Realistic in-medium heavy-quark potential from high statistics lattice QCD simulations

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References:
The two faces of heavy quarkonium

Bottomonium and charmonium probe complementary aspects of a HIC

Sequential suppression of excited states – non-equilibrium probe of the full HIC evolution

(Partial) kinetic equilibration with the bulk: memory loss – window on the late stages
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- Bottomonium and charmonium probe complementary aspects of a HIC

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(Partial) kinetic equilibration with the bulk: memory loss – window on the late stages

- Goal: quantitative and intuitive potential based description from 1\textsuperscript{st} principles QCD

For applications see contributions by e.g. B. Krouppa 15/05/2018, 11:50 and S. Kajimoto 14/05/2018, 17:00
Intuition: Interactions with medium via a non-relativistic potential description

\[ \frac{\Lambda_{QCD}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1 \]
Towards the in-medium potential

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- \( T=0 \) potential well understood from the **Euclidean Wilson loop** in lattice QCD

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- At $T>0$ for a long time: **only potential models** available

- Ad hoc identification of $V(r)$ with the color singlet free energies in Coulomb Gauge

$$F^{(1)}(R) = -\frac{1}{\beta} \log \left[ \langle P(R) P^\dagger(0) \rangle \right]$$

Nadkarni, PRD 34,(1986)

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- Many other proposals:
  - internal energy $U^1$, linear combinations
  - \( C. \ Y. \) Wong, PRC 72, (2005), H. Satz JPG 36, (2009), ...
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- For T>0 Q\bar{Q} real-time evolution via potential: need to derive Schrödinger equation

No Schrödinger equation derived for the model potentials
The era of model potentials is finally over: **genuine QCD definition** is available
Heavy quark potential from EFT

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**pNRQCD effective field theory:**

\[ m_Q \gg m_Q v \gg m_Q v^2 = E_{\text{bind}} \]

- Describes heavy quarkonium as singlet and octet wavefunctions: \( \psi_s(R, t), \psi_o(R, t) \)

\[
i\hbar \frac{d}{dt} \langle \psi_s(t) | \psi_s(0) \rangle = \left( V^{QCD}(R) + \mathcal{O}(m_Q^{-1}) + \Theta(R, t) \right) \langle \psi_s(t) | \psi_s(0) \rangle
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Non-potential corrections
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Derived from QCD: \( V_{QCD} \) as Wilson coefficient determined via matching at \( m=\infty \)

\[
V_{QCD}^{QCD}(R) = \lim_{t \to \infty} \frac{i \partial_t W_{\Box}(R, t)}{W_{\Box}(R, t)}
\]

Non-potential corrections

\[
W_{\Box}(R, t) = \text{Tr} \left( \exp \left[ -i \int dx A_\mu(x) \right] \right)
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**Im[V]:** Laine et al. JHEP03 (2007) 054; Beraudo et al. NPA 806:312,2008

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- Previous real models miss the physics of \( \text{Im}[V] \): Landau-damping & singlet \( \leftrightarrow \) octet

- From perturbation theory: \( \text{Re}[V] \) and \( F^1(r) \) similar only at very high temperatures


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How to connect to Euclidean lattice QCD: spectral functions

\[ W_{\Box}(R, t) = \int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \rho_{\Box}(R, \omega) \quad \leftrightarrow \quad W_{\Box}(R, \tau) = \int_{-\infty}^{\infty} d\omega \, e^{-\omega \tau} \rho_{\Box}(R, \omega) \]
Extracting $V^{QCD}$ from the lattice

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Spectral Decomposition

$V^{QCD}(R) = \lim_{t \to \infty} \frac{\int_{-\infty}^{\infty} d\omega \, \omega \, e^{-i\omega t} \rho(R, \omega)}{\int_{-\infty}^{\infty} d\omega \, e^{-i\omega t} \rho(R, \omega)}$
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Intuitive relation: spectral functions and $V^{QCD}$

$V^{QCD}(R) = \omega_0(R) + i\Gamma_0(R)$

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Spectral Reconstruction (Unfolding)

- In case of sizable $\Delta W/W=10^{-2}$ statistical uncertainty in $W_\square$: **Bayesian inference**
  
  incorporate prior information to regularize the inversion task (BR method)

  Y. Burnier, A.R. PRL 111 (2013) 182003

- In case of small $\Delta W/W=10^{-3}$ statistical uncertainty in $W_\square$ also **Pade approximation**

  exploit the analyticity of the Wilson correlator to extract spectra

  see e.g. A. Tripolts contribution 14/05/2018, 17:50
Extracting $V_{\text{QCD}}$ from the lattice

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Potential models: only static information $\tau=\beta$

Proper potential: full time information in $W(\beta)$


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Proper potential: full time information in $W(\tau)$

- only at $T=0$: $\tau=\beta$ same falloff as at $\tau<\beta$
Pade approximation for $V^{QCD}(R)$

- Pade approximation for spectral reconstruction:
  - **Project** Wilson correlator on an rational function basis
    \[ W(i\omega_n) \approx R(i\omega_n) = \frac{W(i\omega_0)}{1+} \frac{a_1(\omega - \omega_0)}{1+} \ldots \frac{a_N(\omega - \omega_N)}{1+} \]
  - **Analytically continue** the basis functions
    \[ \rho(\omega) = -\frac{1}{\pi} \text{Im}[R(i\omega \rightarrow \omega)] \]

L. Schlessinger, Phys. Rev. 167, 1411 (1968)
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- **Pade requires high precision:** feasibility test with **HTL data** at $N_r=12$ and $dW/W=10^{-2}$

In Wilson approximation
- spectral correlators, $\text{Re}[V], \text{Im}[V]$ known analytically
- Y.Burnier, A.R. PRD87 (2013) 114019
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- Due to simple spectral structure: **Re[V] ok** but **Im[V] systematically underestimated**

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Lattice QCD setup

- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)
  - realistic $m_\pi \sim 161\text{MeV}$ ($T=151-407\text{MeV}$) - slightly higher $m_\pi \sim 300\text{MeV}$ ($T=450-1250\text{MeV}$)
  - fixed box ($N_s=48$ - $N_T=12$, $N_T=16$): vary $T$ via lattice spacing
  - low temperature configurations ($N_T=32-64$) for calibration available

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503
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- \( \text{Re}[V] @ T\approx0 \) consistent in different extractions: Bayes, Pade, correlator fits

\( \rho @ T=0 \quad b=6.740 \quad \text{HISQ Pade} \)

\( \beta=6.740 \quad \beta=7.030 \quad \beta=7.373 \quad \beta=7.595 \quad \beta=7.825 \quad \beta=8.000 \quad \beta=8.400 \)
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- $\text{Re}[V]$ @ $T\approx 0$ consistent in different extractions: Bayes, Pade, correlator fits

- No significant difference between $N_T=12$ and $N_T=16$ – close to continuum already

A. Bazavov et.al. PRD97 (2018) 014510, HotQCD PRD90 (2014) 094503
The finite temperature potential

- Extraction using Bayes and Pade
  - $T \leq 198\text{MeV}$ BR and Pade work, give same result
  - $T > 198\text{MeV}$ only Pade robust, Bayes shows ringing artifacts
- Errorbars from 10-bin Jackknife and varying the # of used correlator points

$\text{Re}[V](r)\ [\text{GeV}]$ (shifted in $y$)

- $T=150\text{MeV}$ (Nt=12)
- $T=159\text{MeV}$ (Nt=12)
- $T=174\text{MeV}$ (Nt=12)
- $T=186\text{MeV}$ (Nt=12)
- $T=198\text{MeV}$ (Nt=12)
- $T=252\text{MeV}$ (Nt=12)
- $T=306\text{MeV}$ (Nt=16)
- $T=354\text{MeV}$ (Nt=16)
- $T=498\text{MeV}$ (Nt=16)
- $T=576\text{MeV}$ (Nt=16)
- $T=1248\text{MeV}$ (Nt=16)

$F^{(1)}(r)$

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- Errorbars from 10-bin Jackknife and varying the # of used correlator points
- \(\text{Re}[V]\) shows **smooth transition** from Cornell to asymptotically flat form
- Comparison to \(F^1(r)\) (gray) shows **agreement** with \(\text{Re}[V]\) within errors
  in particular no indication found that \(\text{Re}[V]\) is steeper than \(F^1\)

A closer look at Re[V]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Re[V](r) [GeV] vs. r [fm] at different temperatures.

- \( T \approx 0 \) \( \beta = 6.740 \) for \( T = 150\text{MeV} \) (Nt=12).
- \( T \approx 0 \) \( \beta = 7.280 \) for \( T = 252\text{MeV} \) (Nt=12).
}
\end{figure}
A closer look at Re[V]

just below $T_c$: no screening Re[V] & $F^1$ agree with $T=0$ Re[V]
A closer look at $\text{Re}[V]$

just below $T_c$: no screening $\text{Re}[V]$ & $F^1$ agree with $T=0$ $\text{Re}[V]$

in QGP phase: $\text{Re}[V]$ weakens compared to $T=0$ case. Still $\text{Re}[V]$ & $F^1$ agree upto uncertainty.
Tentative extraction of $\text{Im}[V]$

- Extraction using BR method, since Pade shown to severely underestimate $\text{Im}[V]$. 

![Graph](image-url)
Tentative extraction of $\text{Im}[V]$

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around $T_c$ compatible with zero, finite values at $T \geq 173 \text{MeV}$
Tentative extraction of $\text{Im}[V]$

- Extraction using BR method, since Pade shown to severely underestimate $\text{Im}[V]$ around $T_c$ compatible with zero, finite values at $T \geq 173 \text{MeV}$

- Improving the extraction of $\text{Im}[V]$ urgently needed, requires larger $N_t \sim 48$
Interpreting the T>0 potential

- Classic idea of screening by Debye only applies to Abelian plasmas
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- Using the generalized Gauss-Law: screening of confining force

V.V. Dixit, Mod. Phys. Lett. A5 (1990) 227
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  \[ \text{Re}[V](R,T) \& \text{Im}[V](R,T) \text{ from one T-dep. parameter Debye mass } m_D \]
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- Combining non-perturbative T=0 Cornell and perturbative medium:
  Using the generalized Gauss-Law: screening of confining force
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Combining non-perturbative T=0 Cornell and perturbative medium:
Re[V] (R,T) & Im[V] (R,T) from one T-dep. parameter Debye mass $m_D$


**Re*[V](r) [GeV] (shifted in *y*)

**β**=

6.740
6.800
6.880
6.950
7.030
7.280
7.373
7.595
7.825
8.000
8.400

Using the generalized Gauss-Law: screening of confining force

$V_{Cornell}(r) = -\frac{\alpha_s}{r} + \sigma r + c$

**Re[V](0)/GeV (shifted in *y*)

**β**=

6.740
6.900
7.030
7.280
7.373
7.595
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fit T=0 Cornell parameters

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- Combining non-perturbative $T=0$ Cornell and perturbative medium:
  $\text{Re}[V](R,T)$ & $\text{Im}[V](R,T)$ from **one $T$-dep. parameter** Debye mass $m_D$


- Lattice $\text{Re}[V]$ well described by tuning $m_D$, reflects smooth onset for $T>T_c$

Fit $T=0$ Cornell parameters

$V_{\text{Cornell}}(r) = -\alpha_s/r + \sigma r + c$
Once $m_D$ is fixed from $\text{Re}[V]$ Gauss-Law parametrization predicts a $\text{Im}[V]$
Once $m_D$ is fixed from $\text{Re}[V]$ Gauss-Law parametrization predicts a $\text{Im}[V]$. Gauss Law parameterization seems to give slightly lower values than extracted just above $T_c$. 

![Graph showing the relationship between $\text{Im}[V]$ and $\Delta r$ for different temperatures.](image-url)
QCD based static potential $V^{QCD}$ for in-medium heavy-QQ available
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Grazie per la vostra attenzione – Thank you for your attention