Quarkonia Production in Heavy Ion Collisions: Coupled Boltzmann Transport Equations

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Introduction

- Debye (static) screening on heavy quark bound state, not enough explain quarkonia production suppression

- Production complicated by many factors:

  - Cold nuclear matter (CNM) initial production

  - Static screening (real part potential suppressed) v.s. dynamical screening (imaginary part potential, related to dissociation)

  - In-medium evolution (dissociation and recombination)

  - Feed-down, etc.

- **Include all factors consistently**

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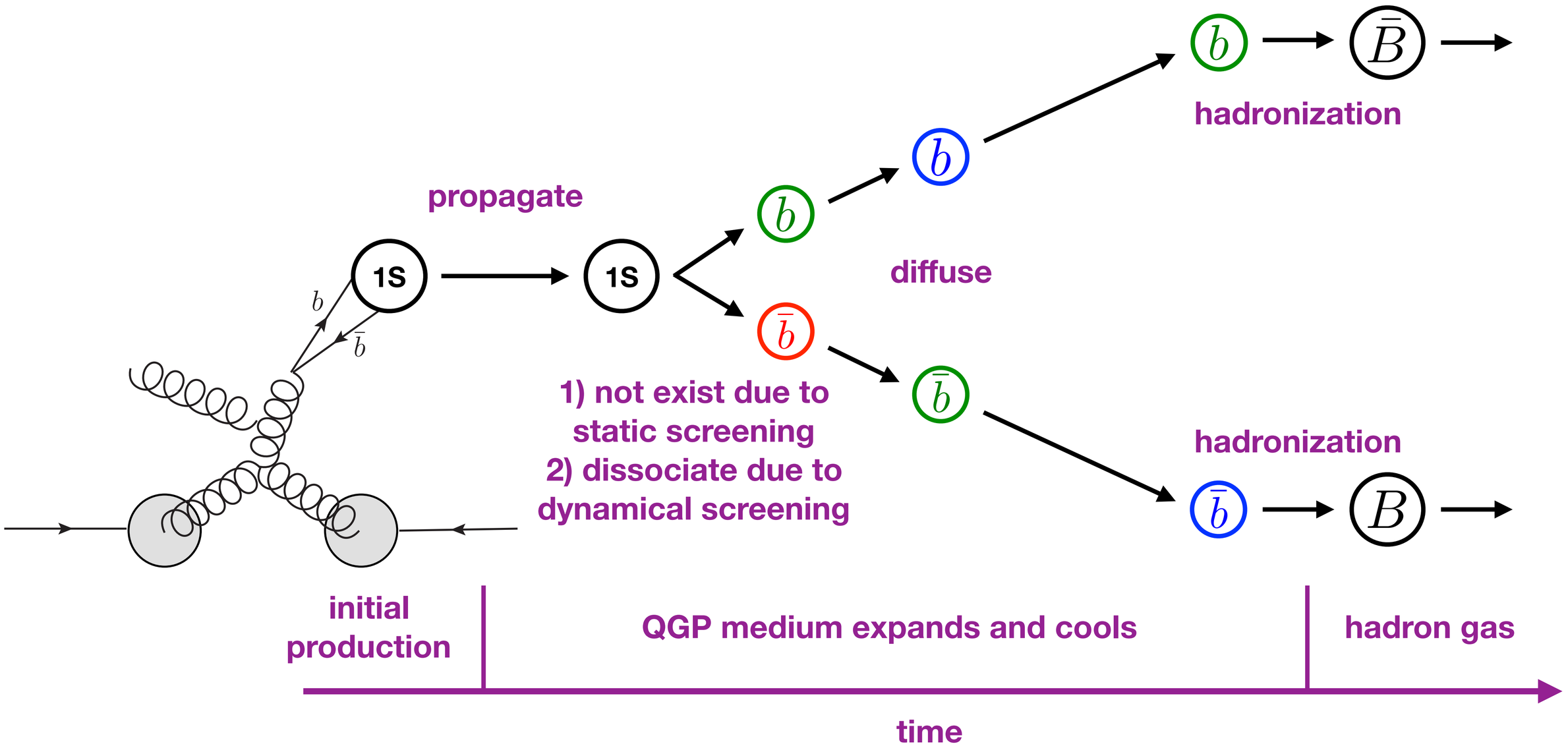
Dynamical Evolution: Dissociation



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Dynamical Evolution: Recombination

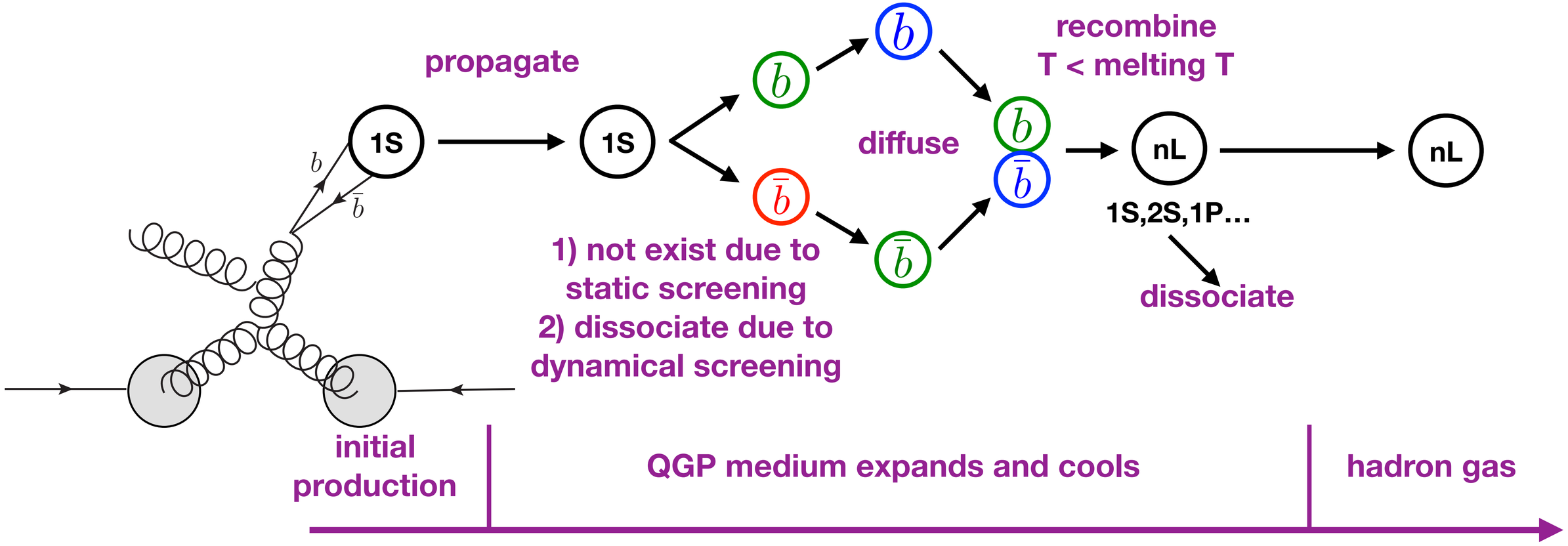
melting temperature: above which a specific bound state

1) ill defined (thermal width too large)

2) not exists (potential not supports bound state)

in-medium formation

RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)



Coupled Boltzmann Equations

heavy quark

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_Q^+ + \mathcal{C}_Q^- + \mathcal{C}_Q$$

anti-heavy quark

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^- + \mathcal{C}_{\bar{Q}}$$

each quarkonium state
nl = 1S, 2S,1P etc.

$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = +\mathcal{C}_{nl}^+ - \mathcal{C}_{nl}^-$$

Coupled Boltzmann Equations

phase space evolution of distribution function

heavy Q energy loss

heavy quark

anti-heavy quark

each quarkonium state
nl = 1S, 2S,1P etc.

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_Q^+ + \mathcal{C}_Q^- + \mathcal{C}_Q$$

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^- + \mathcal{C}_{\bar{Q}}$$

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = +\mathcal{C}_{nl}^+ - \mathcal{C}_{nl}^-$$

recombination **dissociation**

quarkonium gain **quarkonium loss**

heavy quark loss **heavy quark gain**

Heavy Quark Energy Loss: Linearized Boltzmann

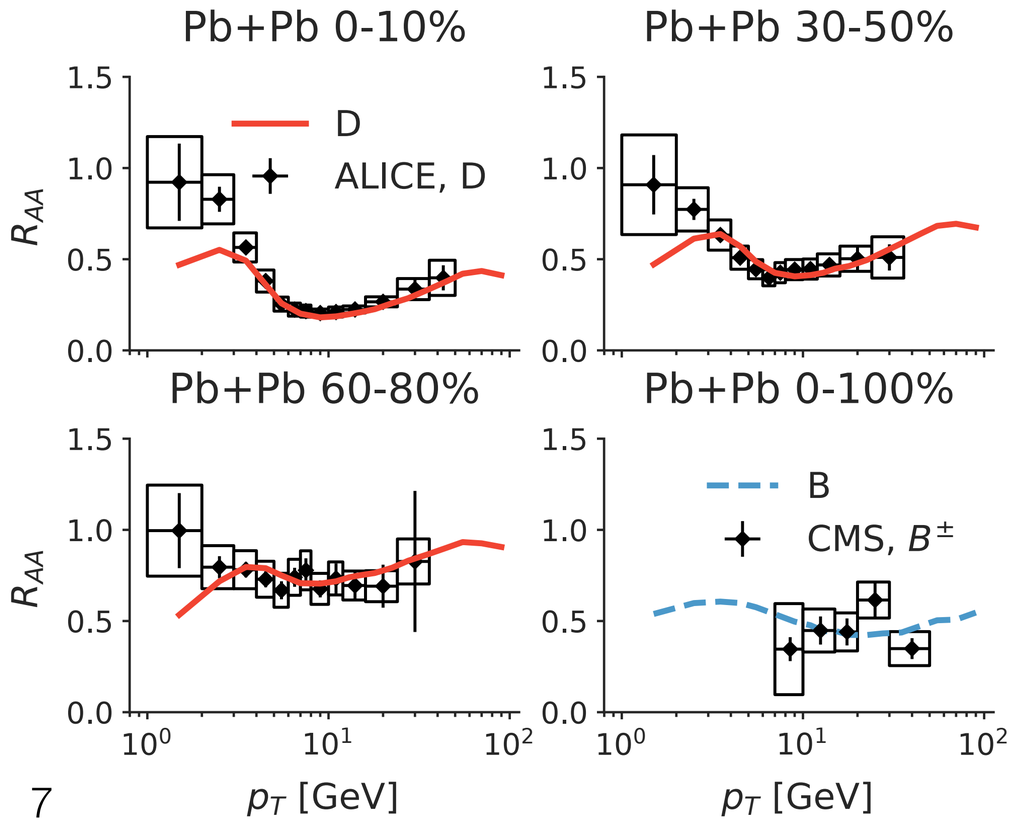
$$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = -\mathcal{C}_Q^+ + \mathcal{C}_Q^- + \boxed{\mathcal{C}_Q}$$

- Collision terms: gQ—>gQ, qQ—>qQ, gQ—>gQg, qQ—>qQg, gQg—>gQ, qQg—>qQ (LPM effect)

Gossiaux, Aichelin Phys.Rev.C78,014904(2008)
Gossiaux, Bierkandt, Aichelin Phys.Rev.C79,044906(2009)
Uphoff, Fochler, Xu, Greiner J.Phys.G42,no.11,115106(2015)

- Specific implementation: Duke LBT describe open heavy R_{AA} and v_2 developed by Weiyao Ke, Yingru Xu, Steffen Bass

- See posters for details
ID: 288 (W. Ke) 300 (Y. Xu)



Dissociation, Recombination, pNRQCD

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}\right) f_{nl}(\boldsymbol{x}, \boldsymbol{p}, t) = \boxed{+\mathcal{C}^{+}_{nl}} \boxed{-\mathcal{C}^{-}_{nl}}$$

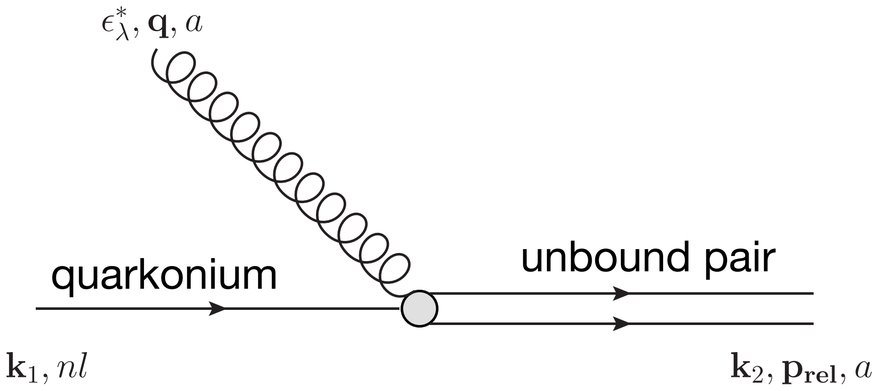
$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + V_A({\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\mathbf{r}\cdot g\mathbf{E}, {\rm O}\} + \cdots \Big)$$

Brambilla, Ghiglieri, Vairo, Petreczky, Phys. Rev. D 78, 014017 (2008)
Brambilla, Escobedo, Ghiglieri, Vairo, JHEP1112,116(2011)JHEP1305,130(2013)

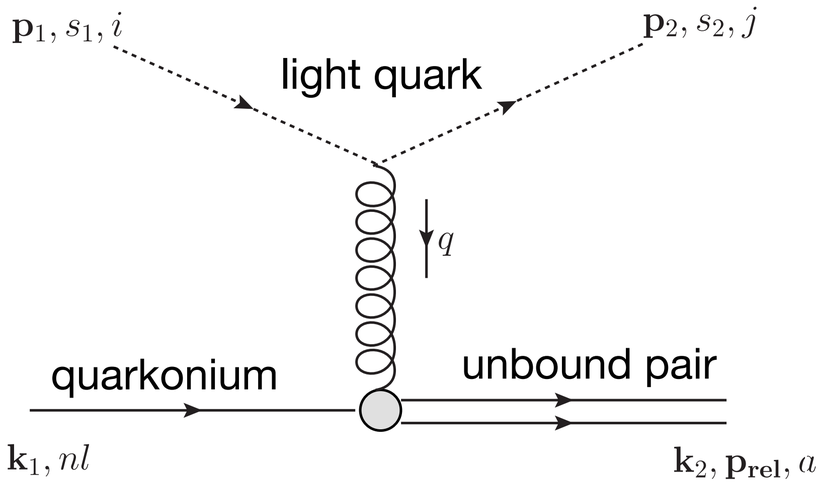
- Separation of scales (bound state exists) $M \gg Mv \gg Mv^2, T, m_D$
- Systematic expansion in $\frac{1}{M}$, $r \sim \frac{1}{Mv}$

$$H_{s,o} = \frac{P^2_{\rm c.m.}}{4M} + \boxed{\frac{p^2_{\rm rel}}{M} + V^{(0)}_{s,o}} + \frac{V^{(1)}_{s,o}}{M} + \frac{V^{(2)}_{s,o}}{M^2} + \cdots$$ virial theorem

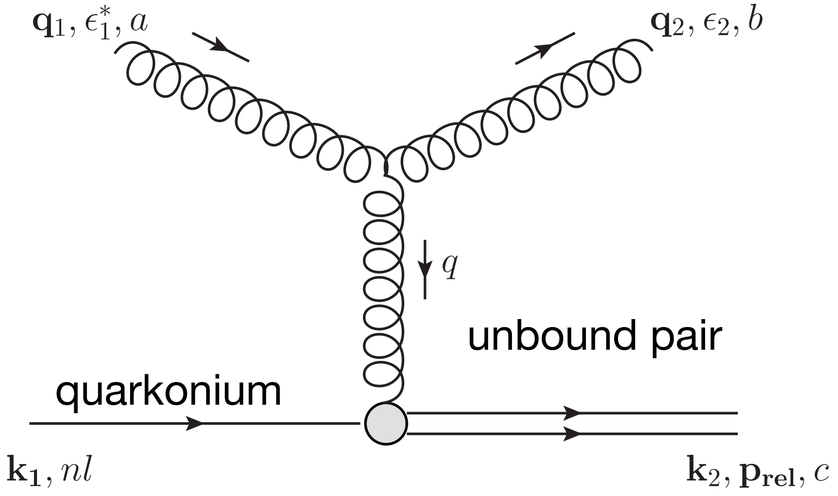
$$V^{(0)}_s = -C_F \frac{\tilde{\alpha}_s}{r} \qquad V^{(0)}_o = \frac{1}{2N_C}\frac{\tilde{\alpha}_s}{r}$$ **no imaginary potential**



gluon absorption/emission



inelastic scattering w/ light quark



inelastic scattering w/ gluon

Approach Equilibrium

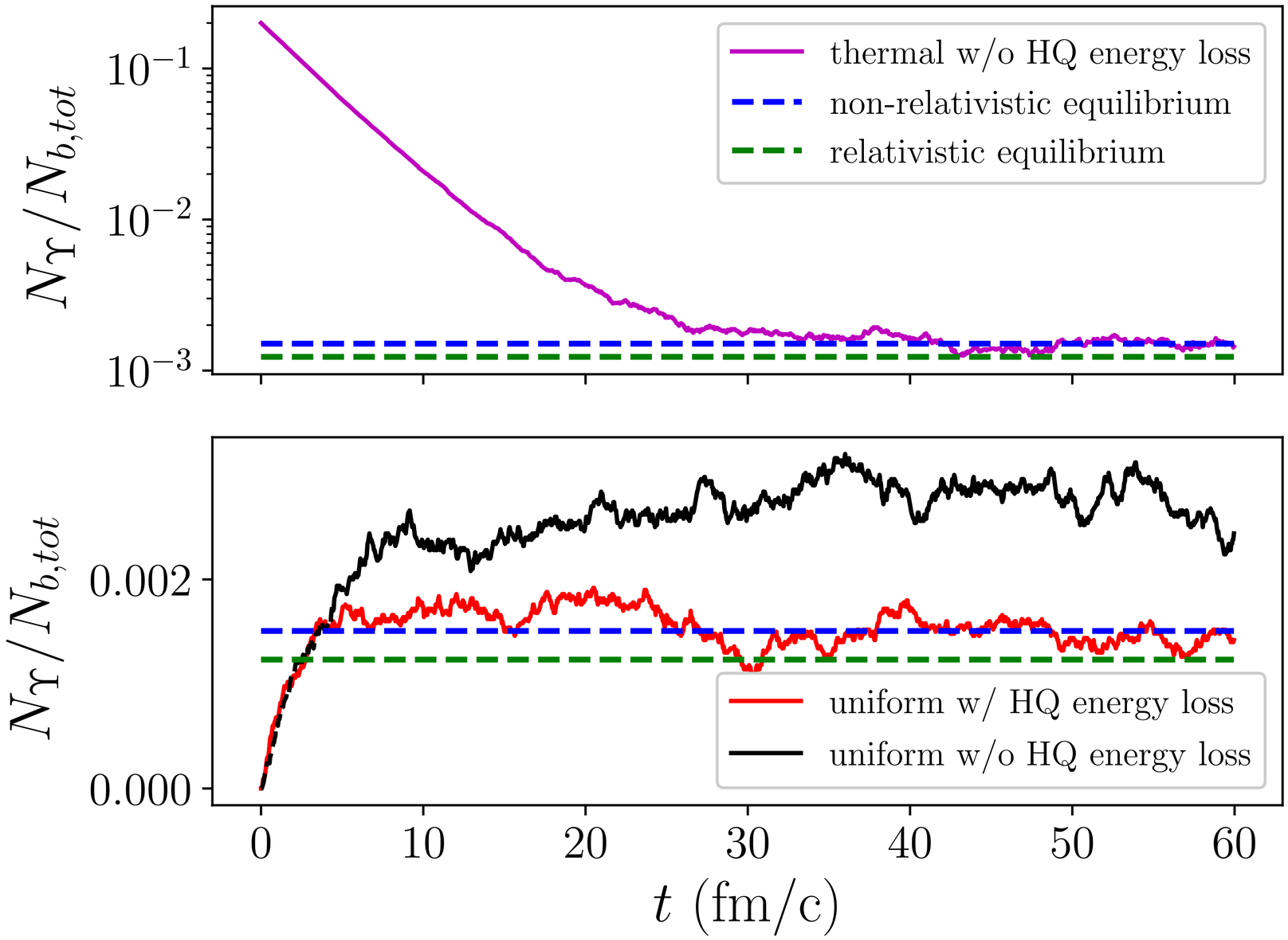
XY, B.Mueller, Phys. Rev. C 97, no. 1, 014908 (2018)

Setup:
QGP box, 1S state, b quark
Total b flavor = 50 (fixed)
Initial momenta thermal or uniform

Recombination from QCD effective field theory

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy loss necessary to drive kinetic equilibrium of quarkonium



Collision Event Simulation

- Initial production:

PYTHIA 8.2 Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159

Nuclear PDF (cold nuclear matter effect) Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

Trento, sample position, hydro. initial condition Bernhard, Moreland, Bass, Liu, Heinz, Phys.Rev.C94,no.2,024907(2016)

- Medium background: 2+1D viscous hydrodynamics Song, Heinz, Phys.Rev.C77,064901(2008) Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)

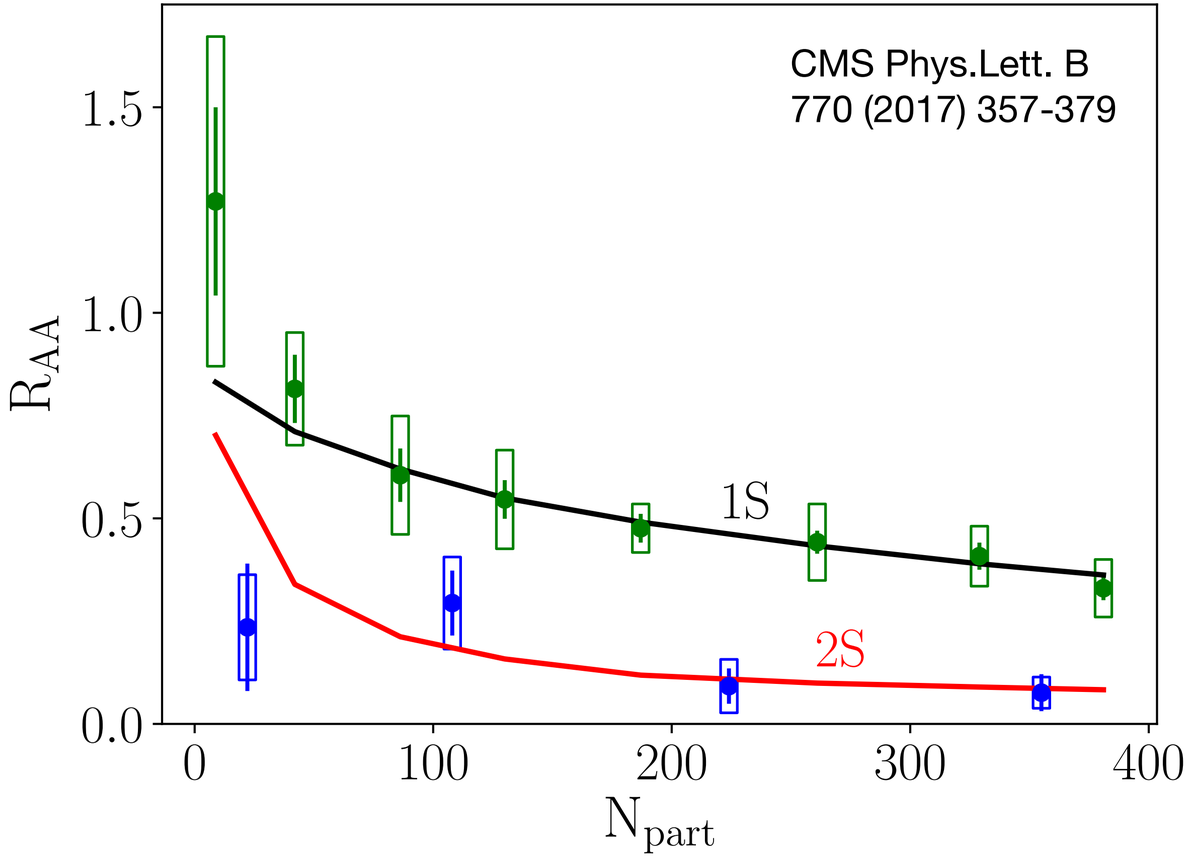
- Include 1S 2S, 2S feed-down 1S ~ 26% (from PDG)

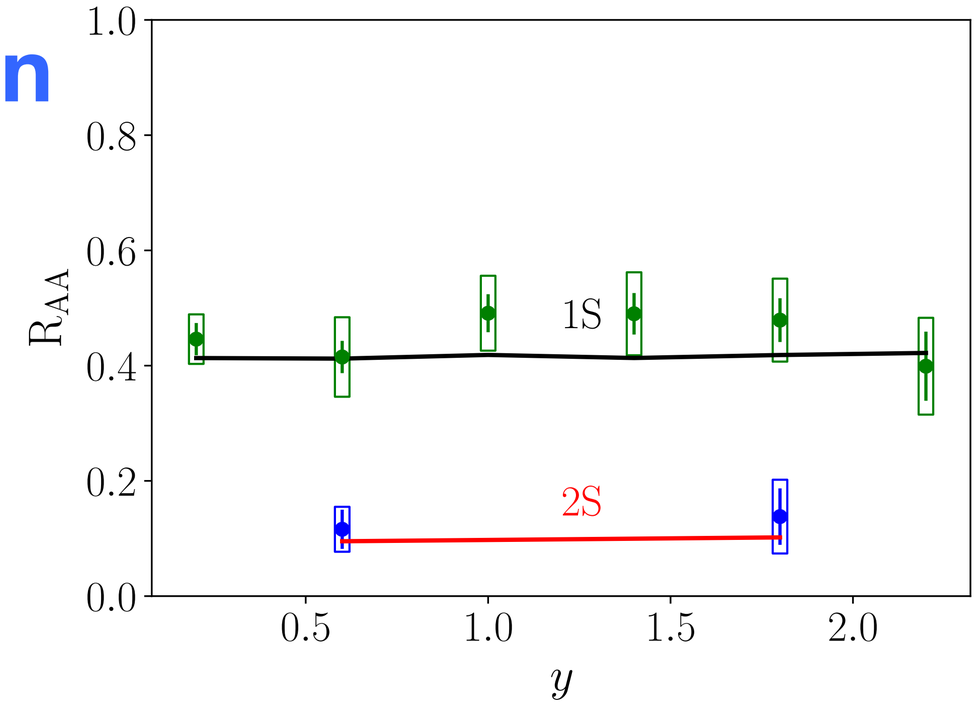
Upsilon in 2760 GeV PbPb Collision

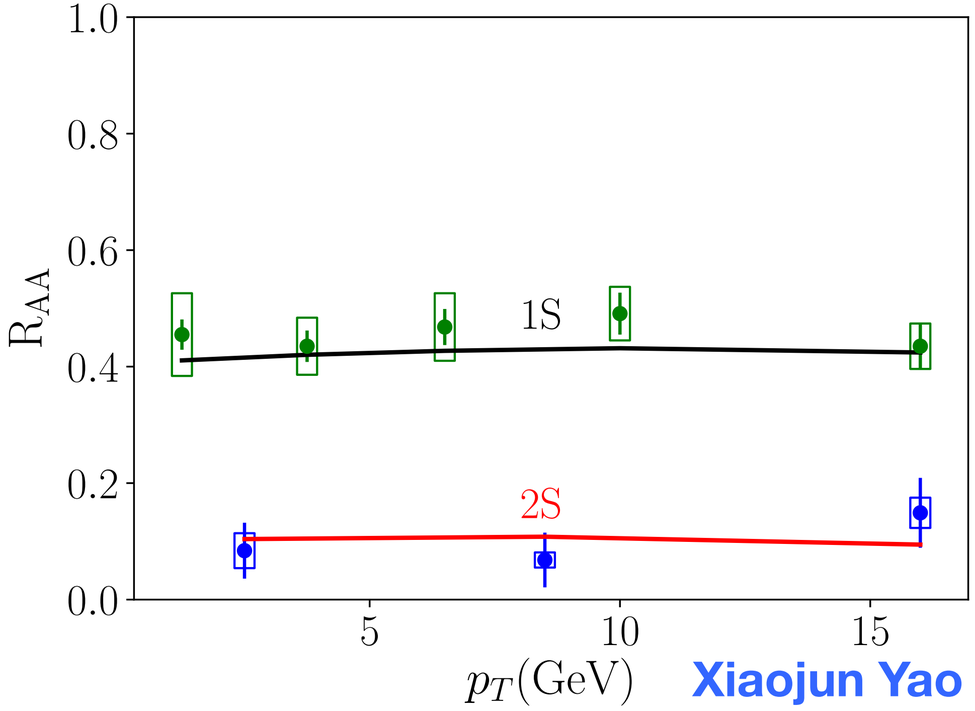
$$\alpha_s = 0.3$$

$$T_{\mathrm{melt}}(2S) = 210\ \mathrm{MeV}$$

$$V_s = -C_F \frac{0.42}{r}$$







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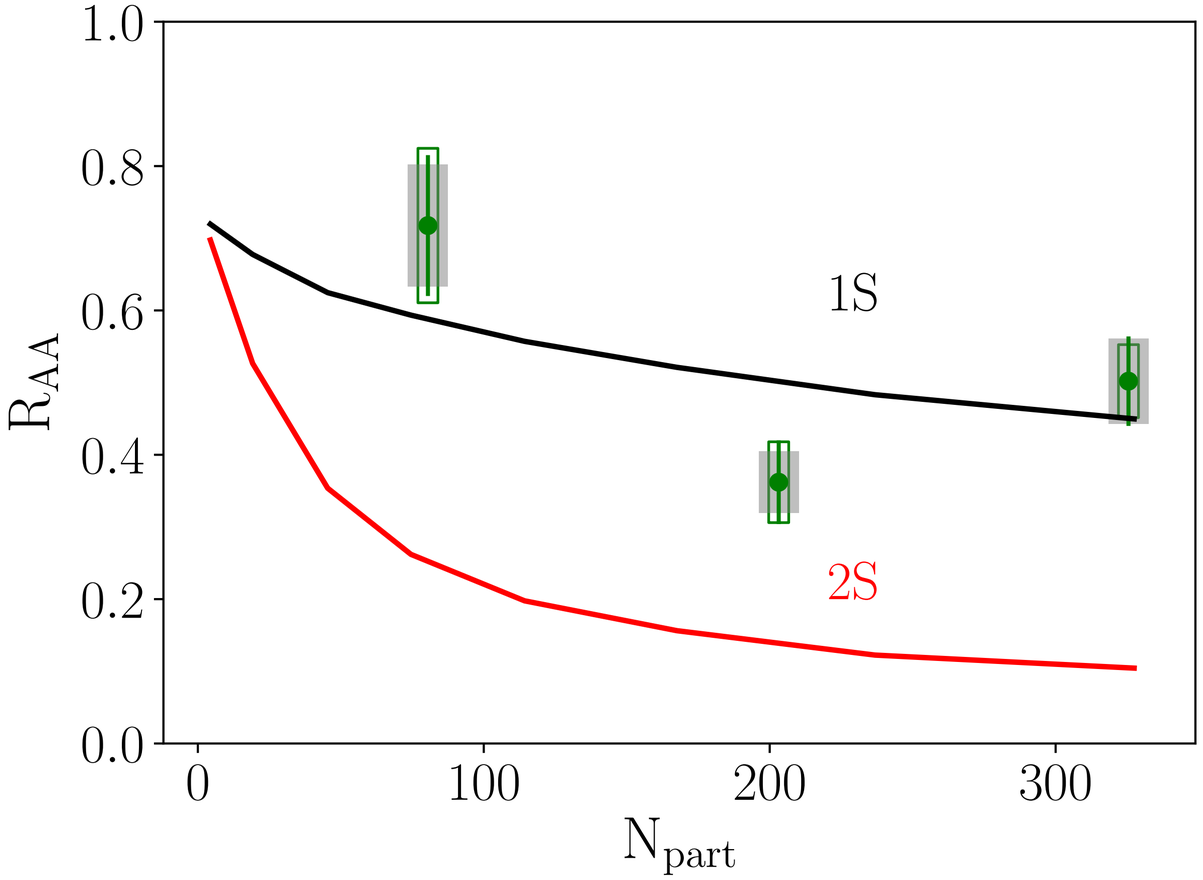
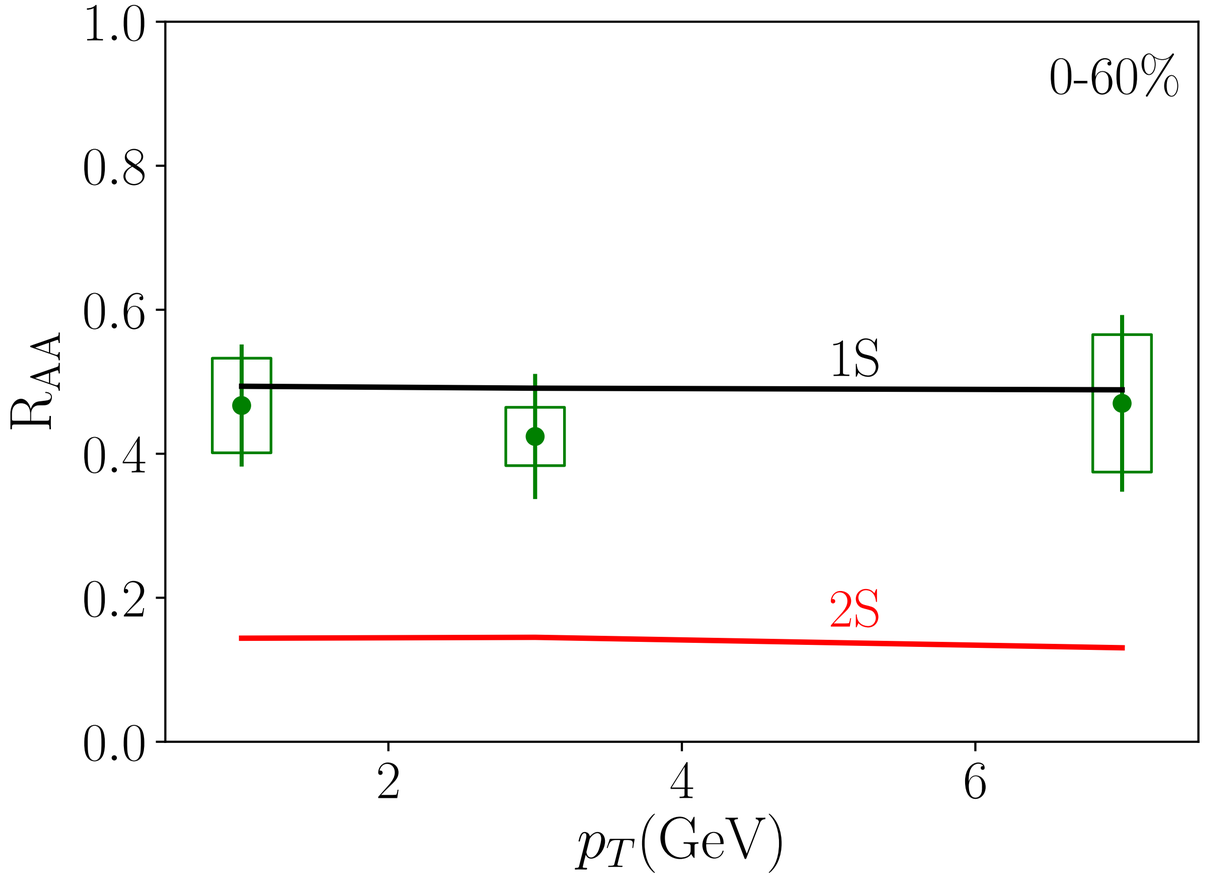
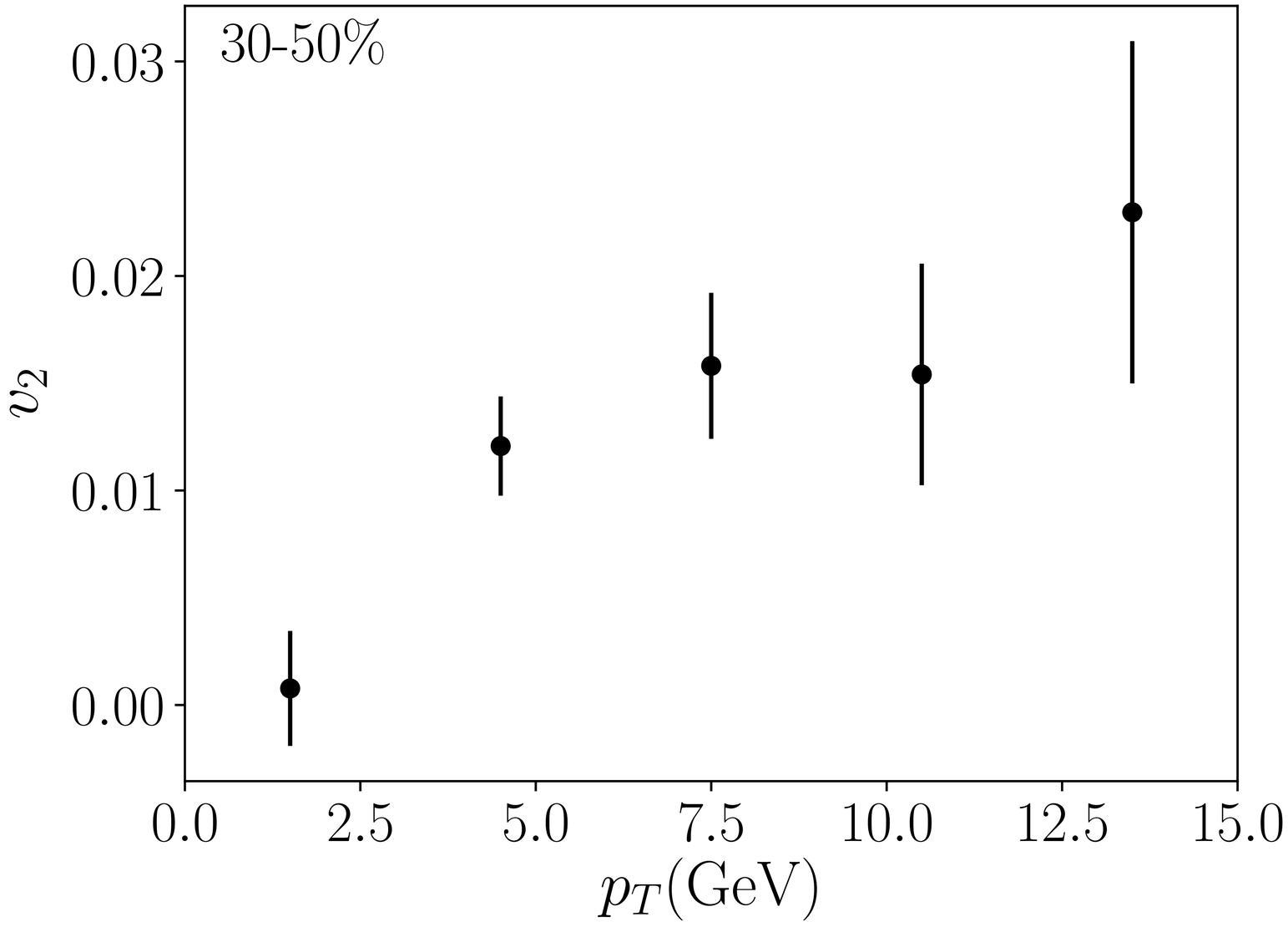
Upsilon in 200 GeV AuAu Collision





STAR Talks at QM 17&18

Upsilon(1S) Azimuthal Anisotropy in 2760 GeV PbPb



Develop azimuthal momentum anisotropy from heavy quark recombination

Better understand recombination from v_2 measurements

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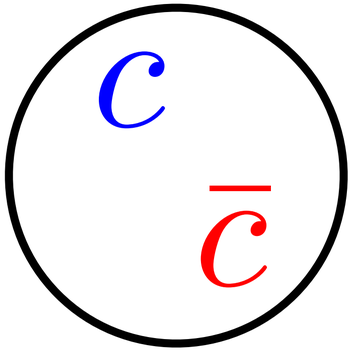
Doubly Charmed Baryon

- LHCb observed a new baryon Ξ_{cc}^{++}(ccu): u bound around cc core

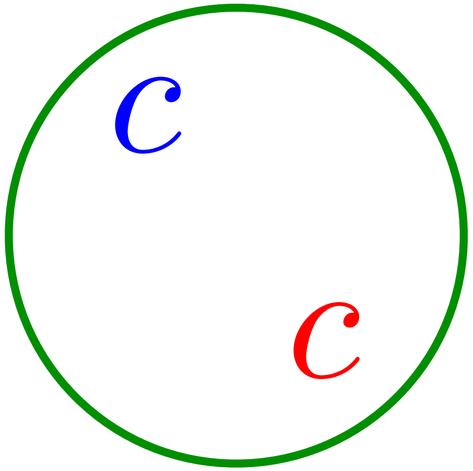
LHCb, Phys. Rev. Lett. 119, no.11,112001 (2017)

- Pair of heavy Q in anti-triplet forms bound state (diquark)



J/ψ

$Q\bar{Q}$ singlet
color neutral
exist in vacuum



QQ anti-triplet
colored
not exist in vacuum
exist in QGP

cc diquark (1S)

- Heavy diquark in QGP: dissociation, recombination (similar to quarkonium), carry color, energy loss different from quarkonium

- Hadronize into doubly charmed baryon

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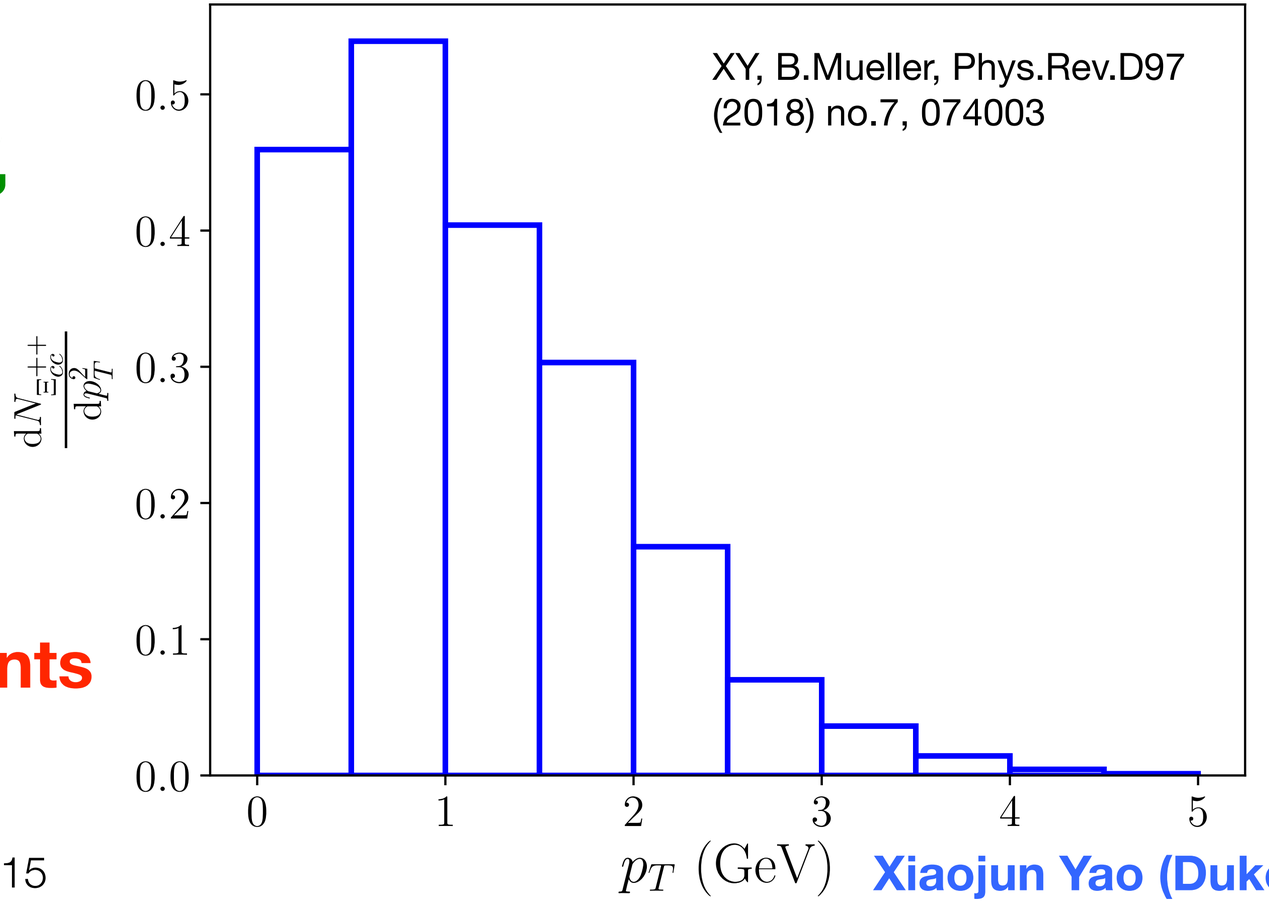
Doubly Charmed Baryon Production in Heavy Ion Collisions

Setup:
coupled Boltzmann for charm quark and diquark (add energy loss of diquark)
assume only charm quark produced initially, diquark comes from (re)combination

Predicted production rate
in 2760 GeV PbPb, -1<y<1, 0<pT<5 GeV,
Ξ_{cc}^{++} 0.02 per collision
with melting temperature = 250 MeV:
Ξ_{cc}^{++} 0.0125 per collision

Study recombination from measurements



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Summary

- Describe both open and hidden heavy flavors: coupled Boltzmann equation
- Consistent dissociation and recombination from pNRQCD
- Extract potential and melting temperature from data
- Future: include 1P 2P 3S states, temperature-dependent potential, systematic extraction procedure (e.g. Bayesian)
- Heavy diquarks and doubly heavy baryons / tetraquarks

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Backup: Thermal Equilibrium

$$N_i^{\rm eq} = g_i {\rm Vol} \int \frac{d^3p}{(2\pi)^3} \lambda_i e^{-E_i(p)/T}$$

$$E_i(p) = \sqrt{M_i^2 + p^2}, \qquad M_i + \frac{p^2}{2M_i}$$

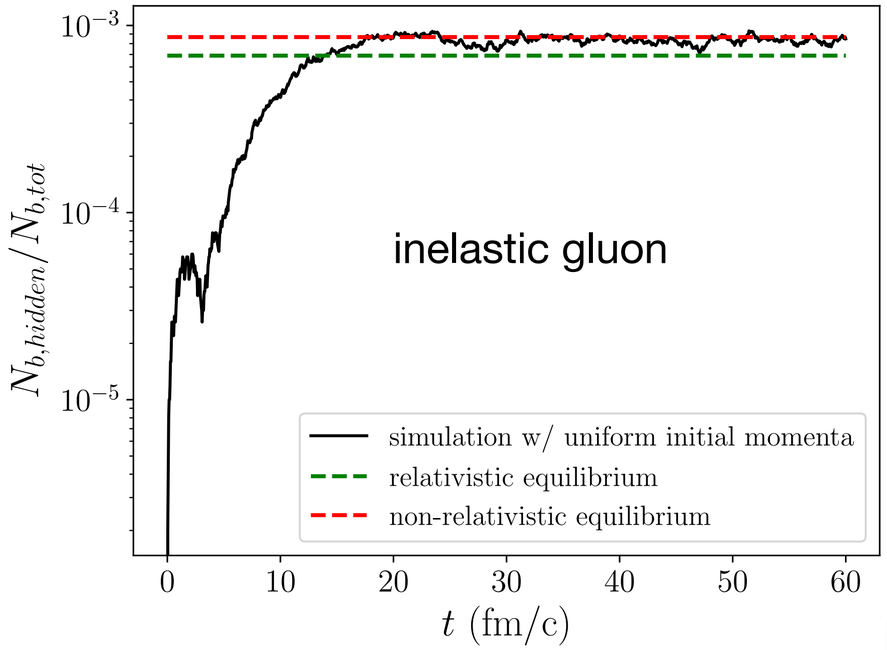
$$g_b = g_{\bar{b}} = 3 \times 2 = 6 \qquad \lambda_b = \lambda_{\bar{b}}$$

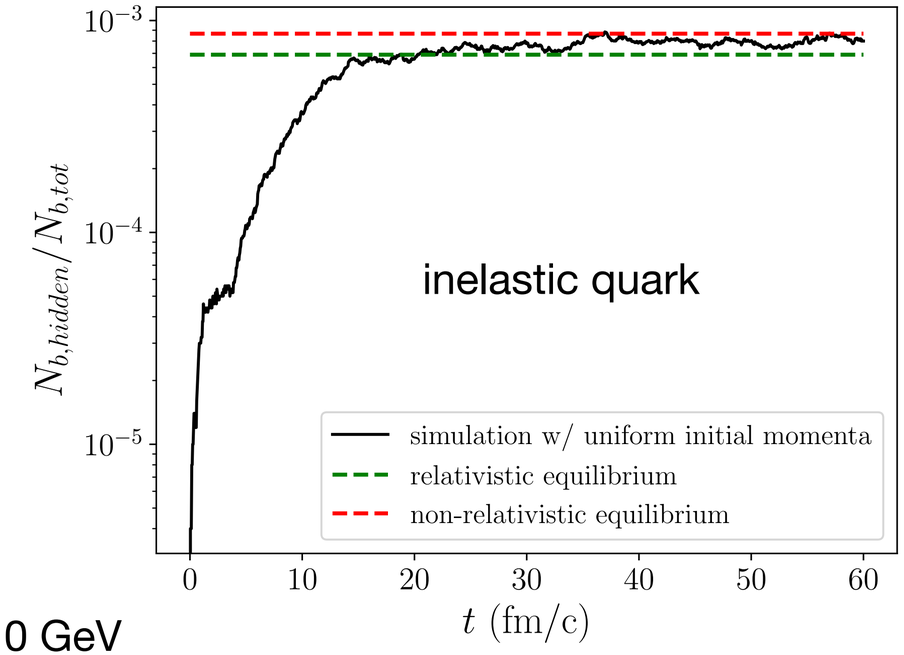
$$g_{\Upsilon(1S)} = 3 + 1 = 4 \qquad \lambda_b^2 = \lambda_{\Upsilon(1S)}$$

hyperfine splitting not considered

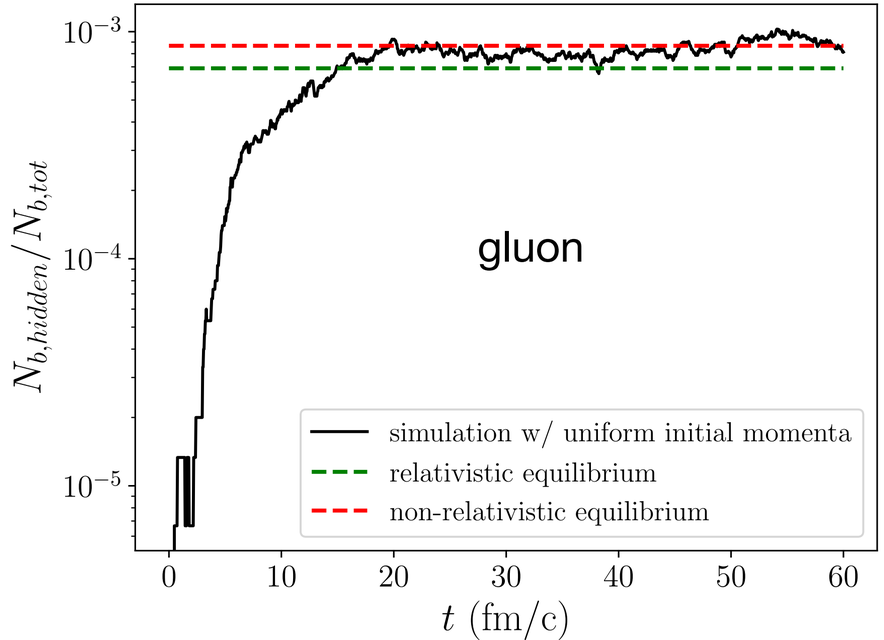
$$N_b^{\rm eq} + N_{\Upsilon(1S)}^{\rm eq} = N_{b,{\rm tot}}$$

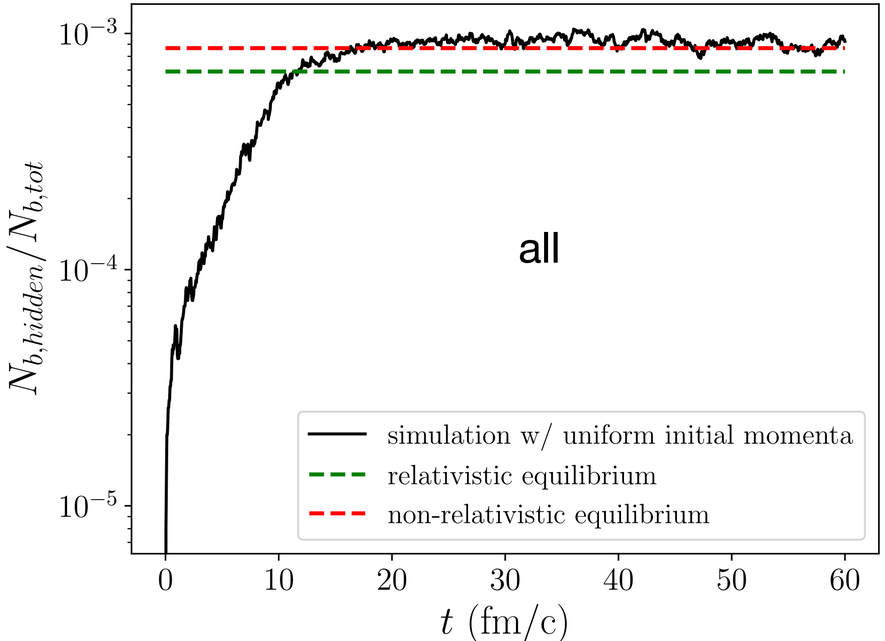
Backup: Thermal Equilibrium





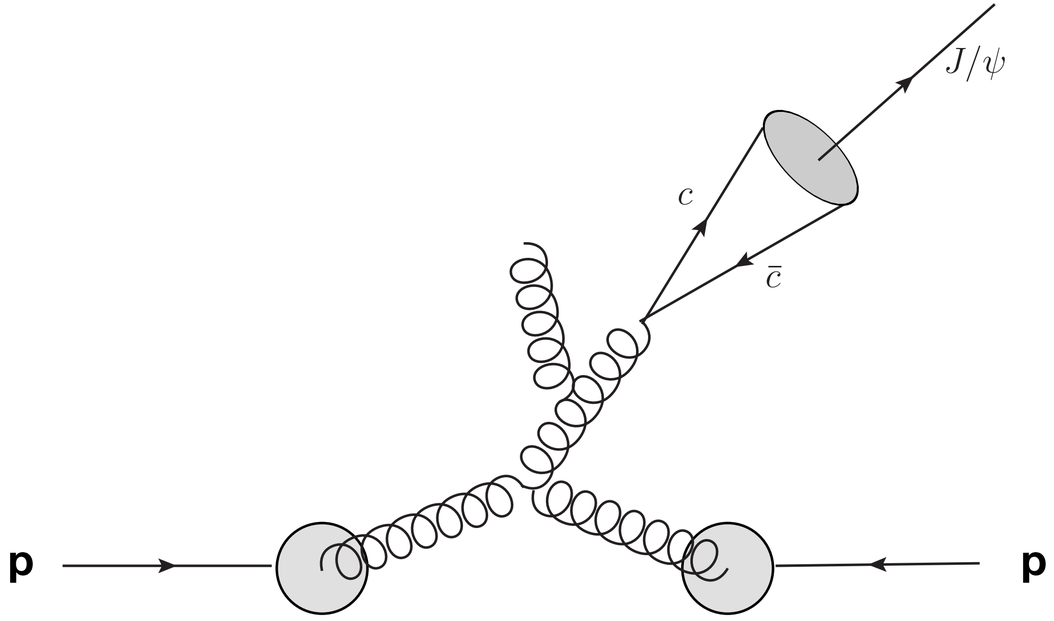
initial p uniform in 10 GeV





Backup: Initial Production

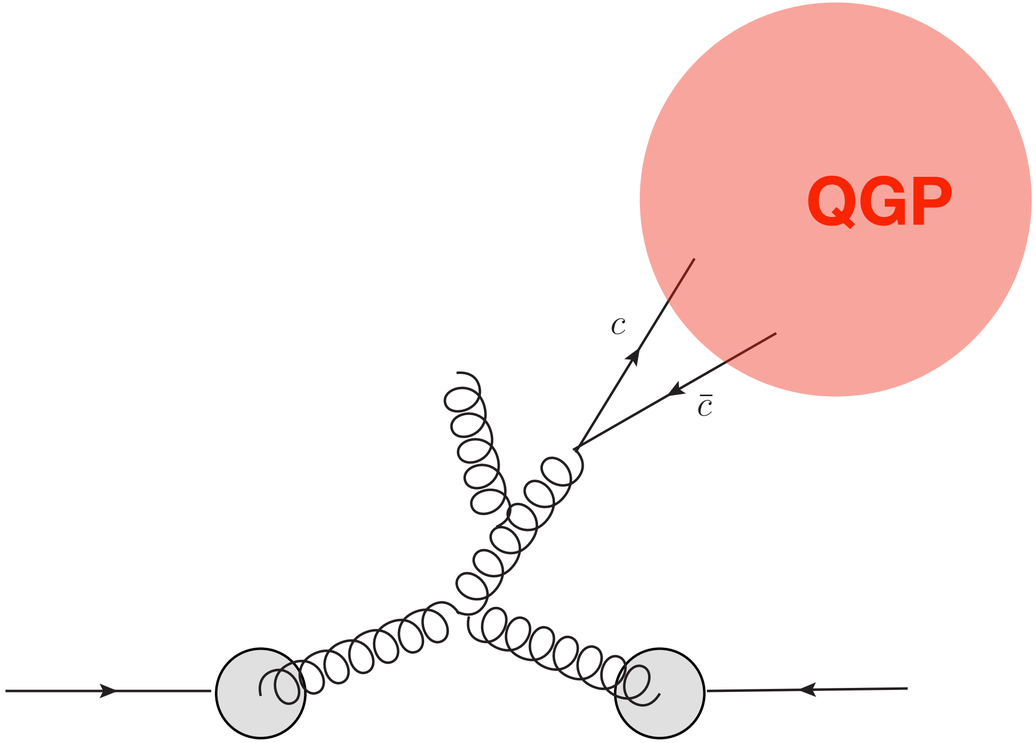
- Quarkonium production in pp collisions NRQCD factorization
  - short-distance production of heavy quarks ~ 1/M
  - long-distance coalescence into quarkonium ~ 1/E



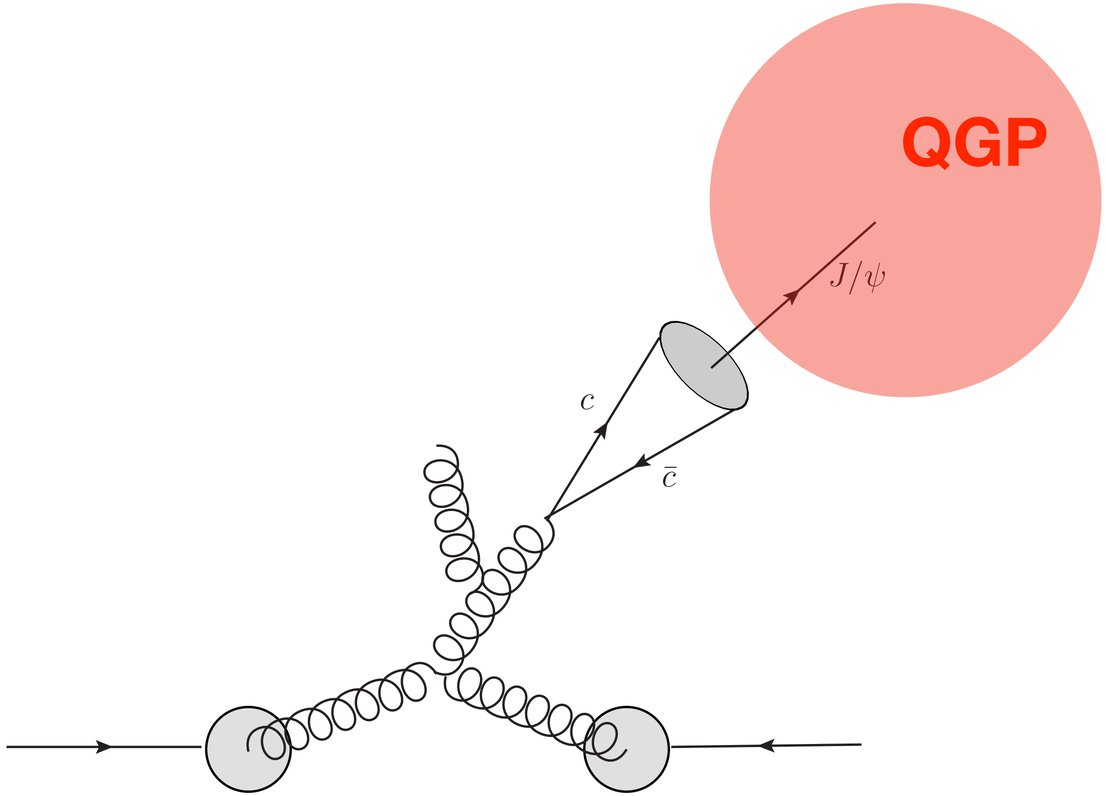
Bodwin, Braaten, Lepage

Phys. Rev. D 51, 1125 (1995)

Backup: Initial Production



Initially no quarkonium enters QGP
quarkonium is formed (recombined)
inside QGP or later
(re)combination dominates



Initially quarkonium is generated
and enters QGP
suppressed due to screening
dissociation dominates

Backup: Numerical Implementation

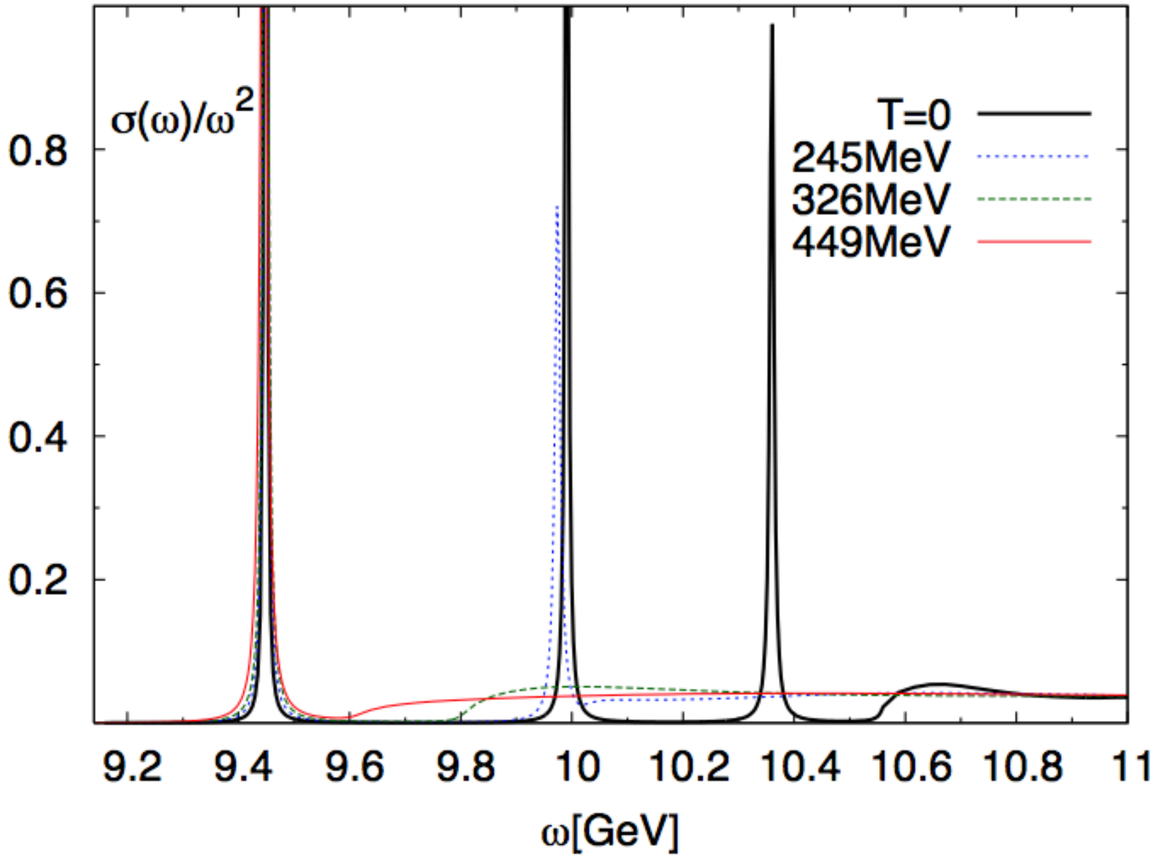
- Test particle Monte Carlo $f(\boldsymbol{x},\boldsymbol{p},t) = \sum_i \delta^3(\boldsymbol{x}-\boldsymbol{y}_i(t))\delta^3(\boldsymbol{p}-\boldsymbol{k}_i(t))$
- Each time step: consider diffusion, dissociation, recombination
- If specific process occurs, sample incoming medium particles and outgoing particles conserving energy momentum
- Recombination term contains $f_Q(\boldsymbol{x},\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x},\boldsymbol{p}_2,t)$

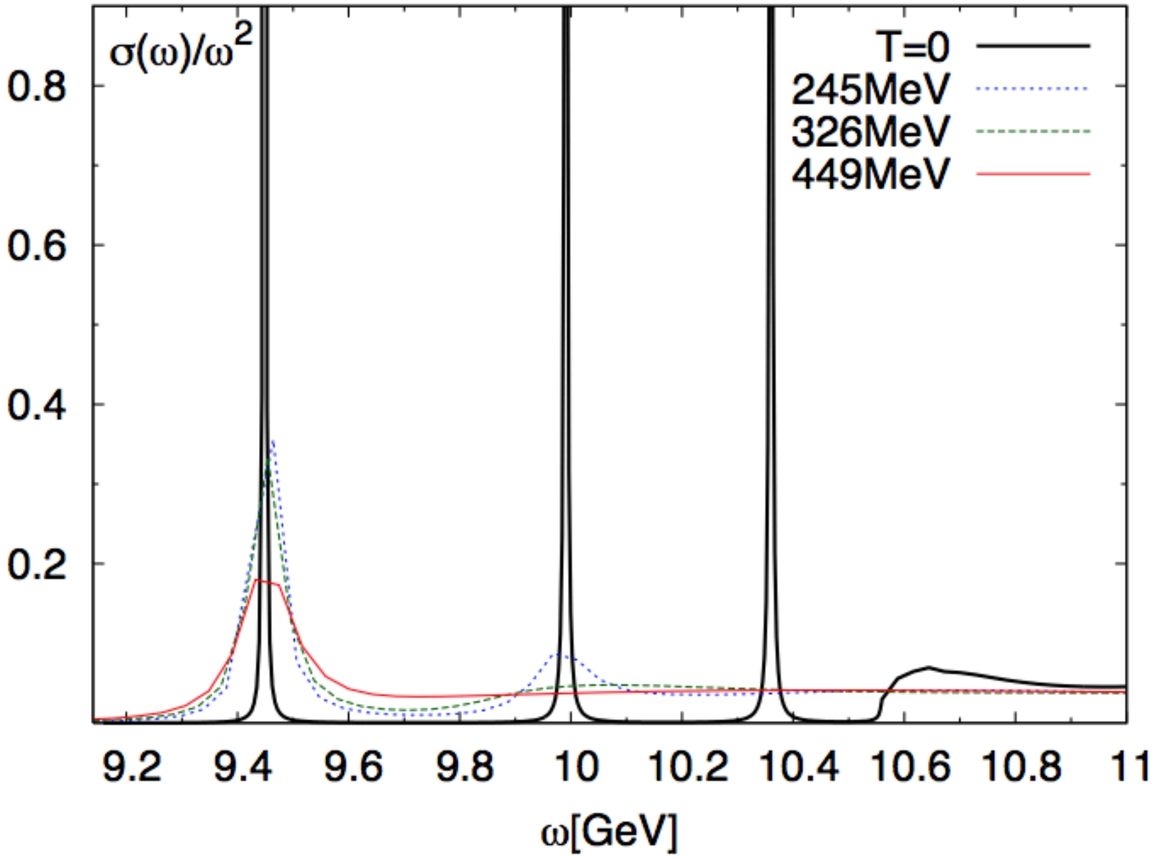
Two delta at same x ill-defined, almost never at same point

Enhance sampling for recombination

$$f_Q(\boldsymbol{x},\boldsymbol{p}_1,t)f_{\bar{Q}}(\boldsymbol{x},\boldsymbol{p}_2,t) \to \sum_{i,j} \frac{e^{-(\boldsymbol{y}_i-\boldsymbol{y}_j)^2/2a_B^2}}{(2\pi a_B^2)^{3/2}} \delta^3\left(\boldsymbol{x}-\frac{\boldsymbol{y}_i+\boldsymbol{y}_j}{2}\right)\delta^3(\boldsymbol{p}_1-\boldsymbol{k}_i)\delta^3(\boldsymbol{p}_2-\boldsymbol{k}_j)$$

Backup: Imaginary Part More Important





C. Miao, A. Mocsy, P. Petreczky arXiv:1012.4433

Backup: Diquark pNRQCD, Rates

$$\mathcal{L}_{\rm pNRQCD} = \int \mathrm{d}^3 r \mathrm{Tr}\Big\{ \mathrm{T}^\dagger (iD_0 - H_T)\mathrm{T} + \Sigma^\dagger (iD_0 - H_\Sigma)\Sigma + \mathrm{T}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\Sigma + \Sigma^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}\mathrm{T}\Big\} + \cdots$$

$$\mathcal{F}^+ \equiv \frac{1}{2} g_+ \int \frac{\mathrm{d}^3 p_1}{(2\pi)^3}\frac{\mathrm{d}^3 p_2}{(2\pi)^3}\frac{\mathrm{d}^3 k_1}{(2\pi)^3}\frac{\mathrm{d}^3 q}{(2\pi)^3 2q}(1+n_B^{(q)}) f_c(\boldsymbol{x},\boldsymbol{p}_1,t) f_c(\boldsymbol{x},\boldsymbol{p}_2,t)(2\pi)^4\delta^3(\boldsymbol{k}_1+\boldsymbol{q}-\boldsymbol{k}_2)\delta(\Delta E)|\mathcal{M}|^2$$

$$\mathcal{F}^- \equiv \frac{1}{2} g_- \int \frac{\mathrm{d}^3 k_1}{(2\pi)^3}\frac{\mathrm{d}^3 k_2}{(2\pi)^3}\frac{\mathrm{d}^3 p_{\rm rel}}{(2\pi)^3}\frac{\mathrm{d}^3 q}{(2\pi)^3 2q} n_B^{(q)} f_{cc}(\boldsymbol{x},\boldsymbol{k}_1,t)(2\pi)^4\delta^3(\boldsymbol{k}_1+\boldsymbol{q}-\boldsymbol{k}_2)\delta(\Delta E)|\mathcal{M}|^2\,,$$

$$\mathcal{C}_c^\pm = \left.\frac{\delta\mathcal{F}^\pm}{\delta\boldsymbol{p}_1}\right|_{\boldsymbol{p}_1=\boldsymbol{p}} + \left.\frac{\delta\mathcal{F}^\pm}{\delta\boldsymbol{p}_2}\right|_{\boldsymbol{p}_2=\boldsymbol{p}}$$

$$\mathcal{C}_{cc}^\pm = \left.\frac{\delta\mathcal{F}^\pm}{\delta\boldsymbol{k}_1}\right|_{\boldsymbol{k}_1=\boldsymbol{p}}\,,$$

$$\frac{\delta}{\delta\boldsymbol{p}_i}\int\prod_{j=1}^n \frac{\mathrm{d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1,\boldsymbol{p}_2,\cdots,\boldsymbol{p}_n)\Big|_{\boldsymbol{p}_i=\boldsymbol{p}} \tag{20}$$

$$\equiv \frac{\delta}{\delta a(\boldsymbol{p})}\int\prod_{j=1}^n \frac{\mathrm{d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1,\boldsymbol{p}_2,\cdots,\boldsymbol{p}_n) a(\boldsymbol{p}_i)$$

$$\int\prod_{j=1,j\neq i}^n \frac{\mathrm{d}^3 p_j}{(2\pi)^3} h(\boldsymbol{p}_1,\boldsymbol{p}_2,\cdots,\boldsymbol{p}_{i-1},\boldsymbol{p},\boldsymbol{p}_{i+1},\cdots,\boldsymbol{p}_n)\,,$$

$$\mathcal{C}_c^+ \equiv \Gamma_f(\boldsymbol{x},\boldsymbol{p},t) f_c(\boldsymbol{x},\boldsymbol{p},t)$$

$$\mathcal{C}_{cc}^- \equiv \Gamma_d(\boldsymbol{x},\boldsymbol{p},t) f_{cc}(\boldsymbol{x},\boldsymbol{p},t)\,.$$