

Equation of state at finite density from the lattice

Szabolcs Borsanyi

Wuppertal-Budapest collaboration.

Z. Fodor, J. Günther, S. D. Katz, A. Pasztor, I. Portelli, C. Ratti, K.
K. Szabo

Bergische Universität Wuppertal

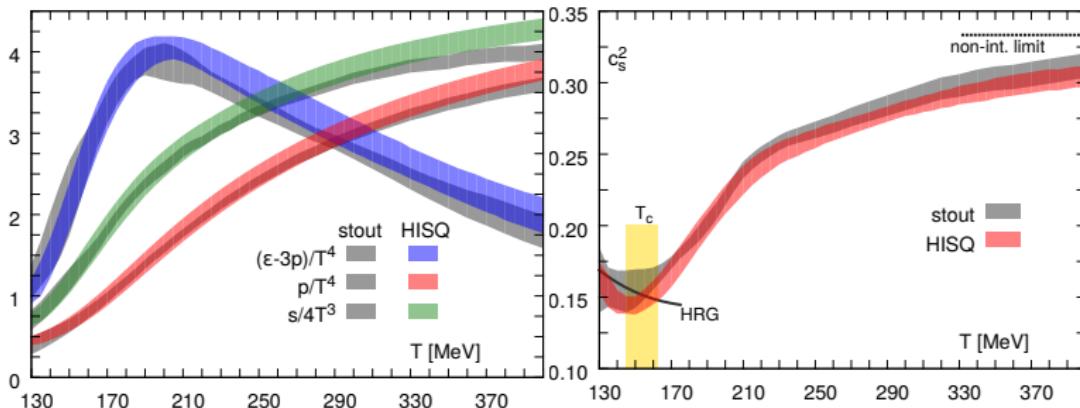
Quark Matter 2018, Venice



Equation of state with up,down and strange quarks

stout result: Wuppertal-Budapest group [1309.5258]

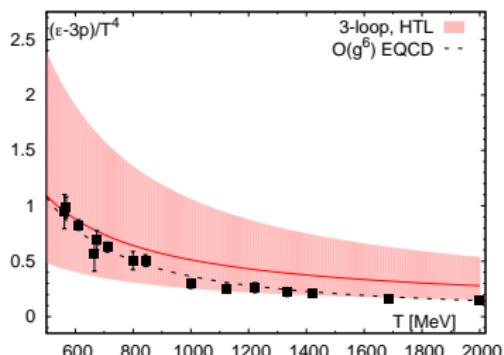
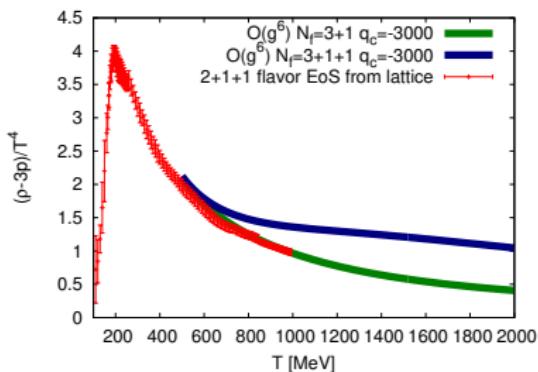
HISQ result: Bielefeld-Brookhaven group [1407.6397]



Around the transition temperature $(\epsilon - 3p)/T^4$ has a steepest point, the speed-of-sound has a minimum

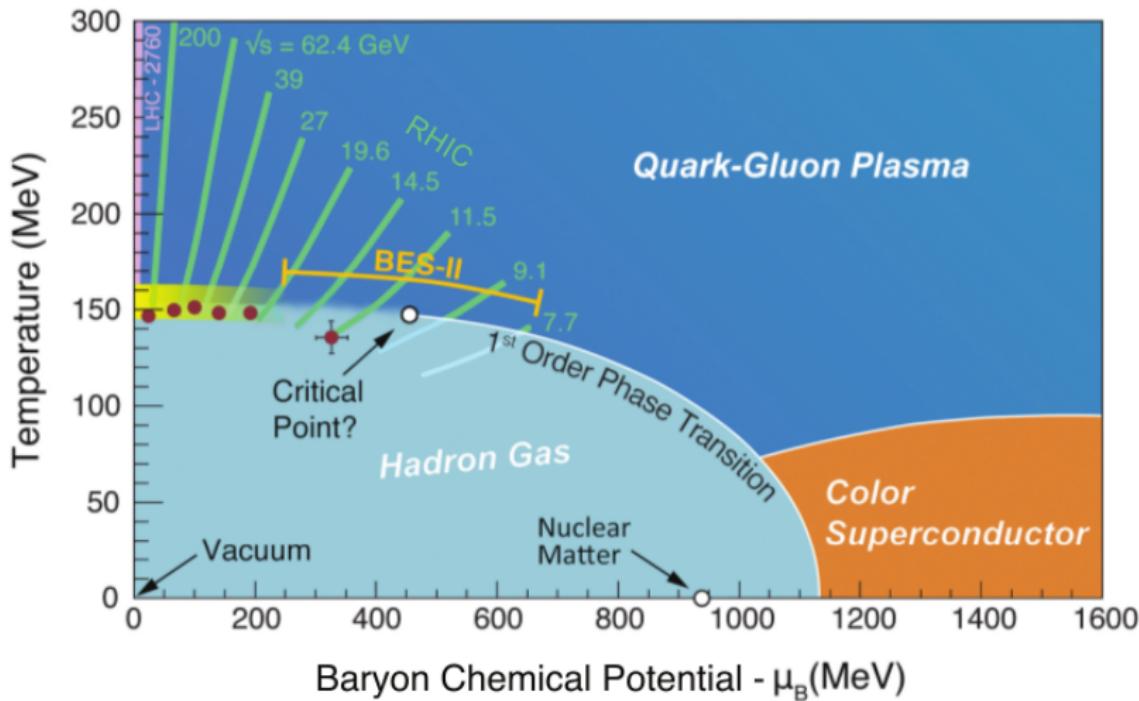
Equation of state at high temperatures

2+1+1 and 2+1+1+1 flavors: Wuppertal-Budapest group [\[1606.07494\]](#)
2+1 HISQ result: Petreczky et al [\[1710.05024\]](#)



Here plotted: $(\epsilon - 3p)/T^4$ at high temperatures

Beam energy scan program



BES-II: chemical potentials of interest: $\mu_B/T = 1.5 \dots 4$

The equation of state in a grand canonical ensemble

At $\mu = 0$ lattice gives the pressure as an integral of the trace anomaly:

$$\frac{p}{T^4} = \int_0^T \frac{\epsilon(T^*) - 3p(T^*)}{T^{*5}} dT^*$$

At finite μ_B/T one writes the pressure in a Taylor series:

$$\frac{p}{T^4}(\mu_B) = \frac{p}{T^4}(0) + \frac{1}{2} \frac{\mu_B^2}{T^2} \chi_2^B(T) + \frac{1}{24} \frac{\mu_B^4}{T^4} \chi_4^B(T) + \frac{1}{720} \frac{\mu_B^6}{T^6} \chi_6^B(T) + \dots$$

or

$$\frac{p}{T^4} = \frac{p}{T^4}(0) + \int_0^{\mu_B/T} \frac{n_B(\hat{\mu})}{T^3} d\hat{\mu}$$

$$\frac{n_B}{T^3}(\mu_B) = +\frac{\mu_B}{T} \chi_2^B + \frac{\mu_B^3}{T^3} \frac{1}{6} \chi_4^B + \frac{\mu_B^5}{T^5} \frac{1}{120} \chi_6^B + \frac{\mu_B^7}{T^7} \frac{1}{5040} \chi_8^B + \dots$$

The trace anomaly, energy, entropy are obtained as

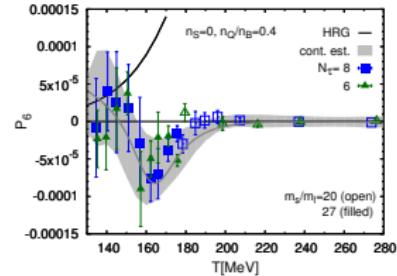
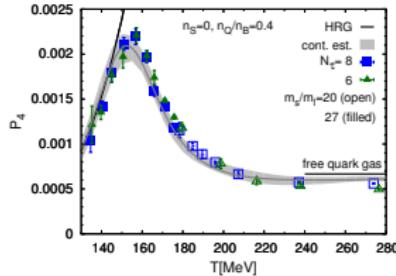
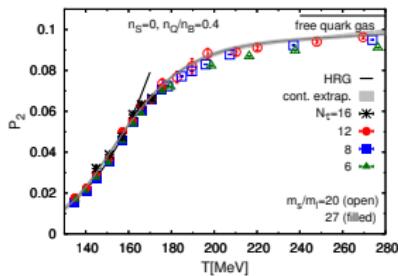
$$\frac{\epsilon - 3p}{T^4} = T \left. \frac{\partial}{\partial T} \frac{p}{T^4} \right|_{\mu/T=\text{const}} \quad sT = \epsilon + 3p - \mu_B n_B$$

Pressure coefficients as of 2017

expansion coefficients assuming strangeness neutrality

Taylor method:

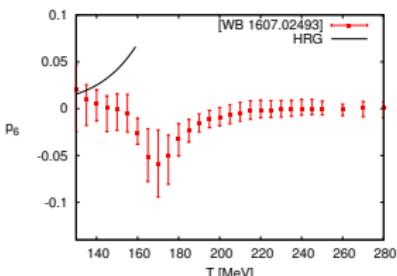
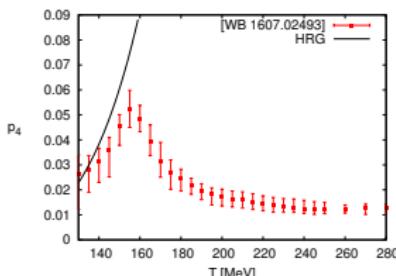
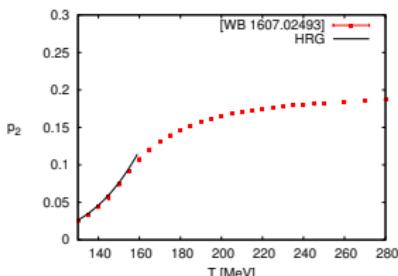
mostly $N_t = 8$, $\mathcal{O}(10^5)$ configurations point, [BNL-Bielefeld-CCNU: 1701.04325]



Analytical method:

continuum, $\mathcal{O}(10^4)$ configurations/point, errors include systematics

[Wuppertal-Budapest 1607.02493]



Strangeness neutrality

There are several options for an extrapolation to finite density:

- Keep $\mu_S = 0$ (each quark with the same chemical potential):

Convenient for theory

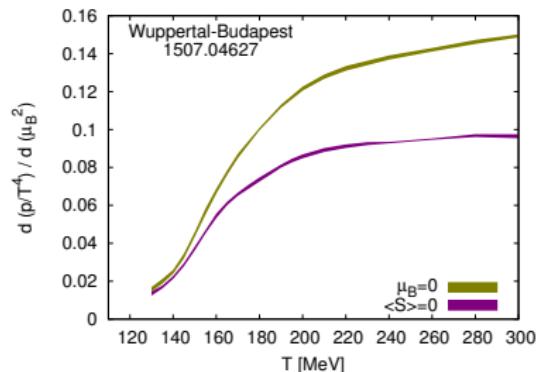
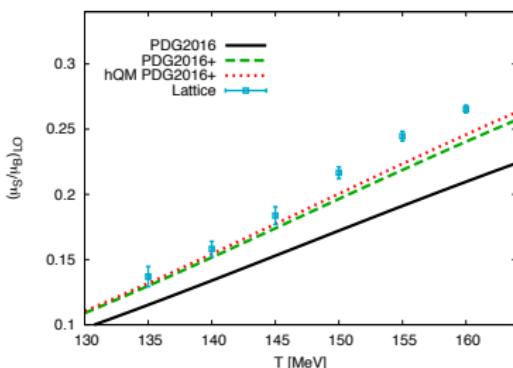
$$\langle S \rangle = \chi_{11}^{BS} \mu_B + \frac{1}{6} \chi_{31}^{BS} \mu_B^3 + \dots$$

- Strangeness neutrality: $\langle S \rangle = 0$; μ_S is a function of T and μ_B .

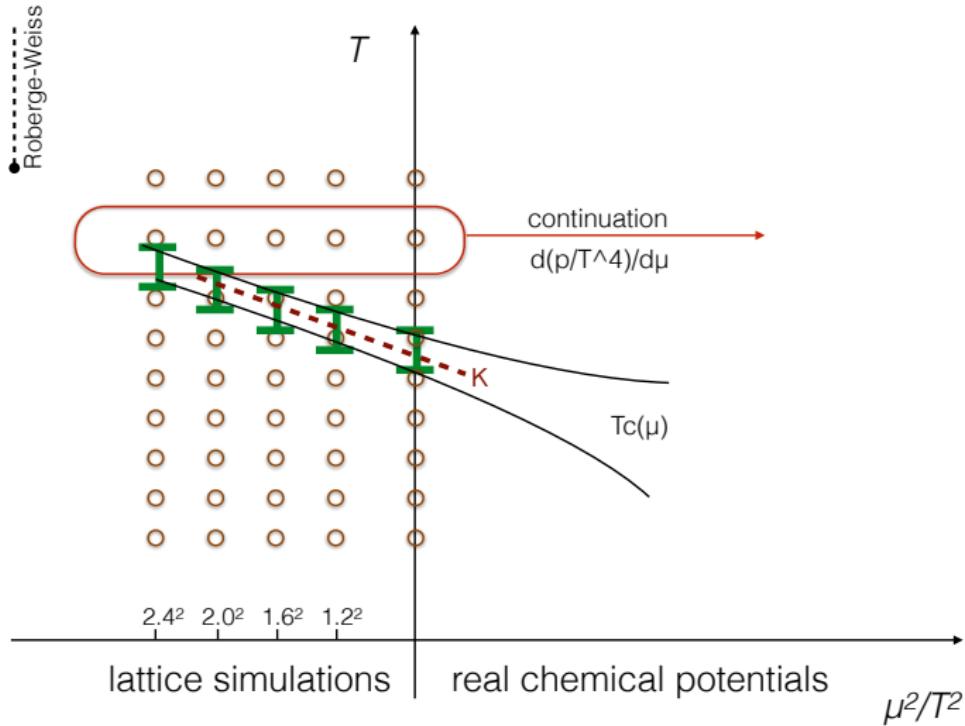
Natural for experiment

- Strangeness neutrality with isospin asymmetry:

$$\langle S \rangle = 0 \quad \langle Q \rangle / \langle B \rangle = Z^{Au} / A^{Au} \approx 0.4$$



Analytic continuation



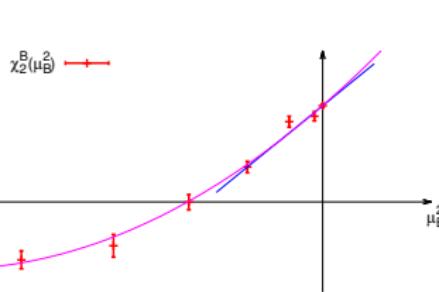
Many exploratory studies: [de Forcrand & Philipsen hep-lat/0205016]

[Philipsen 0708.1293] [Philipsen 1402.0838] [Cea et al hep-lat/0612018,0905.1292,1202.5700]

Higher order χ_B in the μ_B approach

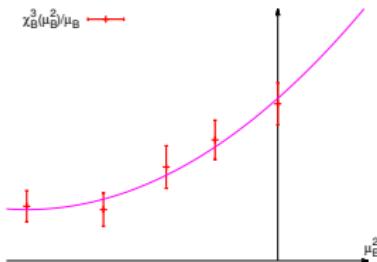
$\chi_2^B(\mu_B^2)$ is determined at several non-zero chemical potentials.

[see also D'Elia et al 1611.08285]



$$\begin{aligned}\chi_2^B(\mu_B^2) &\approx \chi_2^B(0) + \frac{1}{2}\mu_B^2\chi_4^B(0) \\ &+ \frac{1}{24}\mu_B^4\chi_6^B(0) + \dots\end{aligned}$$

If $\chi_3^B(\mu_B^2)$, and even higher derivatives are determined at each non-zero chemical potentials, we have more direct access to χ_6^B .



$$\begin{aligned}\frac{\chi_3^B(\mu_B^2)}{\mu_B} &\approx \chi_4^B(0) + \frac{1}{6}\mu_B^2\chi_6^B(0) \\ &+ \frac{1}{120}\mu_B^4\chi_8^B(0)\end{aligned}$$

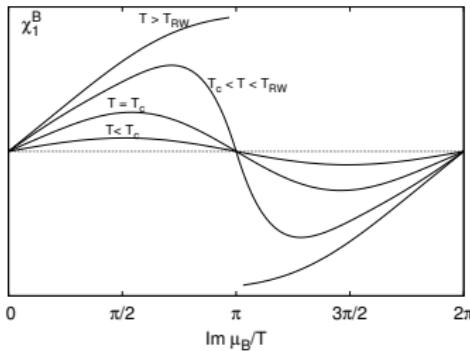
Our strategy: *Use all observables, in a combined, correlated fit.*

Correlated fit of the χ_B coefficients

- For each temperature there are *six* separate simulations:

$$\mu_B = \sqrt{-1} \frac{\pi}{8} j, \quad j = 0, 3, 4, 5, 6, (7) \quad (1)$$

$\chi_{1,2,3,4}^B$ is taken from these six simulations.

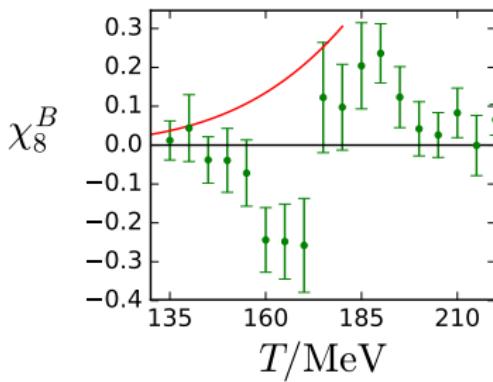
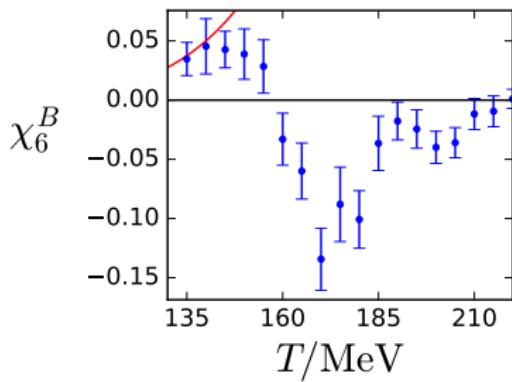
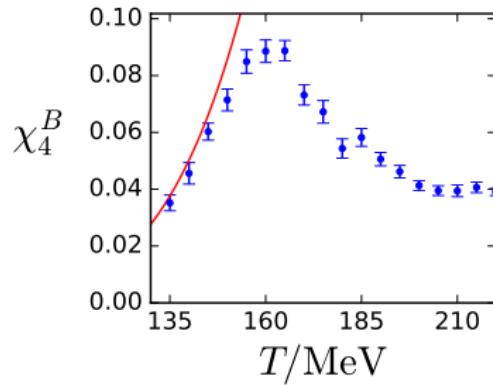
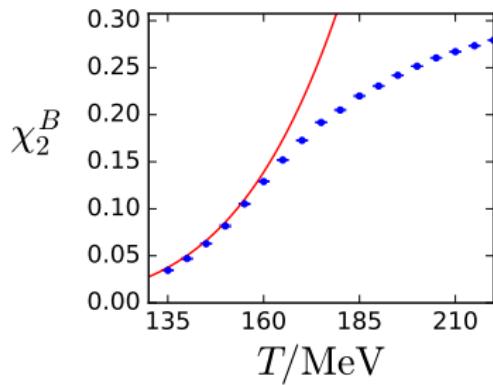


- Independent fit for each separate temperature
- Correlation is kept between coefficients from the same simulation
- A prior distribution is introduced for χ_8^B and χ_{10}^B . E.g.:

$$\chi_8^B(T) = \xi(T)\chi_4^B(T), \quad \xi(T) \approx -1.25 \pm 2.75 \text{ (Gaussian)} \quad (2)$$

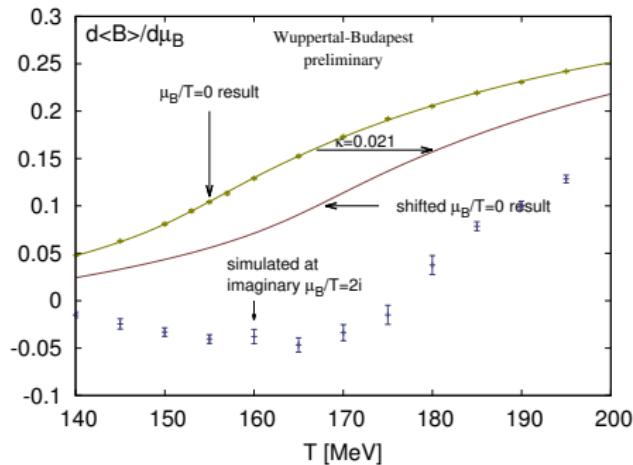
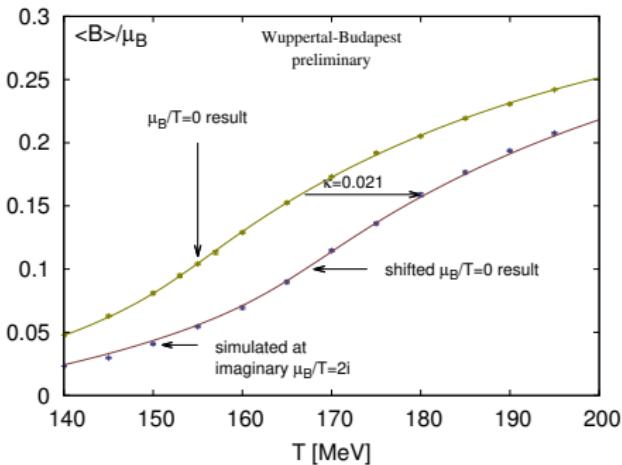
$$\text{HRG : } \xi = 1, \quad \text{high } T : \xi = 0 \quad (3)$$

Results for χ_n^B



A closer look to imaginary μ_B simulation results

The χ_1^B/μ_B quantity at imaginary chemical potential is just the shifted version of the $\mu_B = 0$ response function:

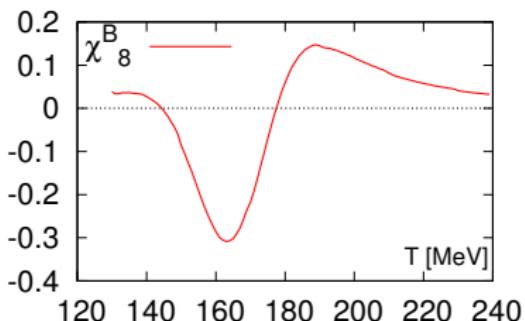
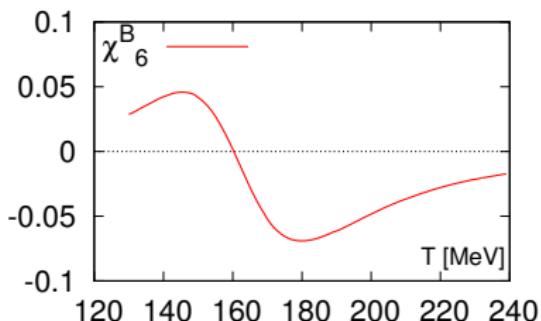
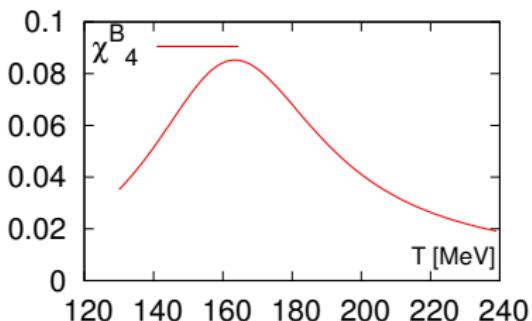
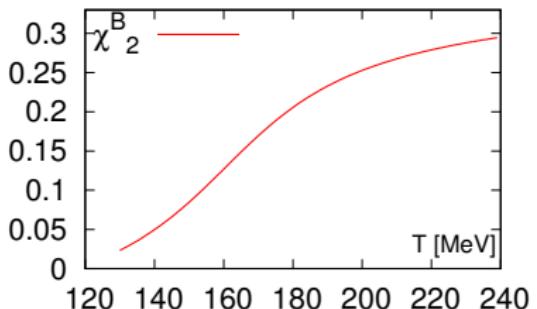


The same is true for many other quantities, but not for $\chi_2^B(T)$.

What would we get if we ignored the lattice simulation, and simply took the shifted version of the $\mu = 0$ result, instead?

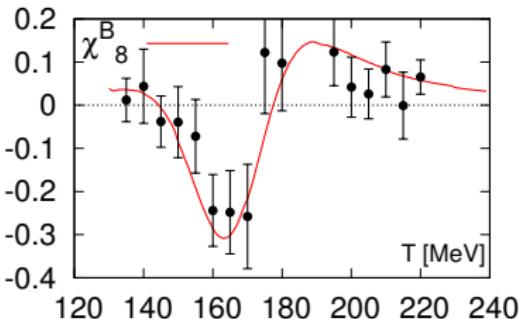
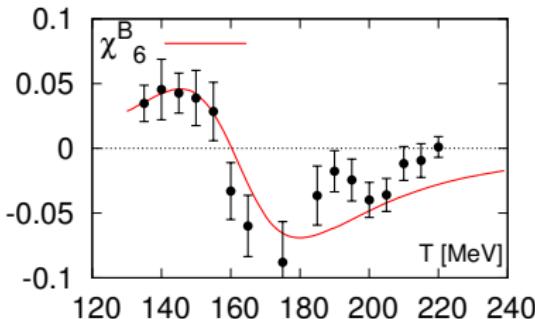
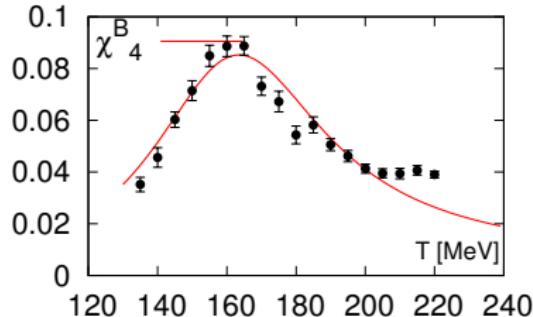
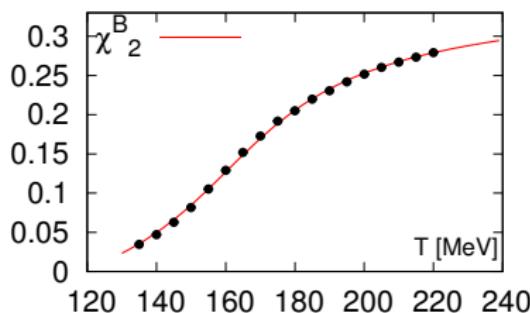
Higher order χ_B in the simple model

Input: $\chi_2^B(T, \mu = 0)$ from Wuppertal-Budapest, and $\kappa = 0.02$ [see 1508.07599]:



Higher order χ_B in the simple model

Comparing the simple model with our lattice result:

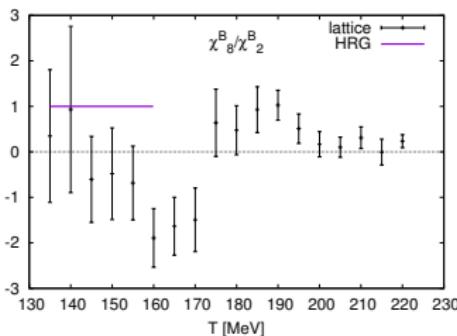
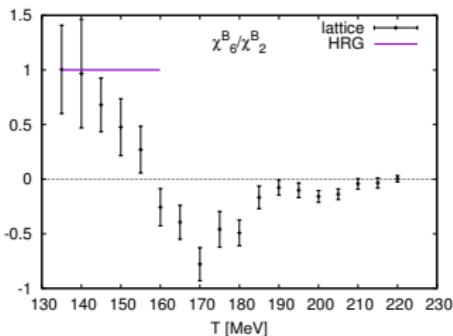
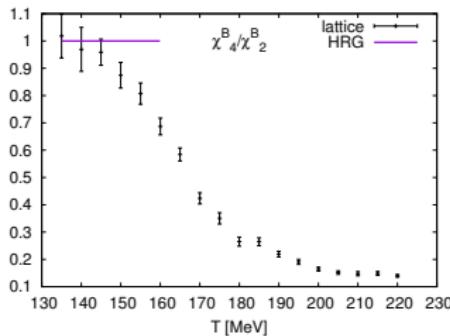
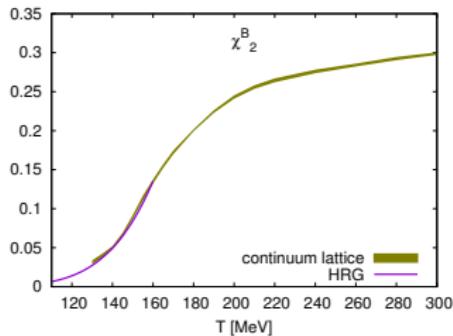


Simple model describes lattice result surprisingly well.

Result is consistent with the no-critical-end-point scenario.

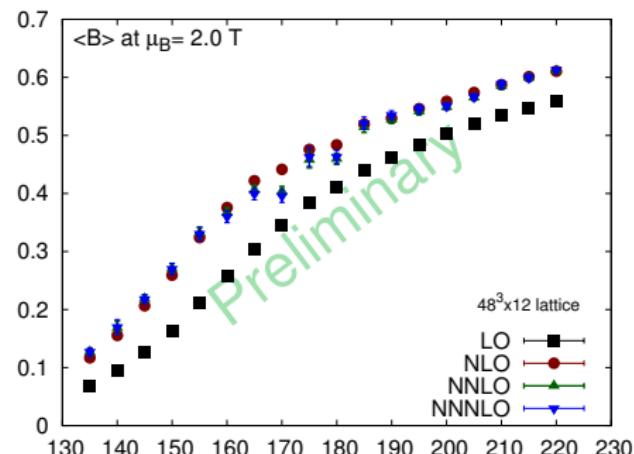
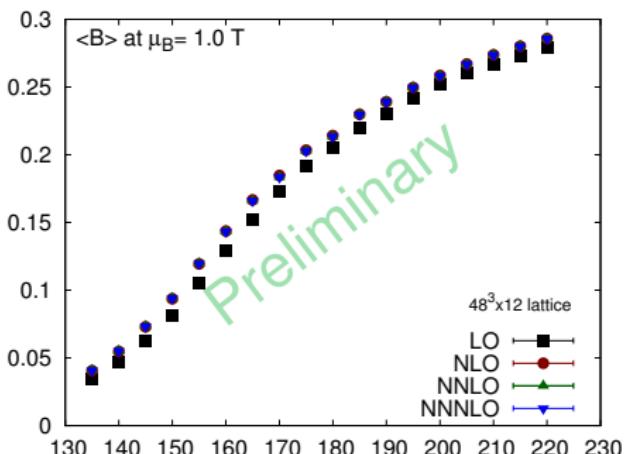
Cumulant ratios

$$\langle B \rangle = \chi_2^B \mu_B \left[1 + \frac{1}{6} \frac{\chi_4^B}{\chi_2^B} \mu_B^2 + \frac{1}{120} \frac{\chi_6^B}{\chi_2^B} \mu_B^4 + \frac{1}{5040} \frac{\chi_8^B}{\chi_2^B} \mu_B^6 + \dots \right]$$



Taylor approach to the equation of state

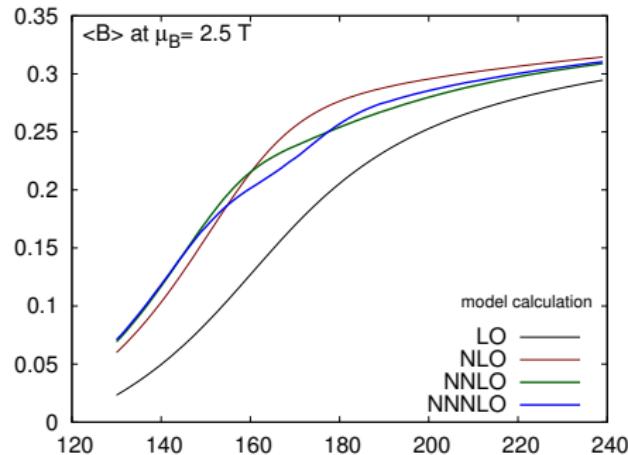
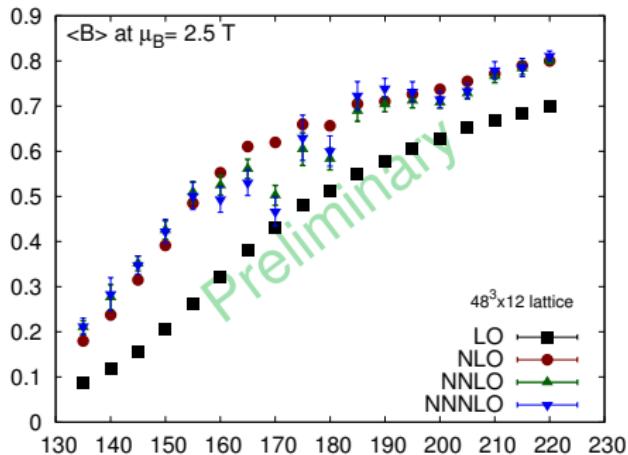
$$\langle B \rangle = \chi_2^B \mu_B + \frac{1}{6} \chi_4^B \mu_B^3 + \frac{1}{120} \chi_6^B \mu_B^5 + \frac{1}{5040} \chi_8^B \mu_B^7 + \dots$$



Between $\mu_B/T = 1\dots 2$ NLO is becoming increasingly important.

Taylor approach to the equation of state

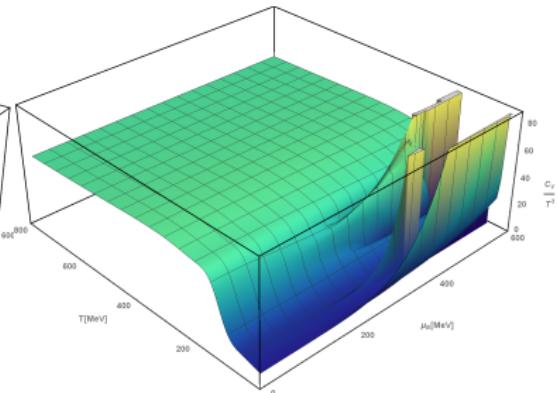
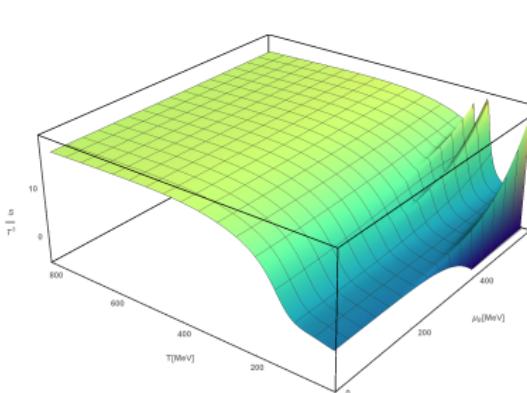
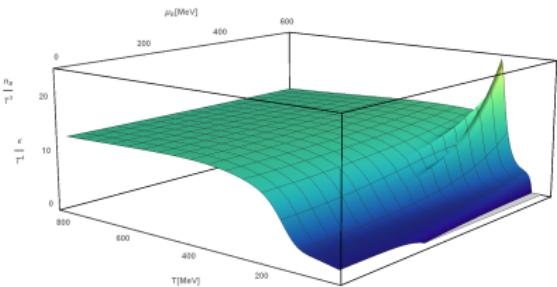
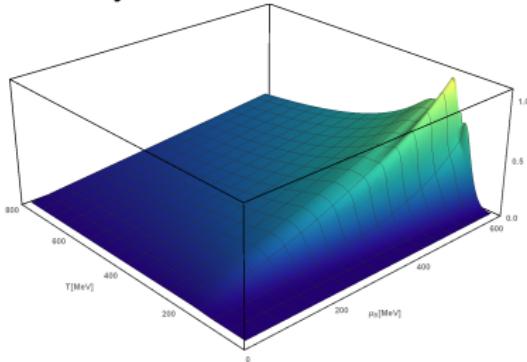
$$\langle B \rangle = \chi_2^B \mu_B + \frac{1}{6} \chi_4^B \mu_B^3 + \frac{1}{120} \chi_6^B \mu_B^5 + \frac{1}{5040} \chi_8^B \mu_B^7 + \dots$$



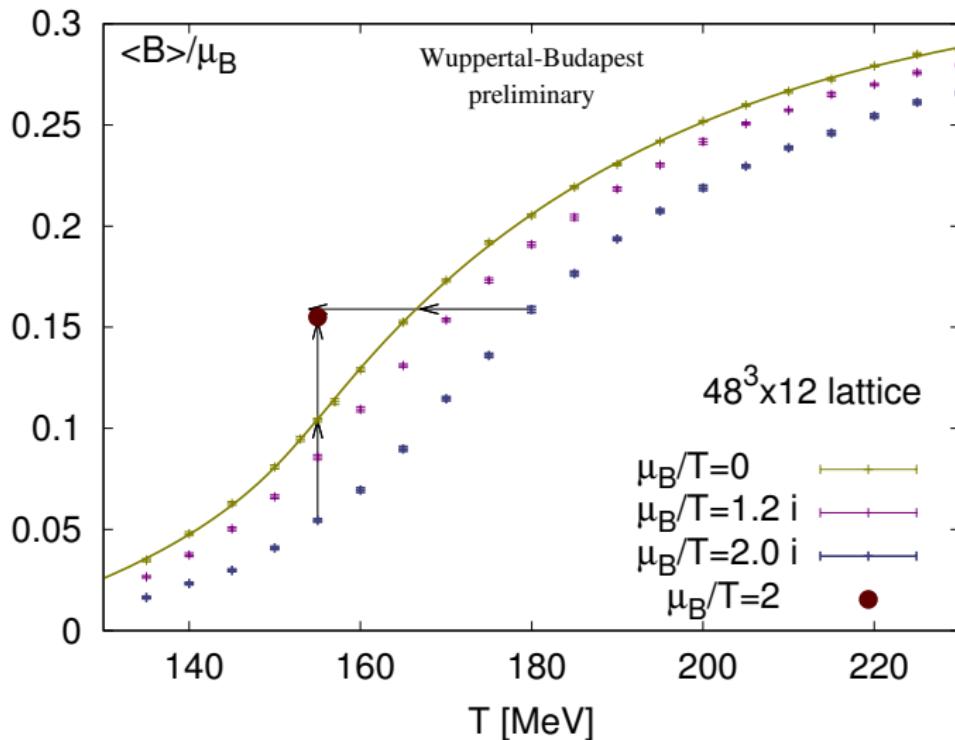
A wiggle is seen near 170–180 MeV at higher chemical potentials, which is present in the simplified model, too.
This translates to thermodynamical instabilities in the equation of state.

The problem also appears in...

other thermodynamic variables, calculated by P. Parotto:



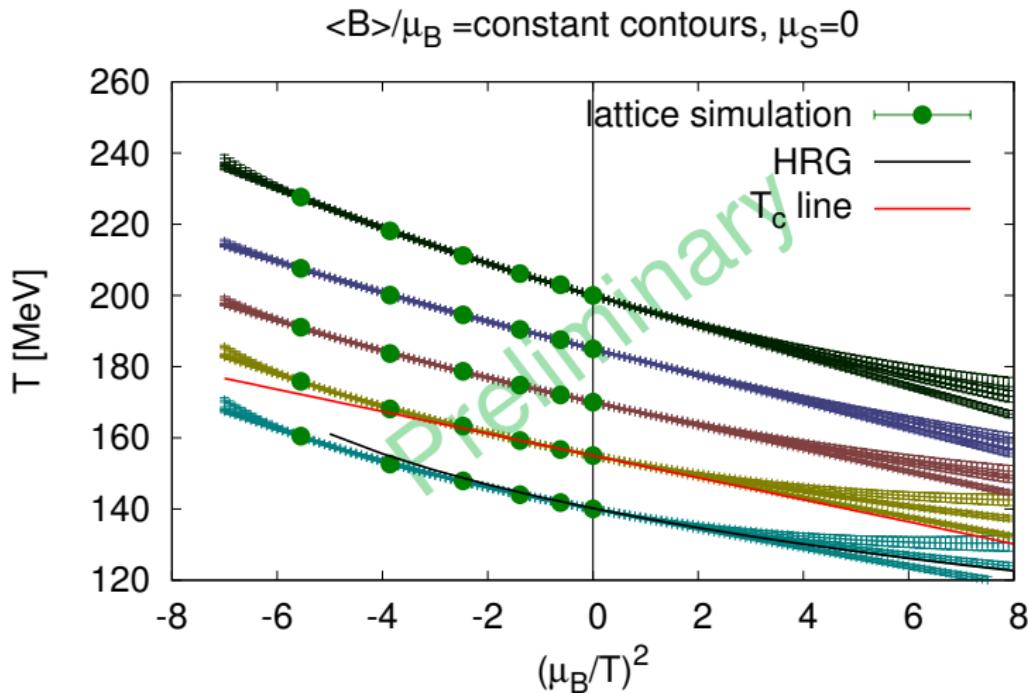
A closer look to imaginary μ_B simulation results



Vertical arrows: Fixed T extrapolation (analogous to Taylor series)

Horizontal arrows: Fixed physics extrapolation

Lines of constant physics

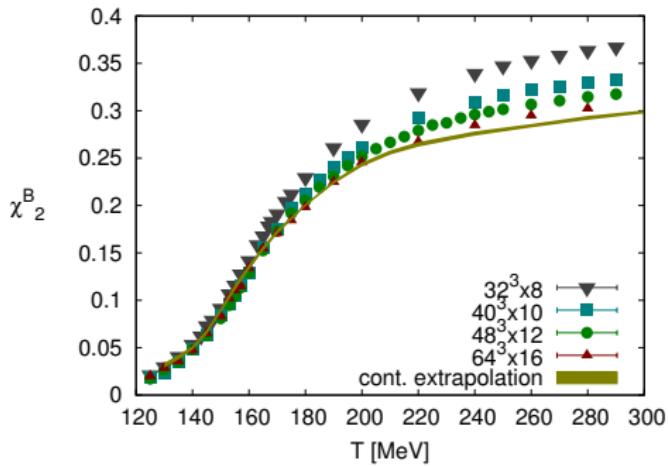
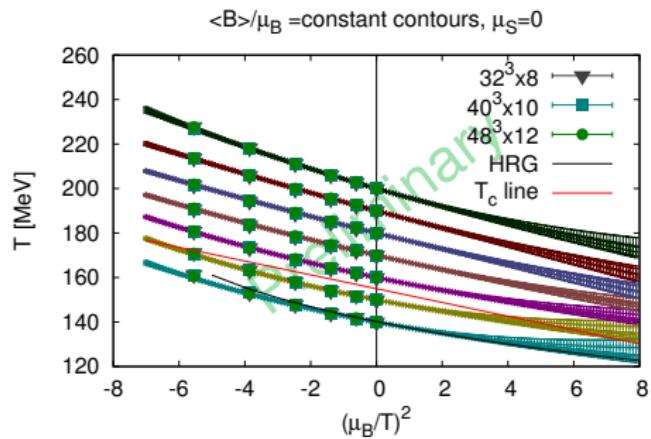


Lines are parallel to cca 5% accuracy, with a very slight preference for a strengthening at $\mu_B > 0$.

Systematic errors are important

Here: quadratic, 1/1 Padé, rational extrapolation from $\mu_B^2 \leq 0$.

Lines of constant physics



In principle, all χ_n^B coefficients follow from:

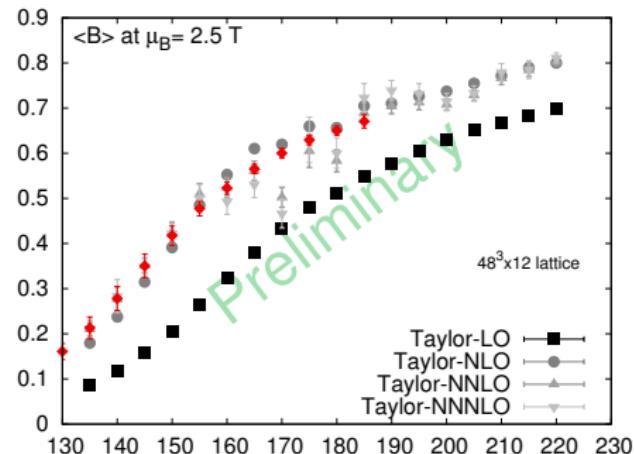
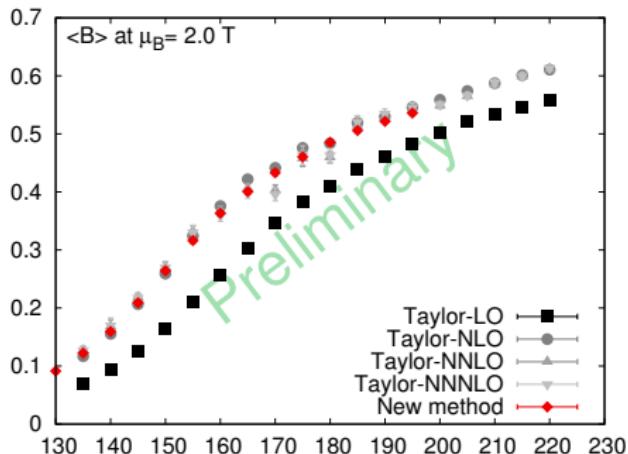
- the contours of fixed χ_1^B / μ_B in the phase diagram
- The $\chi_2^B(T)$ susceptibility at $\mu_B = 0$

The discretization effects are far smaller for the contours than for the $\mu_B = 0$ physics.

In this talk I use $48^3 \times 12$ lattices for the contours with continuum extrapolated $\chi_2^B(T)$.

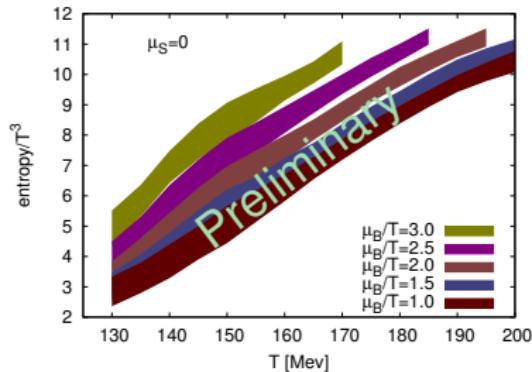
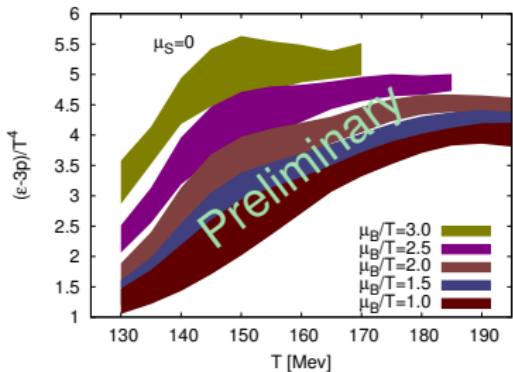
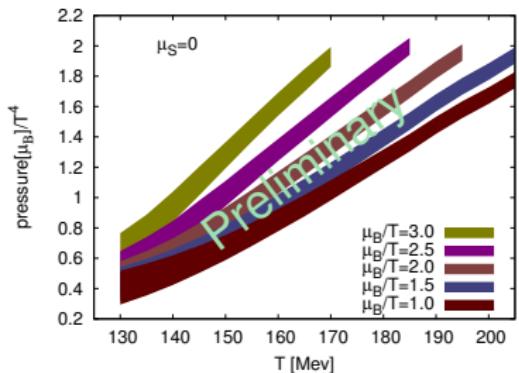
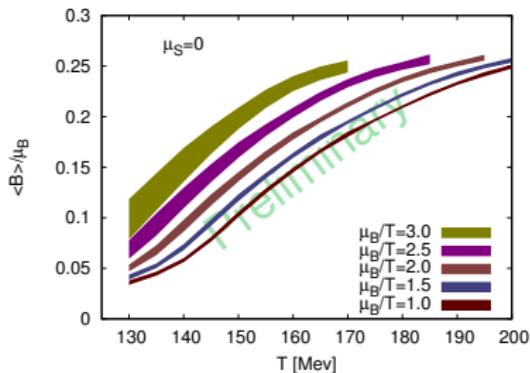
New method vs taylor approach to the equation of state

$$\langle B \rangle = \chi_2^B \mu_B + \frac{1}{6} \chi_4^B \mu_B^3 + \frac{1}{120} \chi_6^B \mu_B^5 + \frac{1}{5040} \chi_8^B \mu_B^7 + \dots$$



Through the extrapolation on the line of constant physics: the wiggles are mostly smoothed out.

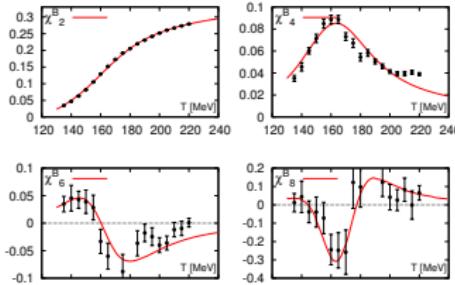
Equation of state using the contours in the phase diagram



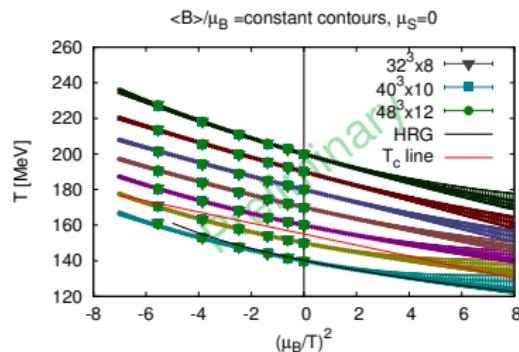
Summary

- Lattice QCD is making progress in exploring deeper into the phase diagram
 - But because of the sign **and** overlap problems this is exponentially difficult,
 - thus results are always somewhat speculative.
 - or will have large systematic errors, reflecting on the ambiguity of the extrapolation.
- Two approaches:

Taylor, with high net-baryon moments:



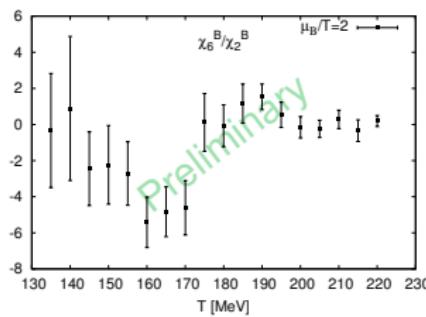
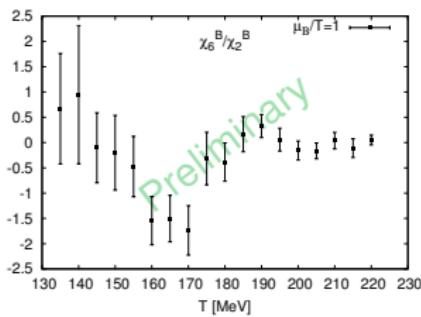
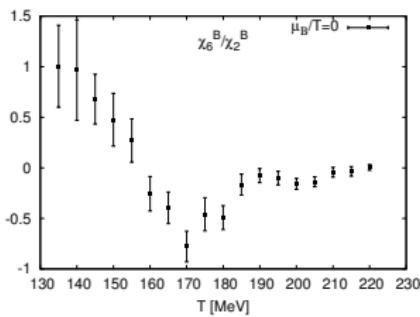
Extrapolating contours of fixed χ_1^B/μ_B :



backup

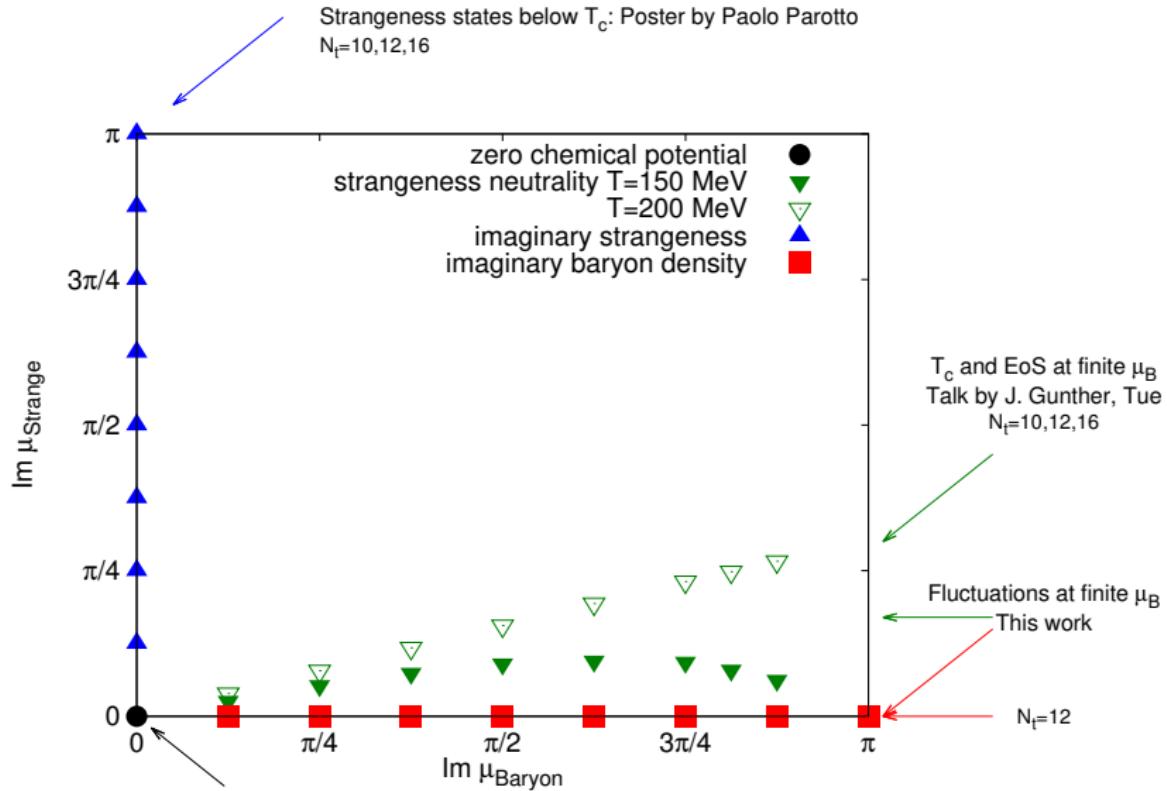
Extrapolation of χ_6^B/χ_2^B

$$\left. \frac{\chi_6^B}{\chi_2^B} \right|_{\mu_B/T} = \frac{\chi_6^B}{\chi_2^B} + \frac{\mu_B^2}{2} \left(\frac{\chi_8^B}{\chi_2^B} - \frac{\chi_4^B}{\chi_2^B} \right) + \dots$$

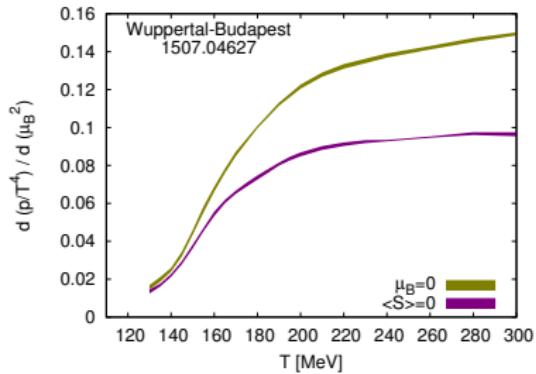
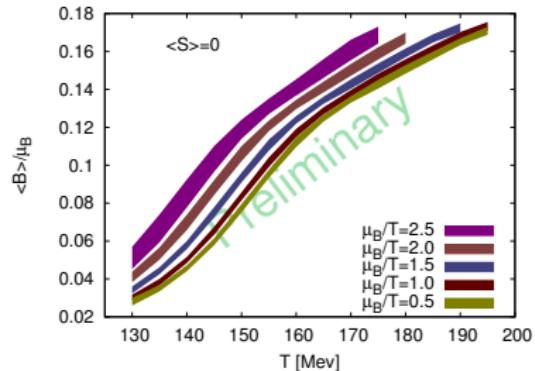
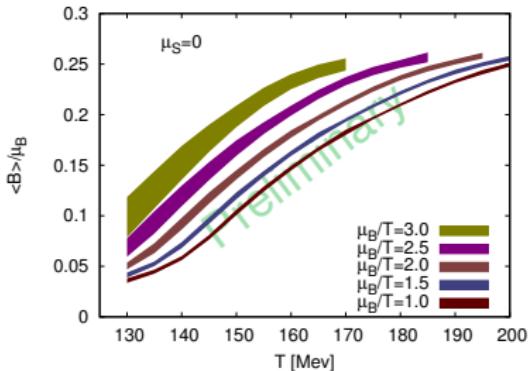


Preliminary

Simulation landscape with imaginary μ



Strangeness neutral or not



Equation of state with strangeness neutrality

