



# *Electric conductivity of hot and dense quark matter in a magnetic field with Landau level resummation via kinetic equations*



Kenji Fukushima

The University of Tokyo  
Fukushima-Hidaka, PRL120, 162301 (2018)

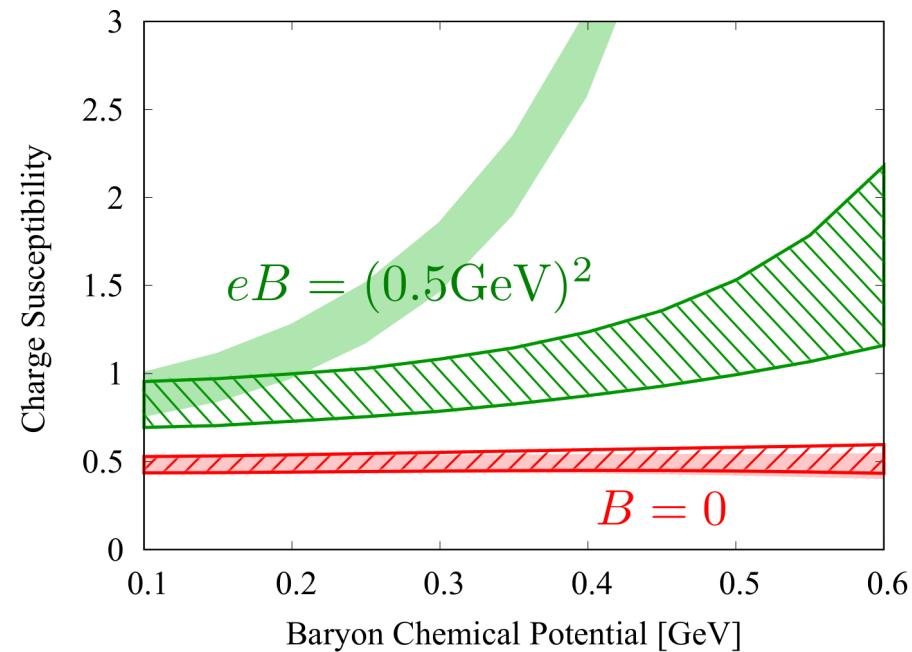
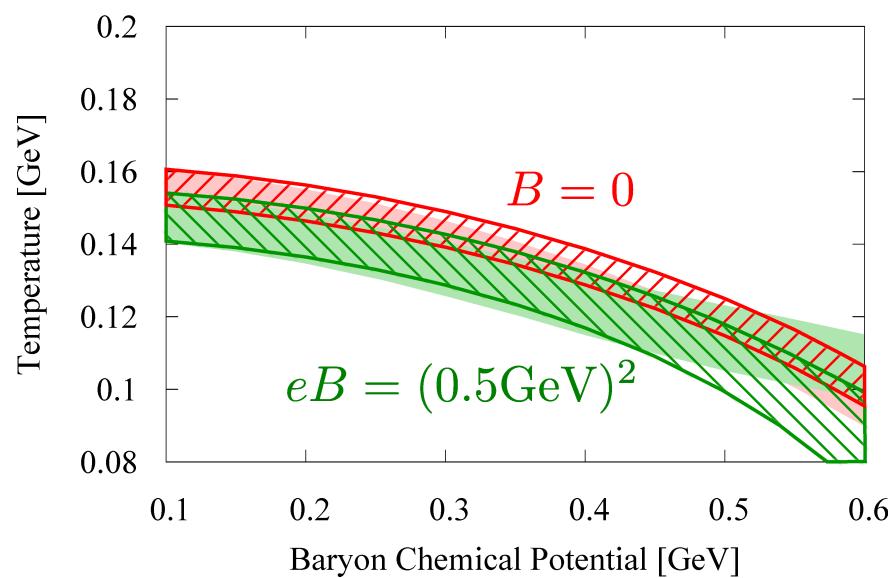
— XXVII International Conference on Ultrarelativistic Nucleus-Nucleus Collisions —

# *Prologue*



**Fukushima-Hidaka, PRL117, 102301 (2016)**

We computed B-dependent freezeout curve and electric charge susceptibility using the HRG model.



**Susceptibility enhanced... electric conductivity also?**

In preceding works (Hattori, Li, Rischke, Satow, Yee)

$$\sqrt{eB} \gg T \gg gT$$

as explained later, very problematic!

In the present work (Fukushima-Hidaka)

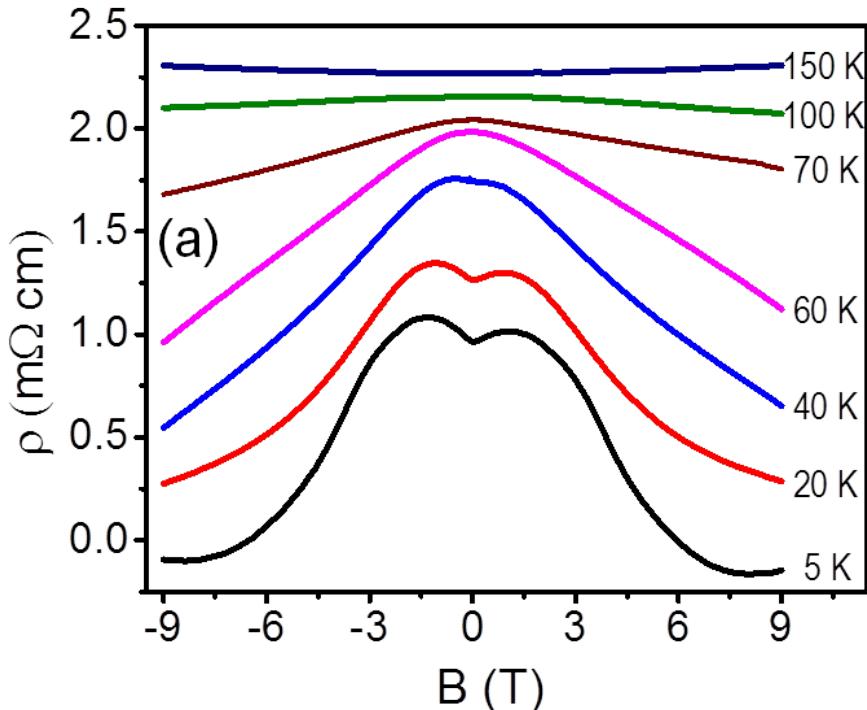
$$T \sim \sqrt{eB} \gg gT$$

technically involved, crucial improvements!

# Physics Goal

## CME Signature with Weyl Semimetals

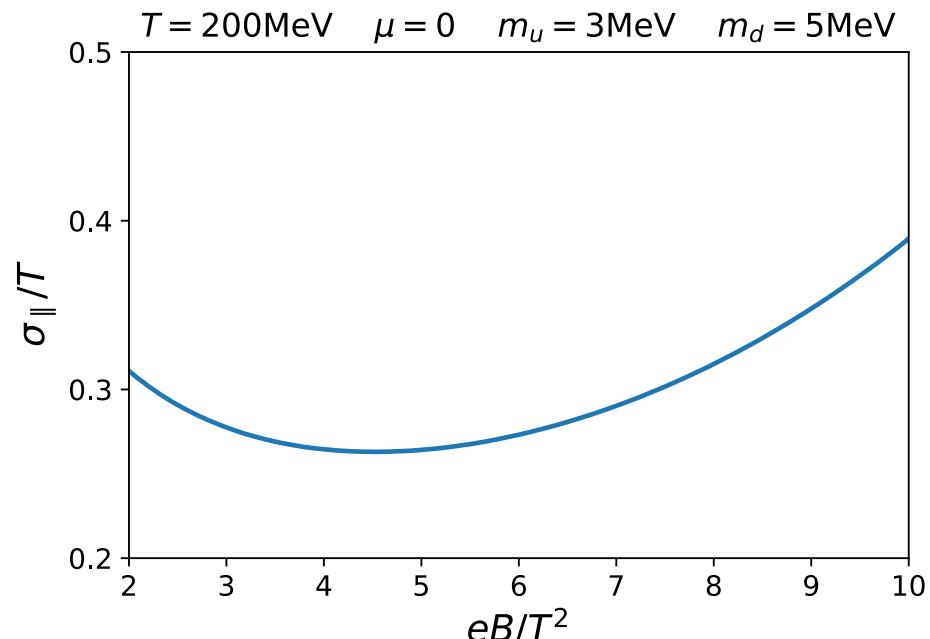
Li et al. Nature Phys. 12, 550 (2016)



$$\rho \propto 1/B^2$$

Negative magnetoresistance

## Our Results (QCD)

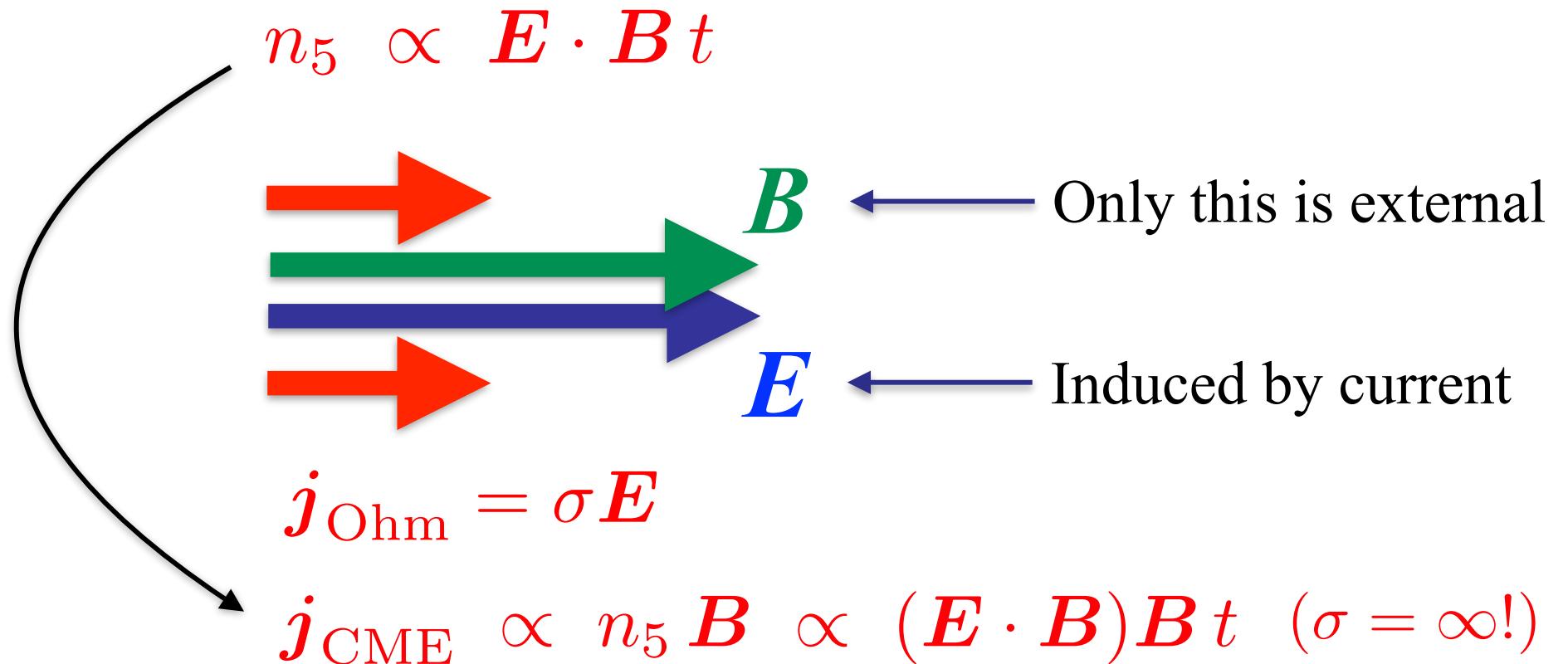


$$\sigma \propto B^2 \sim B$$

$$(\rho \propto 1/B^2 \sim 1/B)$$

Correct asymptotic behavior

# CME – Cond-mat Signature

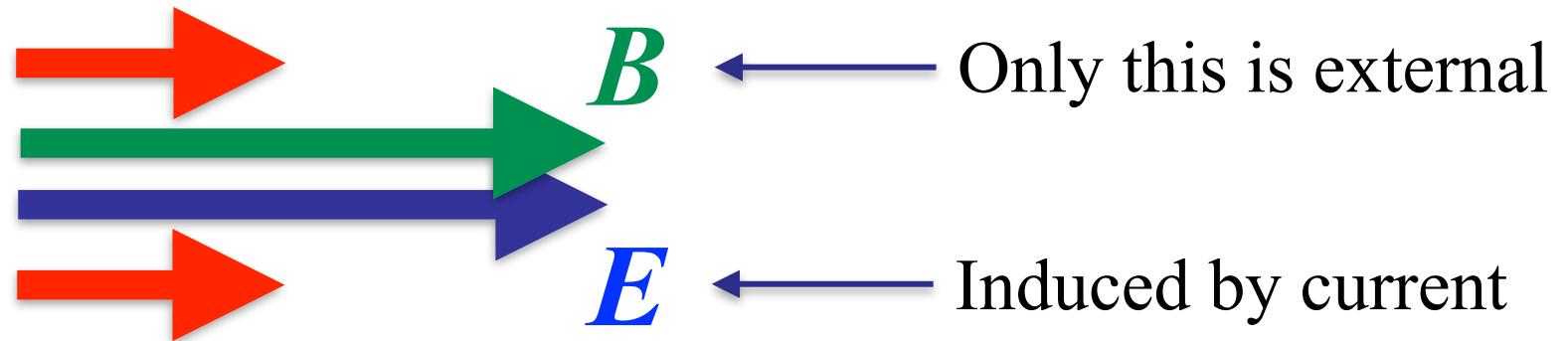


If this linear rise is balanced by “relaxation”  $\sigma_{\text{CME}} \propto B^2$

# CME – Cond-mat Signature



$$j_{\text{CME}} \propto B^2$$



$$j_{\text{Ohm}} = \sigma E$$

$$j = (\sigma_{\text{Ohm}} + \sigma_{\text{CME}})E \quad \sigma_{\text{CME}} \propto B^2$$

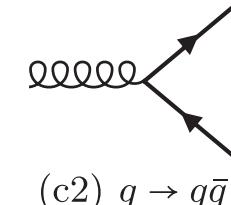
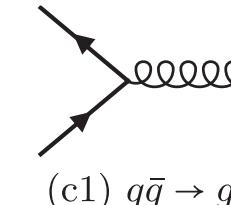
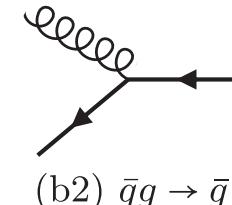
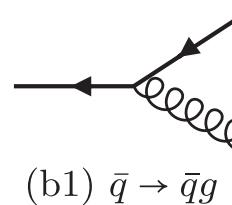
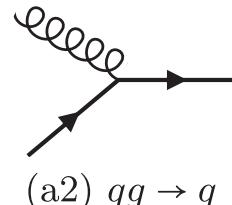
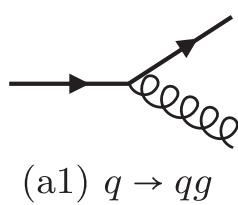
Son-Spivak (2012)

# Caveat



Relaxation time approximation is a very crude approximation violating conservation laws not considering B dependence in the interaction etc etc...

Theorists must do theoretical calculations...



Strong magnetic field limit simplifies theory a lot...

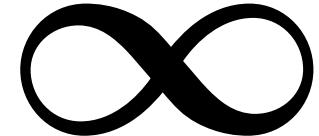
$$\sqrt{eB} \gg T \gg gT$$

**However !**

# Caveat

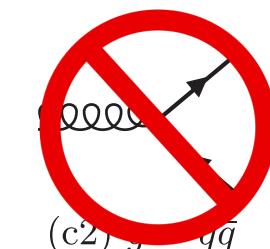
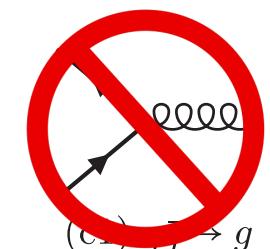
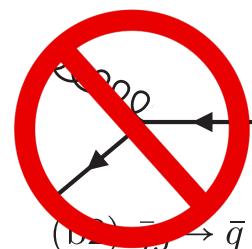
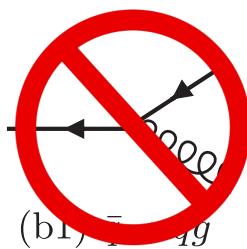
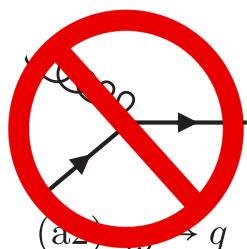
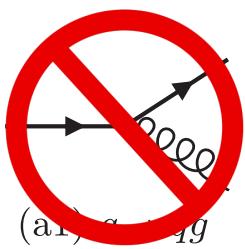


$\sqrt{eB} \gg T \gg gT$  with  $m = 0$  leads to



**Strong magnetic field  $\rightarrow$  Dimensional reduction (1+1)D**

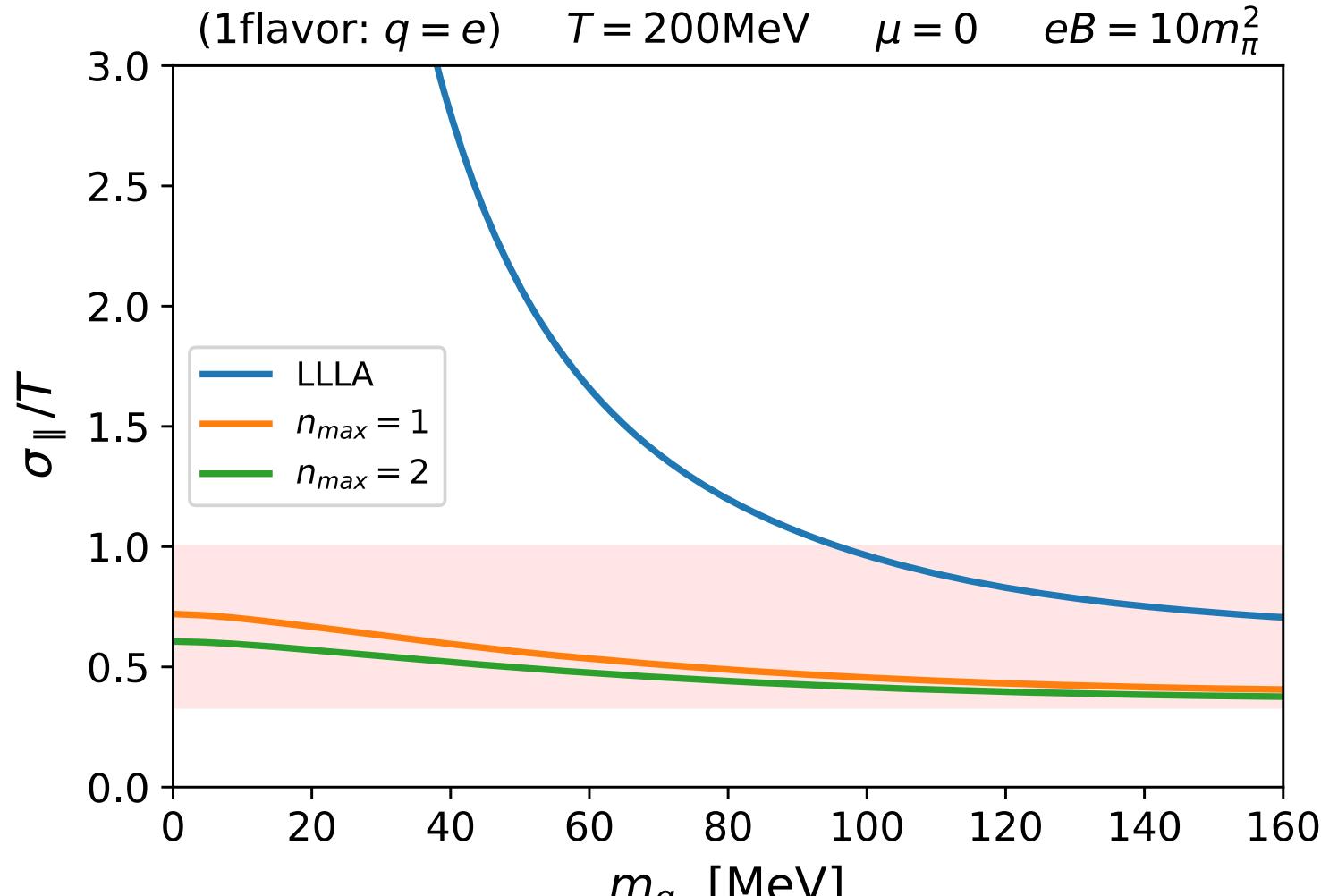
(c.f. High temperature  $\rightarrow$  Dimensional reduction 3D)



**No scattering  $\rightarrow$  Divergent electric conductivity**

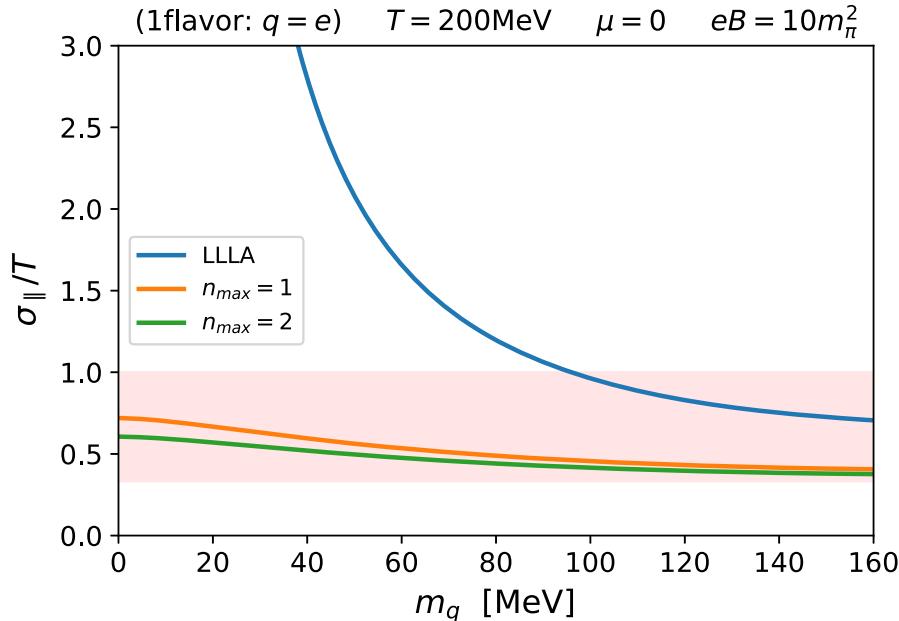
(c.f. Free particles  $\rightarrow$  Divergent viscosities)

# Higher Landau Level Contributions



Fukushima-Hidaka (2018)

# Higher Landau Level Contributions



**Convergence is very fast  
(as soon as including  $n=1$ )**

**Mass dependence is minor  
(once including  $n=1$ )**

**Surprisingly consistent with the lattice-QCD (red-shaded)**

Ding et al. (2011, 2016)

explained by the fact that  $B$ -dependence is mild in this region

# Calculation Details



## Kubo Formula

$$\sigma^{ij} = \lim_{k_0 \rightarrow 0} \lim_{\mathbf{k} \rightarrow \mathbf{0}} \frac{1}{2ik_0} [\Pi_R^{ij}(k) - \Pi_A^{ij}(k)]$$

$$\Pi_R^{\mu\nu}(k) := i \int d^4x e^{ik \cdot x} \theta(t) \langle [j^\mu(x), j^\nu(0)] \rangle,$$

$$\Pi_A^{\mu\nu}(k) := -i \int d^4x e^{ik \cdot x} \theta(-t) \langle [j^\mu(x), j^\nu(0)] \rangle$$

$$j_{\text{em}}^i = \underline{n_0 T^{0i} / (\mathcal{E} + \mathcal{P}_i)}$$



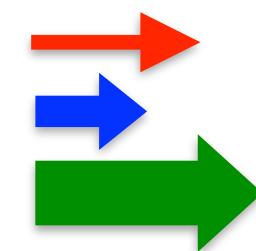
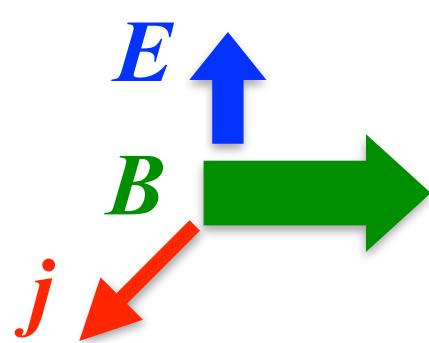
Hydro zero-mode subtracted (which lead to divergence)

# Calculation Details

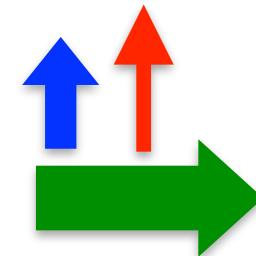


## Tensor Decomposition

$$\sigma^{ij} = \sigma_H \epsilon^{ijk} \hat{B}^k + \sigma_{\parallel} \hat{B}^i \hat{B}^j + \sigma_{\perp} (\delta^{ij} - \hat{B}^i \hat{B}^j)$$



CME



$$\frac{\sigma_{\perp}}{T} \sim \frac{g^2 T^2}{|qB|}$$

$$\sigma_H = \frac{n_0}{B}$$

Hall conductivity

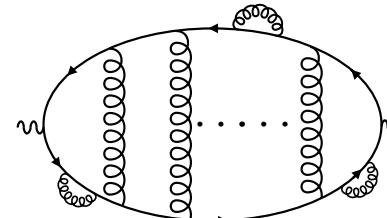
Suppressed  
 $T \gtrsim \sqrt{qB} \gg gT$

# Calculation Details



**Pinch singularities must be avoided by resummation**

Higher order diagrams



generated by

$$\overrightarrow{\text{---}} = \overrightarrow{\text{---}} + \overrightarrow{\text{---}} + \text{higher order terms}$$

$$\overrightarrow{\text{---}} = \overrightarrow{\text{---}} + \text{higher order terms}$$

leading to

$$2P_p^\mu (\partial_\mu + q_f F_{\nu\mu} \partial_{p_\nu}) f_p = -C[f]$$

$$2\bar{P}_{p'}^\mu (\partial_\mu - q_f F_{\nu\mu} \partial_{p'_\nu}) \bar{f}_{p'} = -\bar{C}[f]$$

$$2k^\mu \partial_\mu g_k = -\tilde{C}[f]$$

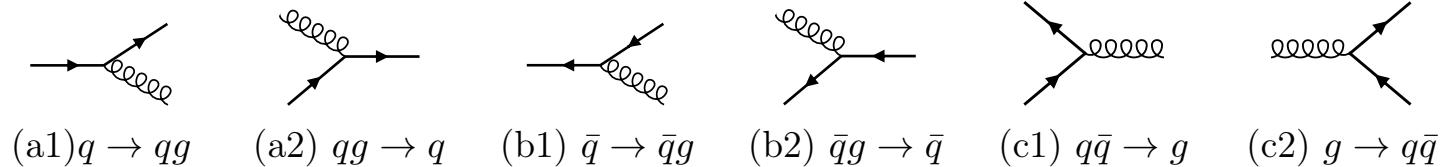
$$2P_p^\mu := \bar{u}(p) \gamma^\mu u(p)$$

$$2\bar{P}_{p'}^\mu := \bar{v}(p') \gamma^\mu v(p')$$

# Calculation Details



## Strategy



Solve the Boltzmann equations for given collision terms

Expand distribution  $f$  around thermal equilibrium  $f_{\text{eq}}$

Linear deviation  $\delta f$  proportional to external  $E$

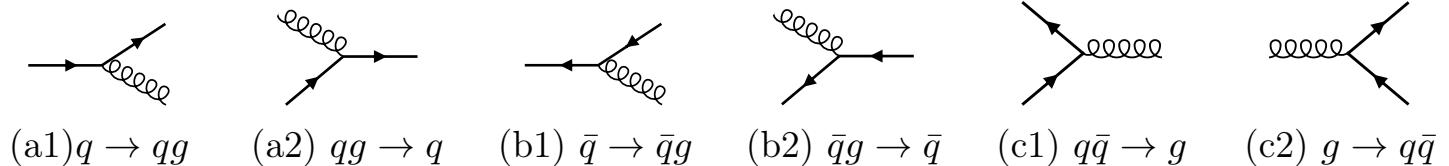
Rescale them to  
remove trivial  
kinematical factors

$$\begin{aligned}\delta f_p &= \beta f_{\text{eq}}(p)[1 - f_{\text{eq}}(p)] E_z \chi_p , \\ \delta \bar{f}_{p'} &= \beta \bar{f}_{\text{eq}}(p')[1 - \bar{f}_{\text{eq}}(p')] E_z \bar{\chi}_{p'} , \\ \delta g_k &= \beta g_{\text{eq}}(k)[1 + g_{\text{eq}}(k)] E_z \tilde{\chi}_k .\end{aligned}$$

# Calculation Details



## Strategy



Express the electric current with the distribution funcs.

Equilibrium has no current

Current proportional to  $\delta f$  and thus  $E$

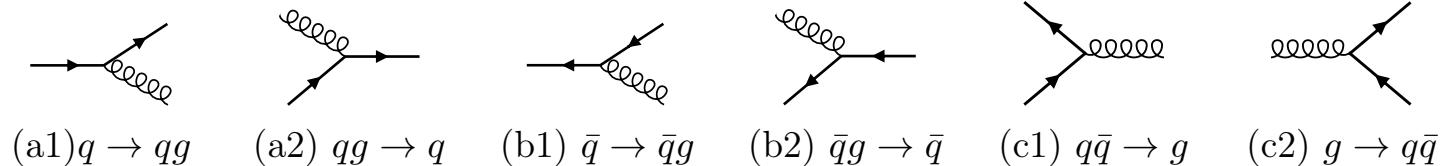
$$j_z = \sigma_{\parallel} E_z = \int_p 2P_p^3 q_f (\delta f_p - \delta \bar{f}_p)$$

Once  $\delta f$  is determined,  $\sigma_{\parallel}$  can be computed

# Calculation Details



## Strategy



$$2P_p^\mu (\partial_\mu + q_f F_{\nu\mu} \partial_{p_\nu}) f_p = -C[f]$$

$$2\bar{P}_{p'}^\mu (\partial_\mu - q_f F_{\nu\mu} \partial_{p'_\nu}) \bar{f}_{p'} = -\bar{C}[f]$$

$$2k^\mu \partial_\mu g_k = -\tilde{C}[f]$$

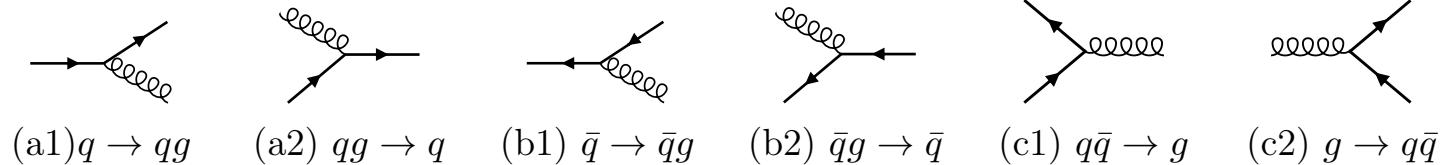
**Linearize and solve them!**

This involves the inversion of a huge matrix.  
(If hydromodes not subtracted, no inversion possible)

# Calculation Details



## Strategy

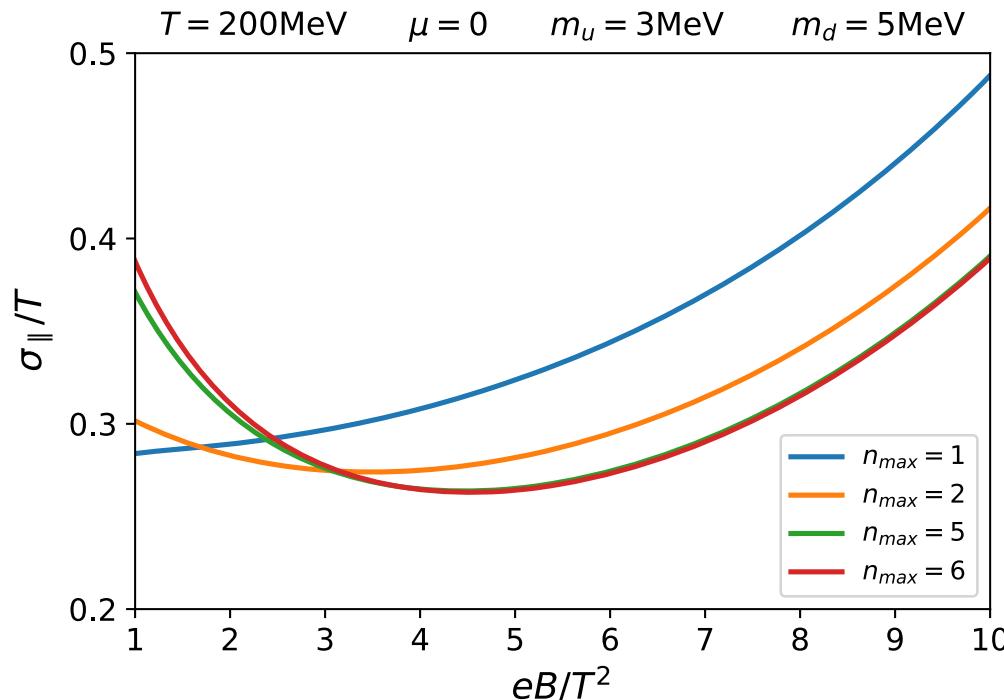


## One example of complication...

$$\begin{aligned}
 & \mathcal{L}_{m_1 l_1; m_2 l_2}^{gg} \\
 &= -\frac{1}{2} \sum_f \sum_{n=1}^{\infty} \sum_{n'=0}^{n-1} \int \frac{dp_z}{2\pi} \frac{1}{2\varepsilon_{fn}} \int' \frac{dp'_z}{2\pi} \frac{1}{2\varepsilon_{fn'}} X(n, n', \xi_-) f_{\text{eq}}(p) [1 - f_{\text{eq}}(p')] [1 + g_{\text{eq}}(k)] \\
 &\quad \times d_{l_1}(k_z) d_{l_2}(k_z) b_{m_1}(k_{\perp}) b_{m_2}(k_{\perp}) \\
 &\quad - \frac{1}{2} \sum_f \sum_{n=1}^{\infty} \sum_{n'=0}^{n-1} \int \frac{dp_z}{2\pi} \frac{1}{2\varepsilon_{fn}} \int \frac{dp'_z}{2\pi} \frac{1}{2\varepsilon_{fn'}} X(n, n', \xi_-) \bar{f}_{\text{eq}}(p) [1 - \bar{f}_{\text{eq}}(p')] [1 + g_{\text{eq}}(k)] \\
 &\quad \times d_{l_1}(k_z) d_{l_2}(k_z) b_{m_1}(k_{\perp}) b_{m_2}(k_{\perp}) \\
 &\quad + \frac{1}{2} \sum_f \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \int \frac{dp_z}{2\pi} \frac{1}{2\varepsilon_{fn}} \int \frac{dp'_z}{2\pi} \frac{1}{2\varepsilon_{fn'}} X(n, n', \xi_+) f_{\text{eq}}(p) \bar{f}_{\text{eq}}(p') [1 + g_{\text{eq}}(k)] \\
 &\quad \times d_{l_1}(k_z) d_{l_2}(k_z) b_{m_1}(k_{\perp}) b_{m_2}(k_{\perp}).
 \end{aligned}$$

This is “just” a  $gg$  diagonal component and more and more...

# Final Results



**Non-monotonic !?**

**For large  $B$  LLL dominant  
and  $\sigma$  grows with  $B$  linearly**

**For small  $B$  the cross section  
increases with  $B$  and thus  
 $\sigma$  decreases with  $B$**

**Quadratic dependence is “accidental”  
Negative magnetoresistance still CME smoking-gun**

**Finite baryon / isospin density, see our full paper (soon!)**

# *Conclusions / Outlooks*



- We have done the electric conductivity calculation at intermediate magnetic field and at finite density.
- We found weak dependence on the quark mass, the magnetic field, and the chemical potential, consistent with the lattice QCD constraints.
- Adjust some parameters corresponding to not QCD but chiral media in the laboratory, and try to fully reproduce the experimental data.

# *Conclusions / Outlooks*



- Remember : not only the longitudinal but also the transverse conductivity arises.

$$\sigma_H = \frac{n_0}{B} j$$

A diagram illustrating the components of a vector field. A green arrow labeled 'B' points vertically upwards. A blue arrow labeled 'E' points horizontally to the right. A red arrow labeled 'j' points diagonally downwards and to the left. The red arrow 'j' is positioned between the green arrow 'B' and the blue arrow 'E'.

**Longitudinal fields  
from longitudinal expansion  
and/or Glasma fluxes**

- Observable effects on the ellipticity? Yes!?