



# Realistic in-medium heavy-quark potential from high statistics lattice QCD simulations

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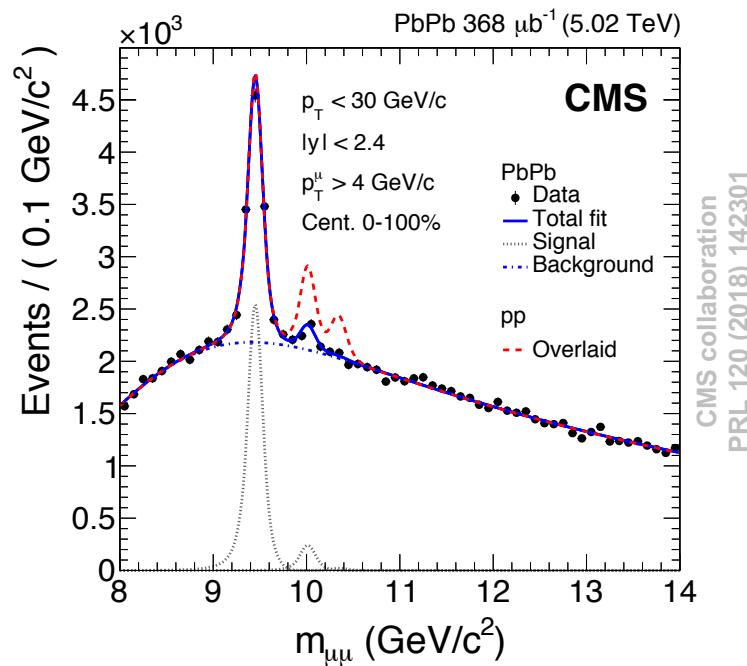
in collaboration with **P. Petreczky, J. Weber**  
for the **TUM-QCD** collaboration

References:

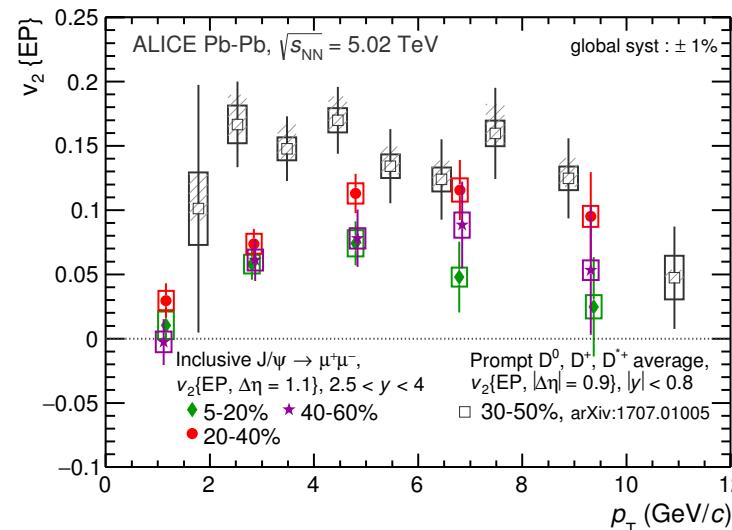
P. Petreczky, A. R., J. Weber, [TUM-QCD] in preparation

# The two faces of heavy quarkonium

- Bottomonium and charmonium probe **complementary** aspects of a HIC



Sequential suppression of excited states –  
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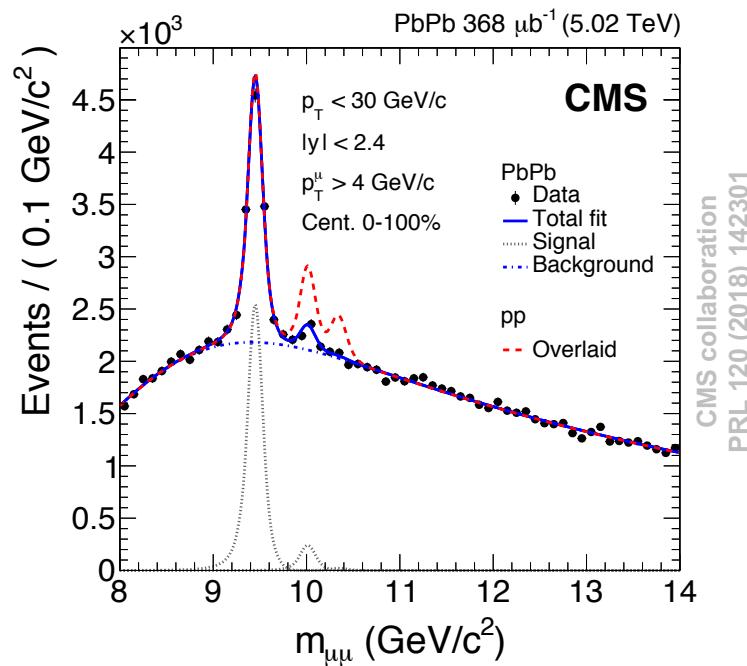


(Partial) kinetic equilibration with the bulk:  
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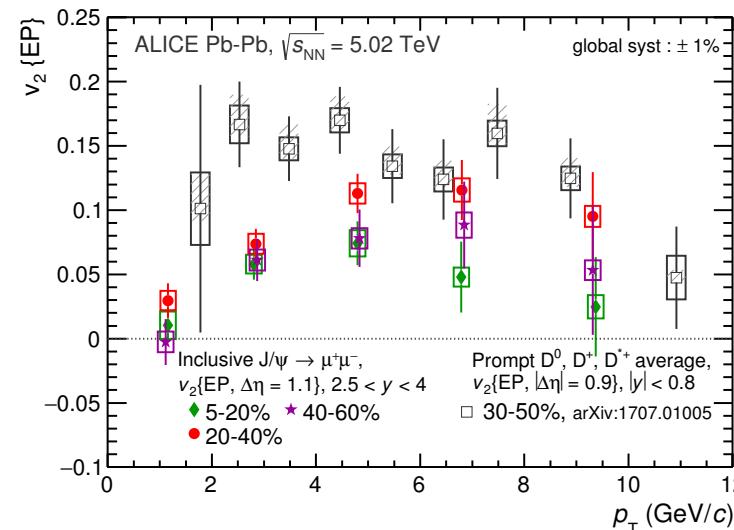
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ALICE collaboration  
PRL 119 (2017) 242301

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- Goal: **quantitative and intuitive potential based description from 1<sup>st</sup> principles QCD**

For applications see contributions by e.g. B. Krouppa 15/05/2018, 11:50 and S. Kajimoto 14/05/2018, 17:00  
see also Y.Burnier, A.R., O.Kaczmarek JHEP 1512 (2015) 101, N. Brambilla et.al. PRD97 (2018) 074009

# Towards the in-medium potential

- **Intuition:** Interactions with medium via a non-relativistic potential description

$$\frac{\Lambda_{\text{QCD}}}{m_Q} \ll 1, \quad \frac{T}{m_Q} \ll 1, \quad \frac{p}{m_Q} \ll 1$$

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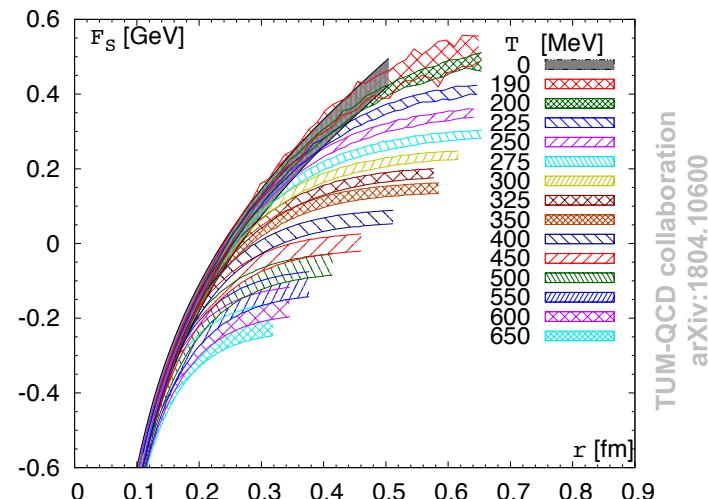
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- At **T>0** for a long time: **only potential models** available
  - Ad hoc identification of  $V(r)$  with the color singlet free energies in Coulomb Gauge

$$F^{(1)}(R) = -\frac{1}{\beta} \log [\langle P(R)P^\dagger(0) \rangle]$$

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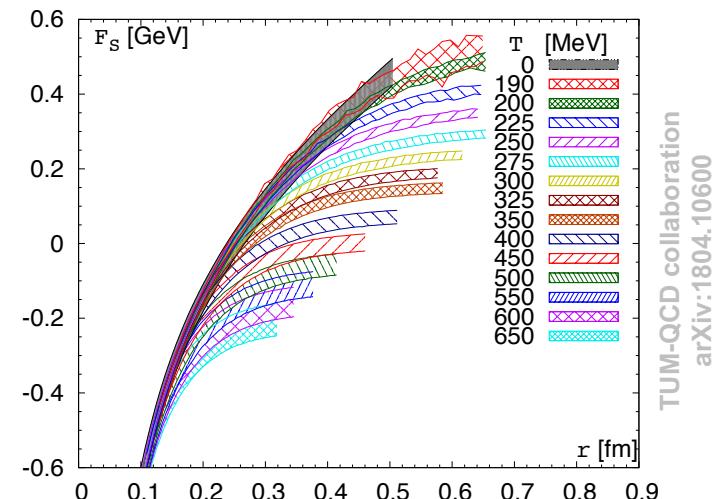
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internal energy  $U^1$ , linear combinations

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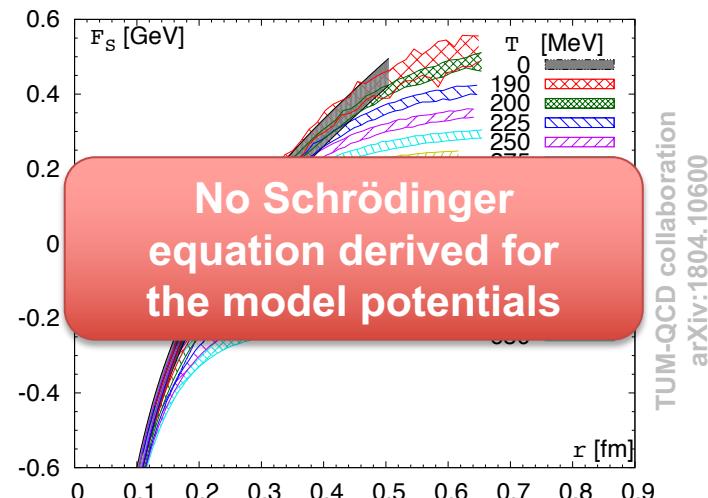
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- For  $T>0$   $Q\bar{Q}$  real-time evolution via potential: need to derive Schrödinger equation



# Heavy quark potential from EFT

- The era of model potentials is finally over: **genuine QCD definition** is available

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Brambilla et. al. Rev.Mod. Phys. 77 (2005) 1423      Brambilla, Ghiglieri, Vairo and Petreczky PRD 78 (2008) 014017

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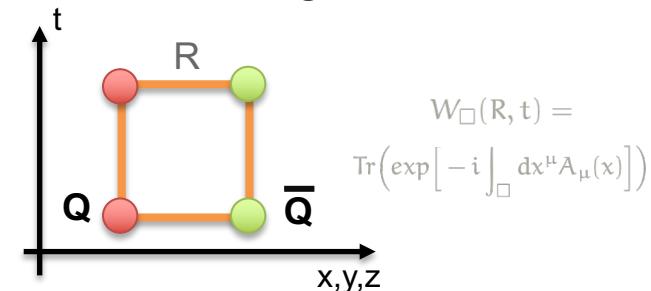
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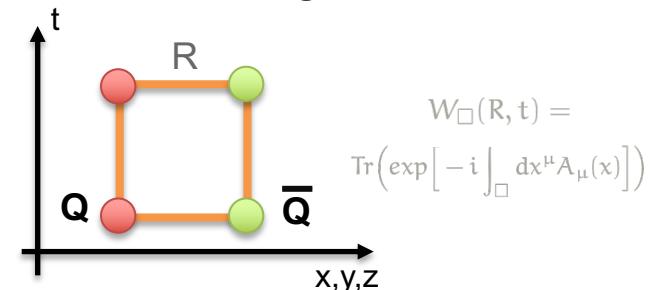
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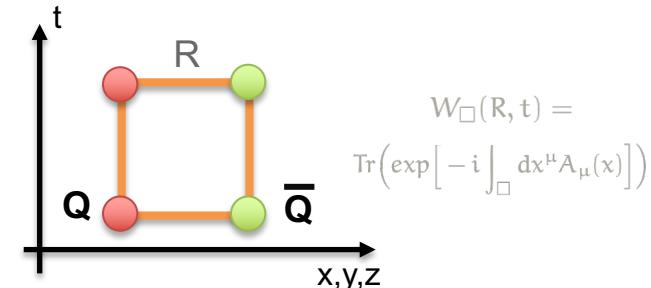
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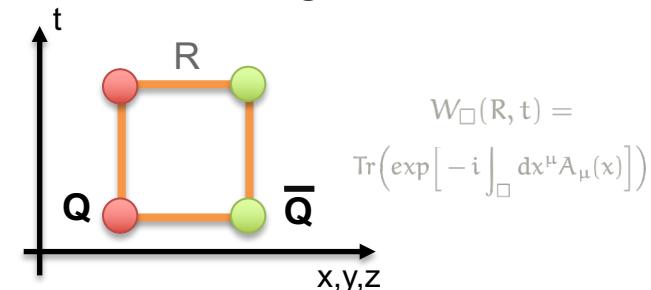
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- Previous real models miss the physics of Im[V]: Landau-damping & singlet  $\leftrightarrow$  octet
- From perturbation theory:  $\text{Re}[V]$  and  $F^1(r)$  similar only at very high temperatures

# Extracting $V^{\text{QCD}}$ from the lattice

- How to connect to Euclidean lattice QCD: **spectral functions**

A.R., T.Hatsuda & S.Sasaki  
PRL 108 (2012) 162001

$$W_{\square}(R, t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \rho_{\square}(R, \omega) \quad \longleftrightarrow \quad W_{\square}(R, \tau) = \int_{-\infty}^{\infty} d\omega e^{-\omega\tau} \rho_{\square}(R, \omega)$$

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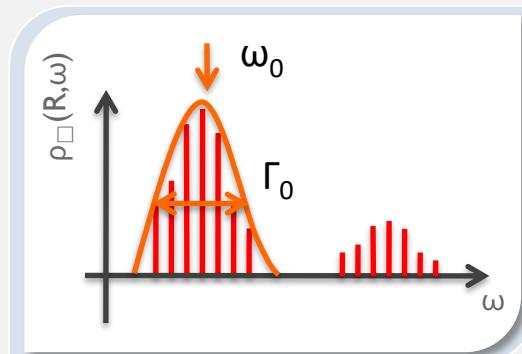
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technical details: Y.Burnier,  
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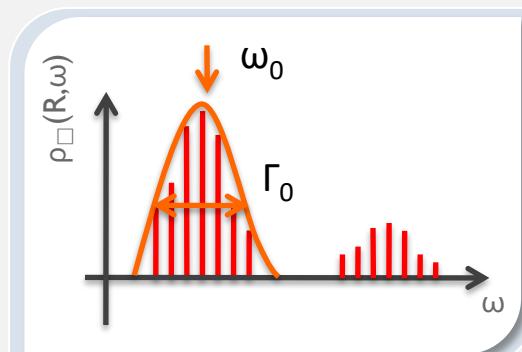
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## Spectral Reconstruction (Unfolding)

- In case of sizable  $\Delta W/W=10^{-2}$  statistical uncertainty in  $W_{\square}$ : **Bayesian inference**

incorporate **prior information** to regularize the inversion task (BR method)

Y. Burnier, A.R. PRL 111 (2013) 182003

- In case of small  $\Delta W/W=10^{-3}$  statistical uncertainty in  $W_{\square}$  also **Pade approximation**

**exploit the analyticity** of the Wilson correlator to extract spectra

see e.g. A. Tripolt's contribution 14/05/2018, 17:50

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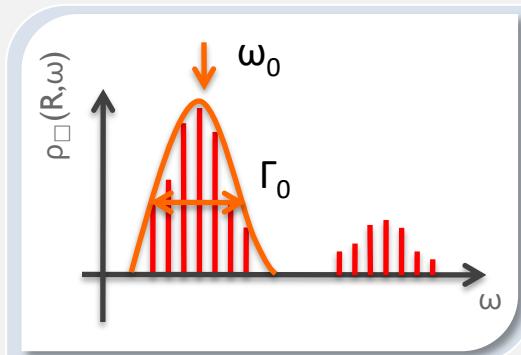
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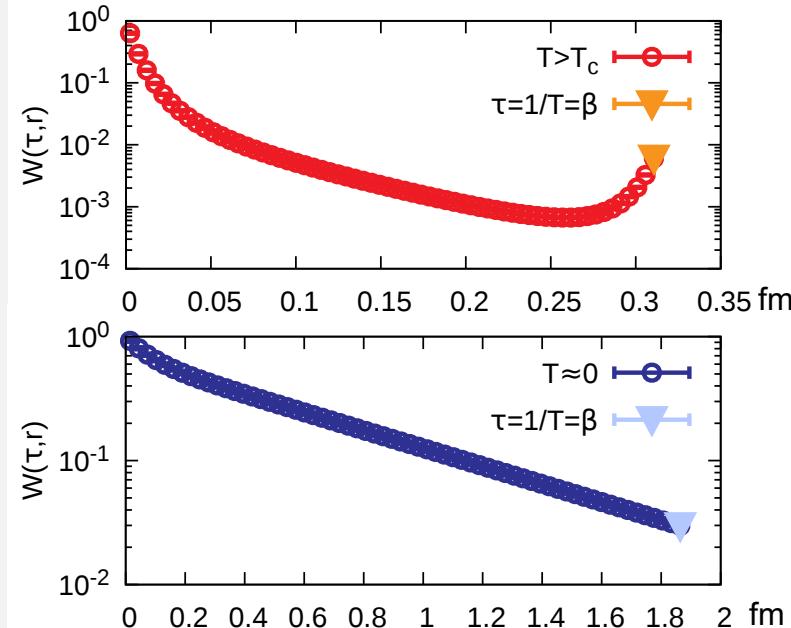
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Proper potential: **full time information in  $W(\tau)$**



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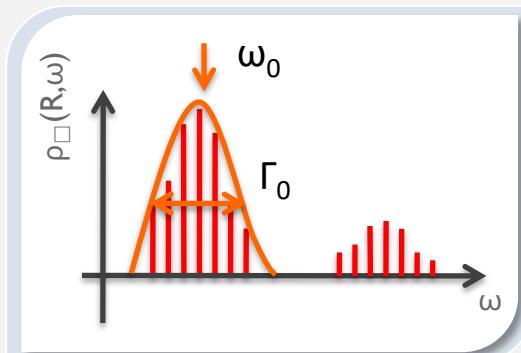
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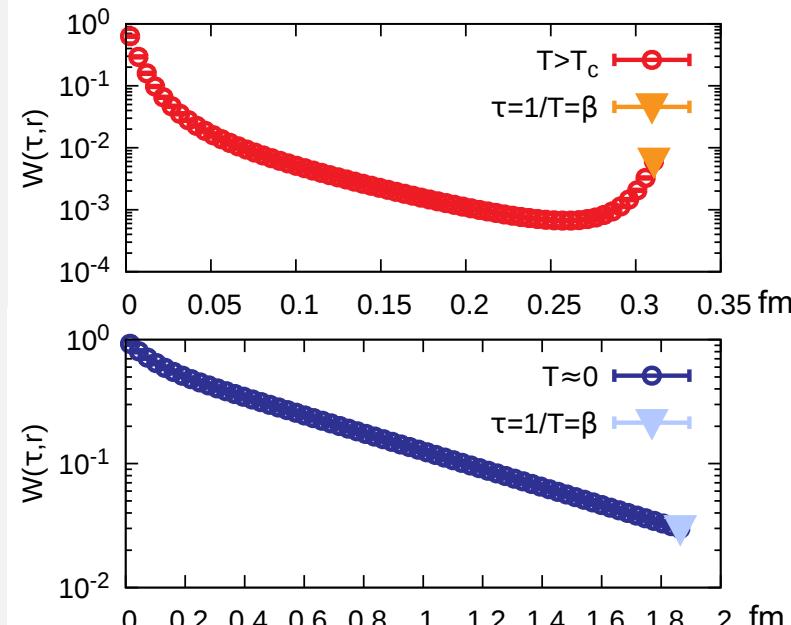
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- only at  $T=0$ :  $\tau=\beta$  same falloff as at  $\tau<\beta$

# Pade approximation for $V_{\text{QCD}}(R)$

- Pade approximation for spectral reconstruction:

- Project Wilson correlator on an rational function basis

$$W(i\omega_n) \approx R(i\omega_n) = \frac{W(i\omega_0)}{1+} \frac{a_1(\omega - \omega_0)}{1+} \dots \frac{a_N(\omega - \omega_N)}{1+}$$

- Analytically continue the basis functions

$$\rho(\omega) = -\frac{1}{\pi} \text{Im}[R(i\omega \rightarrow \omega)]$$

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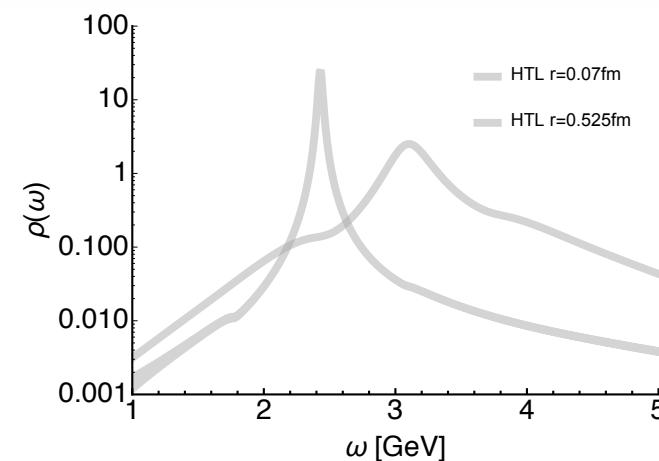
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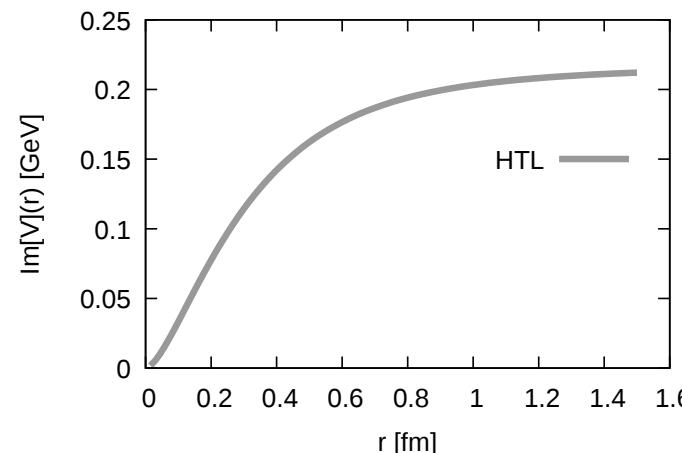
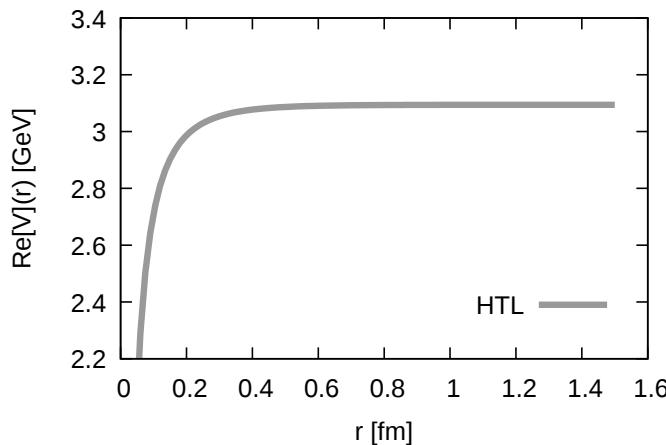
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- Pade requires high precision: feasibility test with **HTL data** at  $N_t=12$  and  $dW/W=10^{-2}$



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Y.Burnier, A.R.  
PRD87 (2013) 114019

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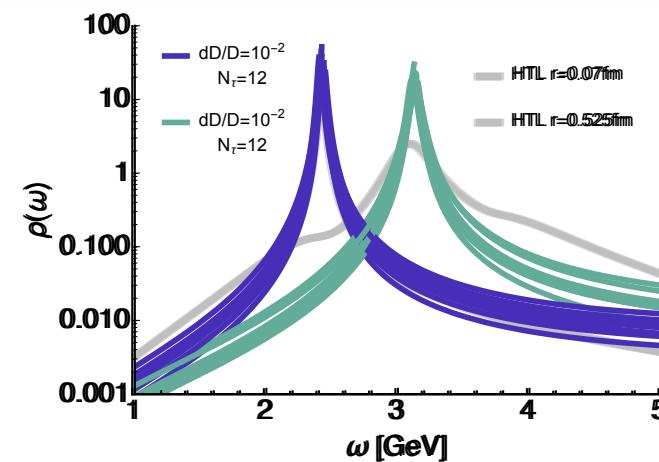
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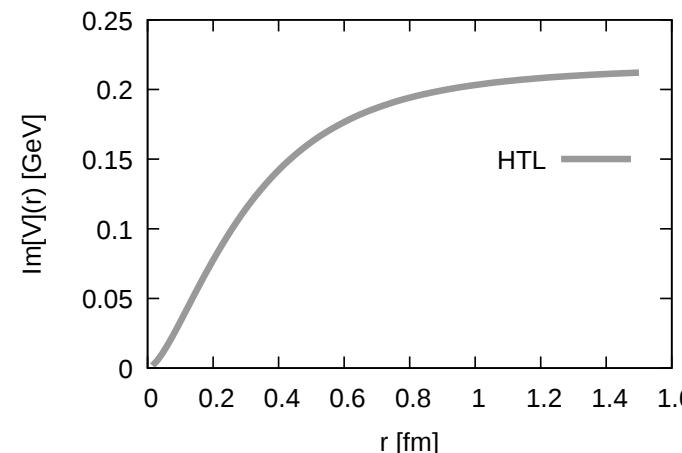
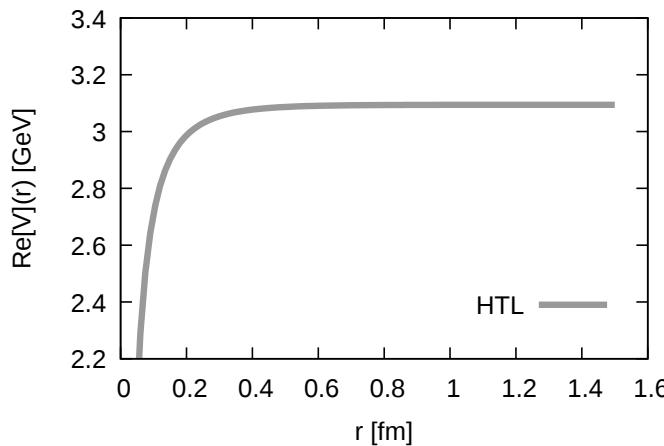
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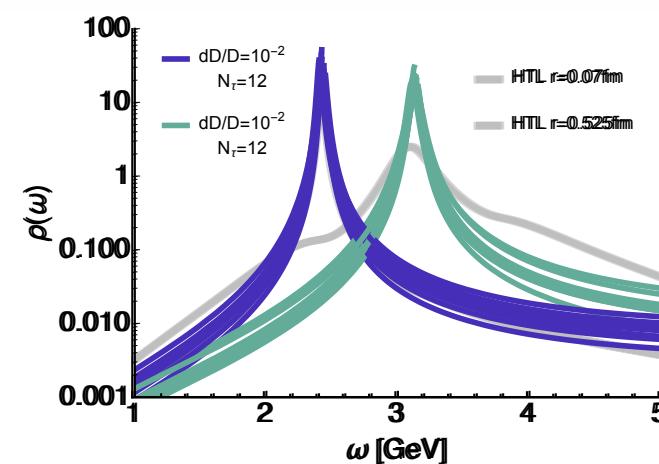
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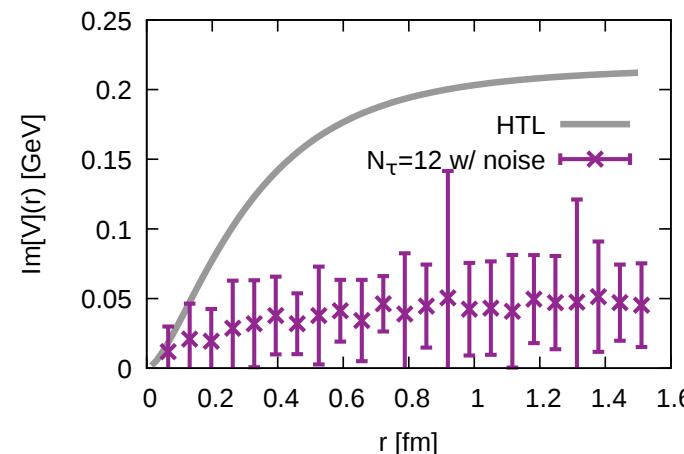
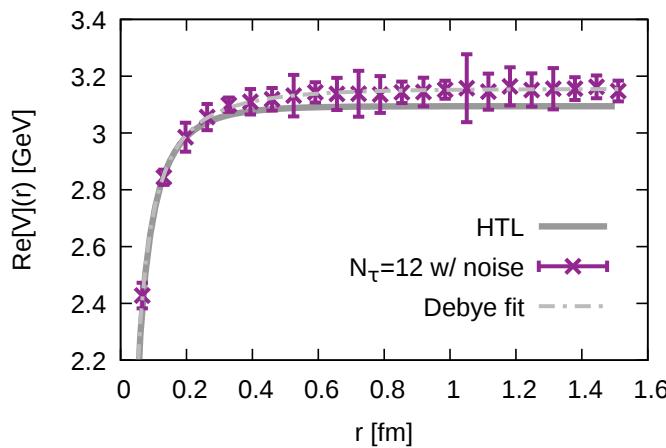
- Analytically continue the basis functions

$$\rho(\omega) = -\frac{1}{\pi} \text{Im}[R(i\omega \rightarrow \omega)]$$

L. Schlessinger,  
Phys. Rev. 167,  
1411 (1968)



- Pade requires high precision: feasibility test with HTL data at  $N_t=12$  and  $dW/W=10^{-2}$



In HTL approximation  
Wilson correlators,  
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 $\text{Re}[V]$   $\text{Im}[V]$  known  
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Y.Burnier, A.R.  
PRD87 (2013) 114019

# Pade approximation for $\nabla\text{QCD}(\mathbf{R})$



- Pade approximation for spectral reconstruction:

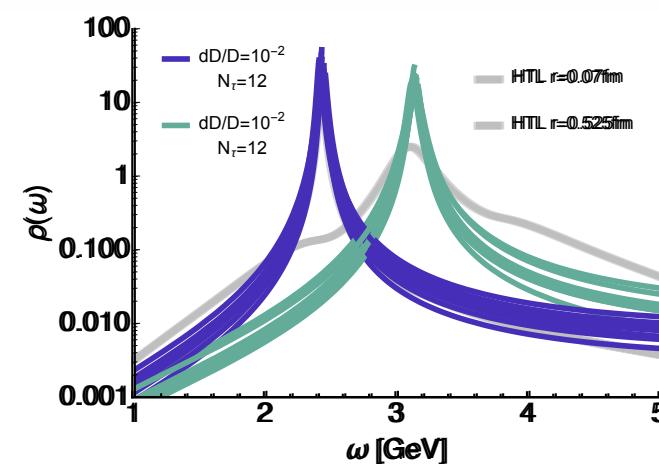
- Project Wilson correlator on an rational function basis

$$W(i\omega_n) \approx R(i\omega_n) = \frac{W(i\omega_0)}{1+} \frac{a_1(\omega - \omega_0)}{1+} \dots \frac{a_N(\omega - \omega_N)}{1+}$$

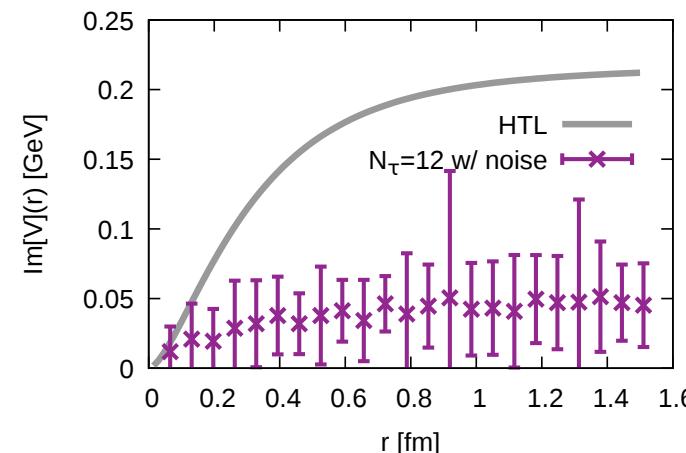
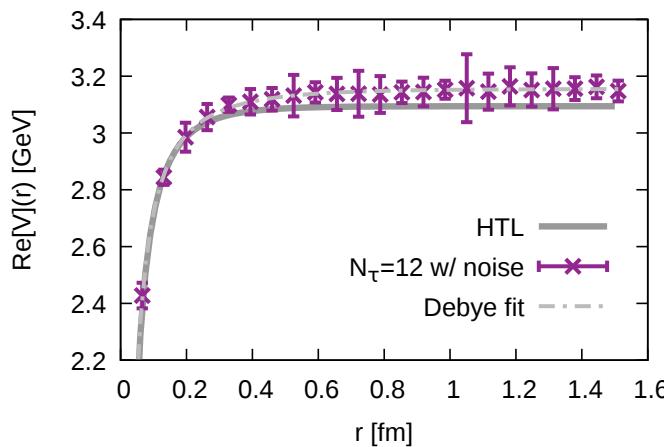
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PRD87 (2013) 114019

- Due to simple spectral structure:  $\text{Re}[V]$  ok but  $\text{Im}[V]$  systematically underestimated

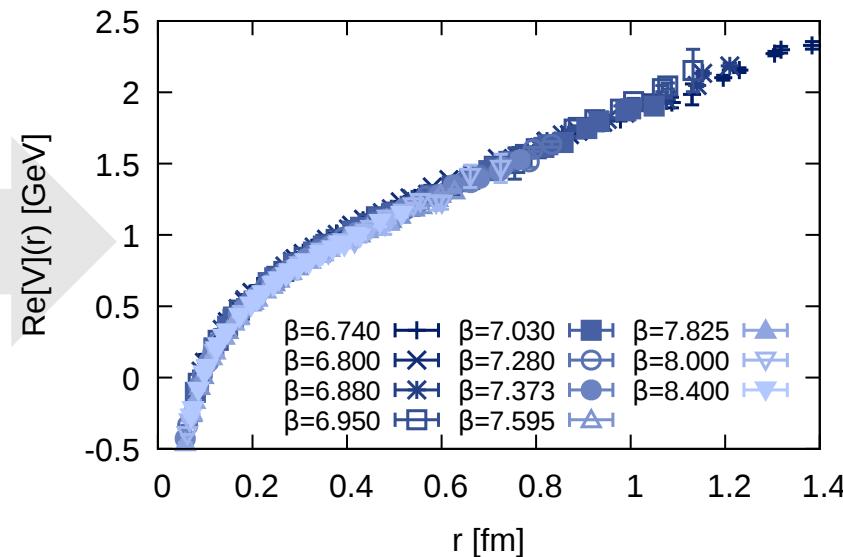
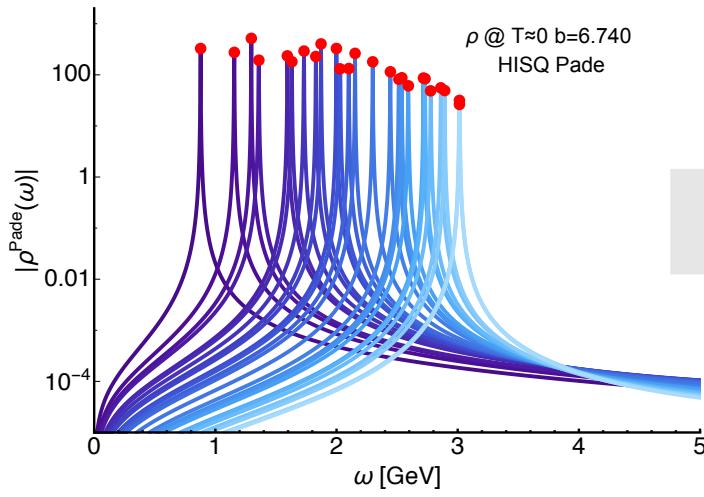
# Lattice QCD setup

- Lattices with dynamical u,d,s quarks (HISQ action, HotQCD & TUMQCD)
  - realistic  $m_\pi \sim 161\text{MeV}$  ( $T=151\text{-}407\text{MeV}$ ) - slightly higher  $m_\pi \sim 300\text{MeV}$  ( $T=450\text{-}1250\text{MeV}$ )
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  - low temperature configurations ( $N_t = 32\text{-}64$ ) for calibration available

A. Bazavov et.al. PRD97 (2018) 014510,  
HotQCD PRD90 (2014) 094503

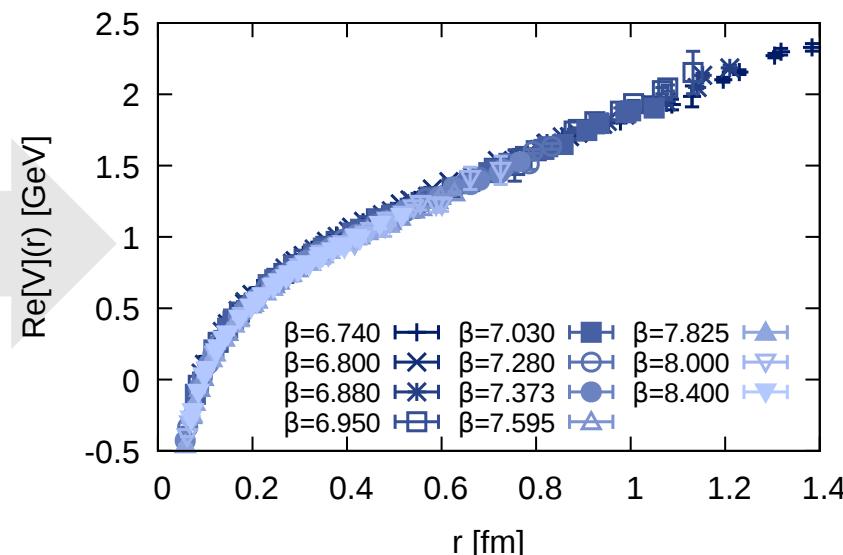
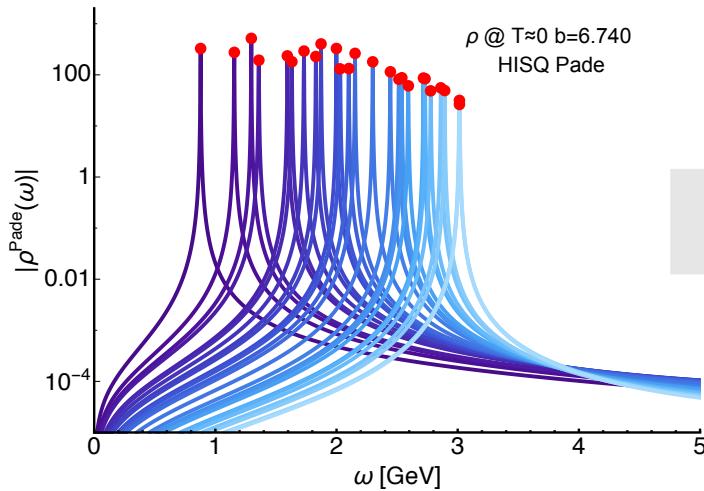
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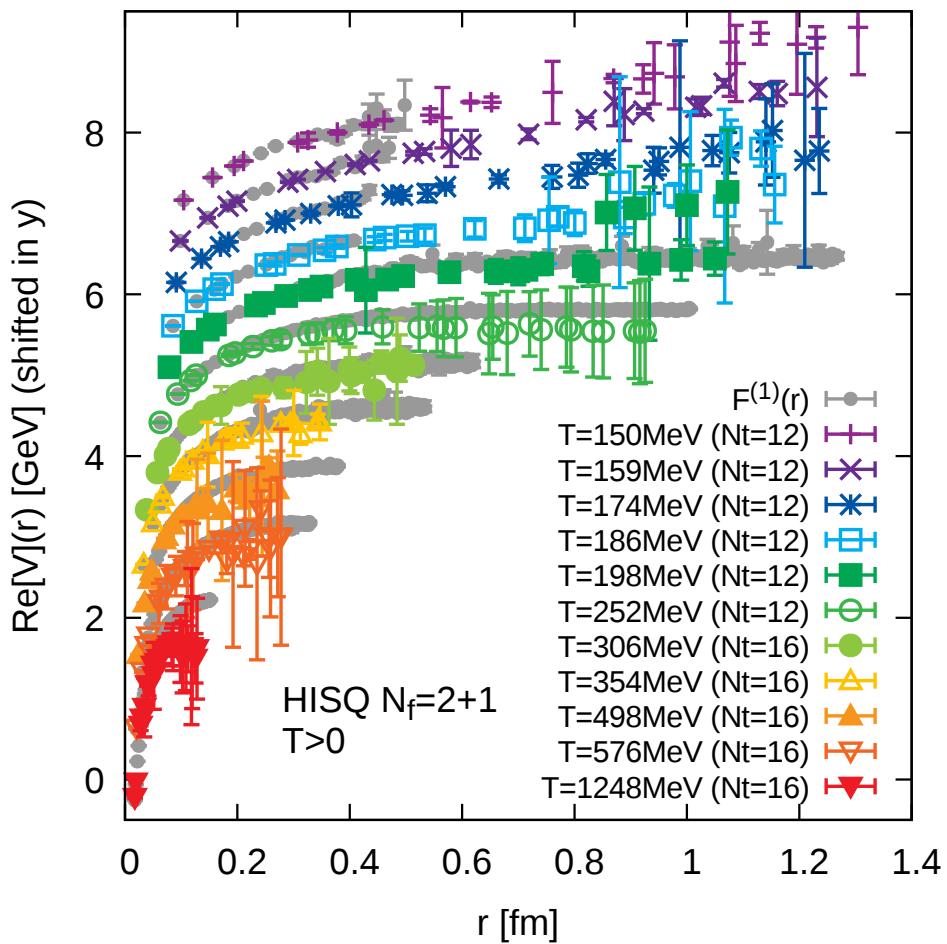
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- No significant difference between  $N_t=12$  and  $N_t=16$  – **close to continuum** already

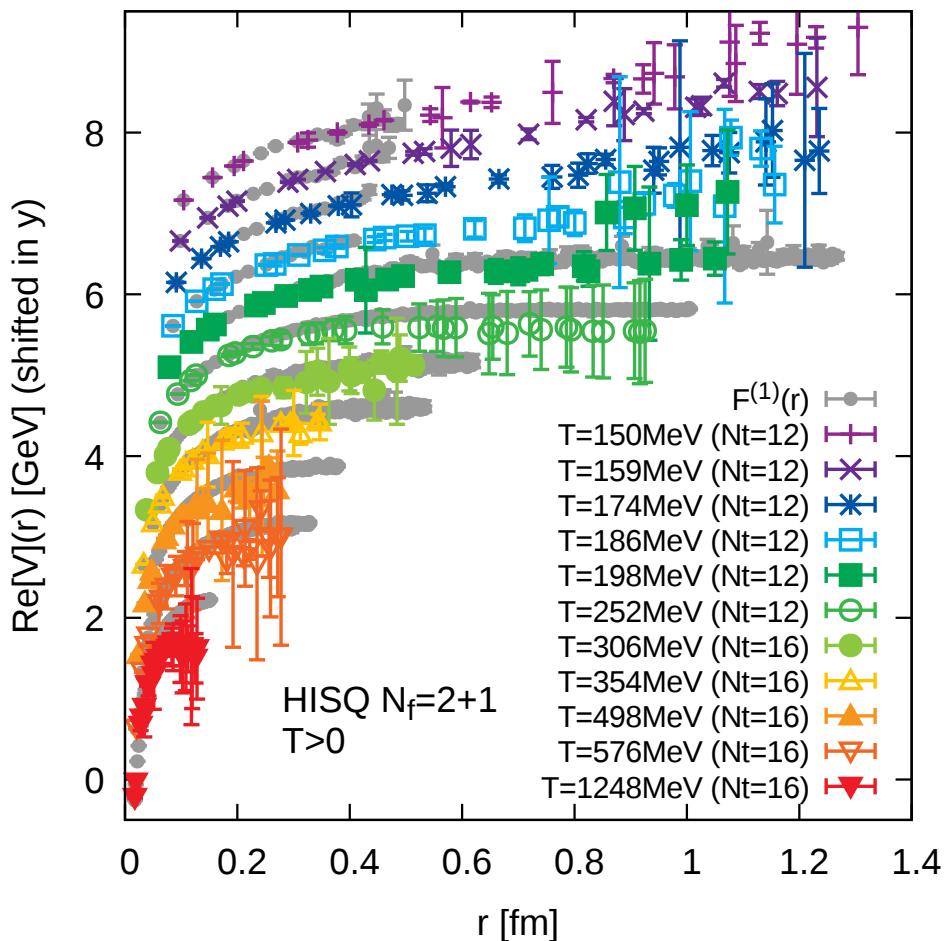
# The finite temperature potential

- Extraction using Bayes and Pade
  - $T \leq 198\text{MeV}$  BR and Pade work, give same result
  - $T > 198\text{MeV}$  only Pade robust, Bayes shows ringing artifacts
- Errorbars from 10-bin Jackknife and varying the # of used correlator points



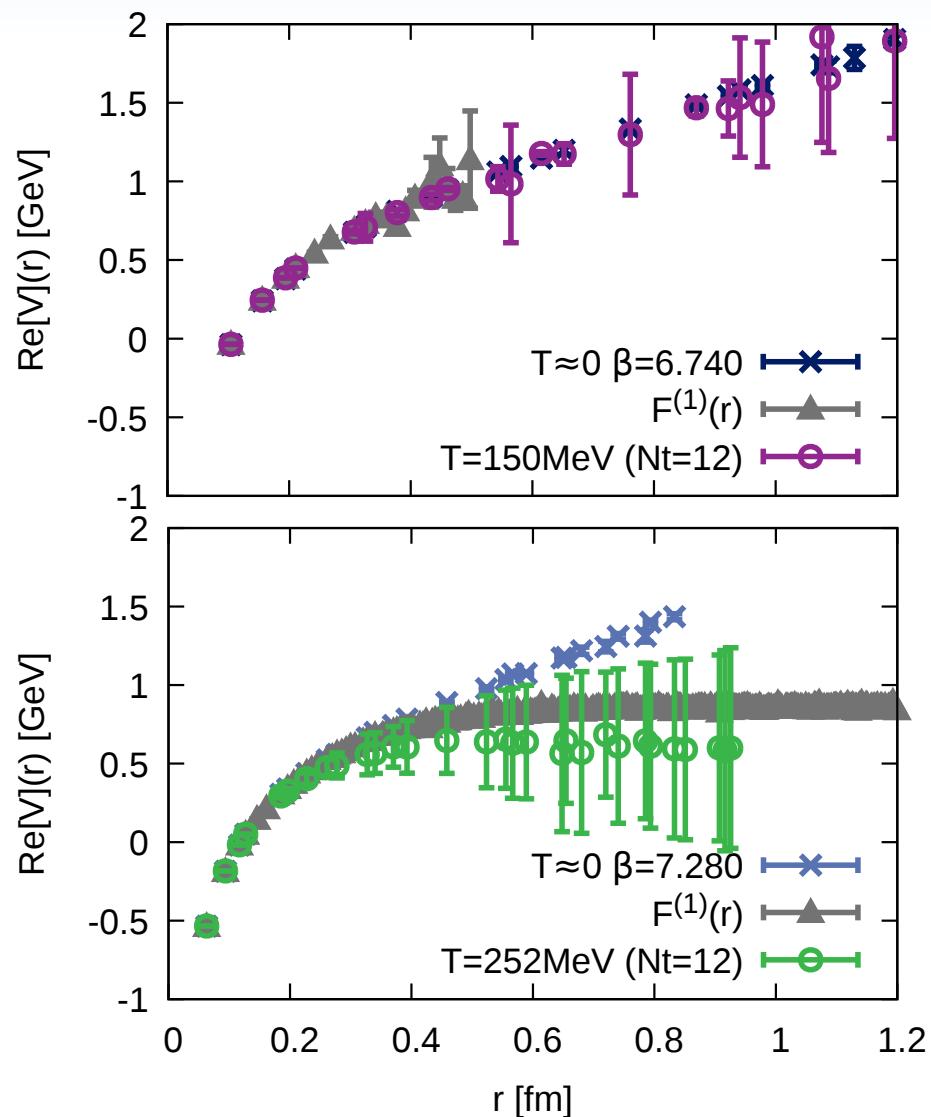
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- $\text{Re}[V]$  shows **smooth transition** from Cornell to asymptotically flat form
- Comparison to  $F^1(r)$  (gray) shows **agreement** with  $\text{Re}[V]$  within errors in particular no indication found that  $\text{Re}[V]$  is steeper than  $F^1$

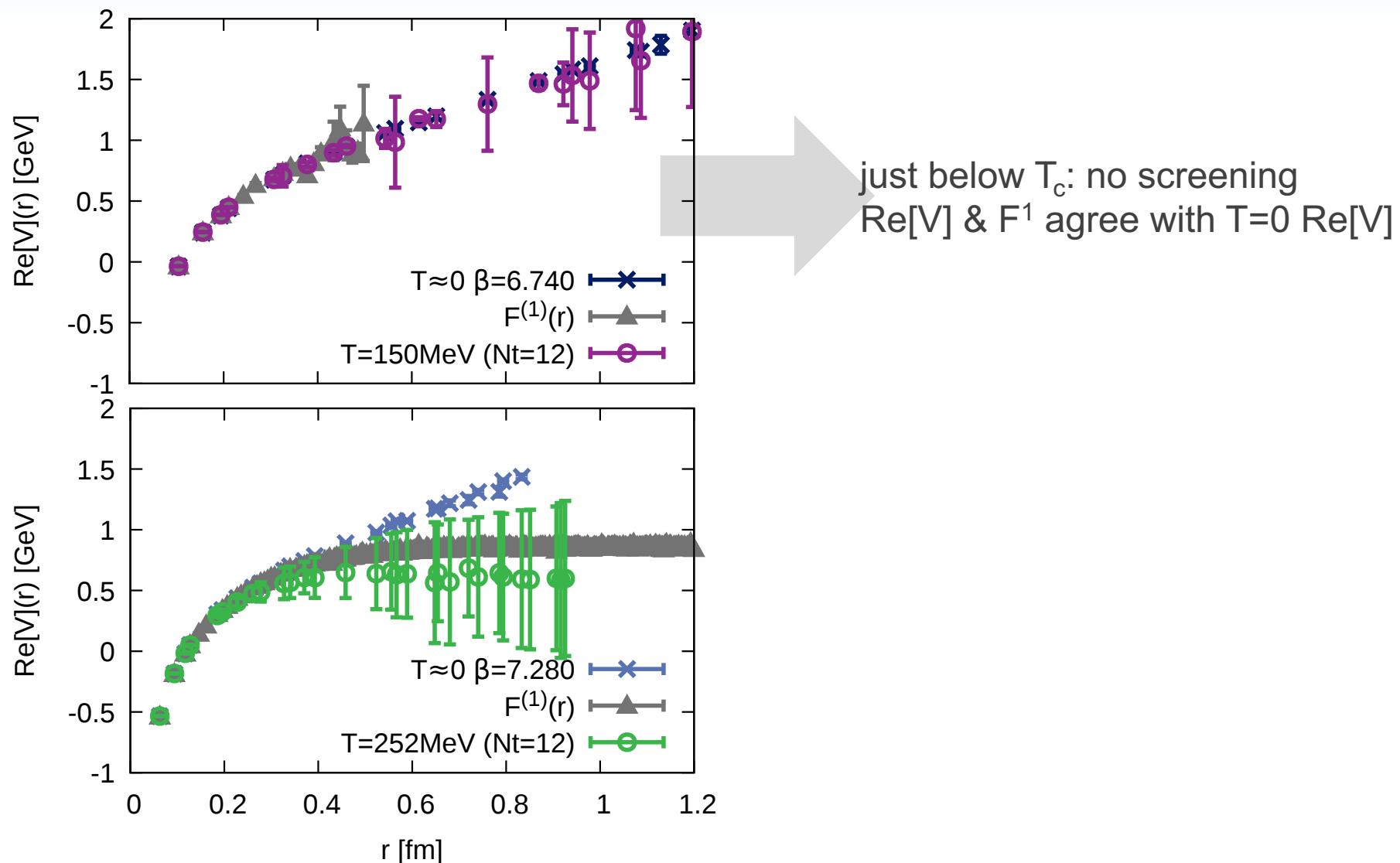


c.f. Y. Burnier, A.R., O. Kaczmarek PRL114 (2015) 082001

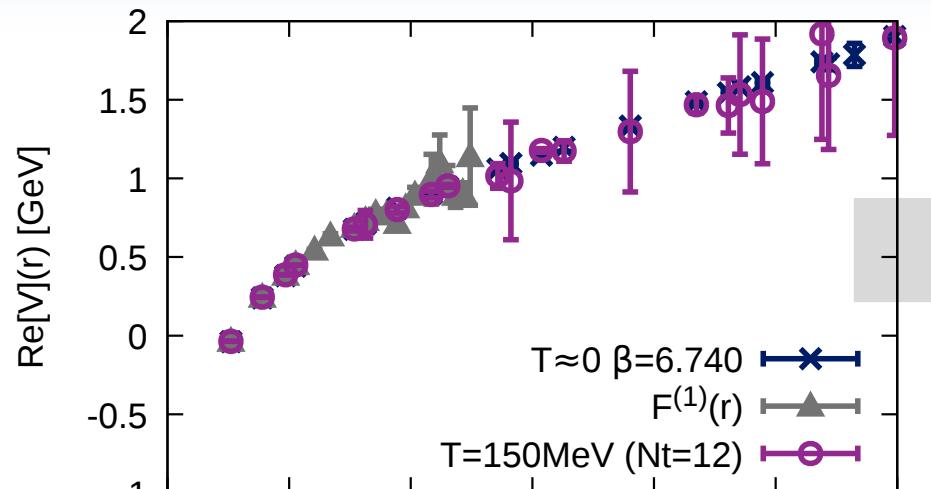
# A closer look at $\text{Re}[V]$



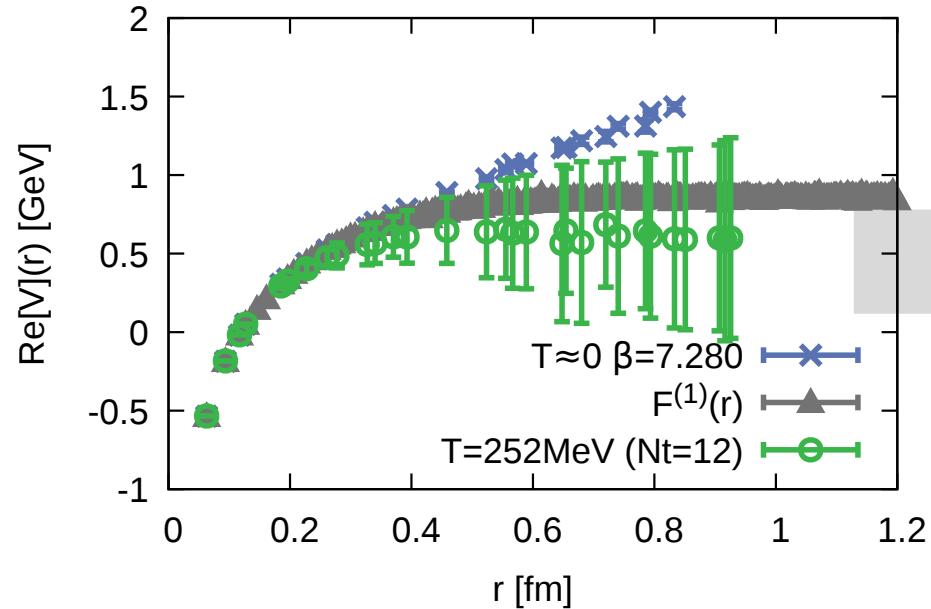
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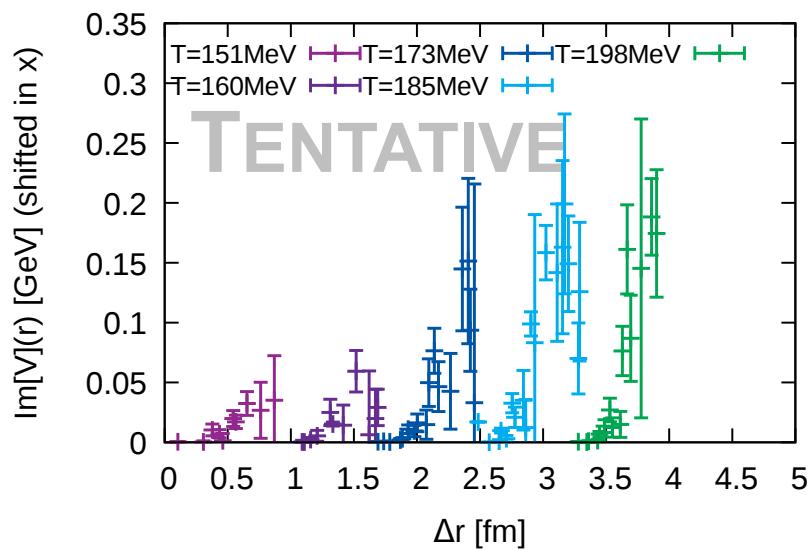
just below  $T_c$ : no screening  
 $\text{Re}[V]$  &  $F^1$  agree with  $T=0$   $\text{Re}[V]$



in QGP phase:  $\text{Re}[V]$  weakens  
 compared to  $T=0$  case. Still  
 $\text{Re}[V]$  &  $F^1$  agree upto uncertainty.

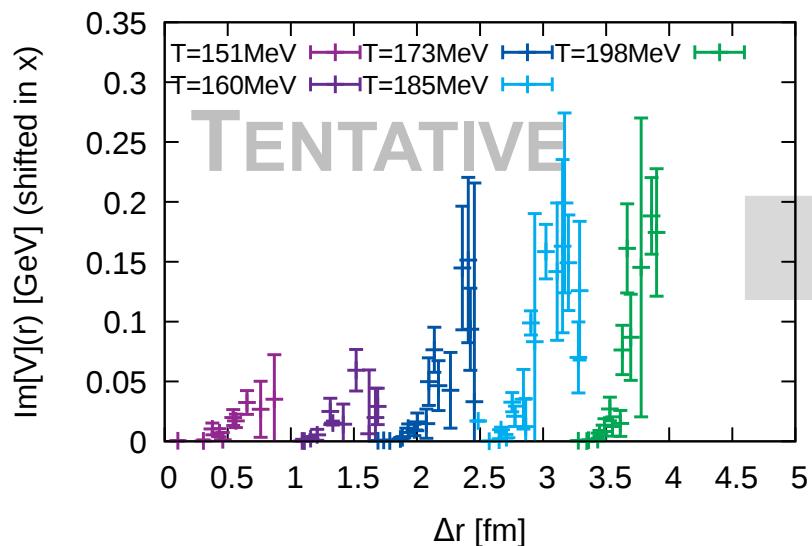
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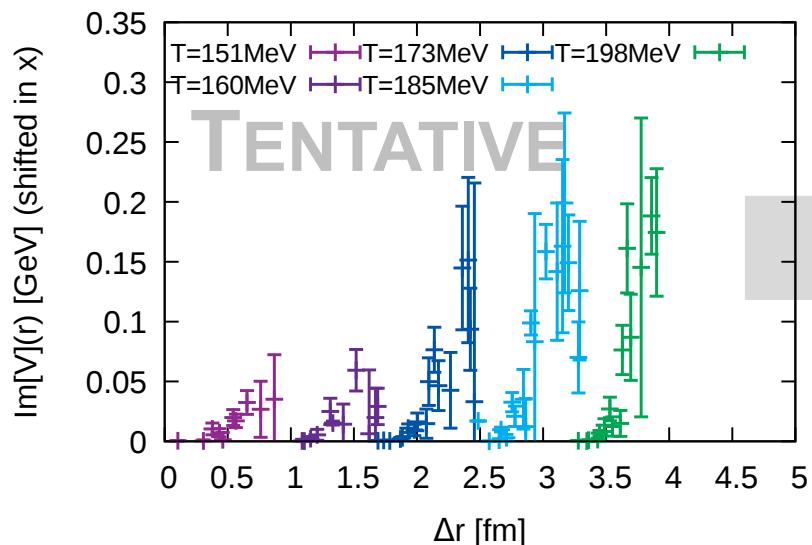
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- Improving the extraction of  $\text{Im}[V]$  urgently needed, requires larger  $N_t \sim 48$

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V.V. Dixit Mod.Phys.Lett. A5 (1990) 227

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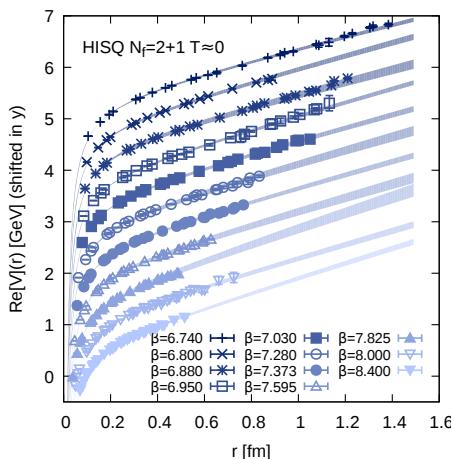
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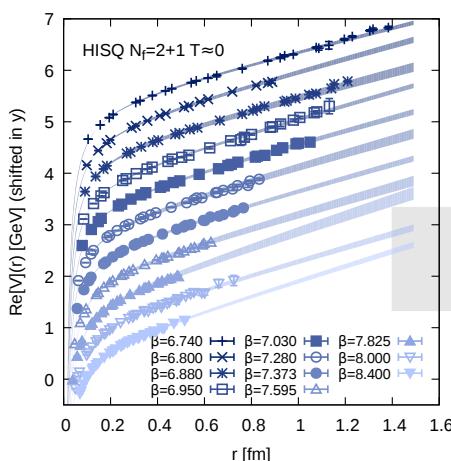


fit T=0 Cornell parameters  
 $V_{\text{Cornell}}(r) = -\alpha_s/r + \sigma r + c$

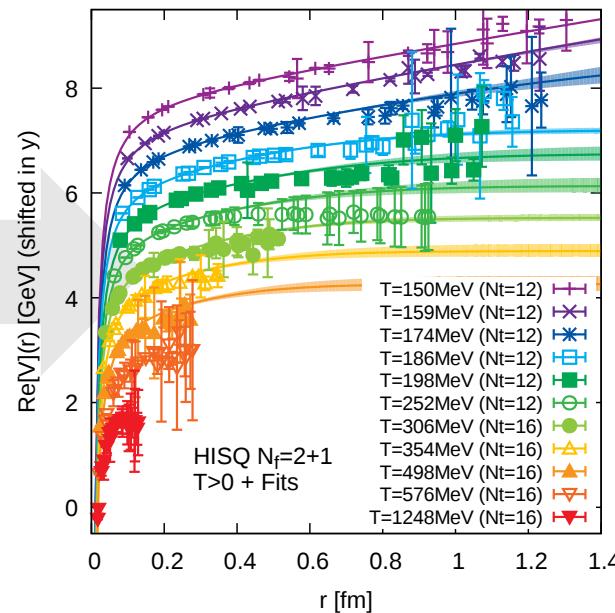
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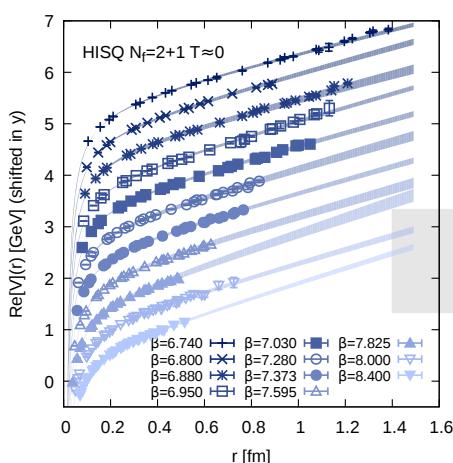


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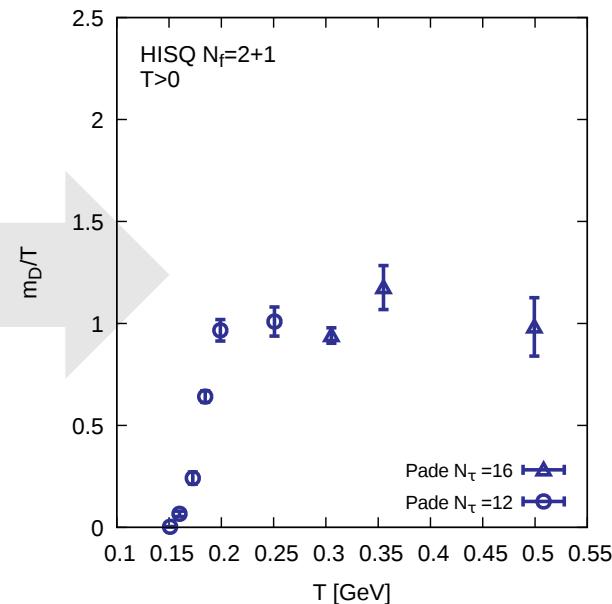
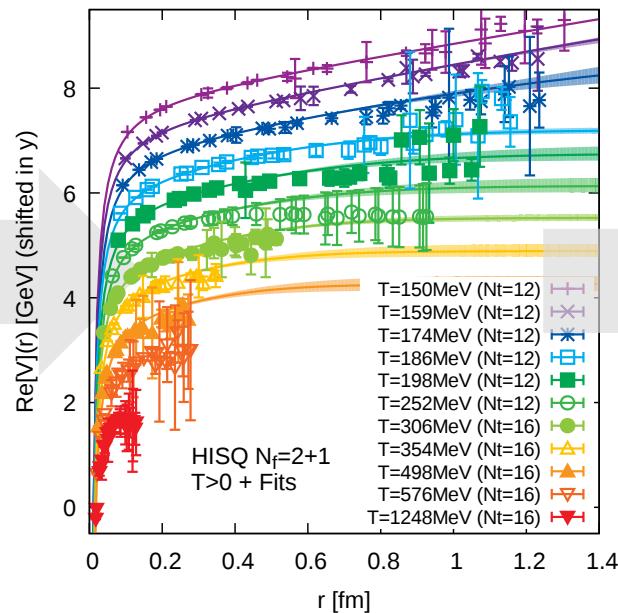


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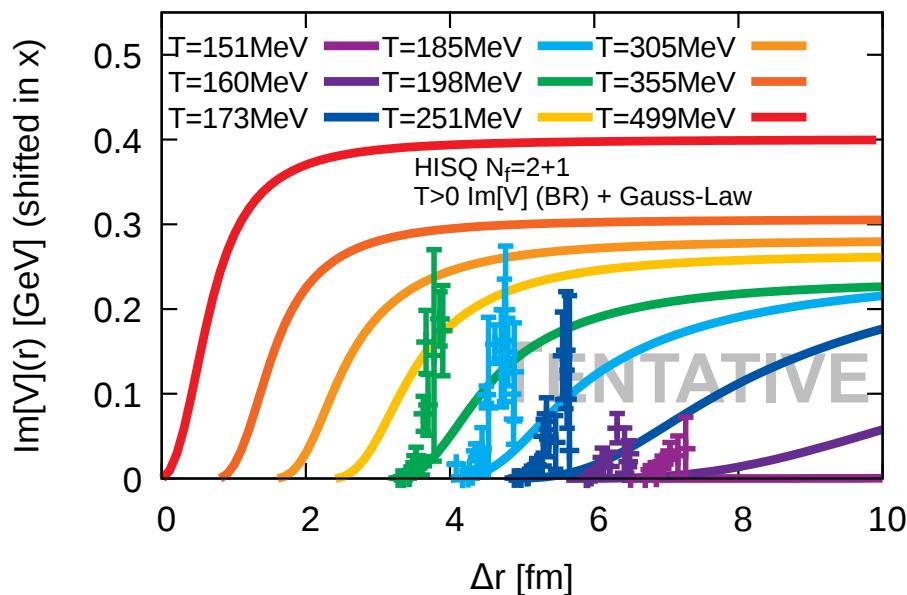
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- Lattice  $\text{Re}[V]$  well described by tuning  $m_D$ , reflects smooth onset for  $T>T_c$

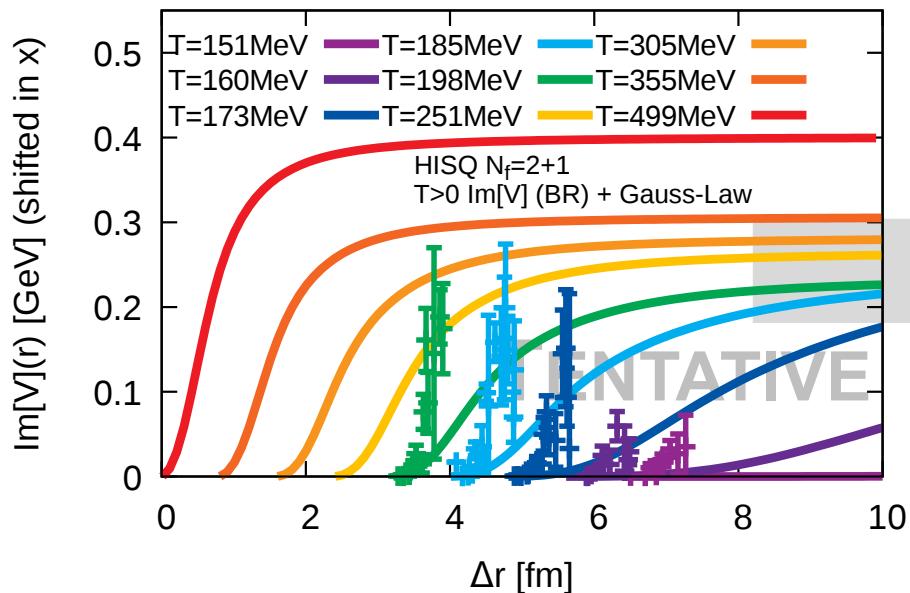
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Gauss Law parameterization seems to give slightly lower values than extracted just above  $T_c$ .

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**Grazie per la vostra attenzione – Thank you for your attention**