The QCD equation of state at high temperatures

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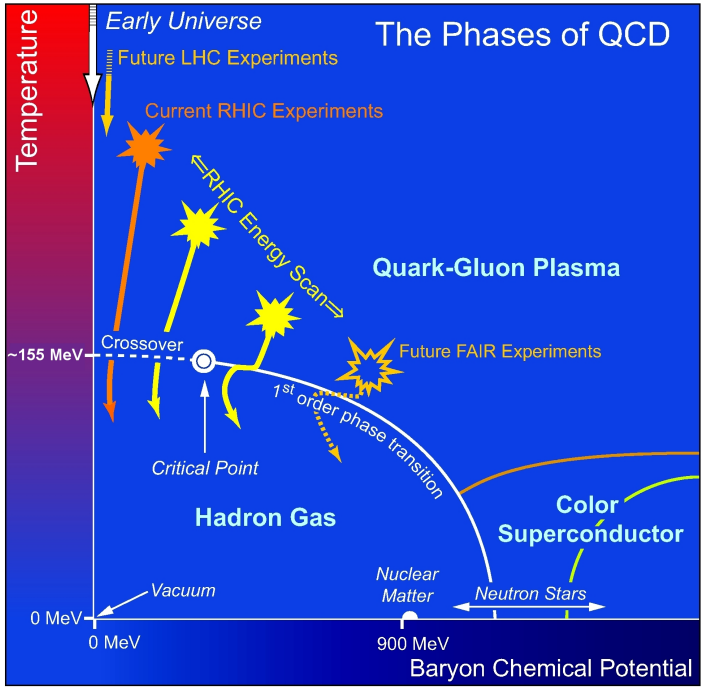
(in collaboration with P. Petreczky and J. Weber)
based on 1710.05024

Michigan State University

May 15, 2018

QCD phase diagram



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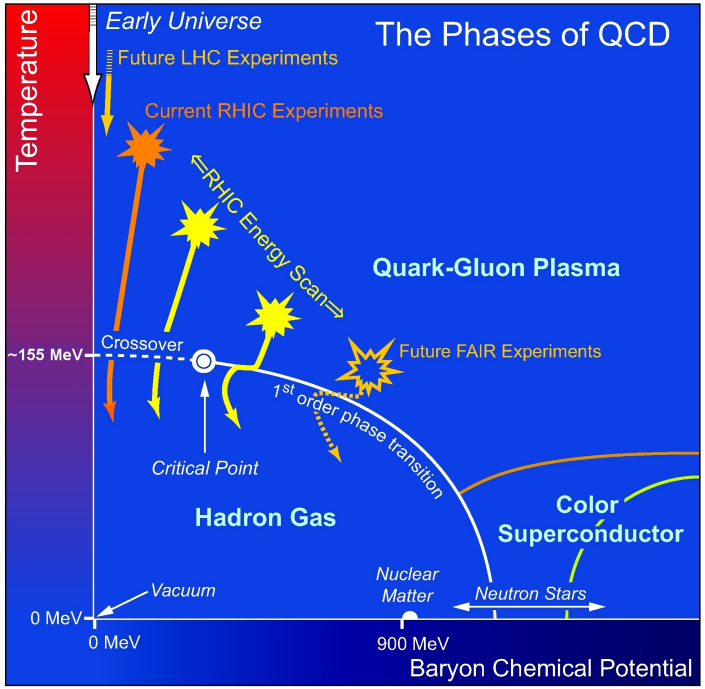

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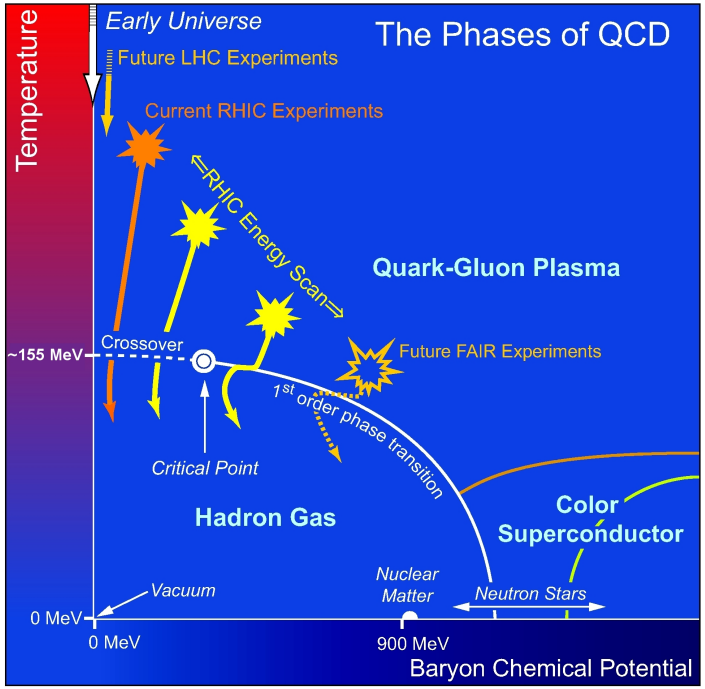

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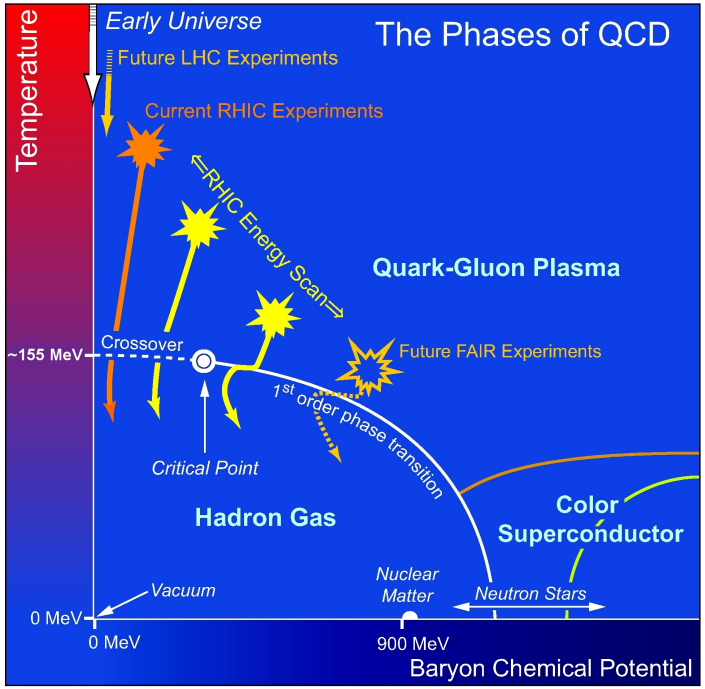

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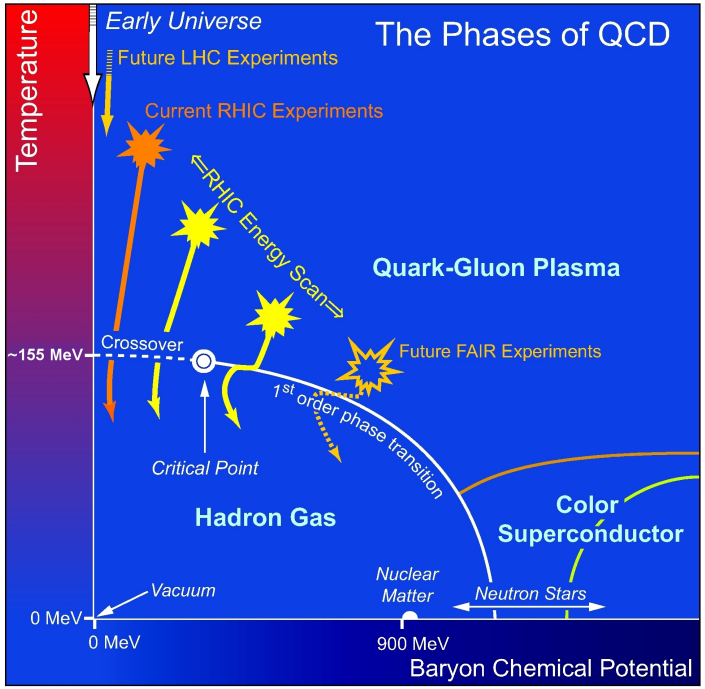

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- High-temperature phase: deconfinement, restoration of chiral symmetry
- 2+1 flavor QCD equation of state at zero baryon density has been recently calculated up to $T = 400 - 500$ MeV

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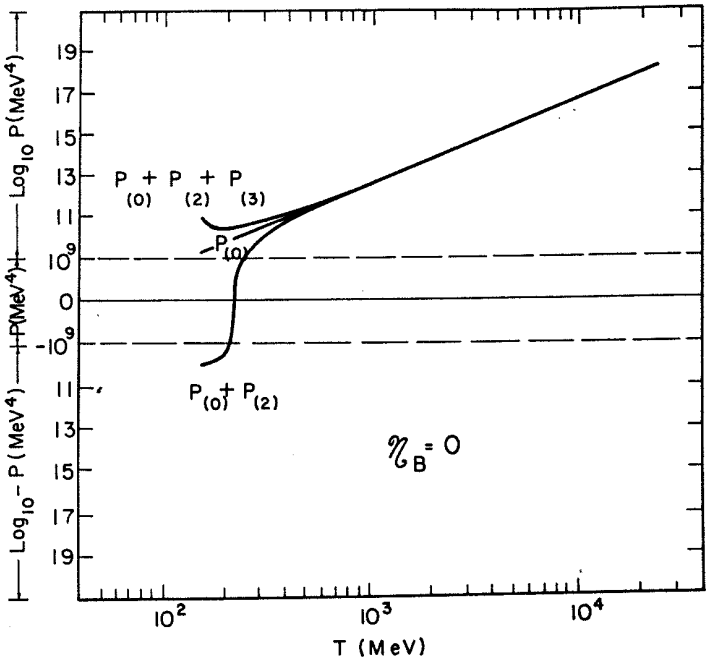
Earlier results on the equation of state



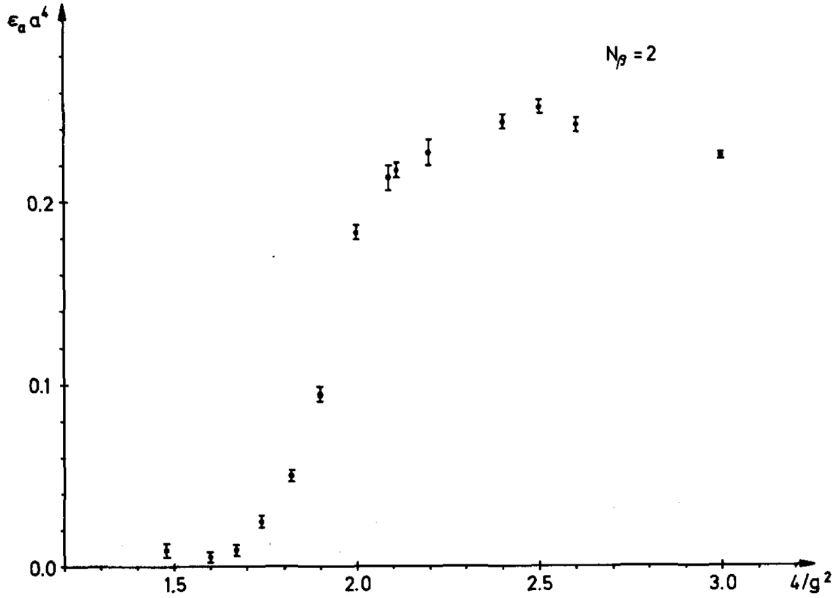
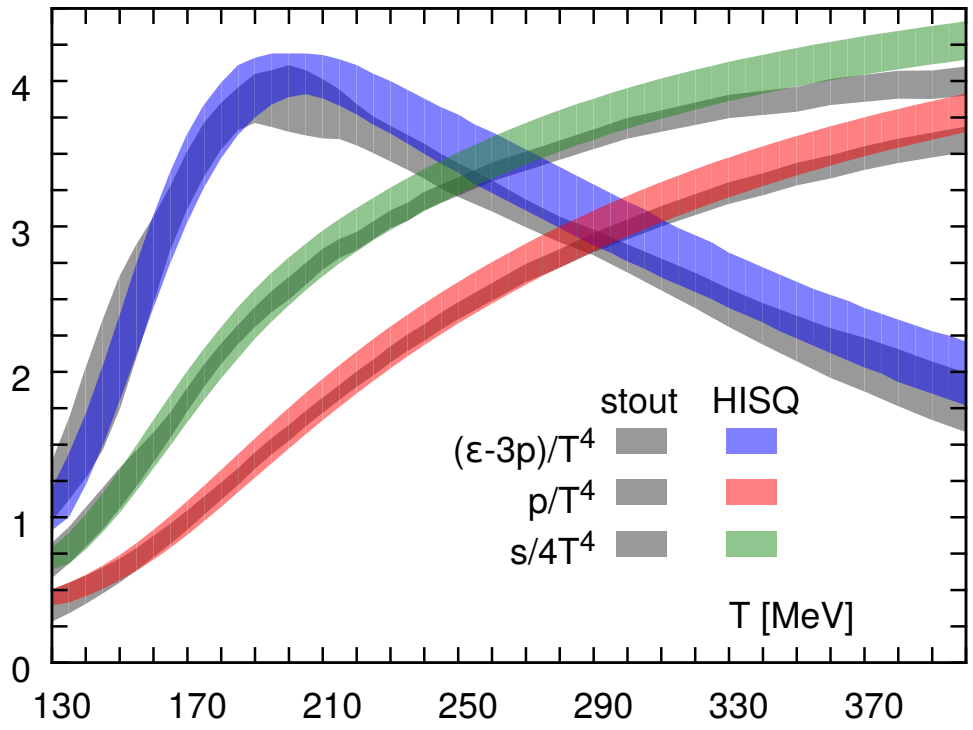


Fig. 3. Energy density of gluon matter versus $4/g^2$, at fixed lattice size $N_\beta = 2$, after about 500 iterations.

- First perturbative EoS calculation[2] (left)
- First lattice pure gauge $SU(2)$ EoS calculation[3] (right)

[2] Kapusta (1979)
[3] Engels et al. (1981)

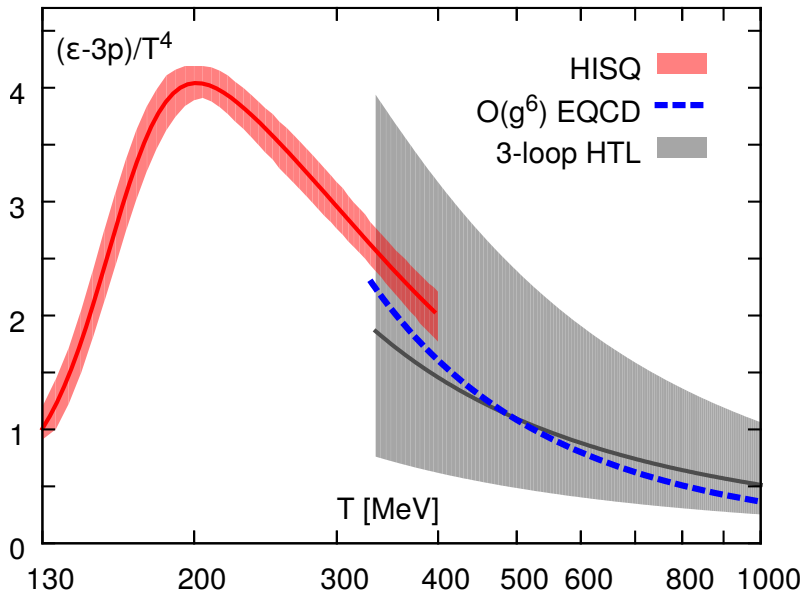
Recent results up to $T = 400$ MeV

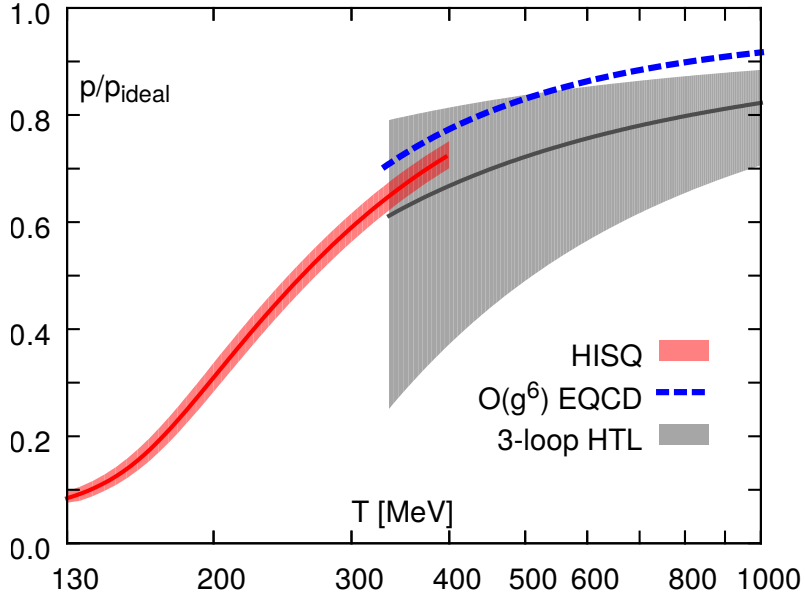


- Comparison of the continuum results with HISQ[4] and stout[5] for the trace anomaly, pressure and entropy density
- About 2σ deviations in the integrated quantities at the highest temperature

[4] Bazavov et al. [HotQCD] (2014)
[5] Borsanyi et al. [WB] (2014)

Approach to the perturbative limit





- The trace anomaly (left) and pressure (right) compared with (HTL)[6] and Electrostatic QCD (EQCD)[7] calculations
- The black line is the HTL calculation with the renormalization scale $\mu = 2\pi T$

[6] Haque et al. (2014)
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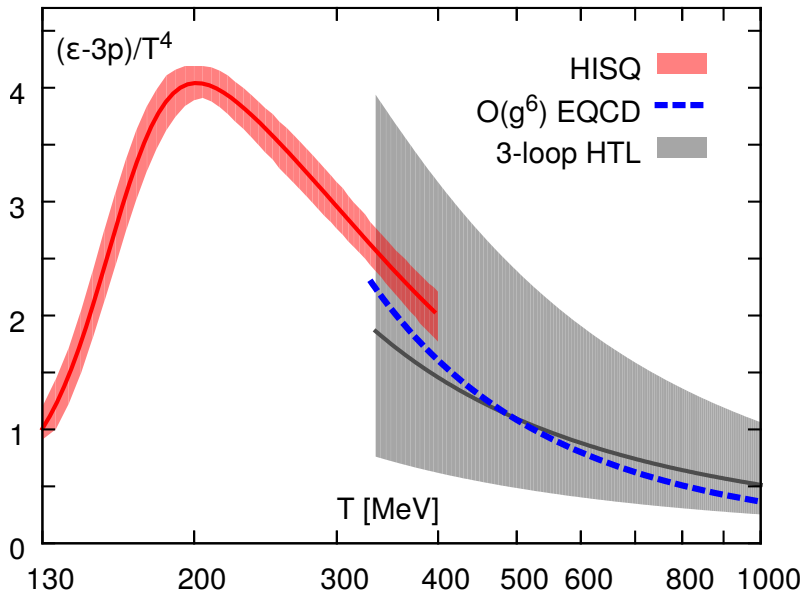

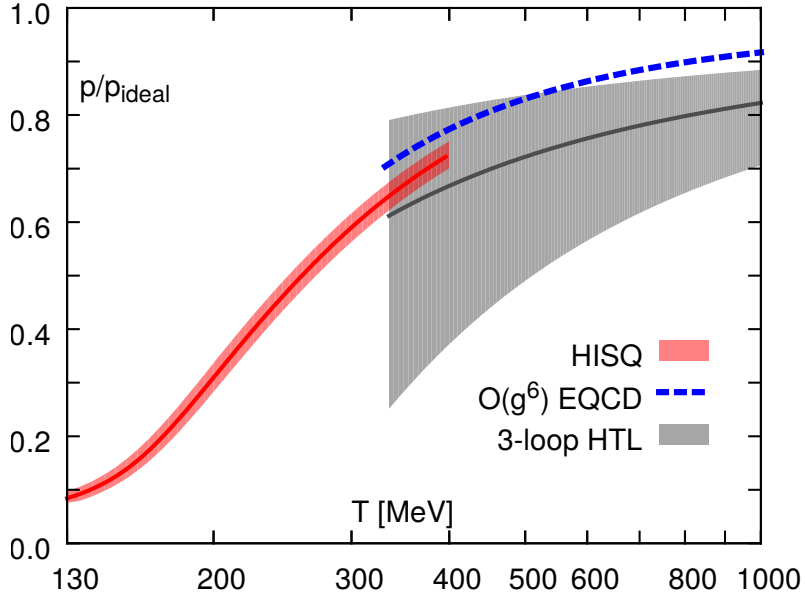

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- THIS TALK: Extension of the 2+1 flavor equation of state to higher temperatures

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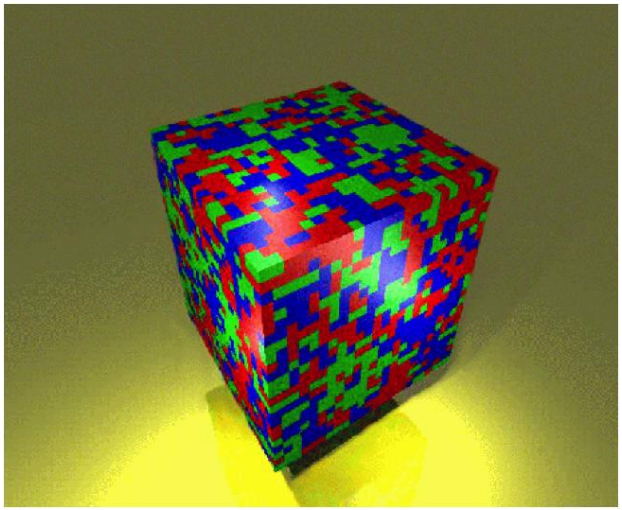
Lattice QCD



- Switch from Minkowski to Euclidean space – imaginary time formalism
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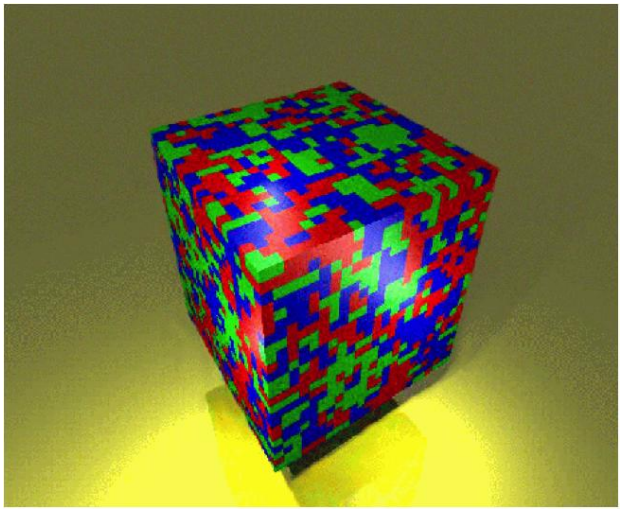

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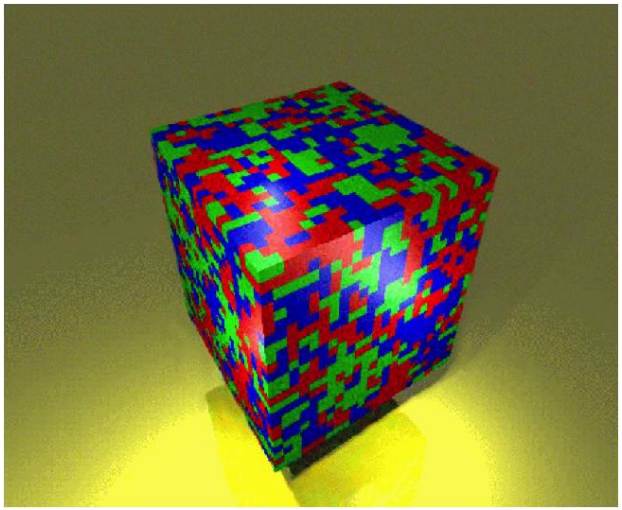

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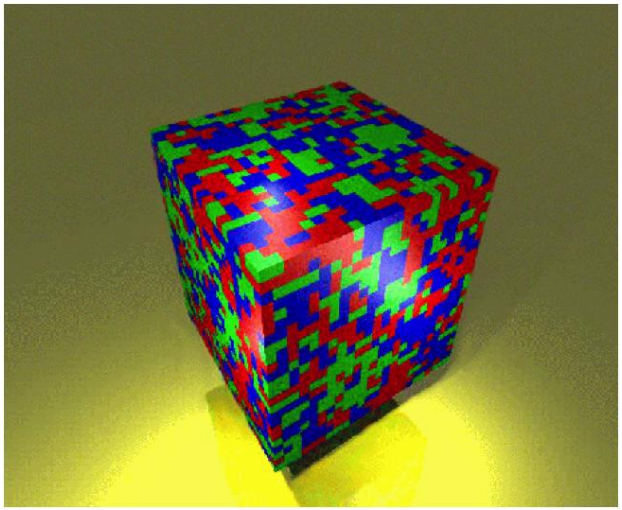

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- Temperature is set as $T = 1/(aN_\tau)$
- Fix N_τ, dial the lattice spacing to cover a temperature range
- The continuum limit is reached as $1/N_\tau \to 0$

Trace anomaly

- The partition function

$$Z = \int DUD\bar{\psi}D\psi \exp\{-S\}, \quad S = S_g + S_f$$

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$$\Theta^{\mu\mu} \equiv \varepsilon - 3p = -\frac{T}{V}\frac{d\ln Z}{d\ln a} \quad \Rightarrow \quad \frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\varepsilon - 3p}{T'^5}$$

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- Requires subtraction of UV divergences (subtract divergent vacuum contribution evaluated at the same values of the gauge coupling):

$$\begin{aligned} \frac{\varepsilon - 3p}{T^4} &= R_\beta[\langle S_G\rangle_0 - \langle S_G\rangle_T] \\ &- R_\beta R_m[2m_l(\langle \bar{l}l\rangle_0 - \langle \bar{l}l\rangle_T) + m_s(\langle \bar{s}s\rangle_0 - \langle \bar{s}s\rangle_T)] \end{aligned}$$

$$R_\beta(\beta) = -a\frac{d\beta}{da}, \quad R_m(\beta) = \frac{1}{m}\frac{dm}{d\beta}, \quad \beta = \frac{10}{g^2}$$

HISQ data sets

- We use the Highly Improved Staggered Quarks[8] action for two degenerate light quarks and physical-mass strange quark and the tree-level Symanzik-improved gauge action
- Previous data set:

$$
\begin{aligned}
m_l &= m_s/20 \\
N_\tau &= 6, 8, 10, 12 \\
\beta &= 5.9, \ldots, 7.825
\end{aligned}
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- New data set:

$$\begin{aligned} m_l &= m_s/5 \\ N_\tau &= 4, 6, 8, 10, 12 \\ \beta &= 8, 8.2, 8.4 \end{aligned}$$

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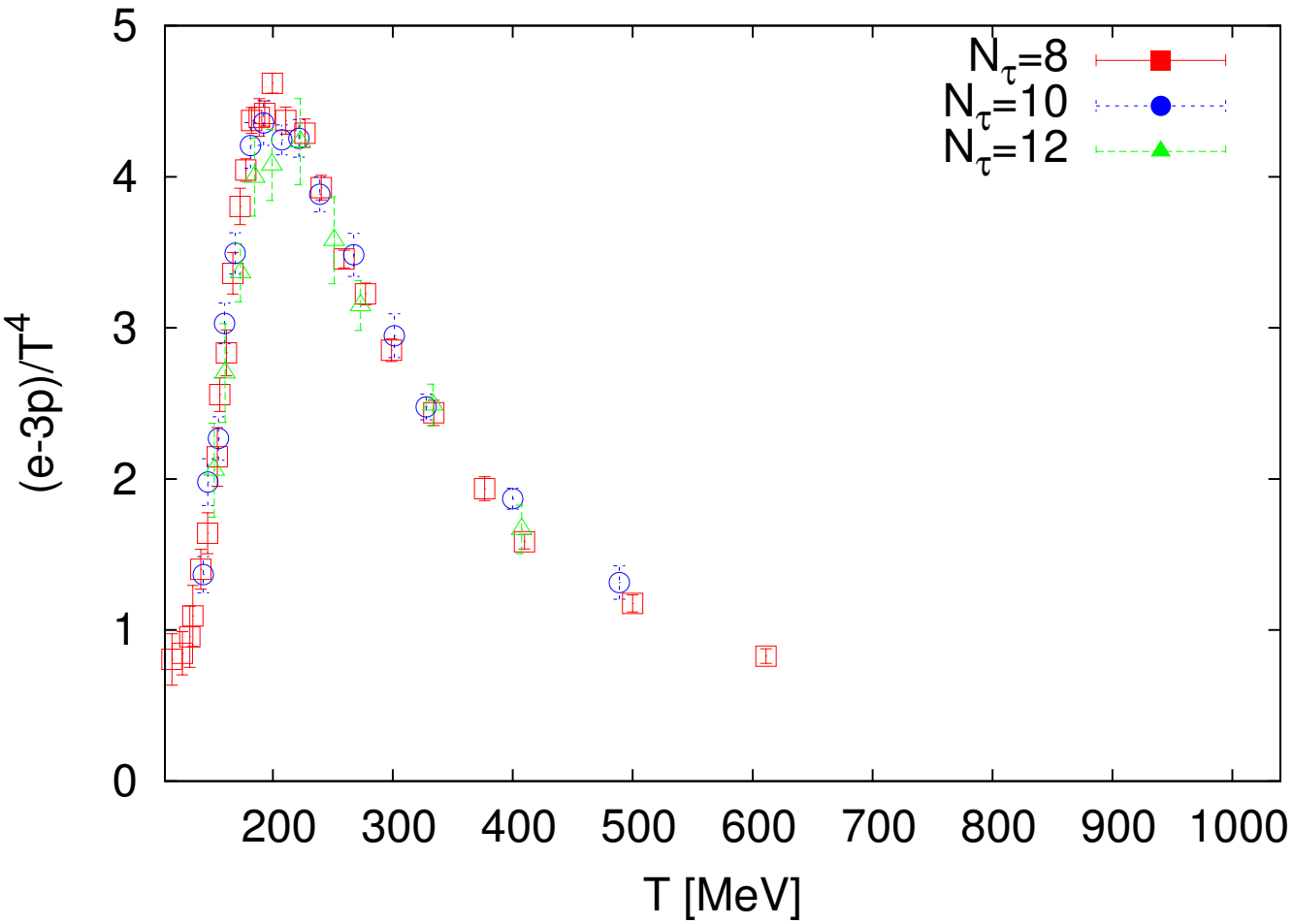
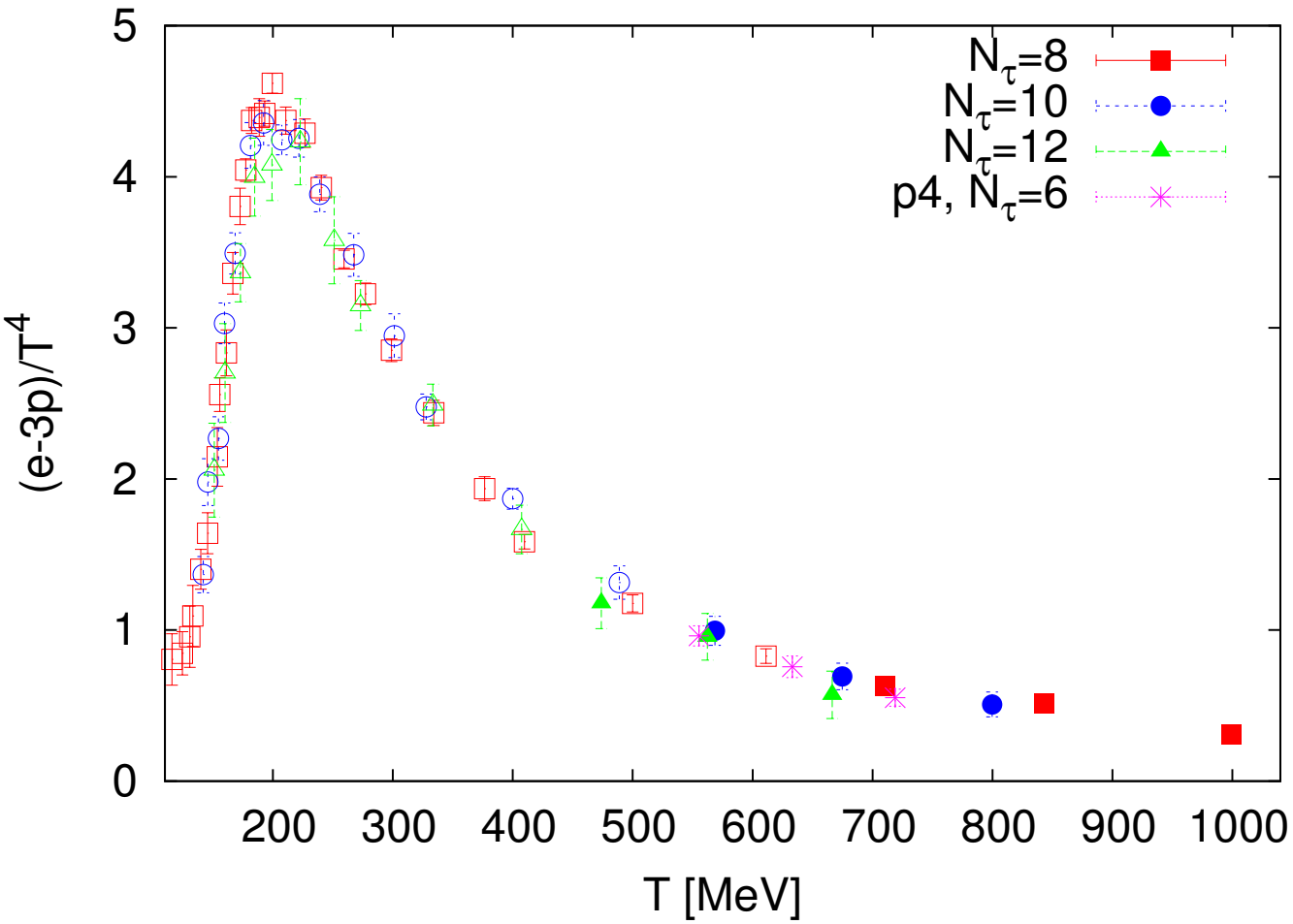
Trace anomaly



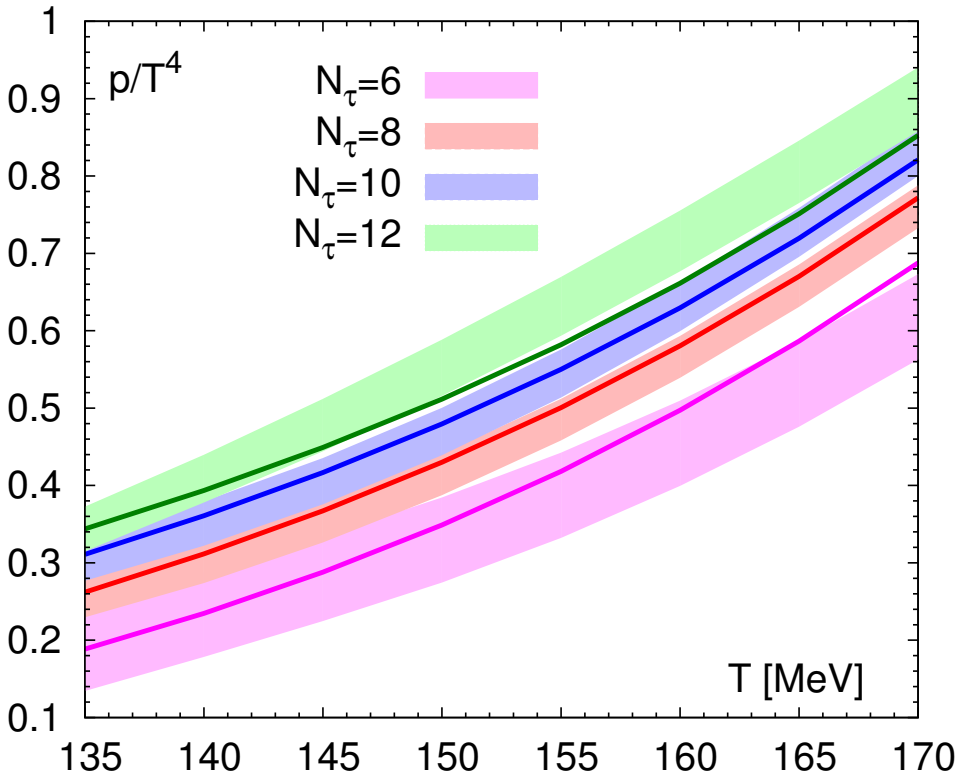
- The trace anomaly with HISQ $m_l = m_s/20$

Trace anomaly



- The trace anomaly with HISQ $m_l = m_s/20$ and $m_l = m_s/5$ at $T > 400 MeV$

Pressure at low temperature



- We have improved the low-temperature region by adding $T = 123$ MeV at $N_\tau = 10$ and $T = 133, 140$ MeV at $N_\tau = 12$

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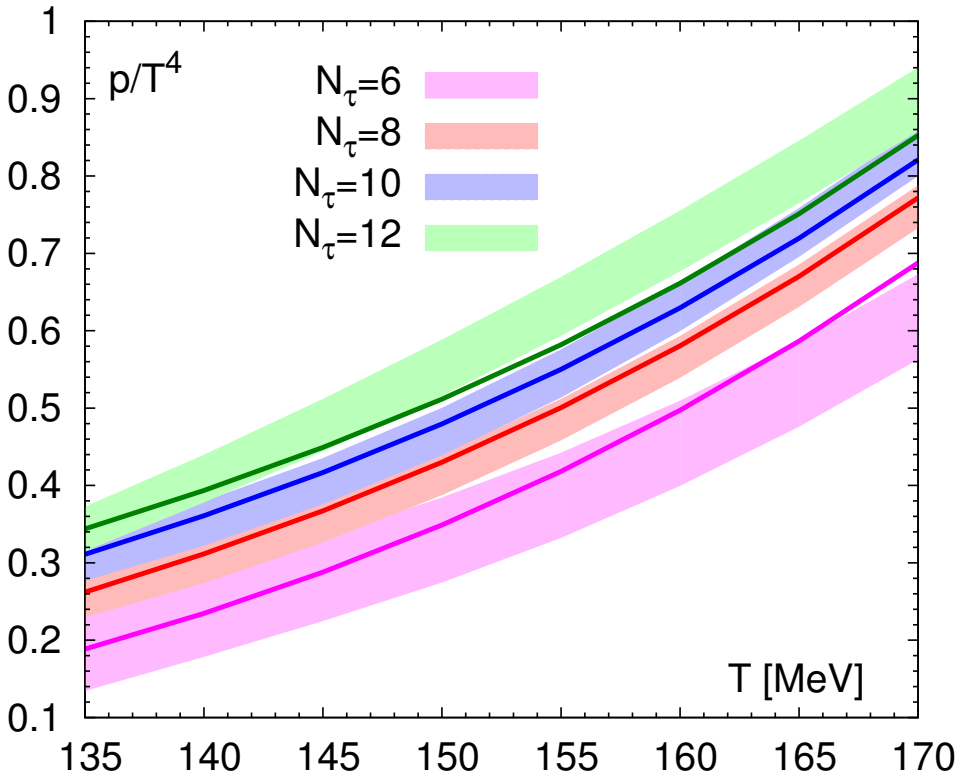

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- Bands are interpolations of the lattice data and the lines are Hadron Resonance Gas model results with the cutoff dependent spectrum

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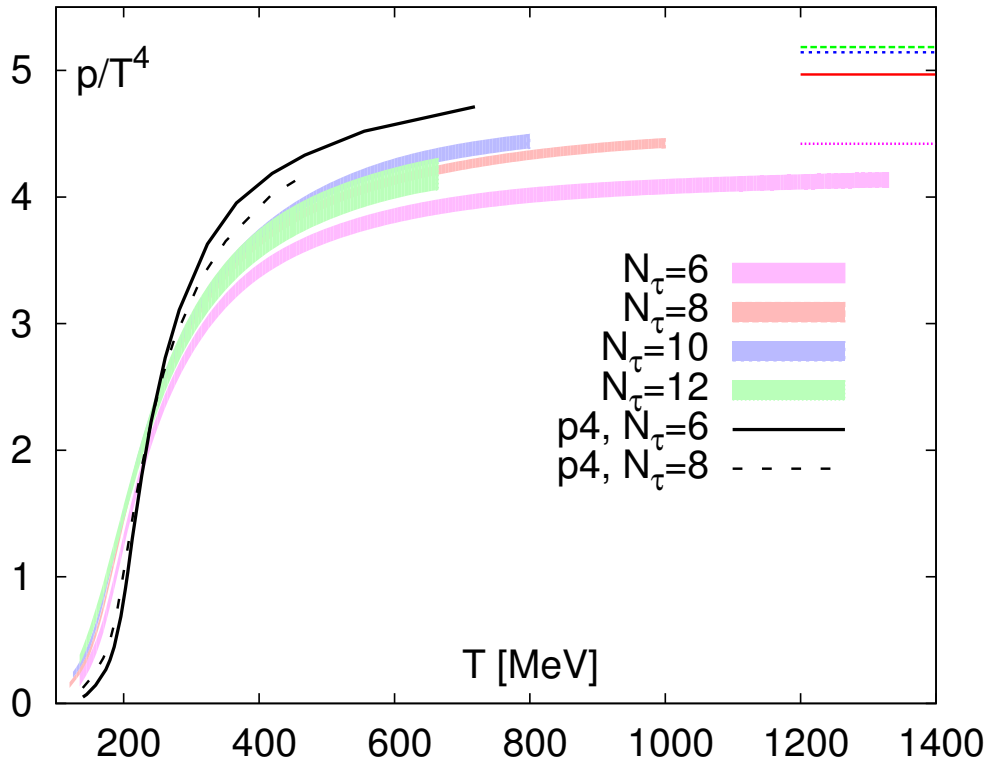

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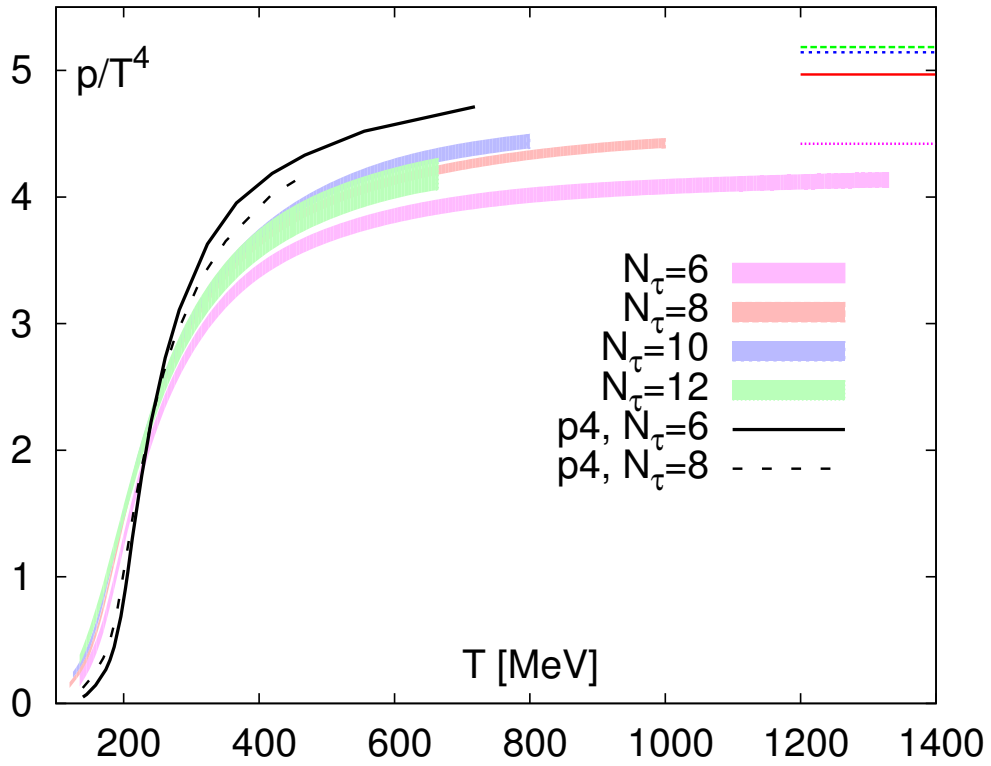

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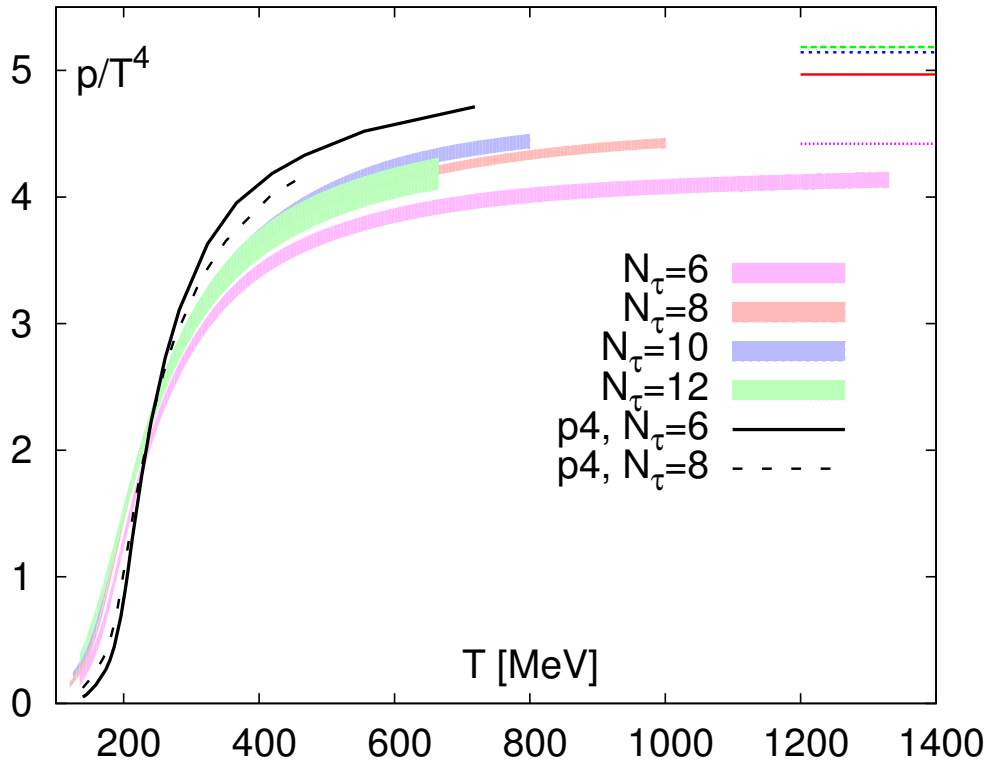

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- The cutoff dependence is similar to the one in free theory
- $N_\tau = 12$ has large statistical uncertainty

Cutoff dependence of the pressure

- The cutoff dependence of the pressure with HISQ and p4 is very similar to the cutoff dependence of quark number susceptibilities:

$$\chi_{2n}^{q} = \frac{\partial^{2n} p(T, \mu_q)}{\partial \mu_q^{2n}}, \; n = 1, 2, \; q = l, s$$

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- At high temperatures where the weak-coupling picture is expected to hold we can write pressure as the sum of the quark and gluon pressures: $p(T) = p^q(T) + p^g(T)$
- The gluonic pressure is known to have negligible cutoff dependence for improved actions, thus we assume

$$\begin{aligned} p(T) &= p(T, N_\tau) + corr(T, N_\tau) \\ corr(T, N_\tau) &= p^q(T)\left(1 - \frac{p^q(T, N_\tau)}{p^q(T)}\right) \end{aligned}$$

Cutoff dependence of the pressure

- We approximate the cutoff dependence of the quark pressure by the one of the second order susceptibilities χ_2^l[9]

$$\frac{p^q(T, N_\tau)}{p^q(T)} \simeq \frac{\chi_2^l(T, N_\tau)}{\chi_2^l(T)}$$

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- The overall estimate

$$corr(T, N_\tau) \simeq p^{q,id-15\%}(T)\left(1 - \frac{\chi_2^l(T, N_\tau)}{\chi_2^l(T)}\right)$$

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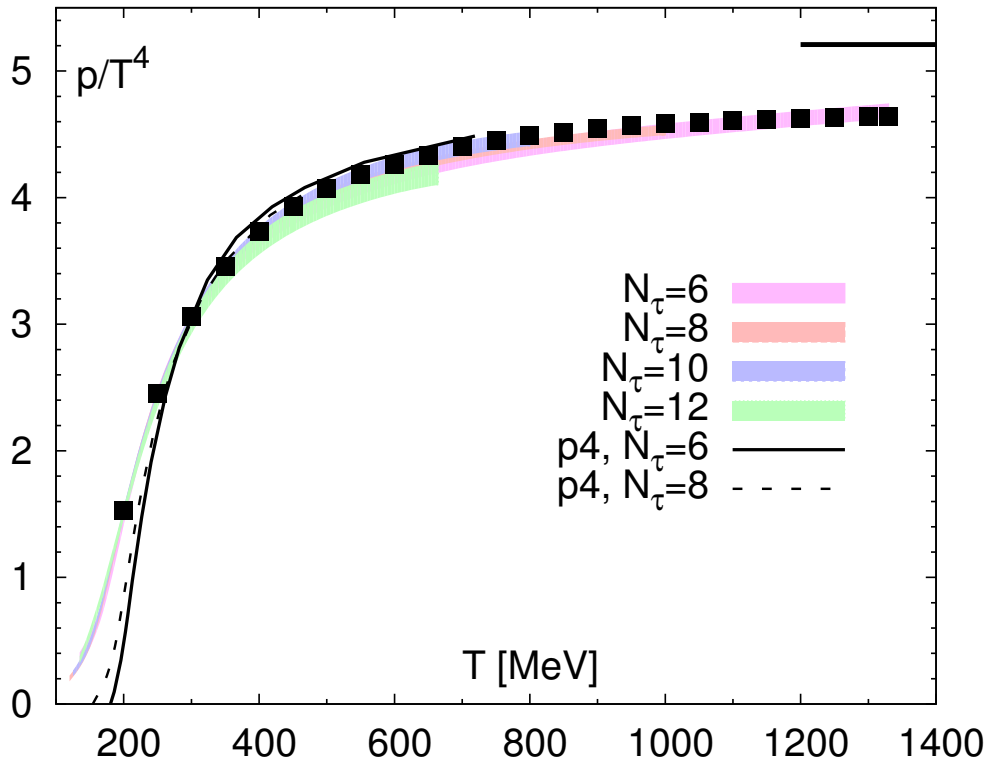
Pressure: correction and continuum



- Corrected pressure at fixed cutoff and the continuum limit/estimate (black boxes)

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- In 200 MeV $< T <$ 660 MeV we have four lattice spacings to perform continuum extrapolations, in 660 MeV $< T <$ 800 MeV – three, and for $T > 800$ MeV we can only provide a continuum estimate
- The continuum result for the pressure agrees with the corrected results: an important cross-check for continuum extrapolations for $T < 1330$ MeV

Trace anomaly above 800 MeV

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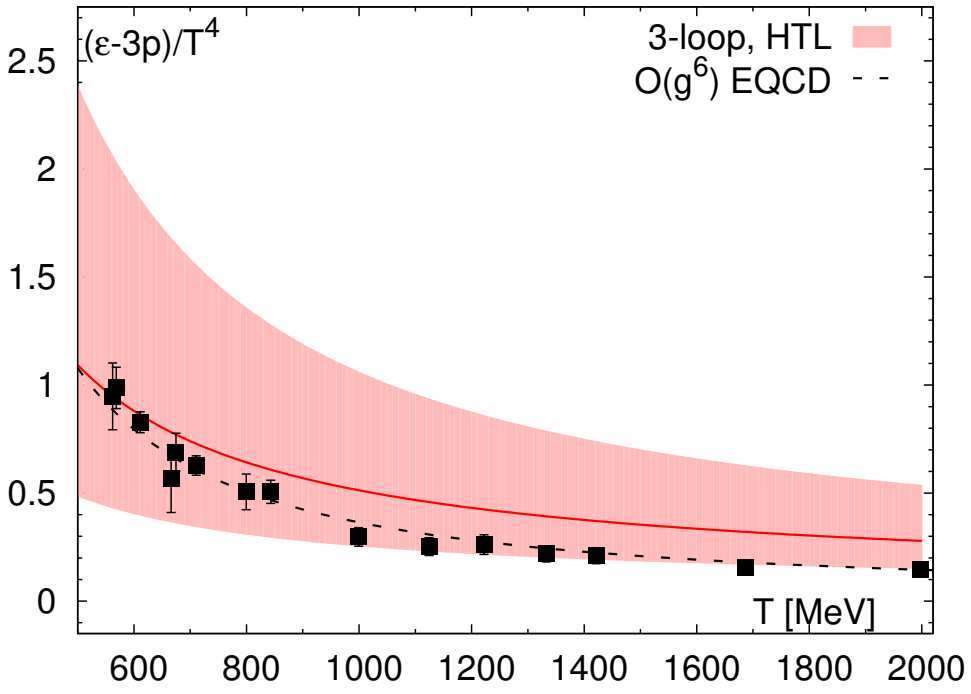
- The $N_\tau = 4$ and 6 results for the trace anomaly lie below this continuum estimate
- We rescale the $N_\tau = 4$ and 6 results on the trace anomaly by factors 1.2 and 1.4, respectively, to bring them in agreement with the continuum estimate for 800 MeV $< T <$ 1000 MeV

Trace anomaly above 800 MeV



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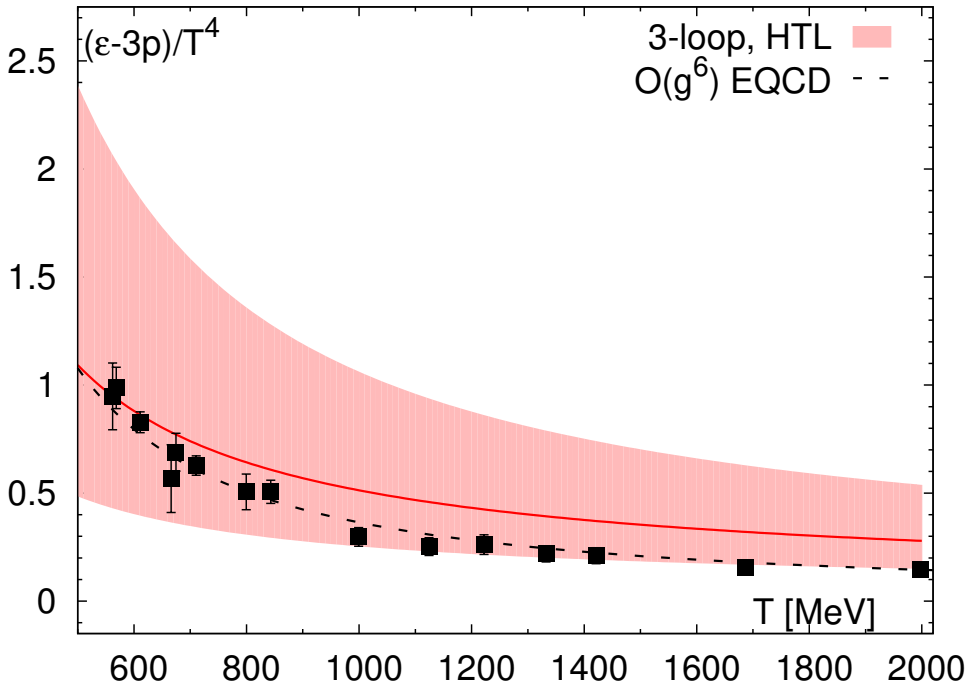

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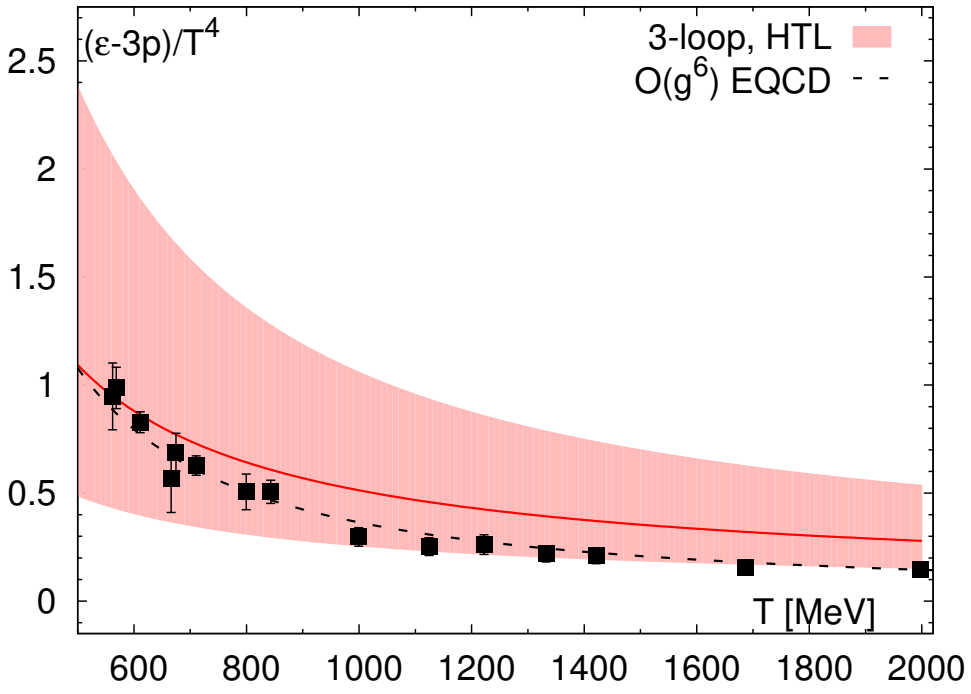
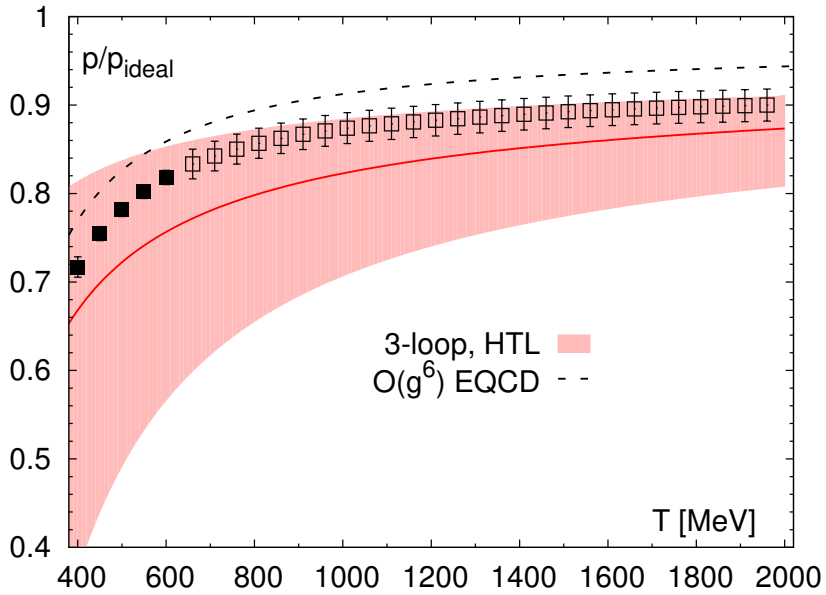
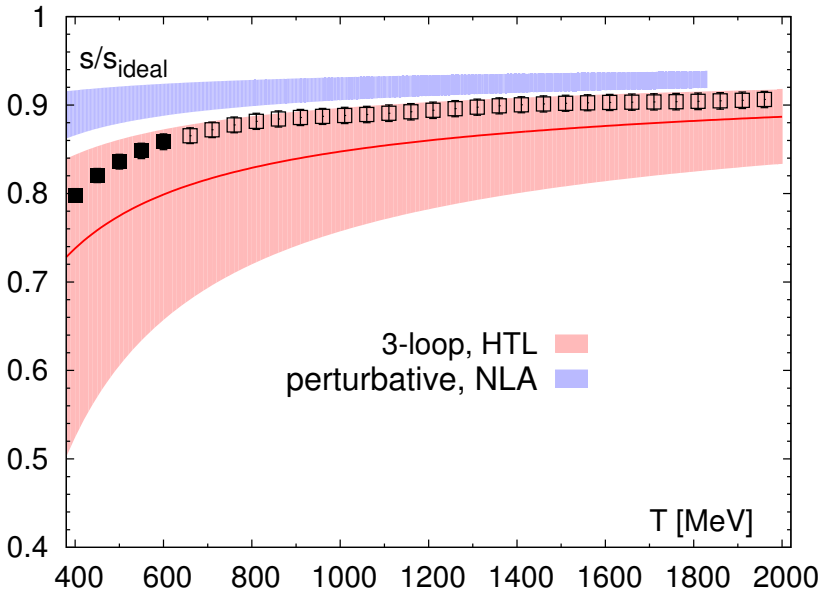


- Perform a spline interpolation of the combined $N_\tau = 12,\ 10,\ 8,\ 6$ and 4 data in the temperature interval 400 MeV $< T <$ 2000 MeV
- Integrate the trace anomaly from $T = 660$ MeV to 2000 MeV to get the pressure and the entropy density

Weak-coupling expansions





- Left: Comparison of the pressure obtained on the lattice with the HTL[10] and EQCD[11] results
- Right: Comparison of the entropy density obtained on the lattice with the HTL and NLA[12] results

[10] Haque et al. (2014)
[11] Laine and Schroder (2006)
[12] Rebhan (2003)

Conclusion

- Previous results by the HotQCD collaboration for the 2+1 QCD equation of state at zero baryon chemical potential have been extended to higher temperatures
- At temperatures above 400 MeV we use ensembles with $m_l = m_s/5$, the quark mass effects below the statistical uncertainties
- Up to 660 MeV we perform the continuum limit with four lattice cutoffs
- At high temperatures the cutoff effects in the pressure are similar to the ones in quark number susceptibilities
- We use two different methods to estimate the continuum pressure up to 1330 MeV
- In the interval 660 to 2000 MeV we provide a continuum estimate based on the rescaled $N_\tau = 4$ and 6 results
- Reasonable agreement between the weak-coupling results and the lattice at high temperature