{ GeV}\)

Gauge Action:
Symanzik-improved, tree-level tadpole

Fermion Action:
clover, stout-links, tree-level tadpole

\begin{tabular}{cccc}
\(N_s\) & \(N_\tau\) & \(T\text{(MeV)}\) & \(T / T_c\) \\
\hline
24, 32 & 16 & 352 & 1.90 \\
24 & 20 & 281 & 1.52 \\
24, 32 & 24 & 235 & 1.27 \\
24, 32 & 28 & 201 & 1.09 \\
24, 32 & 32 & 176 & 0.95 \\
24 & 36 & 156 & 0.84 \\
24 & 40 & 141 & 0.76 \\
32 & 48 & 117 & 0.63 \\
16 & 128 & 44 & 0.24 \\
\end{tabular}

(Hadron Spectrum Collaboration)
Baryon correlators

computed all octet and decuplet baryon correlators

\[ S = 0: \quad N \quad \Delta \]
\[ S = -1: \quad \Lambda \quad \Sigma \quad \Sigma^* \]
\[ S = -2: \quad \Xi \quad \Xi^* \]
\[ S = -3: \quad \Omega \]

For each baryon: positive and negative parity channels

Technical remarks

- studied various interpolation operators
- Gaussian smearing for multiple sources and sinks
- same smearing parameters at all temperatures
Lattice correlators: Nucleon

- euclidean correlator $G_+(\tau)$

- not symmetric around $\tau = 1/2T$ below $T_c$
- more symmetric as temperature increases
Lattice correlators: Nucleon

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Lattice correlators: Nucleon

- $\pm$ parity channels nondegenerate
- more $T$ dependence in negative-parity channel
Lattice correlators: Nucleon

Experiment:

+ve parity: $M_N = 939$ MeV

-ve parity: $M_{N^*} = 1535$ MeV
Lattice correlators: $\Delta$

- at low $T$ parity channels nondegenerate
- more $T$ dependence in negative-parity channel
Lattice correlators: $\Omega$

- at low $T \pm$ parity channels nondegenerate
- more $T$ dependence in negative-parity channel
Baryons in the hadronic phase

- determine masses of $\pm$ parity groundstates
- in-medium effects
## Masses of ± parity groundstates (in MeV)

<table>
<thead>
<tr>
<th>$S$</th>
<th>$T / T_c$</th>
<th>0.24</th>
<th>0.76</th>
<th>0.84</th>
<th>0.95</th>
<th>PDG ($T = 0$)</th>
</tr>
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<tr>
<td>0</td>
<td>$m^N_{++}$</td>
<td>1158(13)</td>
<td>1192(39)</td>
<td>1169(53)</td>
<td>1104(40)</td>
<td>939</td>
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<tr>
<td></td>
<td>$m^N_{--}$</td>
<td>1779(52)</td>
<td>1628(104)</td>
<td>1425(94)</td>
<td>1348(83)</td>
<td>1535</td>
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<td></td>
<td>$m^A_{++}$</td>
<td>1456(53)</td>
<td>1521(43)</td>
<td>1449(42)</td>
<td>1377(37)</td>
<td>1232</td>
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<tr>
<td></td>
<td>$m^A_{--}$</td>
<td>2138(114)</td>
<td>1898(106)</td>
<td>1734(97)</td>
<td>1526(74)</td>
<td>1700</td>
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<tr>
<td>−1</td>
<td>$m^\Sigma^+_{++}$</td>
<td>1277(13)</td>
<td>1330(38)</td>
<td>1290(44)</td>
<td>1230(33)</td>
<td>1193</td>
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<tr>
<td></td>
<td>$m^\Sigma^+_{--}$</td>
<td>1823(35)</td>
<td>1772(91)</td>
<td>1552(65)</td>
<td>1431(51)</td>
<td>1750</td>
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<td>$m^\Lambda^+_{++}$</td>
<td>1248(12)</td>
<td>1293(39)</td>
<td>1256(54)</td>
<td>1208(26)</td>
<td>1116</td>
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<tr>
<td></td>
<td>$m^\Lambda^+_{--}$</td>
<td>1899(66)</td>
<td>1676(136)</td>
<td>1411(90)</td>
<td>1286(75)</td>
<td>1405–1670</td>
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<td>$m^{\Sigma^*+}_{++}$</td>
<td>1526(32)</td>
<td>1588(40)</td>
<td>1536(43)</td>
<td>1455(35)</td>
<td>1385</td>
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<td>$m^{\Sigma^*+}_{--}$</td>
<td>2131(62)</td>
<td>1974(122)</td>
<td>1772(103)</td>
<td>1542(60)</td>
<td>1670–1940</td>
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<tr>
<td>−2</td>
<td>$m^{\Xi^+}_{++}$</td>
<td>1355(9)</td>
<td>1401(36)</td>
<td>1359(41)</td>
<td>1310(32)</td>
<td>1318</td>
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<tr>
<td></td>
<td>$m^{\Xi^+}_{--}$</td>
<td>1917(27)</td>
<td>1808(92)</td>
<td>1558(76)</td>
<td>1415(50)</td>
<td>1690–1950</td>
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<td></td>
<td>$m^{\Xi^*+}_{++}$</td>
<td>1594(24)</td>
<td>1656(35)</td>
<td>1606(40)</td>
<td>1526(29)</td>
<td>1530</td>
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<tr>
<td></td>
<td>$m^{\Xi^*+}_{--}$</td>
<td>2164(42)</td>
<td>2034(95)</td>
<td>1810(77)</td>
<td>1578(48)</td>
<td>1820</td>
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<tr>
<td>−3</td>
<td>$m^\Omega_{++}$</td>
<td>1661(21)</td>
<td>1723(32)</td>
<td>1685(37)</td>
<td>1606(43)</td>
<td>1672</td>
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<td>$m^\Omega_{--}$</td>
<td>2193(30)</td>
<td>2092(91)</td>
<td>1863(76)</td>
<td>1576(66)</td>
<td>2250</td>
</tr>
</tbody>
</table>
Baryons in the hadronic phase

Masses $m_{\pm}(T)$, normalised with $m_+$ at lowest temperature

In each channel:

- emerging degeneracy around $T_c$
- negative-parity masses reduced as $T$ increases
- positive-parity masses nearly $T$ independent
Baryons in the hadronic phase

Findings:

- positive-parity masses nearly $T$ independent
- negative-parity masses reduced as $T$ increases
- characteristic behaviour, modelled by:

$$m_-(T) = w(T, \gamma)m_-(0) + [1 - w(T, \gamma)]m_-(T_c)$$

with one-parameter transition function

$$w(T, \gamma) = \tanh[(1 - T/T_c)/\gamma]/\tanh(1/\gamma)$$

- small (large) $\gamma \leftrightarrow$ narrow (broad) transition region

Fits in each channel

- $0.22 \lesssim \gamma \lesssim 0.35$, mean $\gamma = 0.27(1)$
- $0.85 \lesssim m_-(T_c)/m_+(0) \lesssim 1.1$
Baryons and parity partners

- distinct temperature dependence in hadronic phase
- understand further using
  - effective parity doublet models?
    Mukherjee, Schramm, Steinheimer & Dexheimer, Sasaki [2017]
  - holography?
- relevant for heavy-ion phenomenology?

Application to HRG

- use states in PDG (not QM)
- $T$-dependent groundstates in neg parity channels

\[
m_-(T) = w(T, \gamma)m_-(0) + [1 - w(T, \gamma)]m_-(T_c)
\]

with $\gamma = 0.3$ and $1 < m_-(T_c)/m_+(0) < 1.1$
In-medium HRG: Pressure

Contributions to pressure from baryons with strangeness

Compare with lattice data from Alba, Ratti et al, 1702.01113
In-medium HRG: Strangeness

Fluctuations of strange baryons $\langle BS \rangle$

![Graph showing fluctuation of strange baryons vs. temperature.]

Compare with lattice data from Budapest-Wuppertal

Budapest-Wuppertal (cont.)
QGP: fate of light baryons

Consider now the quark-gluon plasma

- no clearly identifiable groundstates: baryons dissolved

Example: use conventional exponential fits

No clearly defined groundstates above $T_c$
QGP: fate of light baryons

- no clearly identifiable groundstates: baryons dissolved
- chiral symmetry restoration $\Leftrightarrow$ parity doubling
- study correlator ratio \( \text{Datta, Gupta, Mathur et al 2013} \)

\[
R(\tau) = \frac{G_+(\tau) - G_-(\tau)}{G_+(\tau) + G_-(\tau)}
\]

- no parity doubling and \( m_- \gg m_+ \): \( R(\tau) = 1 \)
- parity doubling: \( R(\tau) = 0 \)

by construction: \( R(1/T - \tau) = -R(\tau) \) and \( R(1/2T) = 0 \)
Nucleon channel

- Ratio close to 1 below $T_c$, decreasing uniformly
- Ratio close to 0 above $T_c$, parity doubling
Integrate the $R(\tau)$ ratio

$$R(\tau) = \frac{G_+ (\tau) - G_- (\tau)}{G_+ (\tau) + G_- (\tau)}$$

⇒ quasi-order parameter

$$R = \frac{\sum_n R(\tau_n)/\sigma^2(\tau_n)}{\sum_n 1/\sigma^2(\tau_n)}$$
Quasi-order parameter
parity doubling in the QGP: \( R \sim 1 \rightarrow 0 \)

- crossover behaviour, tied with deconfinement transition and hence chiral transition – note: \( m_q \neq 0 \)
- effect of heavier \( s \) quark visible
Parity doubling

- clear signal for parity doubling even with finite quark masses
- crossover behaviour, coinciding with transition to QGP
- visible effect of heavier s quark
Spectral properties: fermions

\[ G^{\alpha\alpha'}(\tau, p) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho^{\alpha\alpha'}(\omega, p) \]

- fermionic Matsubara frequencies

\[ K(\tau, \omega) = T \sum_n \frac{e^{-i\omega_n \tau}}{\omega - i\omega_n} = \frac{e^{-\omega \tau}}{1 + e^{-\omega/T}} = e^{-\omega \tau} [1 - n_F(\omega)] \]

- kernel not symmetric, instead

\[ K(1/T - \tau, \omega) = K(\tau, -\omega) \]

- positivity: \( \rho_4(p), \pm \rho_{\pm}(p) \geq 0 \) for all \( \omega \)

- \( \rho_m(p) = [\rho_+(p) + \rho_-(p)]/4 \) not sign definite
Spectral functions

Extract same information from spectral functions

\[ G_{\pm}(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} K(\tau, \omega) \rho_{\pm}(\omega) \]

\[ K(\tau, \omega) = \frac{e^{-\omega\tau}}{1 + e^{-\omega/T}} \]

- *ill-posed* inversion problem
- use Maximum Entropy Method (MEM)
- featureless default model
- construct \( \rho_{+}(\omega) \geq 0 \) for all \( \omega \)
- \( \rho_{-}(\omega) = -\rho_{+}(-\omega) \)
Baryon spectral functions: Nucleon

- groundstates below $T_c$
- degeneracy emerging above $T_c$
Baryon spectral functions: $\Delta$

- groundstates below $T_c$
- degeneracy emerging above $T_c$
Baryon spectral functions: $\Omega$

- groundstates below $T_c$
- degeneracy emerging above $T_c$, finite $m_s$
Baryon spectral functions

- all channels: low and high temperature

- groundstates below $T_c$
- degeneracy emerging above $T_c$
Baryon spectral functions

- results consistent with correlator analysis
- effect of heavier $s$ quark visible
Summary

In hadronic phase

► +ve parity groundstates mostly $T$ independent
► characteristic $T$ dependence in -ve parity groundstates
  reduction in mass, near degeneracy close to $T_c$

Application

► heavy-ion phenomenology: in-medium HRG

In quark-gluon plasma

► $\pm$ parity channels degenerate: parity doubling
► linked to deconfinement transition and chiral symmetry restoration
► effect of heavier $s$ quark noticeable