Chiral phase transition of (2+1)-flavor QCD

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for
HotQCD collaboration

Universität Bielefeld

CRC-TR 211
Strong-interaction matter under extreme conditions

Overview

Overview

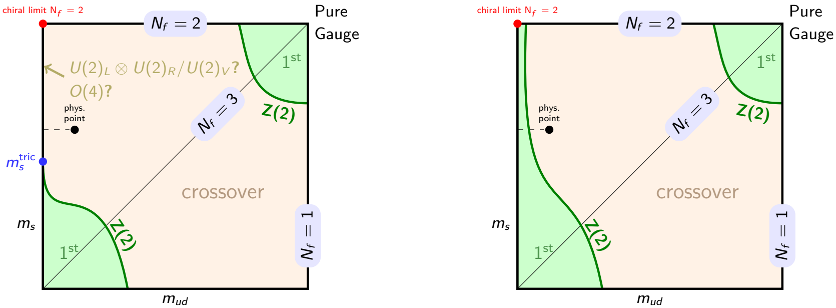
Introduction?

- In QCD, transition for physical quark masses and vanishing or small baryon density is a crossover.
- T_{pc} for physical mass QCD has been determined with very good accuracy from various observables : $T_{pc}(\mu_B = 0) = 156.5 \pm 1.5$ MeV $\Rightarrow$ "QCD transition at zero and non-zero baryon densities" by Patrick Steinbrecher on 16 May 2018, 11:30 for recent HotQCD results.
- Important question : How do singular terms of the chiral phase transition at vanishing quark mass affect observables at physical values of the quark mass? $\Rightarrow$ How far the physical regime is from chiral limit w.r.t. scaling window?
- Key question : What is the chiral transition temperature, T_c^0?

Introduction?

In this talk, we are trying to address the followings :

- Key question : What is the chiral transition temperature, T_c^0?
- Possibly another question : What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios : [O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]



- We are trying to pin down the correct one.

Overview

Basic quantities

In terms of temperature T and symmetry breaking field $H = m_l/m_s$ the scaling variables are defined as :

$$t = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0} \qquad \text{and} \qquad h = \frac{1}{h_0}\frac{m_l}{m_s} = \frac{1}{h_0}H$$

Scaling variable :

$$z = \frac{t}{h^{\frac{1}{\beta\delta}}} = z_0\left(\frac{T - T_c^0}{T_c^0}\right)\left(\frac{1}{H^{1/\beta\delta}}\right); \quad z_0 = \frac{h_0^{\frac{1}{\beta\delta}}}{t_0}$$

Chiral condensate : $\langle\bar{\psi}\psi\rangle_f = \frac{T}{V}\frac{\partial \ln Z}{\partial m_f}$

Chiral susceptibility : $\chi_m^{fg} = \frac{\partial}{\partial m_g}\langle\bar{\psi}\psi\rangle_f$

Scaling relations

Renormalization group invariant (RGI) definition of order parameter :

$$M = \frac{m_s}{f_K^4}\left(\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) - \frac{m_u + m_d}{m_s}\langle\bar{\psi}\psi\rangle_s\right)$$

RGI definition of order parameter susceptibility :

$$\chi_M = \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right) M$$

Close to chiral limit, singular part behaves as :

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

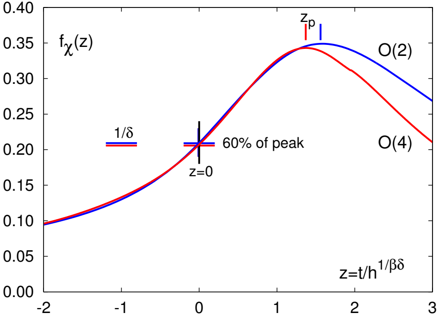
Scaling functions : Some intriguing facts

Conventional approach to determine T_c^0 : determine pseudo-critical temperature $T_{pc}(H)$ from the peak location of $\chi_M(T,H)$ and

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{1/\beta\delta}\right)$$

Our approach in this work : determine temperature $T^-_{60\%}(H)$ at 60% of peak height of $\chi_M(T,H)$ and

$$T^-_{60\%}(H) = T_c^0 \left(1 + \frac{z^-_{60\%}}{z_0} H^{1/\beta\delta}\right)$$



Advantage : $z^-_{60\%} \simeq 0 \Rightarrow$ reduces influence of regular terms, simplifies scaling analysis.

Dependence on quark mass (H) reduced by two orders of magnitude $\Rightarrow$

	z_p	$z^-_{60\%}$
$O(2)$	1.56	-0.009
$O(4)$	1.37	-0.01

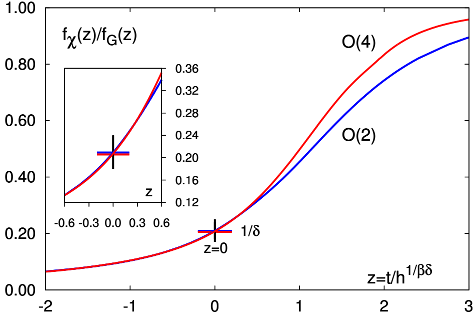
Scaling functions : Some intriguing facts

- Other approach used for consistency check : Crossing points of

$$\frac{H\chi_M}{M} = \frac{f_\chi(z)}{f_G(z)} + \text{regular terms.}$$

  is unique for $H \to 0$.

- Ratio has a curvature in $z \Rightarrow$ Non-linear part of scaling function is important.

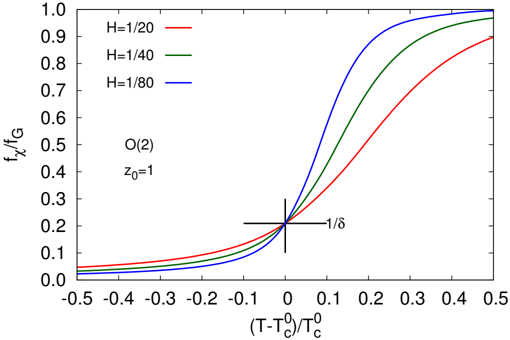


	$1/\delta$
$O(2)$	0.209
$O(4)$	0.207

Difference between $O(4)$ and $O(2)$ is negligible in the z range we are working.

Scaling functions : Some intriguing facts

$$\frac{f_\chi(z)}{f_G(z)} = \begin{Bmatrix} 0 & , & z \to -\infty \\ 1/\delta & , & z = 0 \\ 1 & , & z \to +\infty \end{Bmatrix}$$



- Ratio is expected to have a constant value at the crossing point, $z = 0$, *i.e.* in chiral limit at T_c^0.
- Uniqueness of the crossing point get spoiled in presence of regular terms.
- Around T_c^0 : slope $\propto H^{-1/\beta\delta}$ and curvature $\propto H^{-2/\beta\delta}$.

- Determine temperature $T_\delta(H)$ which satisfies :

$$\frac{H\chi_M\left(T_\delta, H\right)}{M\left(T_\delta, H\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} T_\delta(H)$$

.

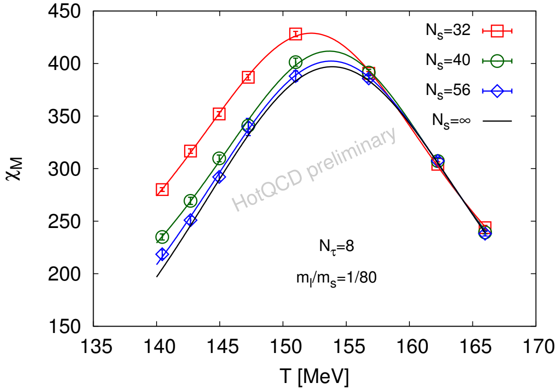
Disclaimer

- Here we will mainly show results for $N_\tau = 8$ only.
- Same analyses also carried out for $N_\tau = 6$.
- Same analyses for $N_\tau = 12$ are still at preliminary stage.
- Our goal is to find out the chiral transition temperature, T_c^0, in continuum limit.
- More lattices are still being generated for low masses and the analyses are not settled yet for those parameter sets.
- We are working on the systematic error estimation for various analyses and most of the results shown here are just indicative.

Overview

No evidence for 1^{st} order transition

- Volume dependence of χ_M is studied for $H = 1/80$ which corresponds to $m_\pi = 80$ MeV in continuum.
- χ_M^{max} is not proportional to volume.



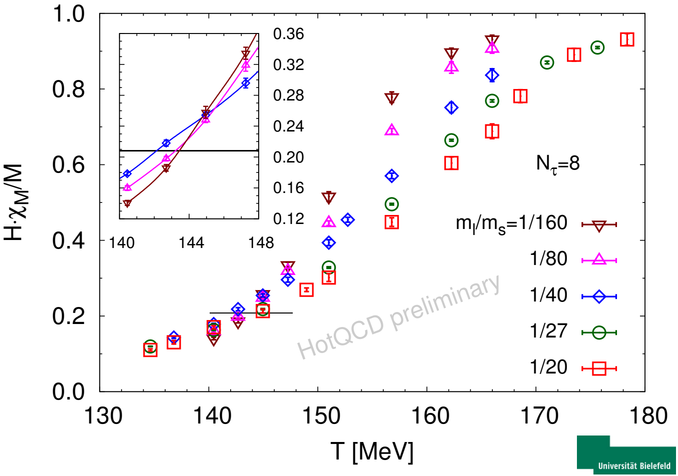
- Possibility for 1^{st} order phase transition can be ruled out at $m_\pi = 80$ MeV for $N_\tau = 8$.
- We only focus on $O(2)$, $O(4)$ and $Z(2)$.
- Similar results are also obtained for $N_\tau = 6$ and 12.
- $N_\tau = 12$: $42^3 \times 12$ and $60^3 \times 12$ lattices have been used for $m_l/m_s = 1/40$.

Estimation of T_c^0 from $H\chi_M/M$

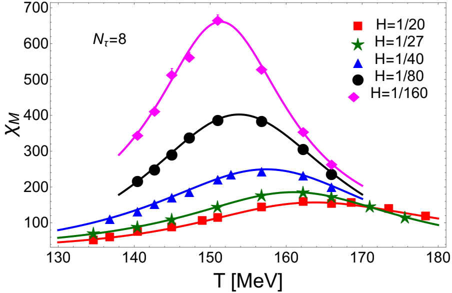
- Behavior of the ratio is like Binder cumulant at critical point.

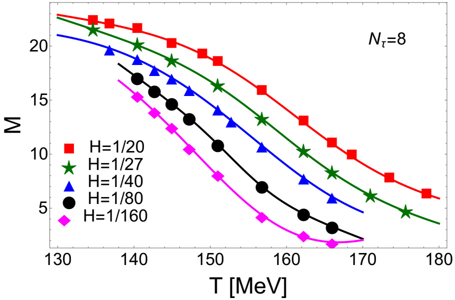
  [F. Karsch and E. Laermann. Phys. Rev. D50, 6954, 1994.]

- For $H = 1/27$ and $1/20$, regular contributions may be larger than lowest three masses.
- Crossing points for three lowest masses gives an estimate $T_c^0 \sim 144$ MeV for $N_\tau = 8$.



Fitting $\chi_M^{f_K}$ and M^{f_K} with rational functions





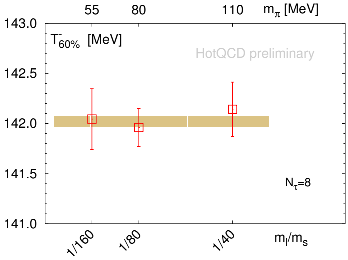
- χ_M for different H are fitted with rational functions :
  1. peak location : $T_{\rm pc}(H)$.
  2. peak height : $\chi_M^{\rm max}(H)$.
  3. $T^-_{60\%}$, defined by :

$$\chi_M\left(T^-_{60\%}\right) = \frac{60}{100}\chi_M^{\rm max}$$

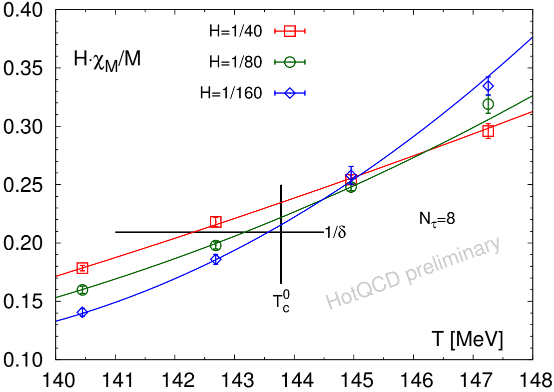
- M at T_{pc} and $T^-_{60\%}$ are calculated for different H by fitting with rational functions.

Sanity checks for T_c^0

- $T^-_{60\%}$ for three lowest masses indicates $T_c^0 \sim 142$ MeV.



- Joint scaling fit of M and χ_M to MEOS with $O(2)$ exponents also gives $T_c^0 \sim 145$ MeV for $N_\tau = 8$.

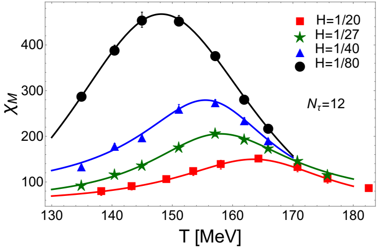


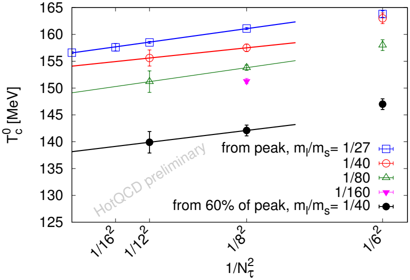
- Simultaneous fit (with regular term) of $H \cdot \chi_M/M$ to the three lowest masses gives $T_c^0 \sim 144$ MeV for $N_\tau = 8$.

Current estimate of T_c^0 for $N_\tau = 8$ is 144(2) MeV.

Towards continuum T_c^0

- Current estimate of T_c^0 is 147(2) MeV for $N_\tau = 6$ and 138(3) MeV for $N_\tau = 12$.
- $T^-_{60\%}$ is approximately independent of H establishes it as a good estimator of T_c^0.





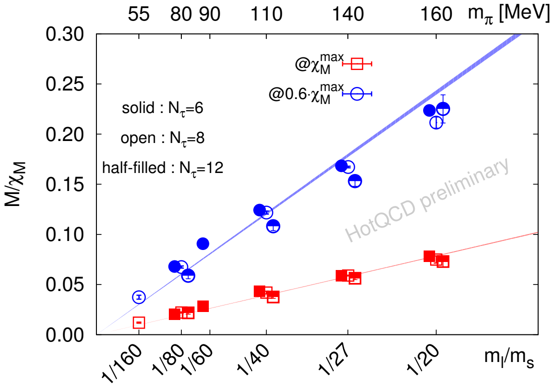
- Calculations at different m_l/m_s suggest that cut-off effects are only weakly (not) dependent on quark mass.
- Experience from $m_l/m_s = 1/27$ calculations suggest that extrapolation to continuum limit based on $N_\tau = 8$ and 12 is safe.

We find $T_c^0 = 138(5)$ MeV in continuum.

Order of chiral transition : work in progress....

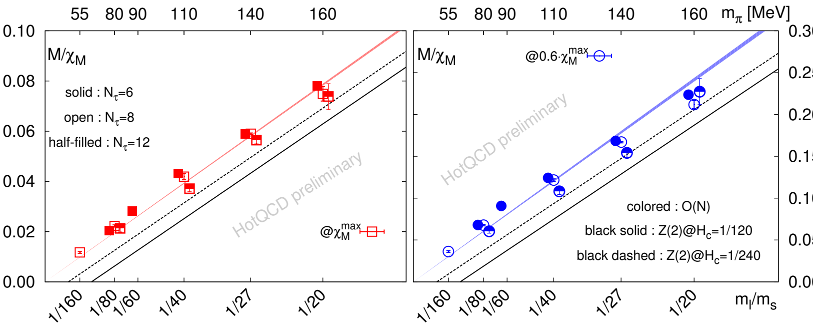
$$\frac{M}{\chi_M} = H \frac{f_G(z)}{f_\chi(z)}$$

- For small H the data seems to be linear.
- Lines are NOT fitted curves rather expectations for $O(2)$ and $O(4)$.



- $Z(2)$ transition, at some finite H_c, will results into a strong V dependence and sudden drop in the ratio $\Rightarrow 1^{st}$ order transition is unlikely for $m_\pi > 55$ MeV.
- Additional low H measurements : slope can be directly determined as fit parameter.

Order of chiral transition : work in progress....



- $Z(2)$ lines are schematic.
- If M is not exactly order parameter then the $Z(2)$ lines will have a curvature.
- Mixing becomes weak as H_c becomes small.
- Data seems to favor $O(N)$ compared to $Z(2)$.

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Summary and Outlook

- Summary :
  1. Measurements are according to the expectations even without being specific about the universality class.
  2. Proposed method can distinguish between $Z(2)$ and $O(N)$.
  3. Scaling fits with $O(2)$ exponents worked reasonably.
  4. Current estimate of T_c^0, in continuum, is 138(5) MeV.
- Outlook :
  1. Finer lattices are being generated for $H = 1/40$ and $1/80$.
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