Quantum and Classical Dynamics of Heavy Quarks in a Quark-Gluon Plasma

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Work in collaboration with Jean-Paul Blaizot. ArXiv:1711.10812 and 1803.07996 (submitted to JHEP).

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Plan

The original idea of Matsui and Satz (1986)

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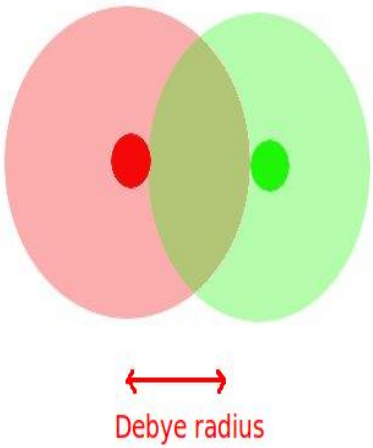
- Quarkonium is quite stable in the vacuum.
- Phenomena of colour screening, quantities measurable in Lattice QCD at finite temperature (static) support this. For example Polyakov loop.
- Dissociation of heavy quarkonium in heavy-ion collisions due to colour screening signals the creation of a quark-gluon plasma.

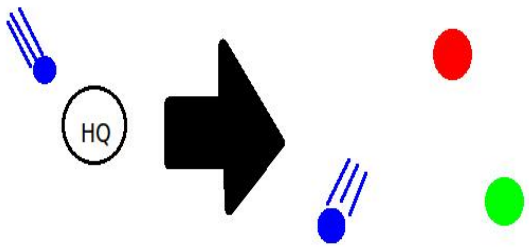
Colour screening

$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature

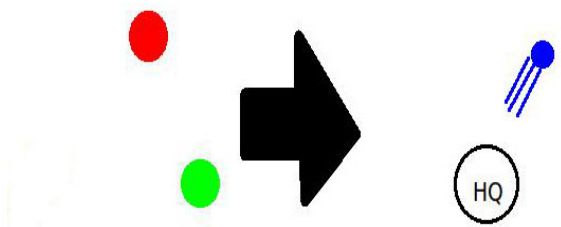


Another mechanism, collisions



A singlet can decay into an octet. Interaction with the medium changes the color state. Dissociation without screening. This is the mechanism behind the imaginary part of the potential (First found by Laine et al. (2007)).

Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

Evolution of quarkonium in a fireball

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Aim

To obtain a well motivated evolution equation that can describe quarkonium in a fireball with a small computational cost.

Plan

Setting

We treat heavy quarks in first quantization

$$H = H_{\rm pl} + H_Q + H_1$$

- $H_{\rm pl}$ is the Hamiltonian of the medium.
- H_Q is the Hamiltonian of quarkonium. s=singlet and o=octet.

$$H_Q = H_{s,o} = -\frac{\Delta_r}{M} - \frac{\Delta_R}{4M} + V_{s,o}(r)$$

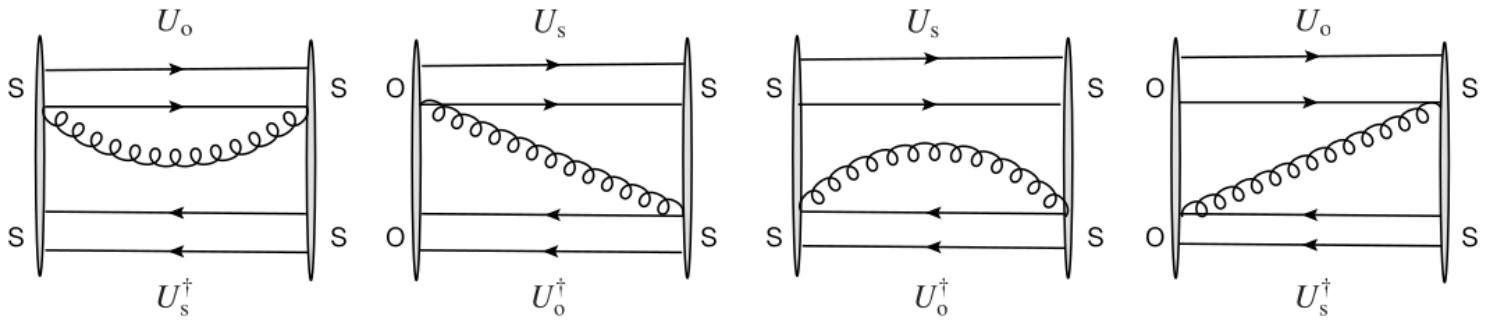
- H_1 is the interaction between the two.

$$H_1 = -g \int_r A_0^a(r) n^a(r)$$

A_0^a is the temporal component of the gluon field and n^a is the color current.

The evolution of the density matrix

4 diagrams that connect any state at time t with a singlet at time $t + dt$.



These diagrams represent the evolution of the density matrix

$|\psi(t)\rangle$ ——→—— $|\psi(t+dt)\rangle$

$\langle\phi(t)|$ ——←—— $\langle\phi(t+dt)|$

In the case of the octet, also octet to octet transitions are possible.

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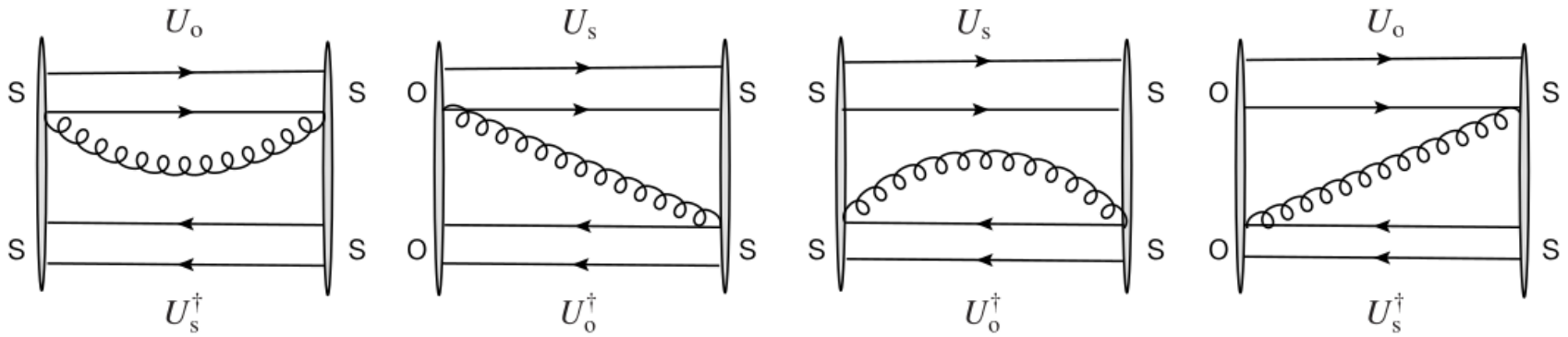


The instantaneous gluon exchange approximation means that

$$U_{s,o} = 1$$

We can do a little bit better by using

$$U_{s,o} = 1 - iH_{s,o}(t - t')$$

In fact, we are going to see that in QED substituting H by the kinetic term K implies an huge improvement

$$U_{s,o} = 1 - iK(t - t')$$

The evolution of the density matrix $\mathcal{D}_Q$

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx -\frac{i}{2}\int_{\mathbf{x}\mathbf{x}'} V(\mathbf{x}-\mathbf{x}')[n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q]$$
$$+\frac{1}{2}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left(\{n^a_{\mathbf{x}} n^a_{\mathbf{x}'}, \mathcal{D}_Q\} - 2n^a_{\mathbf{x}}\mathcal{D}_Q n^a_{\mathbf{x}'}\right)$$
$$+\frac{i}{4T}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left([n^a_{\mathbf{x}}, \dot{n}^a_{\mathbf{x}'}\mathcal{D}_Q] + [n^a_{\mathbf{x}}, \mathcal{D}_Q \dot{n}^a_{\mathbf{x}'}]\right)$$

- Unitary evolution. Includes screening and conserves entropy.
- The different transitions present in the exact instantaneous gluon exchange approximation.
- Influence of the kinetic energy inside $U_{s,o}$, we have used fluctuation-dissipation theorem to obtain this. [1]
- In our notation, the potential of the corresponding Schrödinger equation is $V(r) + iW(r)$.

[1] A similar equation can be found in D. de Boni, JHEP 1708 (2017) 064

The classical limit

We define x and y such that

$$\langle x + \frac{y}{2} | \mathcal{D}_Q | x - \frac{y}{2} \rangle$$

at large times $y \sim \frac{1}{\sqrt{MT}}$ because it is the conjugate of the momentum

$$e^{-\frac{p^2}{2MT}}$$

which in this case will be much smaller than x. This implies that at large times the system is in a approximately well-defined position at a given time.

There is a systematic way to obtain the classical equations using the Wigner transform.

The classical limit. QCD

In QED the strict $y = 0$ limit gives

$$\frac{\partial D}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D$$

In QCD we get

$$\frac{\partial D_s}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_s - 2C_F\Gamma(\mathbf{r})(D_s - D_o)$$

$$\frac{\partial D_o}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_o + \frac{1}{N_c}\Gamma(\mathbf{r})(D_s - D_o)$$

Next we are going to consider two options.

Langevin equation with a random color force. Option 1

One option is to diagonalize the system of equations

$$\frac{\partial D_0}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_0$$

$$\frac{\partial D_8}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_8 - N_c\Gamma(\mathbf{r})D_8$$

where

$$D_0 = \frac{1}{N_c^2}(D_s + (N_c^2 - 1)D_o)$$

$$D_8 = \frac{2}{N_c}(D_s - D_o)$$

D_0 represents the state of maximum entropy in color while D_8 represents the short lifetime fluctuations around it.

Langevin equation with a random color force

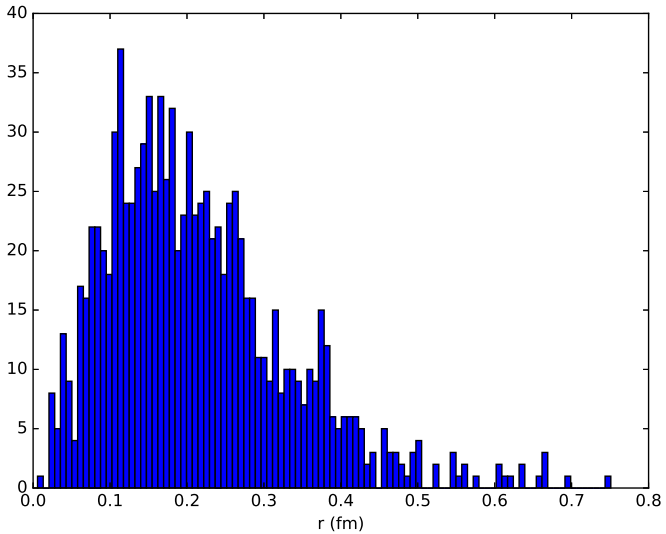
Now we diagonalize the system but instead of doing it in the strict $y = 0$ limit we do it at order y^2.

$$\partial_t D_0' + \frac{2\mathbf{p}\cdot\nabla}{M} D_0' - \frac{C_F}{4}\mathcal{H}_{ij}(0)\Delta_p^{ij} D_0' - \frac{2C_F F^i(\mathbf{r})F^j(\mathbf{r})}{N_c^2\Gamma(\mathbf{r})}\Delta_p^{ij} D_0'$$
$$- \frac{C_F}{2MT}\mathcal{H}_{ij}(0)\nabla_p^i(p^j D_0') = 0$$

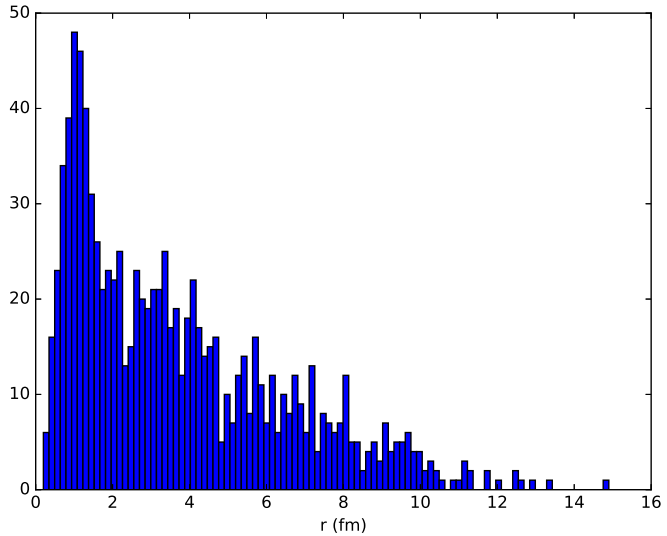
- Vlasov equation with zero force.
- Stochastic force and drag corresponding to the interaction of a heavy quark with the medium.
- Stochastic force corresponding to the attractive/repulsive force between heavy quarks that is averaged to zero due to the color state.

Heavy quarkonium. Histogram of distance after $\Delta t = 5 fm$ for initial 1S

$t = 0$

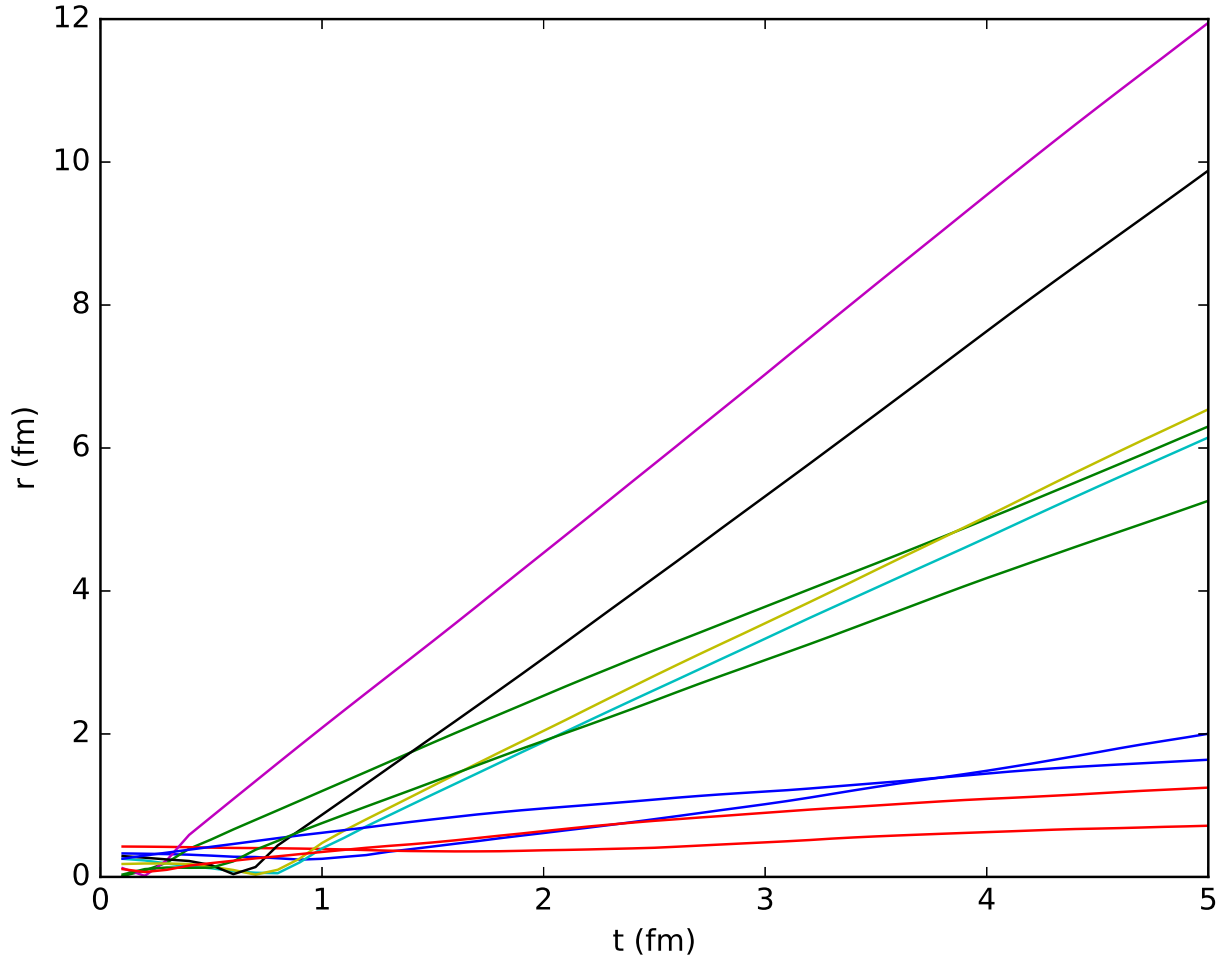


$t = 5 fm$



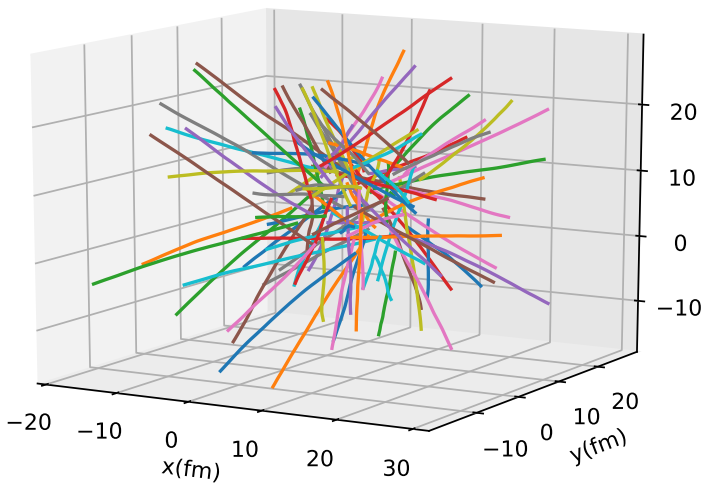
Parameters: $M_b = 4881 MeV$, $\gamma = \frac{0.25 T^2}{2 M_b}$, $T = 350\, MeV$. α_s evaluated at $2\pi T$ inside the Debye mass and at $\frac{1}{a_0} = 1348 MeV$ in the Coulombic part.

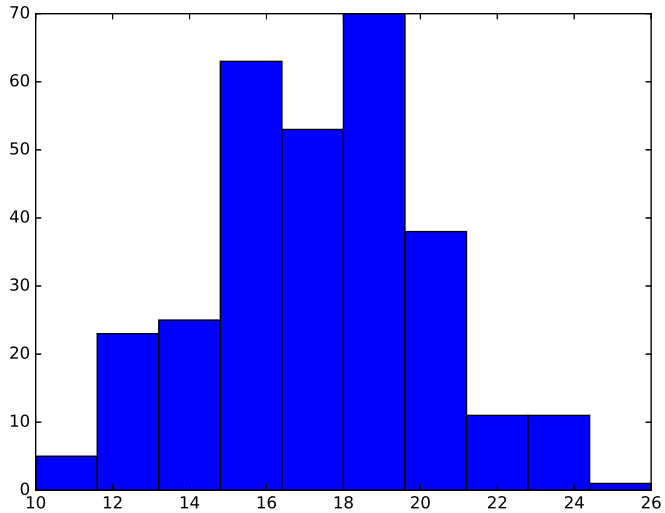
Distance as a function of time for 10 arbitrary trajectories



Problem not found in QED. For small values of r the random force can get unphysically big.

Simulating many particles





We can simulate 50 pairs and make an histogram of how many of them will form a bound state at the end of the evolution (recombination).

Boltzmann equation. Option 2

The convergence radius of the small y expansion is much smaller than in QED. The reason could be the diagonalization procedure. Let us go back to linear order in y but without diagonalizing (Now $P_o = (N_c^2 - 1)D_o$)

$$\left[\partial_t + \frac{2\mathbf{p}\cdot\nabla_\mathbf{r}}{M} + C_F\mathbf{F}(\mathbf{r})\cdot\nabla_\mathbf{p}\right] P_s = -2C_F\Gamma(\mathbf{r})\left(P_s - \frac{P_o}{N_c^2-1}\right)$$

$$\left[\partial_t + \frac{2\mathbf{p}\cdot\nabla_\mathbf{r}}{M} - \frac{1}{2N_c}\mathbf{F}(\mathbf{r})\cdot\nabla_\mathbf{p}\right] P_o = -\frac{1}{N_c}\Gamma(\mathbf{r})(P_o - (N_c^2-1)P_s)$$

This is a Boltzmann equation. $\Gamma(r) = W(r) - W(0)$

Boltzmann-Langevin equation

If terms of order y^2 are included the Fokker-Planck equation can be written as a mixture between a Boltzmann and Langevin equations as long as terms that go like $y^2((N_c^2-1)P_s - P_o)$ are neglected. We need that the only term with non-diagonal elements is the collision term.

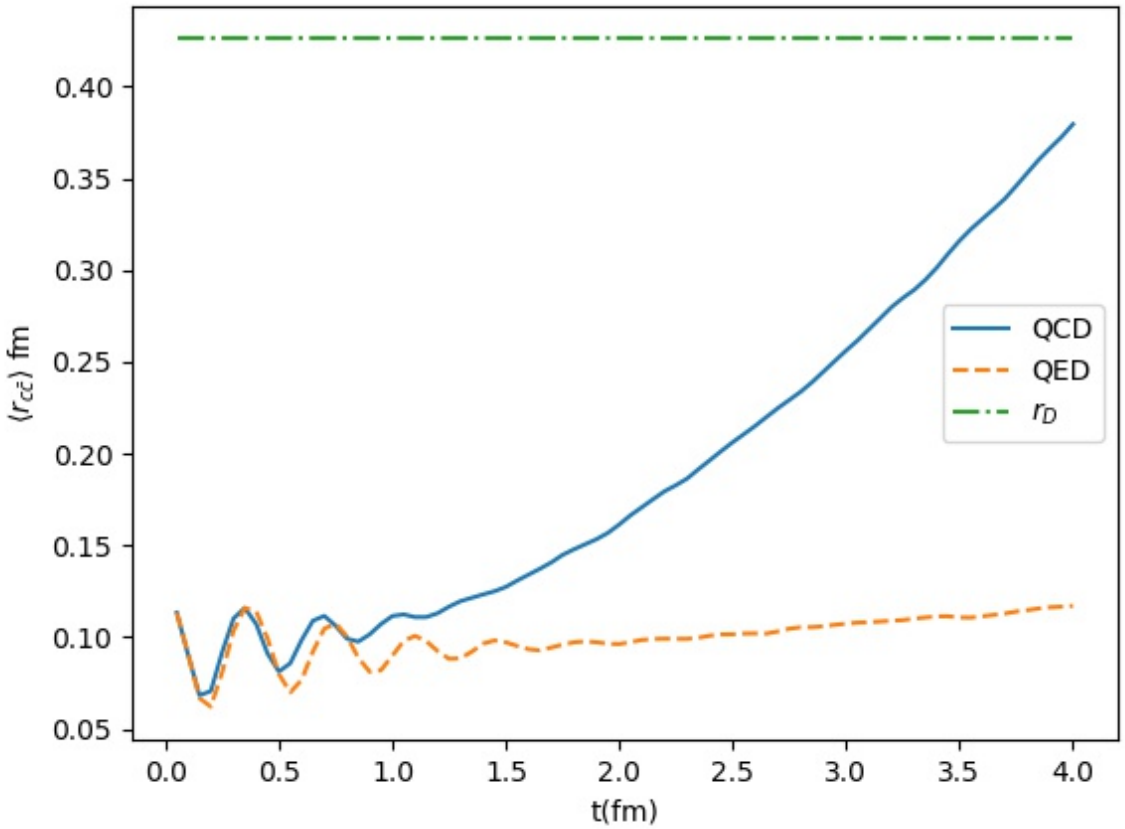
$$\partial_t P_s + \frac{2p^i \nabla_r^i}{M} P_s + C_F F^i(\mathbf{r}) \nabla_p^i P_s - \frac{C_F}{2MT}(\mathcal{H}_{ij}(0) + \mathcal{H}_{ij}(\mathbf{r}))\nabla_p^i (p^j P_s)$$
$$= -2C_F \Gamma(\mathbf{r}) \left(P_s - \frac{P_o}{N_c^2 - 1} \right) + \frac{C_F}{4}(\mathcal{H}_{ij}(0) + \mathcal{H}_{ij}(\mathbf{r}))\Delta_p^{ij} P_s$$

Exactly the same Langevin equation as in QED plus collision term that changes color state

Boltzmann-Langevin equation for the octet

$$\partial_t P_o + \frac{2p^i \nabla_r^i}{M} P_o = -\frac{1}{N_c}\Gamma(\mathbf{r})(P_o - (N_c^2-1)P_s) + \frac{1}{2N_c}F^i(\mathbf{r})\nabla_p^i P_o$$
$$+ \frac{1}{4}\left(C_F\mathcal{H}_{ij}(0) - \frac{1}{2N_c}\mathcal{H}_{ij}(\mathbf{r})\right)\Delta_p^{ij} P_o$$
$$+ \frac{1}{2MT}\left(C_F\mathcal{H}_{ij}(0) - \frac{1}{2N_c}\mathcal{H}_{ij}(\mathbf{r})\right)\nabla_p^i(p^j P_o)$$

Study of J/Ψ at $T = 160\ MeV$



- Better result that the Langevin equation with the random color.
- At large times it tends to the maximum color entropy state, but it does not start from there.

Pros and Cons of the two methods

Random color force

- There are unphysical large hits.
- It can be used to study many heavy particles with a low computational cost.

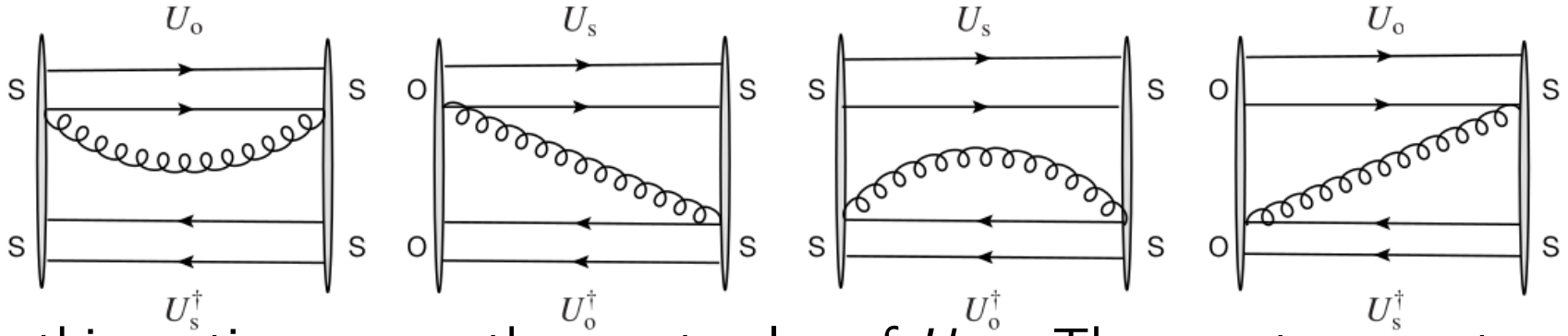
Boltzmann-Langevin equation

- Gives more realistic results.
- Difficult to generalize to many particles. We would need to classify all possible color states and compute transition rates.

Plan

The evolution of the density matrix

Reminder of the 4 diagrams that connect whatever state at time t with a singlet at time $t + dt$.



In this section we use the exact value of $U_{s,o}$. The quantum master equation can be written schematically as

$$\frac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] =$$
$$- \int_{\mathbf{x}\mathbf{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [n^A_{\mathbf{x}}, U_Q(\tau) n^A_{\mathbf{x}'} \mathcal{D}_Q(t-\tau) U_Q^\dagger(\tau)] \, \Delta^>(\tau; \mathbf{x}-\mathbf{x}'))$$
$$- \int_{\mathbf{x}\mathbf{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [U_Q(\tau) \mathcal{D}_Q(t-\tau) n^A_{\mathbf{x}'} U_Q^\dagger(\tau), n^A_{\mathbf{x}}] \, \Delta^<(\tau; \mathbf{x}-\mathbf{x}'),$$

where we have set $t - t' = \tau$.

$U_{s,o}$ and energy conservation

The role of $U_{s,o}$ is to inform the master equation of the size and the sign of the energy different between the state at t' and the state at t.

- If we make the approximation $U_{s,o} = 1$ all transitions are equally likely.

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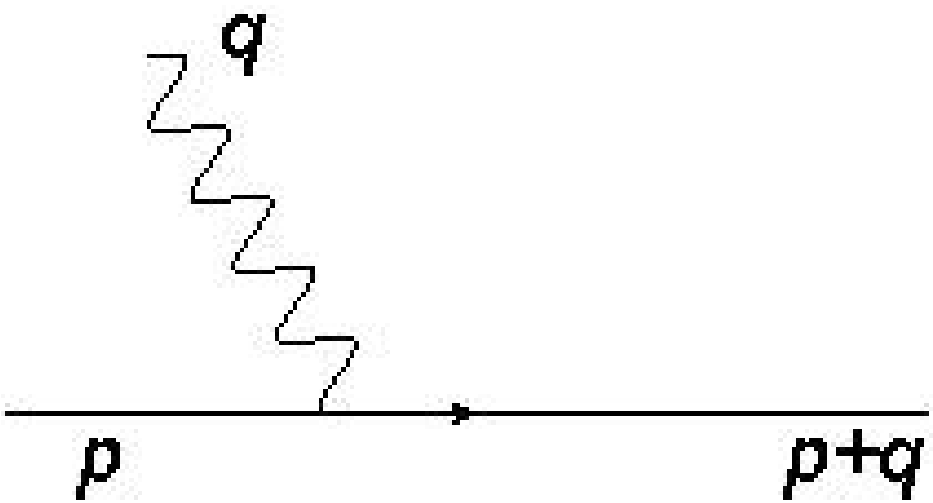
- This is very important to understand QCD. An octet to singlet transition is always energetically favorable while the opposite happens for singlet to octet.

Why does the Langevin equation work so well in QED while the analogous equation in QCD does not?



In QED

- The position will not change after the absorption of the gluon $\rightarrow$ same potential energy.
- If the electrons is closed to be thermalized then $p \sim \sqrt{MT}$. q can be at most of order T. Then

$$\frac{(\mathbf{p}+\mathbf{q})^2 - p^2}{2MT} \sim \sqrt{\frac{T}{M}}$$

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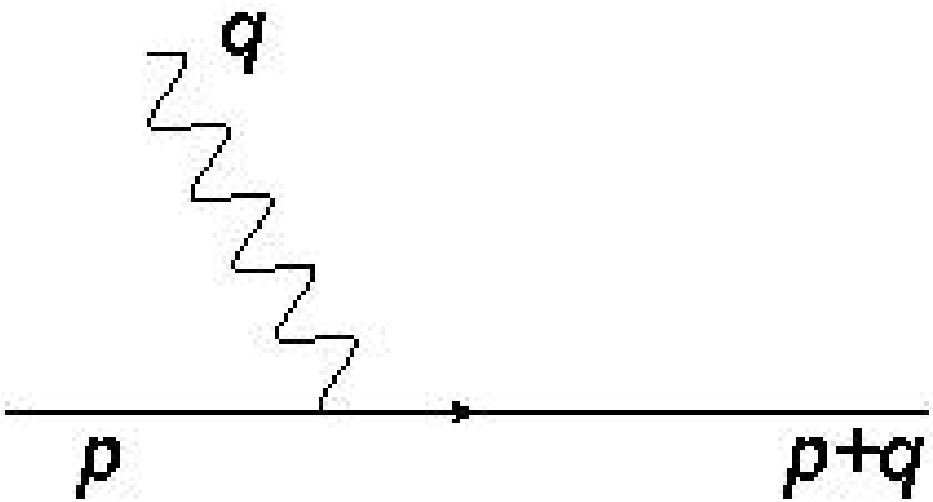

In QCD, even if position doesn't change there will be a change in the potential energy

$$\frac{\Delta E}{T} \sim \frac{\Delta V}{T}$$

Combination of rate equation + Langevin equation

- There is no gap in octet to octet transitions, therefore this can be approximated by a Langevin equation (same case as QED).
- In the large N_c limit the octet potential is zero.
- Singlet to octet transitions can be described as a rate equation. As an illustration we consider a toy model in which there is only one singlet eigenvalue (the 1S).

$$\frac{\mathrm{d}p^{\mathrm{s}}}{\mathrm{d}t} = g^2 C_F \int_{\mathbf{p}} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s}|\mathcal{S}_{\mathbf{q}\cdot\hat{\boldsymbol{r}}}|\mathrm{o}, \mathbf{p}\rangle|^2 ,$$

and

$$\frac{\partial p^{\mathrm{o}}_{\mathbf{p}}}{\partial t} - \gamma \nabla(\mathbf{p} p^{\mathrm{o}}_{\mathbf{p}}) - \frac{T\gamma M}{2} \Delta^2 p^{\mathrm{o}}_{\mathbf{p}} =$$
$$- \frac{g^2}{2N_c} \frac{1}{\Omega} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s}|\mathcal{S}_{\mathbf{q}\cdot\hat{\boldsymbol{r}}}|\mathrm{o}, \mathbf{p}\rangle|^2 ,$$

Toy model. Results

	$\Omega = 1\,\mathrm{fm}^3$			$\Omega = 100\,\mathrm{fm}^3$		
	$5\,\mathrm{fm/c}$	$100\,\mathrm{fm/c}$	eq.	$5\,\mathrm{fm/c}$	$100\,\mathrm{fm/c}$	eq.
$T = 200\,\mathrm{MeV}$	0.86	0.136	0.0814	0.85	0.0438	0.00089
$T = 400\,\mathrm{MeV}$	0.39	0.0515	0.0175	0.36	0.0002	0.00018

Table: p^s as obtained by solving eqs. in previous slide

Remarks

- The gap $E^o_{\mathbf{p}} - E^s$ suppresses the singlet decay width, helping to make the results compatible with experimental observations.
- E^s depends on the real part of the potential. Screening reduces the binding energy and this increases the decay width.
- The volume of the medium Ω suppresses the decay of octets into bound singlets.

Plan

We have learned...

- Memory effects inside the master equation are encoded in the time evolution operator $U_{s,o}$. We have discussed the consequences of different ways of approximating it.
- We can find Langevin-like equations in QCD but their region of validity is smaller than in QED. We understand why.
- Screening and the decay width are not completely separated phenomena. Screening influences the binding energy and the binding energy influences the decay width.
- A combination of a rate equation for singlet to octet transitions and a Langevin equation for octet to octet transition reproduces the important physical aspects with a very small computational cost.