Second-order transport coefficients at NLO in pQCD

Jacopo Ghiglieri, CERN

QM2018, Venice, May 15 2018

Second-order transport coefficients at NLO in pQCD (cinemascope edition)

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My "QCD at high T" trilogy

First installment, QM2015.

(full) NLO (AMY) kinetic theory for jets

pedagogical review of light-cone techniques in JG Teaney **1502.03730 QGP5**

gritty details in JG Moore Teaney **JHEP1603** (2016)

Second installment, QM2017.

(almost) NLO results for first-order transport coefficients

JG Moore Teaney, **JHEP1803** (2018)

Third (and final?) installment, QM2018.

second-order transport coefficients: NLO results and bounds

JG Moore Teaney, **1805.02663**

Previously…

The effective kinetic theory

- A direct evaluation of the Kubo formulae is usually not the simplest path to a perturbative determination of transport coefficients
- A determination from kinetic theory is in general simpler and equivalent
  Jeon, Aarts Martinez-Resco, Gagnon Jeon, Czajka Jeon
- Effective Kinetic Theory (EKT) for the phase space density of quarks and gluons

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}}\right) f(\mathbf{p}) = C$$

Transport coefficients from the EKT

- To obtain the transport coefficients linearize the theory

$$f(\mathbf{p}) = f_{\mathrm{EQ}}(\mathbf{p}) + \sum_{\ell} \delta f_{\ell}(\mathbf{p})$$

- Source term equates linearized collision operator

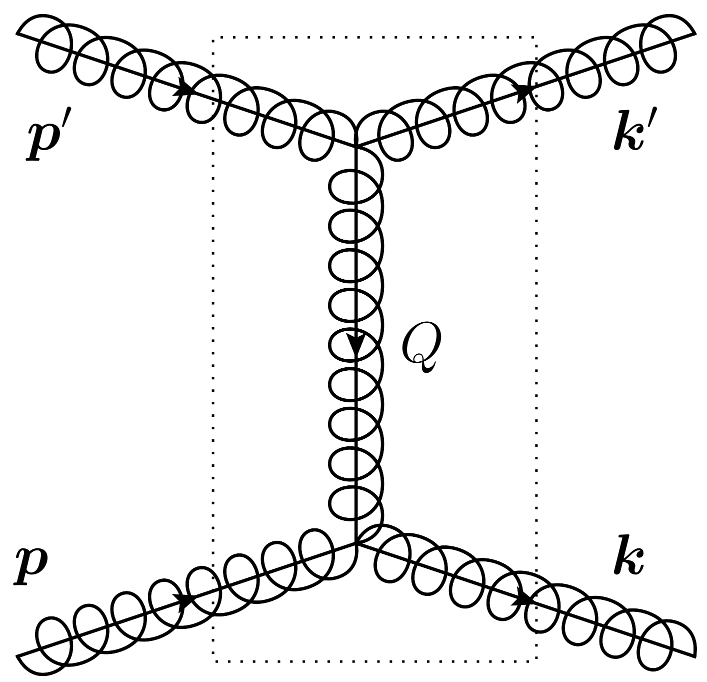
$$\mathcal{S}_{\ell} = \mathcal{C}\delta f_{l}$$

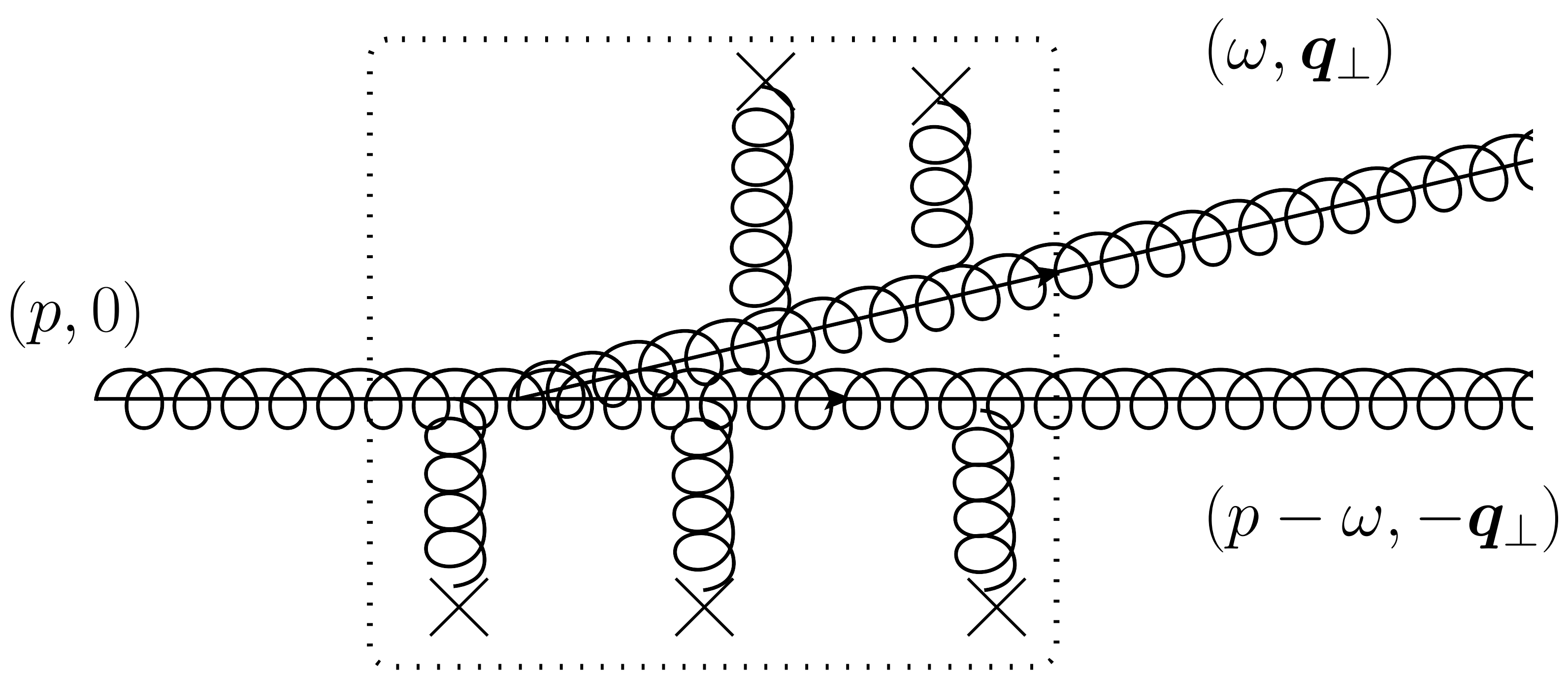
$$\mathcal{S}_{\ell} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} \right) f_{\mathrm{EQ}}(\mathbf{p}, u, \beta, \mu)$$

- Since $\langle T^{i\neq j} \rangle \propto \eta$, $\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$ (light flavor diffusion) η requires ℓ=2, D_q ℓ=1
- Transport coefficients obtained by the kinetic theory definitions of T, J once δf_{ℓ} has been obtained

The effective kinetic theory of QCD

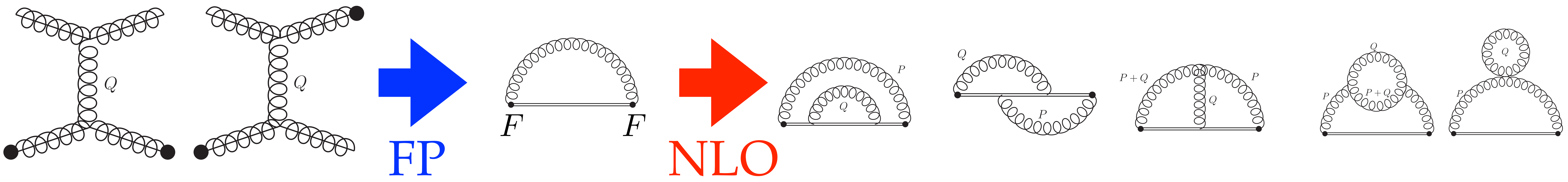
- At **leading order**: elastic, number-preserving 2↔2 processes and collinear, number-changing 1↔2 processes (LPM, HTL, all that) AMY (2003)





The effective kinetic theory of QCD

- At **(almost) NLO**: (almost) all $O(g)$ corrections from intermediate or external soft gluons. JG Moore Teaney (2015-18)
  - Soft-gluon exchange: Fokker-Planck equation with longitudinal and transverse momentum broadening (known at NLO through light-cone techniques), complemented by gain terms for energy-momentum conservation
  - NLO qhat (Caron-Huot 2008) numerically largest contribution



The effective kinetic theory of QCD

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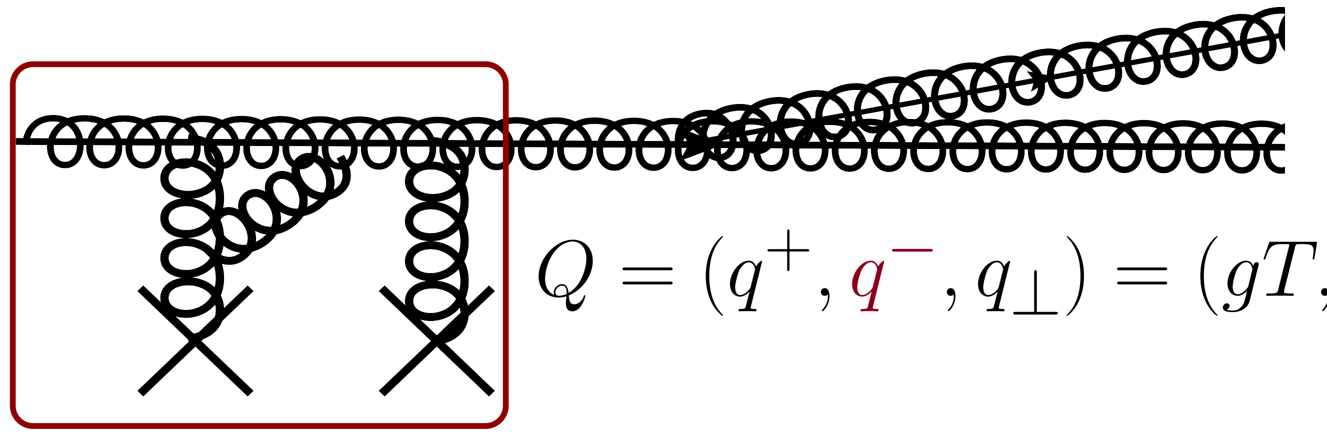
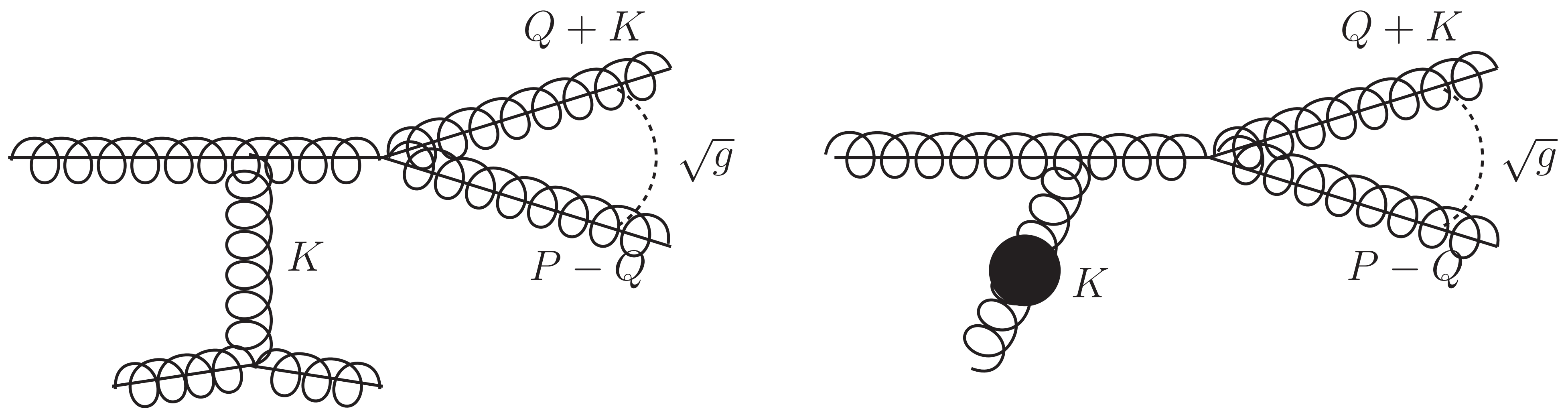
  - Soft-loop corrections to collinear processes



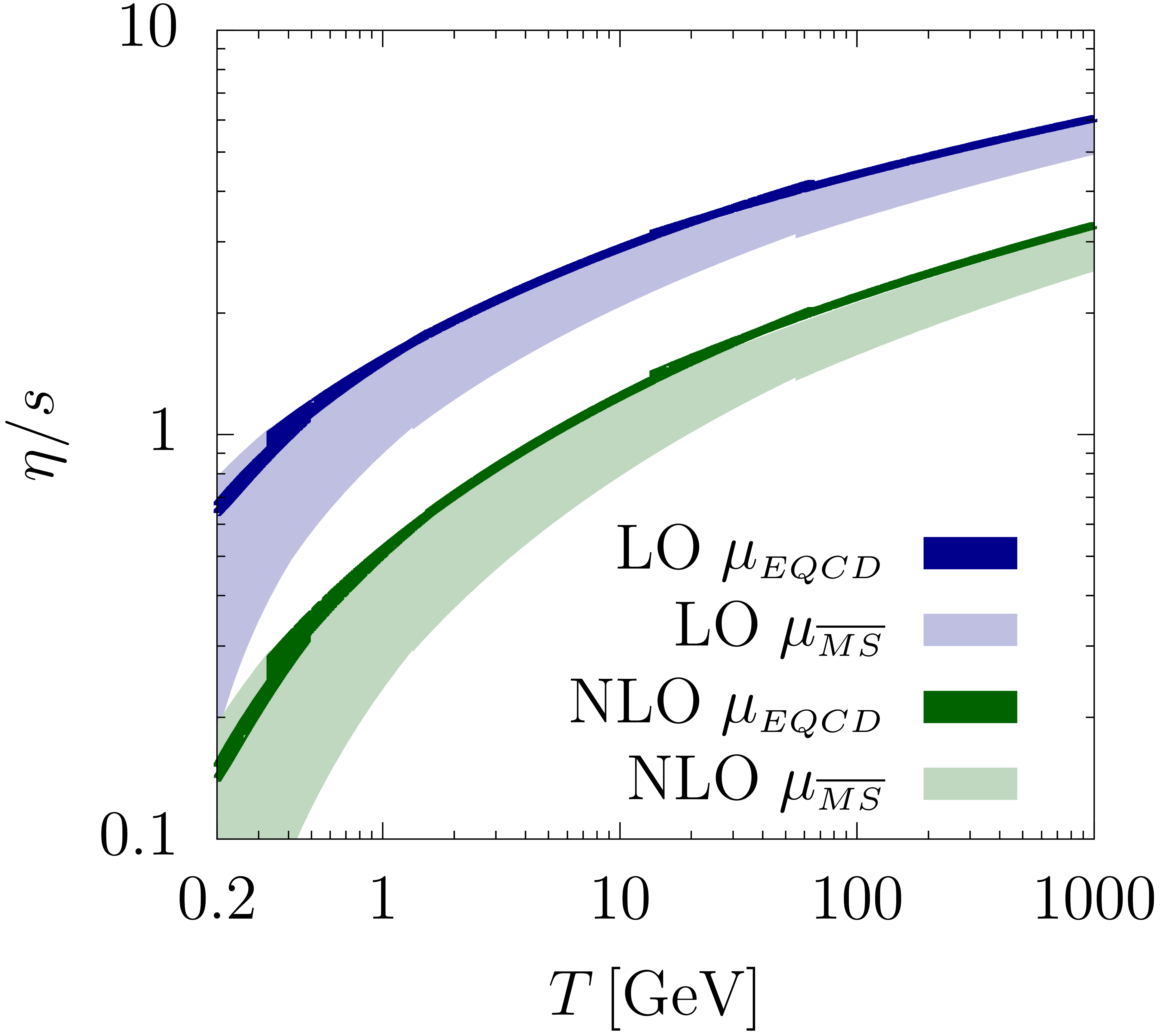
$Q = (q^+, q^-, q_\perp) = (gT,\ g^2T,\ gT)$

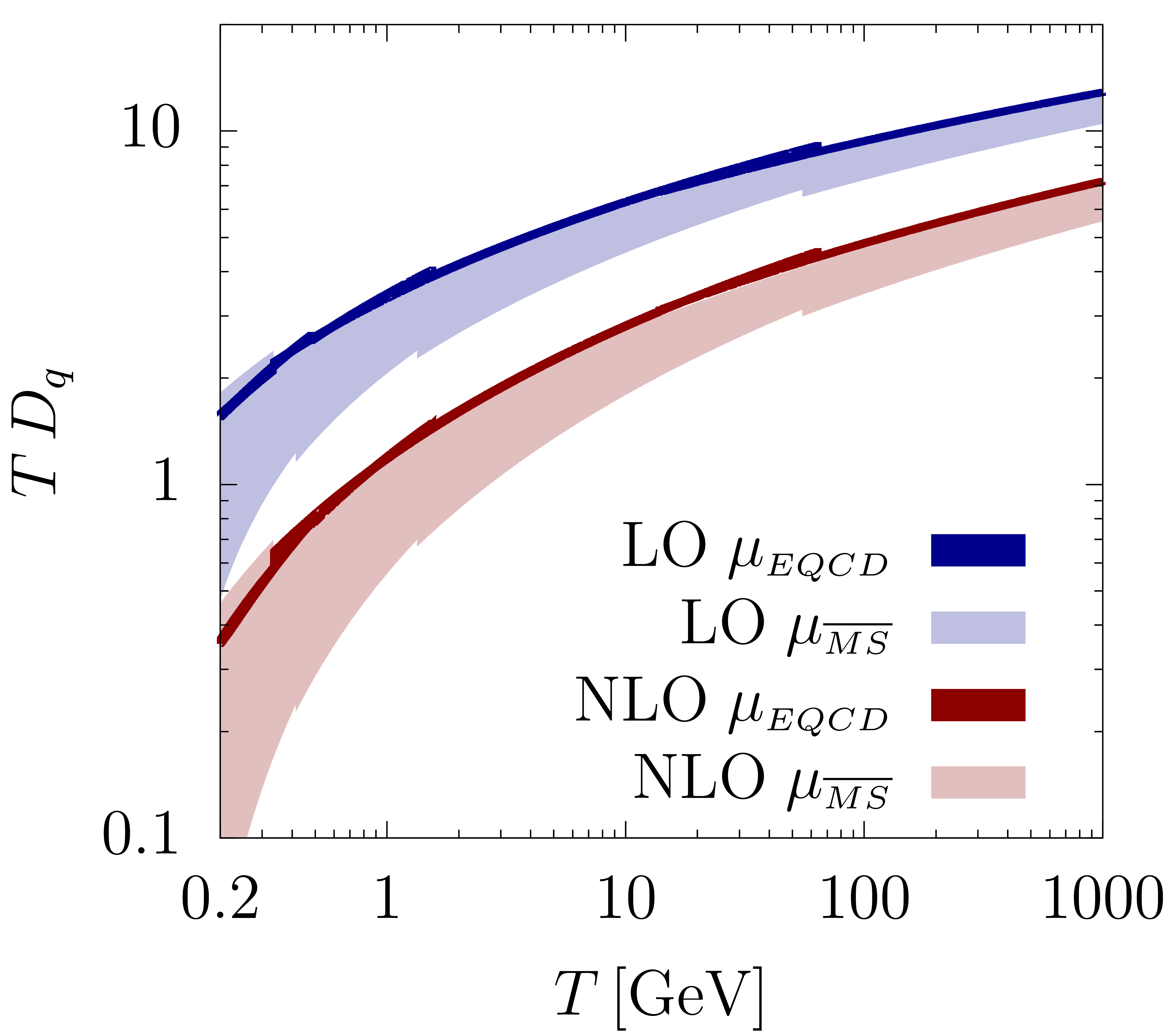
  - Not-so-collinear processes: semi-collinear processes



  - All dealt with using light-cone techniques

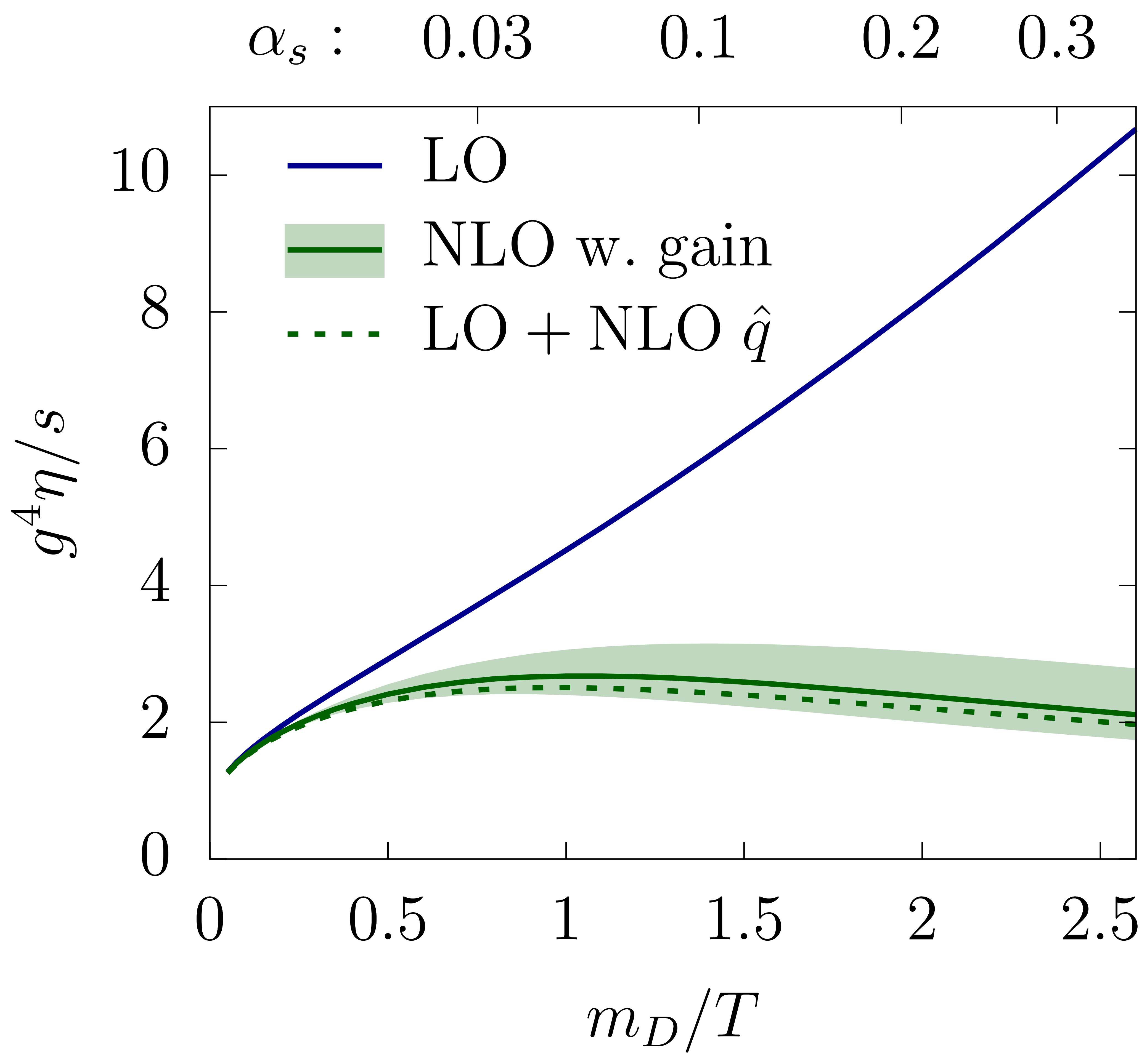
First-order coefficients of QCD

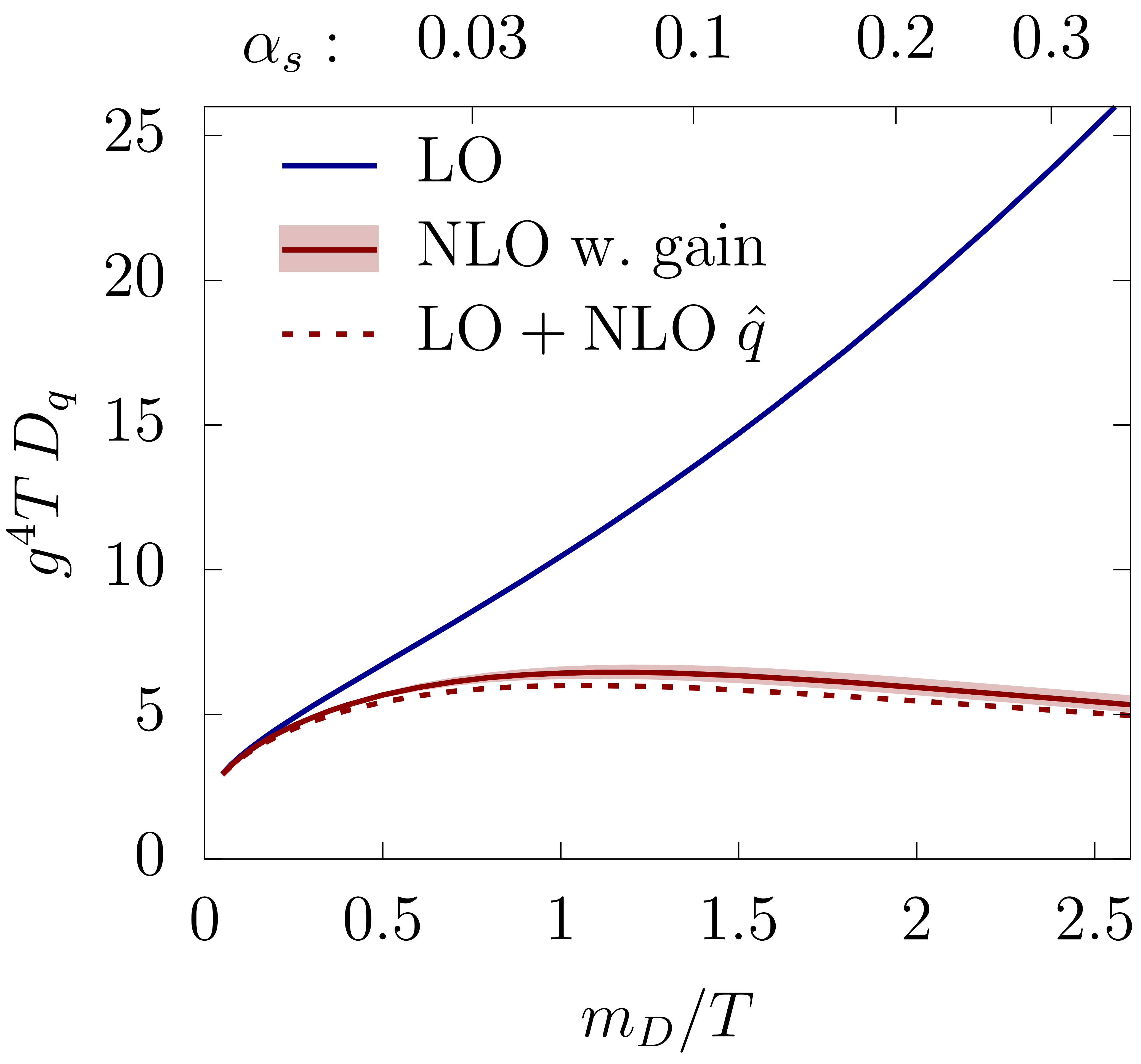




JG Ghiglieri Teaney **JHEP1803** (2018)

First-order coefficients of QCD





- Convergence realized at m_D~$0.5T$. NLO qhat dominates corrections

Back to the present

Second-order relaxation

- To be causal and stable, second order in the gradients is required
- At second order in the gradients, lot of work on writing down (and computing with different methods) all the new transport coefficients that pop up
  Baier *et al.* **JHEP0804** (2007), Denicol *et al.* **PRD85** (2012), Battacharyya *et al.* **JHEP0802** (2007)
- We look at the second-order relaxation τ_π of the shear stress tensor to its Navier-Stokes form
$$\tau_\pi \partial_t \pi^{ij} = \pi_1^{ij} - \pi^{ij}$$
- Similarly for a flavor current $\tau_j \partial_t \boldsymbol{j} = \boldsymbol{j}_1 - \boldsymbol{j}$
  recalling that $\langle \mathbf{J}_q \rangle = -D_q \boldsymbol{\nabla} \langle n_q \rangle$
- Hydro practitioners usually fix second-order transport coefficients by fixing their ratio to the first-order ones. What can we say about that?

Second-order relaxation

- Recall that we had $\mathcal{S}_\ell = \mathcal{C}\delta f_l$ at first order
- In the kinetic theory, second-order coefficients require second-order expansion of f. τ_π obtained from first-order δf acting as source term
- Rewrite first-order δf as $\delta f_l(\mathbf{p}) \equiv f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,\chi_l(\mathbf{p})$
  and introduce an inner product $(g,h) \equiv \int_{\mathbf{p}} f_{\rm EQ}(\mathbf{p})(1+f_{\rm EQ}(\mathbf{p}))\,g(\mathbf{p})\,h(\mathbf{p})$
  Then Moore York **PRD79** (2009)
  $$\eta\tau_\pi = \frac{1}{15T}(\chi,\chi)$$
- Two consequences
  - Can obtain (a)NLO results from the same setup (that gave us χ at (a)NLO)
  - Can think a second more about this inner product

Second-order relaxation: bounds

- The shear viscosity and enthalpy density can be written as

$$\mathcal{S}_\ell = \mathcal{C}\delta f_l \qquad \eta\tau_\pi = \frac{1}{15T}(\chi,\chi) \qquad \eta = \frac{1}{15}(\chi,1) \qquad e+p = \frac{T}{3}(1,1)$$

- The linearized collision operator is symmetric with respect to the inner product and is positive-definite (in the ℓ=1,2 channels), as dictated by stability

- We have all the spectral ingredients for a triangular inequality

$$\frac{\tau_\pi}{\eta/(e+p)} = 5\frac{(\chi,\chi)\,(1,1)}{(\chi,1)^2} \geq 5 \qquad \frac{\tau_j}{D_q} \geq 3$$

- Generic bounds in **any kinetic theory**, as long as enthalpy/charge susceptibility determined consistently within it

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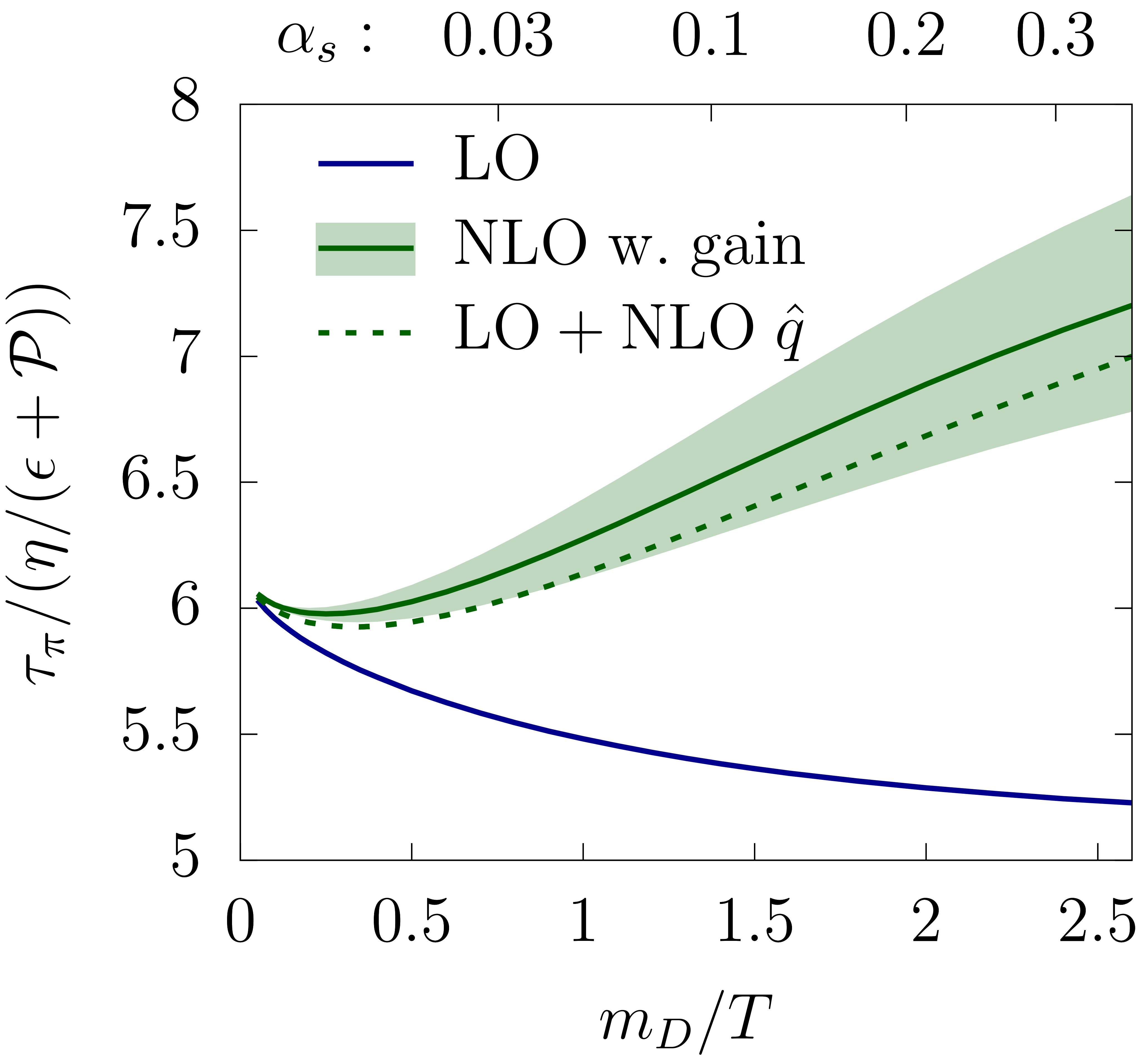
- At infinite coupling in $\mathcal{N}=4$ (finite coupling correction also known for τ_π)

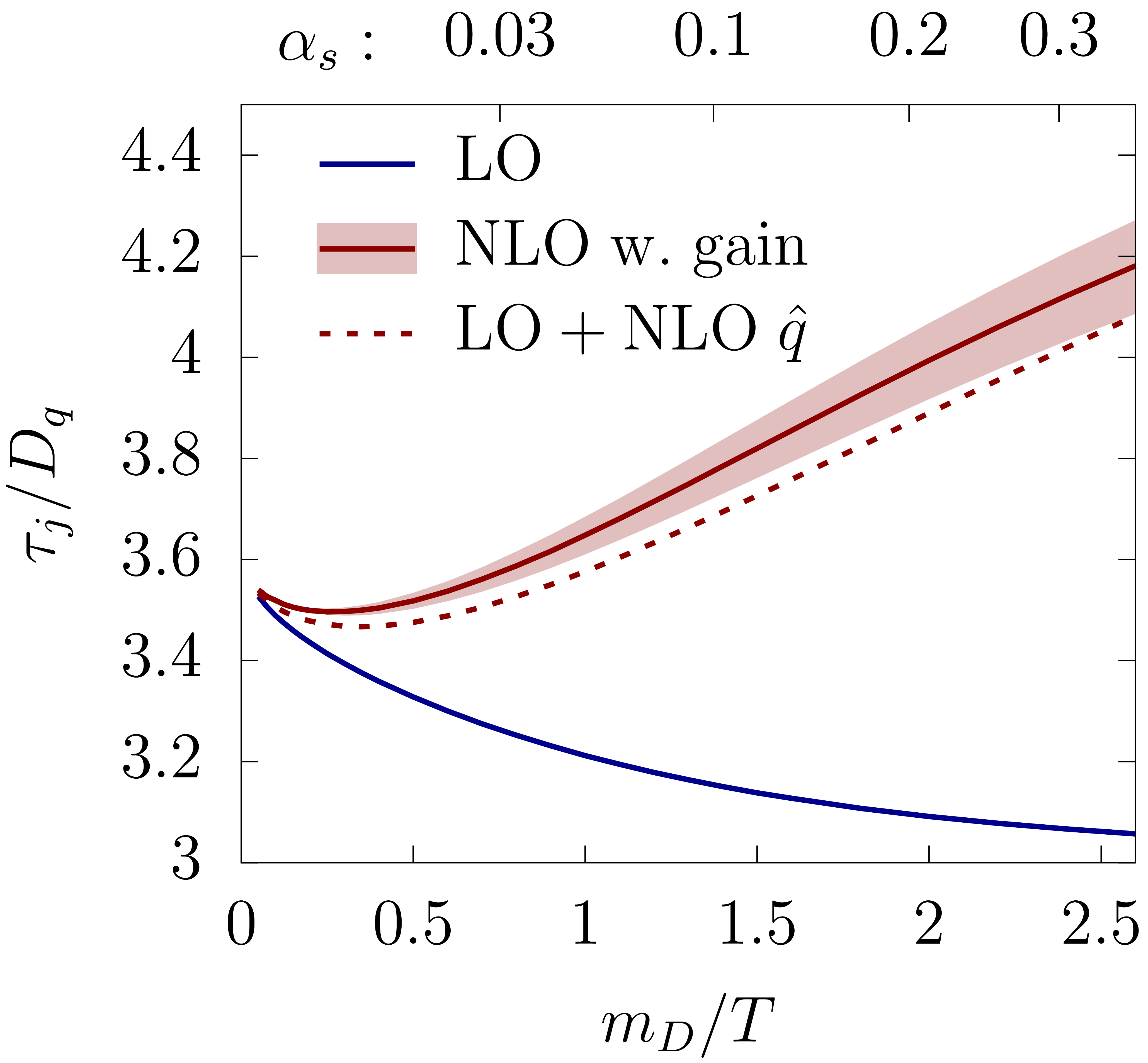
$$\frac{\tau_\pi}{\eta/(e+p)} = 4 - 2\ln(2) \approx 2.6 \qquad \frac{\tau_j}{D_{U(1)}} = \frac{\pi}{2}$$

Baier *et al* **JHEP0804** (2007), Bu Lublinsky Sharon **JHEP1604** (2016)

- Strongly-coupled holographic theories are **very far** from having a kinetic quasiparticle description

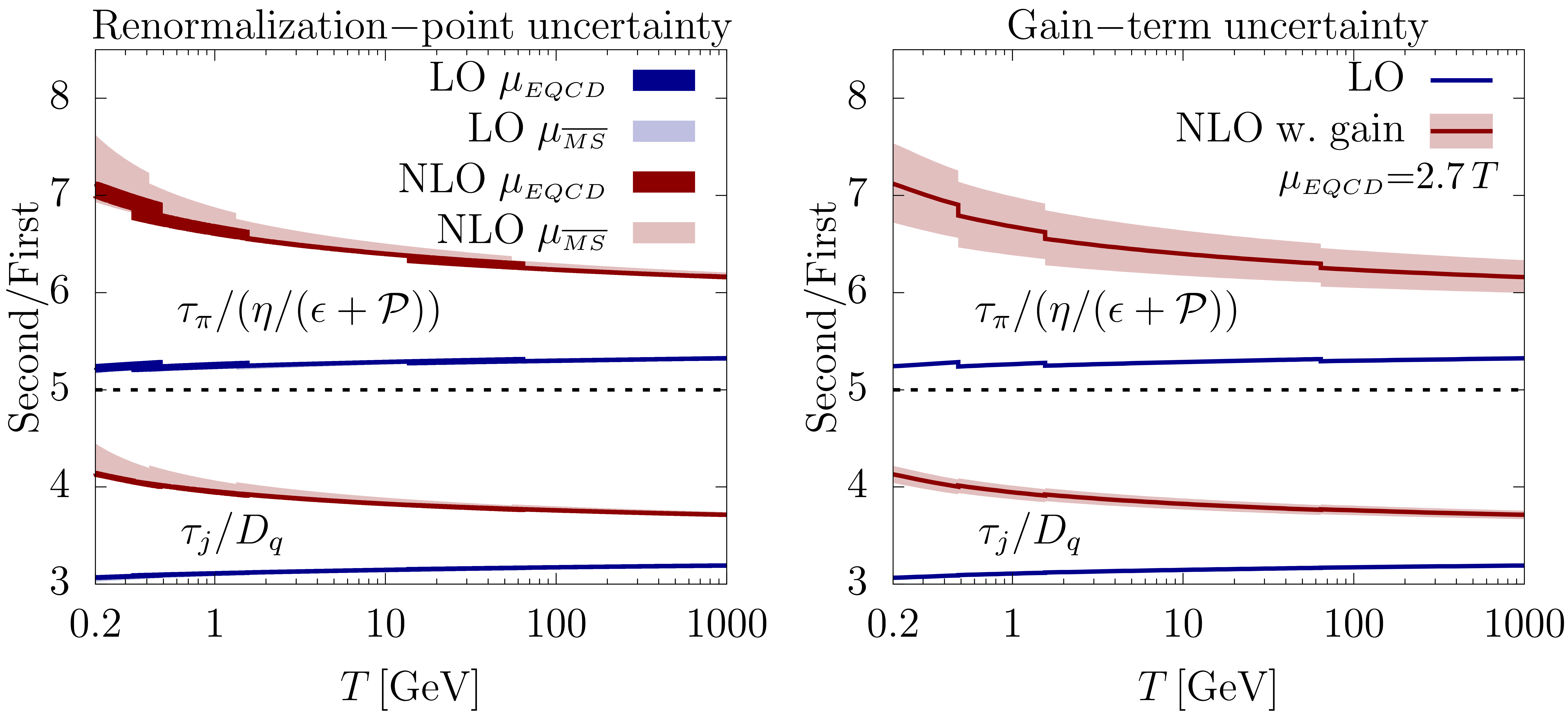
Second-order relaxation: (a)NLO results





- ~40% increase from LO and robustness of these second / first-order coeffs. ratios

Second-order relaxation: (a)NLO results



- Temperature dependence much milder than for the first- or second-order coeffs alone

Conclusions

- We have studied the second-order relaxation coefficients τ_π and τ_j. They are important to make relativistic hydrodynamics causal
- The ratio of these coefficients to their respective first-order relaxation coefficients $\eta/(e+p)$ and D_{q} is an important input for hydrodynamic simulations
- We have shown that within kinetic theory this ratio is bounded from below
- Strongly coupled theories with gravity duals violate these bounds: they are thus very far from having a kinetic, quasiparticle-based description
- (almost) NLO results for these ratios show a modest increase

Backup

Euclideanization of light-cone soft physics

- For t/x_z =0: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n,p)e^{i\mathbf{p}\cdot\mathbf{x}}$$

Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

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$$G_{rr}(t=0,\mathbf{x}) = \SumInt_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

- Consider the more general case $|t/x^z| < 1$

$$G_{rr}(t,\mathbf{x}) = \int dp^0 dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp - p^0 x^0)} \left(\frac{1}{2} + n_{\mathrm{B}}(p^0)\right)\left(G_R(P) - G_A(P)\right)$$

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- Change variables to $\tilde{p}^z = p^z - p^0(t/x^z)$

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- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x}) = T\sum_n \int dp^z d^2p_\perp e^{i(p^z x^z + \mathbf{p}_\perp\cdot\mathbf{x}_\perp)} G_E(\omega_n, p_\perp, p^z + i\omega_n t/x^z)$$

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- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

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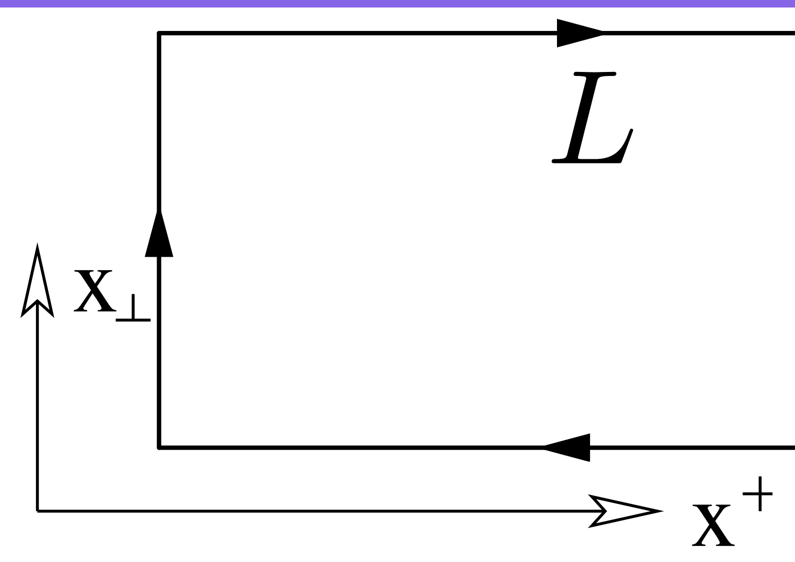
- Retarded functions are analytical in the upper plane in any timelike or lightlike variable => G_R analytical in p^0

$$G_{rr}(t,\mathbf{x})_{\rm soft} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n = 0, \mathbf{p})$$

- Soft physics dominated by $n=0$ (and t-independent) =>EQCD!

Caron-Huot **PRD79** (2009)

LPM resummation



$$\propto e^{\mathcal{C}(x_\perp)L}$$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine **EPJC72** (2012) Laine Rothkopf **JHEP1307** (2013) Panero Rummukainen Schäfer **1307.5850**

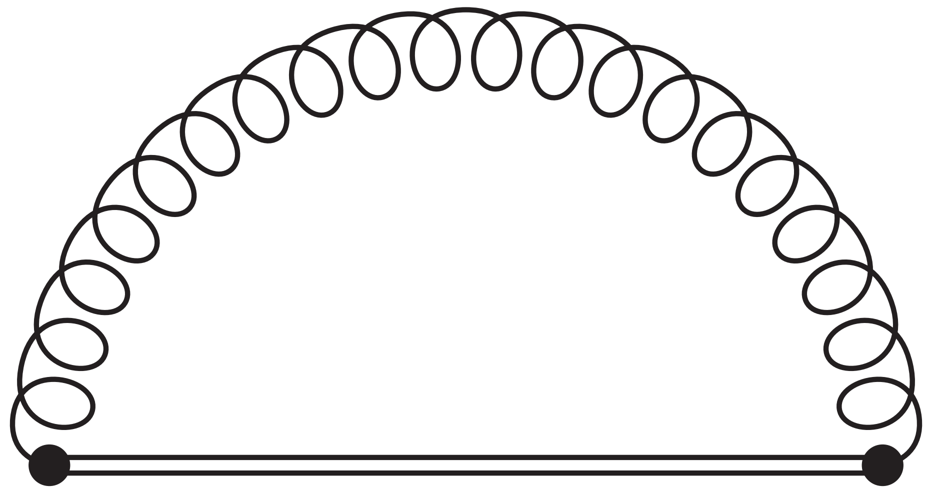
Longitudinal momentum diffusion

- Field-theoretical lightcone definition (justifiable with SCET)

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

$F^{+-}=E^z$, longitudinal Lorentz force correlator

- At leading order

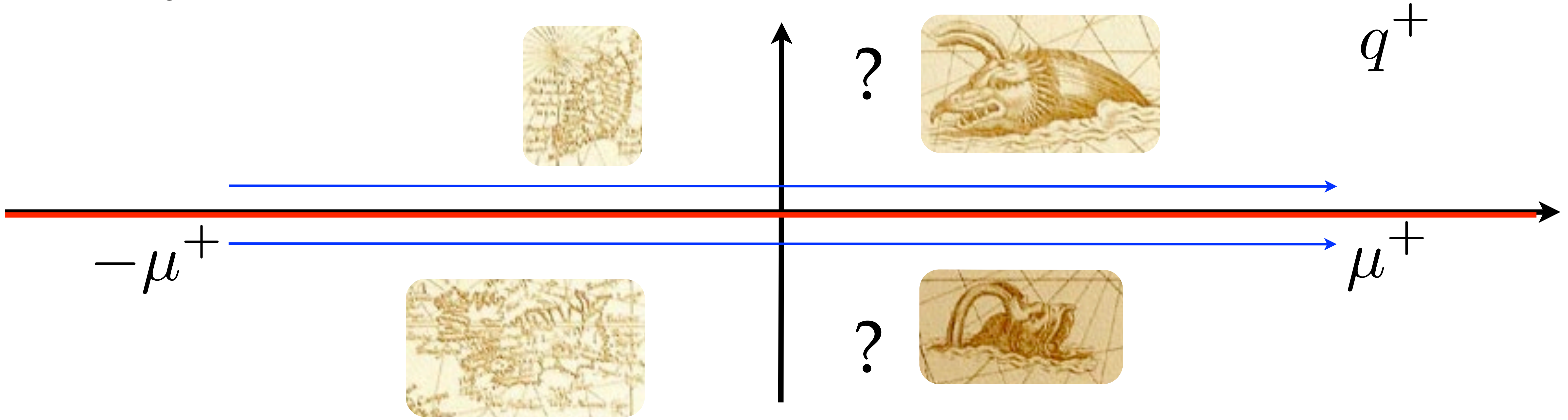


$$\hat{q}_L \propto \int \frac{dq^+ d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$$

$$= \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ (G^R_{++}(q^+,q_\perp,0) - G^A)$$

Longitudinal momentum diffusion

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} T q^+ \left(G_R^{--}(q^+, q_\perp) - G_A^{--}(q^+, q_\perp)\right)$$



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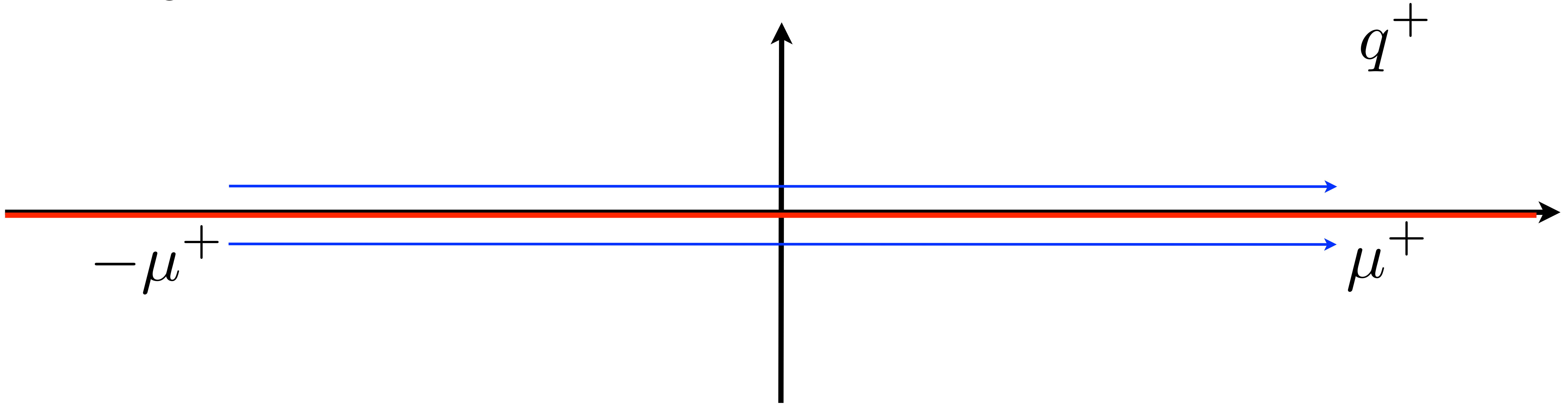

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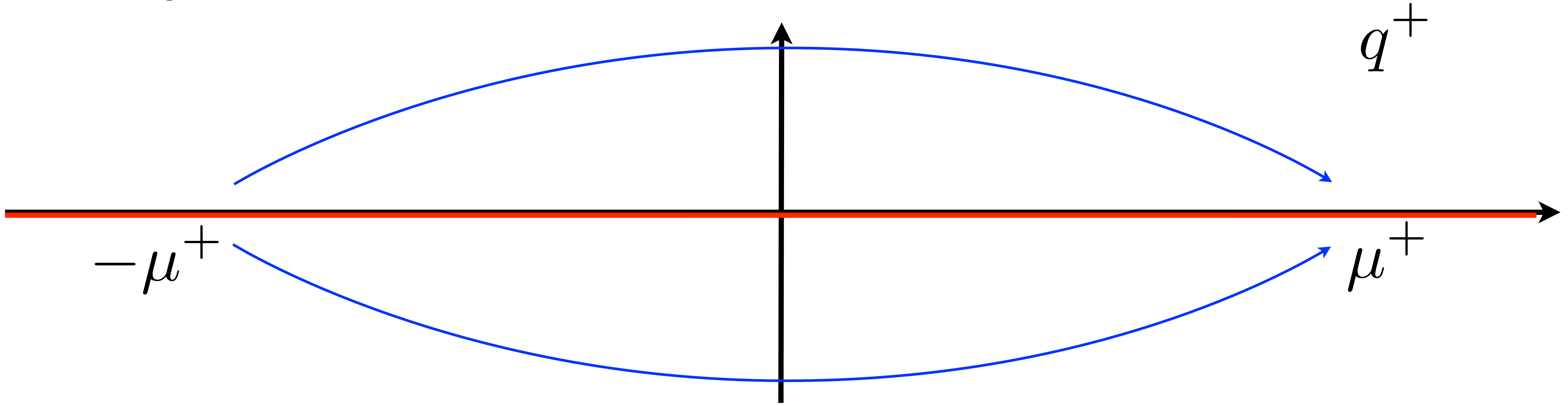

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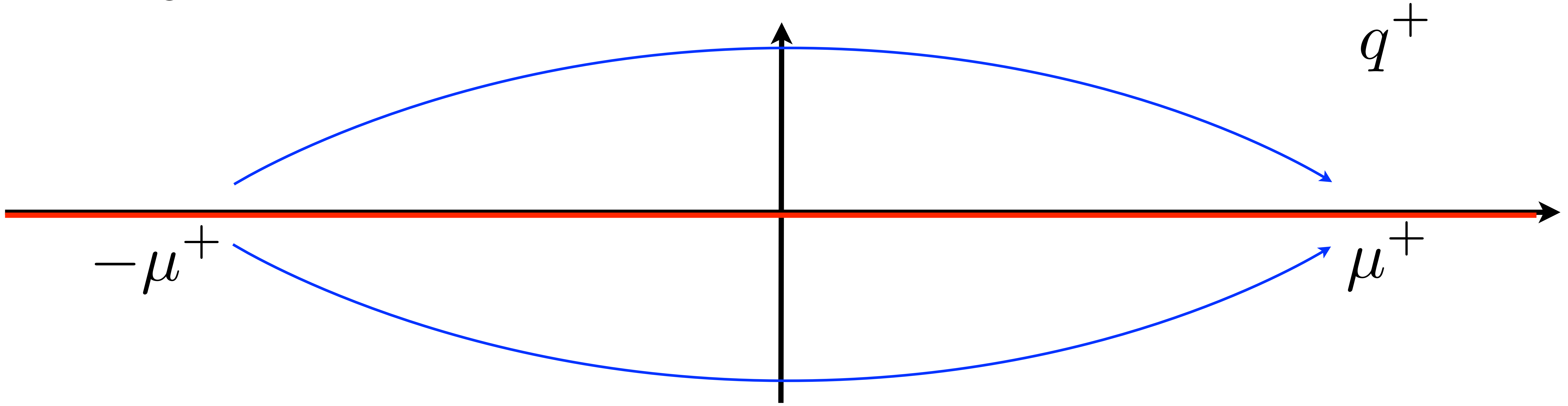

- Use analyticity to deform the contour away from the real axis and keep *1/q*⁺ behaviour

$$\hat{q}_L\Big|_{\mathrm{LO}} = g^2 C_R T \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{M_\infty^2}{q_\perp^2 + M_\infty^2}$$